Global CKM Fits

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On behalf of M. Bona, G. Eigen and R. Ithoh
E. Kou for new physics models

Chapter Outline

- 22 Global interpretation . . . .
- 22.1 Global CKM fits . . . .
- 22.1.1 Methodology . . . .
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- 22.1.3 Theoretical Inputs (lattice) . . . . ✔
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  - Lattice QCD inputs
- 22.1.4 Results from the global fits . . . . ✖
- 22.1.5 Conclusions . . . . ✖
- 22.2 Benchmark “new physics” models . . . . . . ✖
- New Physics model ?

Chapter Writing Assignments

- Description of the fit methods:
  - CKMfitter: Itoh san
  - UTfit: Marcella
  - Scan Method: Gerald

- We assigned a maximum of 2 pages for each of these

- The editors ask us to be very didactic in our explanations to be coherent with the scope of the book

- We refer to publications for a more detailed description, but the ~two pages will contain the core and simplified concepts

- Experimental inputs
  - B factory results (mostly refs. to the rest of the book): Marcella
  - B_s (Delta m_{Bs}) (ref to the mixing part in the book as well): Gerald
  - ε_K: Itoh san
  - V_{us}: Itoh san (what should we use, Flavianet? 2x2 matrix?)

Chapter Writing Assignments

- Theoretical inputs
  - Derivation of hadronic observables: Marcella

  ➔ some text introducing the role of the hadronic parameters but link it to the previous experimental section

- Outstanding issue of lattice QCD inputs is solved (last week)
  - Like Marcella, Itoh san and I finally agreed to use the lattice averages by Laiho, Lunghi and van de Water

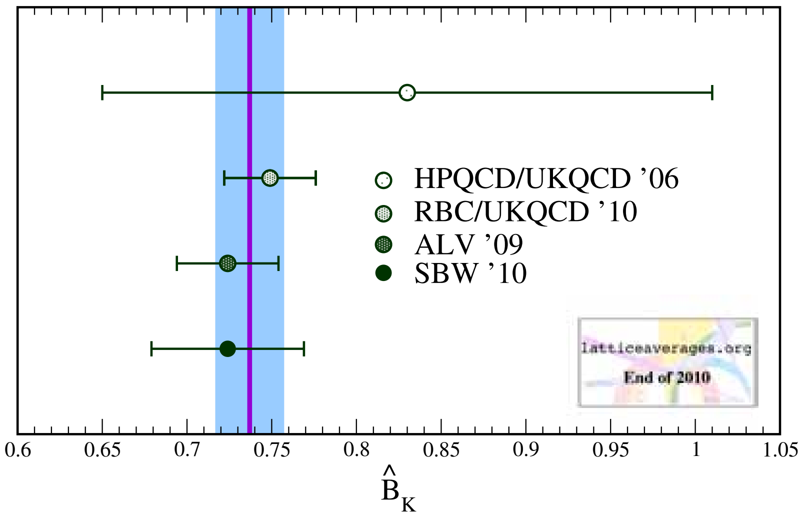
  ➔ Itoh san had some discussions with Bruce

  ➔ After some discussion with the lattice group, 2 rounds of questions and answers, I communicated with Andreas Kronfeld, who convinced me

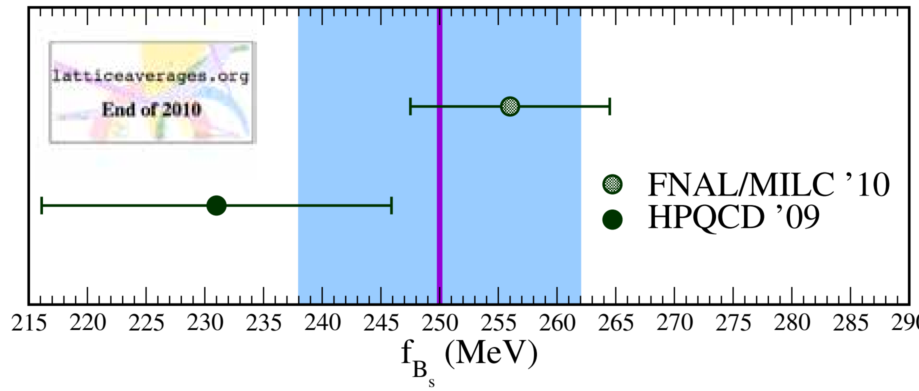
  ➔ Itoh san and I will treat will treat systematic uncertainties of lattice QCD parameters in 2 ways, as Gaussian and non-Gaussian, and compare the results

Lattice Averages 2011

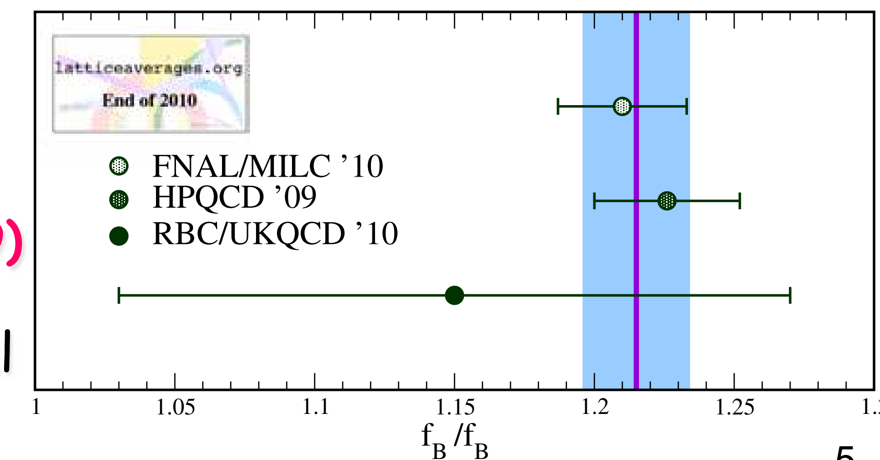
- B_K=0.737±0.0056±0.020



- f_{Bs}= (250±5.4±10.7) MeV



- f_{Bs}/f_{bd}=1.215±0.012±0.015



- Im[A_2]=-(7.9±1.6±3.9)x10^{-13} GeV
- B_{Bs}=1.33±0.06$_{total}$ (HOQCD '09)
- B_{Bs}/B_{Bd}=1.05±0.07$_{total}$ (HPQCD '09)

➔ error decomposition into statistical and systematic?

Chapter Status

- We have written about 6 pages on
  - Different methodologies
  - Experimental inputs
  - Theory inputs

- Itoh san will commit his contribution in a few days (fit method, ε_K and V_{us})

- Complete the experimental input section
  - ➔ waiting for final results in other section of the book
  - ➔ do some coordination with the inputs in other sections to avoid redundancy
  - ➔ perhaps, summarize all measurements from book in big table

- Do some fine tuning of the text

- Complete the references

- Insert tables with updated Lattice QCD inputs

- Add other inputs: B masses, f_π, f_K, α_s, etc

Global Fit Results

- Once we have all the inputs we can run the global fits,
- UTfit estimates 1 day for running plus 2 days for checks
- CKMfitter estimates results in a few days
- Scan method can get results also in a day plus ~2 days for checks (at least the first time
  - We parameterize branching fractions and CP asymmetries in terms of amplitudes in order of λ, (up to λ^4),
  - ➔ Here, we will go up to λ^2
  - ➔ For $K\pi$, $K^*\pi$, $K\rho$, $K^*\rho$, SU(2) breaking is included next order in λ
  - ➔ For modes with η, η', ω … include singlet penguin
- Then we need a few days to add the results into the book, do a comparison among the three fit results & write the conclusion section
- So if all inputs are ready by Christmas, we can have our chapter completed early January, before the editors meet at Aimes

Chapter Status

22.1 Global CKM fits

Editors:
Gerald Eigen (BABAR)
Ryosuke Itoh (Belle)
Marcella Bona (theory)

This section will use all the results produced by the B factories and detailed in the previous chapters to extract the fundamental SM parameters of the CKM matrix (Cabibbo, 1963; Kobayashi and Maskawa, 1973). The determination of the CKM parameters is done with a global fit including the experimental results and the theoretical quantities connecting the observables to the CKM formulation.

22.1.1 Methodology

Three approaches have been identified in the community: below the three methods are described in their techniques and specificities in a general way. More detailed references to the various global fit analyses will be given in each of the following sections regarding the experimental and theoretical inputs.

22.1.1.1 CKMfitter

Here goes the CKMfitter short description.

22.1.1.2 Scan Method

The extraction of CKM parameters from measurements involves the knowledge of theory input which is affected by uncertainties. Since some of these uncertainties are generally not Gaussian distributed, the standard approach of adding these uncertainties with experimental uncertainties in quadrature is not valid. Thus, the scan method was developed that treats Gaussian and non-Gaussian uncertainties differently. Each theory parameter is represented by a term in the χ^2 using the central value and its associated Gaussian error. The non-Gaussian uncertainties are accounted for by selecting a specific value for each theory parameter within the range of the non-Gaussian uncertainties that are used in a specific fit. The theory parameters are either selected by scanning through a fine multi-dimensional grid of fixed points or by specifying them via random numbers. Presently, the theory parameters that are scanned are the non-Gaussians errors in the extraction of CKM parameters $|V_{ub}|$ and $|V_{cb}|$, f_{B_s}, B_{B_s}, f_{B_s}/f_{B_d}, B_{B_s}/B_{B_d}, and B_K. The QCD parameters $\eta_{cc}, \eta_{tt}, \eta_{ct}$, and η_B are presently not scanned, where η_{cc} and the associated error is parameterized in terms of $\overline{m_c}(\overline{m_c})$ and α_s.

Thus, for each set of theory parameters

$$\mathcal{M} = \{V_{ub}, V_{cb}, f_{B_s}, B_{B_s}, f_{B_s}/f_{B_d}, B_{B_s}/B_{B_d}, B_K\}, \quad (1)$$

the function

$$\chi^2_{\mathcal{M}}(\bar\rho, \bar\eta, A; \mathcal{S}_|) = \sum_i \left[\frac{E_i - \mathcal{E}_i(\bar\rho, \bar\eta, A; \mathcal{S}_j; C_k; \mathcal{M}}{\sigma_{E_i}}\right]^2 \quad (2)$$

is constructed and minimized, where $\mathcal{S}_j$ represents other parameters that are minimized, C_k denotes measured quantities that posses experimentally derived or other probabilistic uncertainties such as masses and lifetimes, and σ_{E_i} denote all measurement uncertainties contributing to both E_i and $\mathcal{E}_i$ including uncertainties of theory parameters that are statistical in nature.

The minimization solution $(\bar\rho, \bar\eta, A; \mathcal{S}_|)_{\mathcal{M}}$ for a particular set of theory parameters $\mathcal{M}$ incorporates no prior distribution for non-probabilistic uncertainties of the theoretical parameters and meets the frequency interpretation. All uncertainties depend only on measurement errors and other probabilistic uncertainties including any probabilistic component of the uncertainties on the theoretical parameters relevant to each particular measurement. For practical reasons, the comparatively small uncertainties arising from $\eta_{cc}, \eta_{ct}, \eta_{tt}$, and η_B are treated as probabilistic.

The best-fit solution of a set of theory parameters $\mathcal{M}$ is kept only if the probability of the fit satisfies $\mathcal{P}(\chi^2_{\mathcal{M}}) > \mathcal{P}_{min}$, which is typically chosen to be 5%. For such a fit the 95% *C.L.* contour in the $(\bar\rho, \bar\eta)$ plane is drawn. The 95% *C.L.* contours of all kept fits are overlaid. With a sufficient fine scanning grid or a sufficiently large number of random fits the complete parameter space specified by the theoretical uncertainties is taken into account. This procedure is derived from the technique originally described in **?**. The χ^2 minimization thus serves three purposes:

– 1. If a set of theory parameters $\mathcal{M}$ is consistent with the data, the best estimates for the three CKM parameters are obtained and 95%*C.L.* contours are determined

– 2. If a set of theory parameters $\mathcal{M}$ is inconsistent with the data, the probability $\mathcal{P}(\chi^2_{\mathcal{M}})$ will be low. Thus, the requirement of $\mathcal{P}(\chi^2_{\mathcal{M}}) > 5\%$ provides a test of compatibility between data and its theoretical description.

– 3. By varying the theoretical parameters beyond their specified range correlations on them imposed by the measurements can be studied.

If no set of theory parameters were to survive we would have evidence of an inconsistency between data and theory, independent of the calculations of the theoretical parameters or the choices of their uncertainties. Since the goal of the CKM parameter fits is to look for inconsistencies of the different measurements within the Standard Model, it is important to be insensitive to artificially produced effects and to separate the non-probabilistic uncertainties from Gaussian-distributed errors.

In the standard fit, the parameter set $\mathcal{S}$ includes B masses, B lifetimes, the lattice parameters, $\overline{m}_t(m_t)$, and $\overline{m}_c(m_c)$. Measurements of the Unitarity Triangle α are parameterized in terms of tree amplitudes, color-suppressed tree amplitudes and $B \to c$ penguin amplitudes with corresponding phases. They are defined individually for $B \to \pi\pi$, $B \to \rho\pi$, $B \to \rho\rho$, and $B \to a_1\pi$ modes. Measurements of the Unitarity Triangle γ are parameterized in terms of a ratio of $b \to u$ penguin amplitude to a $b \to c$ tree amplitude and the phase between them. Included are $B \to DK$, $B \to D^*K$, and $B \to DK^*$ modes in which these parameters are extracted in the GLW analysis, ADS analysis and in Dalitz plot decay analyses.

22.1.1.3 UTfit

The UTfit Collaboration is a phenomenological collaboration performing the Unitarity Triangle (UT) analysis following the method described in refs. **??**. In this Section we recall the basic ingredients of the UTfit analysis method that is developed in the framework of the Bayesian approach.

In the following sections, we will find several equations relating a constraint c_j (where c_j stands for one of the n constraints like $|V_{ub}/V_{cb}|$, Δm_d, Δm_s, $|\varepsilon_K|$ and so on, for $j = 1, \ldots, n$) to the unitarity triangle parameters $\bar\rho$ and $\bar\eta$, via a set of ancillary parameters $\boldsymbol{x}$, where $\boldsymbol{x} = \{x_1, x_2, \ldots, x_N\}$ stand for all experimentally determined or theoretically calculated quantities from which the various c_j depend:

$$c_j \equiv c_j(\bar\rho, \bar\eta; \boldsymbol{x}).$$

In an ideal case of exact knowledge of c_j and $\boldsymbol{x}$, each of the constraints provides a curve in the $(\bar\rho, \bar\eta)$ plane. In such a case, there would be no reason to favour any of the points on the curve, unless we have some further information or physical prejudice, which might exclude points outside a determined physical region, or, in general, assign different weights to different points. In a realistic case, we suffer from several uncertainties on the quantities c_j and $\boldsymbol{x}$. Uncertainty does not imply, however, that we are absolutely ignorant about a given quantity. First of all, there are values which, to the best of our knowledge, we consider ruled out (for example a value of m_t of 100 GeV or 500 GeV). Second, we assign different probabilities to the values within the *almost certain range*, 144 GeV $< m_t <$ 184 GeV say[1].

In the m_t case, for example, we think that it is much more probable that the value of m_t lies between 154 and 174 GeV rather than in the rest of the interval, in spite of the fact that the two sub-intervals have the same widths. This means that, instead of a single curve (6) in the $(\bar\rho, \bar\eta)$ plane, we have a family of curves which depends on the distribution of the set $\{c_j, \boldsymbol{x}\}$. As a result, the points in the $(\bar\rho, \bar\eta)$ plane get different weights (even if they were

[1] In this example m_t is the $\overline{MS}$ top mass of Equation , $m_t = (164 \pm 1)$ GeV

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taken to be equally probable a priori) and our confidence on the values of $\bar{\rho}$ and $\bar{\eta}$ clusters in a region of the plane.

The above arguments can be formalised by using the Bayesian approach: the uncertainty is described in terms of a probability density function f(·), which quantifies our confidence on the values of a given quantity. The inference of $\bar{\rho}$ and $\bar{\eta}$ becomes then a straightforward application of probability theory. The simplest way to implement the probabilistic reasoning discussed above is to define an idealised p.d.f. for each constraint:

$$f(\bar{\rho},\bar{\eta}|c_j,\boldsymbol{x}) \propto \delta(c_j - c_j(\bar{\rho},\bar{\eta};vecx)), \qquad (22.1.3)$$

where δ is the Dirac delta distribution. This p.d.f. is a distribution in a mathematical sense, which is to be taken as the limit of a very narrow p.d.f. with values different from zero only along a curve. The p.d.f. which takes into account the full uncertainty about c_j and $\boldsymbol{x}$ is obtained from **??** by making use of the standard probability rules. [to be continued]

22.1.2 Experimental Inputs

22.1.2.1 V_{us}

22.1.2.2 B-factory results

The experimental inputs from the B Factories are:

- Sides
  - $|V_{ub}|$: this element of the CKM matrix can be linked to semileptonic B decays as described in Sec. 14.1. The global fits use the final value of xx.xx ± xx.xx as input, averaging inclusive and exclusive analyses.
    Comment: there is a "Lattice QCD form factor calculations" section (no label) in Sec. 14.1.
  - $|V_{cb}|$: similarly to the above case, this element of the CKM matrix is linked to semileptonic B decays and the analyses are described in Sec. 14.1. The global fits use the final value of xx.xx ± xx.xx as input, averaging inclusive and exclusive analyses.
- B_d Mixing
  - Δm_d: this represents the oscillation frequency of B meson decays and its extraction is described in Sec. **??** and it is derived in Chapter 7 It is linked to the CKM matrix parameters via the expression in Eq. **??**. The inputs needed to connect the experimental value to the fundamental parameters $|V_{td}V_{tb}^*|$ are: the Inami-Lim function $S_0(x_t)$ with $x_t = m_t^2/M_W^2$, m_t taken as the $\overline{MS}$ top mass, $m_t^{\overline{MS}}$, and the perturbative QCD short distance NLO correction η_B. The remaining factors encode the information of non-perturbative QCD and they are f_{B_d} and B_{B_d}.
- Angles:
  - ϕ_1: it is defined in function of the CKM matrix elements in Equation **??**. The analyses contributing to its measurement are described in detail in Sec. 14.6. The global fits use the final value of xx.xx ± xx.xx as input, where the ambiguities from the $\sin(2\phi_1)$ measurements are resolved by the $\cos(2\phi_1)$ analyses.
    Comment: theory uncertainties should be addressed in the relative chapter and if not, they should be discussed here.
  - ϕ_2: it is defined in function of the CKM matrix elements in Equation **??**. The analyses contributing to its measurement are described in detail in Sec. 14.7. The global fits use the final value of xx.xx ± xx.xx as input, where the following final states are included: $\pi\pi$, $\rho\rho$, and $\rho\pi$
    Comment: references to be added but no labels in the phi2 chapter.
  - ϕ_3: it is defined in function of the CKM matrix elements in Equation **??**. The analyses contributing to its measurement are described in detail in Sec. 14.8. The global fits use the final value of xx.xx ± xx.xx as input.
  - $2\phi_1+\phi_3$: this combination of two of the angles defined above can be measured through $B \to D^{(*)\mp}h^{\pm}$ decays as described in Sec. 14.8.
    Comment: are the Phi3 authors going to give a value also for this combination? Also: the $2\phi_1+\phi_3$ section needs a label.
- Leptonic decays
  - $BR(B\to\tau\nu)$: this branching fraction is linked to CKM matrix elements via the formula in Eq. **??**. The final experimental value used as input is $xx \pm xx \pm xx$ as in Sec. **??**.
  - radiative penguins: do we want to include them? if yes, how?

To summarise the inputs used from the B factories, please see the table 1 (still just a place holder to see if this is the preferred way).

Input	value	reference
$\phi_1[°]$	xx ± xx	14.6
$\phi_2[°]$	xx ± xx	14.7
$\phi_3[°]$	xx ± xx	14.8
$\Delta m_d[\mathrm{ps}^{-1}]$	xx ± xx	14.8
$V_{ub}[10^{-3}]$	xx ± xx	14.1
$V_{cb}[10^{-2}]$	xx ± xx	14.1

Table 1. Input values for the global fit.

22.1.2.3 Other results

(briefly on their treatment): ε_K, Δm_s

- top mass m_t:
- $B_s-\bar{B}_s$ Mixing: The measurement of $B_s-\bar{B}_s$ mixing provides a determination of the CKM matrix element $|V_{ts}|$, which constrains the length of the side AB in the Unitarity Triangle, R_t, since the t quark dominates in the electroweak box diagrams. The $B_s-\bar{B}_s$ oscillation frequency corresponds to the mass difference between the two weak B_s^0 mass eigenstates. It was first measured by the CDF experiment yielding $\Delta m_{B_s} = 17.77 \pm 0.10\ (stat) \pm 0.07\ (syst)\ ps^{-1}$ **?** and has been confirmed by LHCb measuring $\Delta m_{B_s} = 17.725 \pm 0.041\ (stat) \pm 0.026\ (syst)\ ps^{-1}$ **?**. The prediction for Δm_{B_s} is obtained from the $\Delta B = 2$ effective Hamiltonian, yielding

$$\Delta m_{B_s} = \frac{G_F^2}{6\pi^2}\eta_B m_{B_s} m_W^2 f_{B_s}^2 B_{B_s} S_0(x_t)|V_{ts}V_{tb}^*|^2, \qquad (22.1.4)$$

where where $G-F$ is the Fermi constant, η_B is a QCD correction factor calculated in NLO **?**, m_{B_s} is the B_s mass, m_W is the W mass, f_{B_s} is the B_s-decay constant, B_{B_s} parameterizes the value of the hadronic matrix element, the Inami-Lim function $S_0(x_t)$ **?** gives the electroweak loop contribution of the top quark without QCD corrections and $x_t = \overline{m}_t{}^2/m_W^2$. The numerical values of the theoretical input parameters are summarized in Table **??**.

- K system:

more:
quarks masses (top mass m_t, charm mass m_c, up mass m_u, strange mass m_s, bottom mass m_b) lifetimes (neutral B_d lifetime, charged B lifetime, neutral B_s lifetime)

Input	value	reference
m_t [GeV/c²]	164.1 ± 0.9	?
m_c [GeV/c²]	1.3 ± 0.1	?
m_u [GeV/c²]	0.0037 ± 0.0004	?
m_s [GeV/c²]	0.10 ± 0.01	?
m_b [GeV/c²]	4.21 ± 0.08	?
τ_{B_d} [ps]	1.525 ± 0.009	?
τ_{B^+} [ps]	1.638 ± 0.011	?
τ_{B_s} [ps]	1.43 ± 0.09	?

Table 2. Input values for the global fit.

22.1.3 Theoretical Inputs

22.1.3.1 Derivation of hadronic observables

22.1.3.2 Lattice QCD inputs

Note 1: UTfit average on f_{Bs}/f_{Bd} is taken from the two collaborations **??**. Since no average for this ratio is available from Laiho, Lunghi, and Van de Water (2010), the uncertainty 0.03 is taken equal to the smaller of the two errors.

Note 2: UTfit gets the B_{Bs}/B_{Bd} average starting from Laiho, Lunghi, and Van de Water (2010) that gives the averages for B_{Bs} and B_{Bd} separately. Being **?** the only one that contributes to that ratio, UTfit would be fine with moving to the HPQCD number.

Note 3: for UTfit uses Lubicz's talk at Lattice 2009: V. Lubicz, arXiv:1004.3473 [heplat]. There is a new average result available for the K parameters coming from a wider lattice community called FLAG. For the moment they do not use the result ETMC result **?** because it was not published yet, but a ETMC article **?** has been submitted for publication so FLAG is going to include it in the published version of their article and UTfit will be moving to this reference from the FLAG collaboration: G. Colangelo *et al.*, arXiv:1011.4408 [hep-lat].

22.1.4 Results from the global fits

22.1.4.1 B to $\tau\nu$

From the global fit rerun without including the BR($B\to\tau\nu$) experimental input, we can also extract the most accurate determinations for the SM expectation value of this branching ratio. It is the most accurate expectation value because it is extracted using all the parameters (V_{ub} and f_B in particular) as obtained from the global fits.

22.1.5 Conclusions

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Table 3. f_{Bs}

Collaboration	value (stat)(syst) (MeV)	UTfit	CKMfitter	Scanning method
FNAL/MILC '08 **?**	243(6)(9)	yes	?	?
HPQCD '09 **?**	231(5)(14)	yes	?	?
average		239(10) **?**	?	?

Table 4. f_{Bs}/f_{Bd}

Collaboration	value (stat)(syst)	UTfit	CKMfitter	Scanning method
FNAL/MILC '08 **?**	1.245(43)	yes	?	?
HPQCD '09 **?**	1.226(26)	yes	?	?
average		1.23(3) (see Note 1)	?	?

Table 5. B_{Bs}

Collaboration	value (stat)(syst)	UTfit	CKMfitter	Scanning method
HPQCD '09 **?**	1.33(6)	yes	?	?
average		1.33(6)	?	?

Table 6. B_{Bs}/B_{Bd}

Collaboration	value (stat)(syst)	UTfit	CKMfitter	Scanning method
HPQCD '09 **?**	1.05(7)	yes	?	?
average		1.06(4) (see Note 2)	?	?

Table 7. B_K

Collaboration	value (stat)(syst)	UTfit	CKMfitter	Scanning method
ALVdW 09 **?**	0.724(8)(28)	yes	?	?
RBC/UKQCD **?**	0.738(8)(25)	yes	?	?
ETMC **?**	0.730(30)(30)	yes	?	?
average		0.731(36) (see Note 3)	?	?

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Bibliography: BaBar Publications

285 **Bibliography: Belle Publications**

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Conclusion

- We finally have a consensus on the lattice QCD input parameters
- Note that we are in a special situation with our chapter, since we have to wait for the other chapters to be finished producing averages that are the inputs for our fits
- It is much easier to write up the chapter if you have all "ducks in a row" otherwise it is like moving in the dark ➔ this is wasting time
- We still have a lot of work ahead of us and I am looking forward to the Christmas break to complete my part of this obligation