Chapter 13

The CKM Matrix and the Kobayashi-Maskawa Mechanism

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Status

- Three Iterations among the Sec. Eds. done
- Section is almost finalized

Progress since last meeting

- Added an example for „the GIM mechanism at work“ in the Kaon system

FCNCs are suppressed by this GIM mechanism. In fact, FCNC's in the kaon system involve a transition of an s quark into a d quark. This can be achieved by a two subsequent charged current processes involving (in the two family picture) the up and the charm quark as an intermediate state. Taking into account Cabibbo mixing these amplitudes are

$$\begin{aligned} \mathcal{A}(s \to d) &= \mathcal{A}(s \to u \to d) + \mathcal{A}(s \to c \to d) \\ &= \sin\theta_C \, \cos\theta_C [f(m_u) - f(m_c)] \end{aligned} \qquad (1)$$

Hence, if the up and charm quark masses were degenerate, no kaon FCNC processes, including K - $\overline{K}$ mixing could occur.

However, the up and charm masses are not degenerate and thus K - $\overline{K}$ can occur. Neglecting the small up-quark mass, the amplitude for K - K mixing turns out to be

$$\mathcal{A}(K \to \overline{K}) \propto \sin^2\theta_C \, \cos^2\theta_C \frac{m_c^2}{M_W^2} \qquad (2)$$

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from which Gaillard and Lee (1974) could extract an estimate for the charm-quark mass of $m_c \sim 1.5$ GeV by comparison with data. It was one of the great triumphs in particle physics when a few months later narrow resonances with masses of about 3 GeV were discovered which were identified as bound state of c and $\bar{c}$. This discovery completed the second particle family and introduced a 2×2 quark mixing matrix into the phenomenology of

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- Added an illustration of the Sacharov conditions

In order to illustrate the first two Saharov conditions, we employ a very simplistic example. Assume that in the early universe existed a particle X that can decay to only two final states $|f_1\rangle$ and $|f_2\rangle$. These two state have the baryon numbers $N_B^{(1)}$ and $N_B^{(2)}$, and the decay rates are

$$\Gamma(X \to f_1) = \Gamma_0 r \quad \text{and} \quad \Gamma(X \to f_2) = \Gamma_0(1-r)\,, \tag{4}$$

where Γ_0 is the total width of X Of course, there is also the CP conjugate situation in which the particle $\overline{X}$ decays to the state $\bar{f}_1$ with the baryon number $-N_B^{(1)}$ and $\bar{f}_2$ with the baryon number $-N_B^{(2)}$. The rates are

$$\Gamma(\overline{X} \to \bar{f}_1) = \Gamma_0 \bar{r} \quad \text{and} \quad \Gamma(\overline{X} \to \bar{f}_2) = \Gamma_0(1-\bar{r}) \tag{5}$$

where Γ_0 is the same as for X due to CP invariance.

The overall change ΔN_b in baryon number induced by the decay of an equal number of X and $\overline{X}$ particles is

$$\begin{aligned} \Delta N_B &= r N_B^{(1)} + (1-r) N_B^{(2)} - \bar{r} N_B^{(1)} - (1-\bar{r}) N_B^{(2)} \\ &= (r-\bar{r}) \left(N_B^{(1)} - N_B^{(2)} \right) \end{aligned} \tag{6}$$

Thus ΔN_B non-zero means that we have to have CP violation ($r \neq \bar{r}$) and a violation of baryon number ($N_B^{(1)} \neq N_B^{(2)}$), illustrating the first two conditions.

- Added a section on the basics of CP violation

13.3 CP Violation in a lagrangian field theory

The standard model is formulated as a quantum field theory based on a Lagrangian derived from symmetry principles. To this end, the (hermitean) Lagrangian of the SM is given in terms of scalar operators $\mathcal{O}_i$ with couplings a_i

$$\mathcal{L}(x) = \sum_i \left(a_i \mathcal{O}_i(x) + a_i^* \mathcal{O}_i^\dagger(x) \right) , \tag{7}$$

where the $\mathcal{O}_i$ are composed of the SM quark, lepton and gauge fields. It is straightforward to verify that CP conservation implies that all couplings a_i can be made real by suitable phase redefinitions of the fields composing the $\mathcal{O}_i$. In turn, CP is violated in a lagrangian field theory, if there is no choice of phases that renders all a_i real.

In the SM there are in principle two sources of CP violation. The so-called "strong CP violation" originates from special features of the QCD vacuum, resulting in a contribution of the form

$$\mathcal{L}_{\text{strong CP}} = \theta \, \frac{\alpha_s}{8\pi} G^{\mu\nu,a} \tilde{G}^a_{\mu\nu} \tag{8}$$

where $G_{\mu\nu}$ ($\tilde{G}_{\mu\nu}$) is the (dual) field strength of the gluon field. This term is P and CP violating due to its pseu-

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- Added a section on CP Phenomenology, explaining the various kinds of CPV and introducing some basic notions

13.6 CP violation phenomenology

Since CP violation is due to irreducible phases of coupling constants, it becomes observable through interference effects. The simplest example is an amplitude consisting of two distinct contributiuons

$$A(B \to f) = \lambda_1 \langle f|O_1|B\rangle + \lambda_2 \langle f|O_2|B\rangle \qquad (26)$$

where $\lambda_{1/2}$ are (complex) coupling constants (in our case combinations of CKM matrix elements) and $\langle f|O_{1/2}|B\rangle$ are matrix elements of interaction operators between initial and final state.

The CP image is the process $\overline{fB} \to \overline{f}$ yielding

$$A(\overline{B} \to \overline{f}) = \lambda_1^* \langle \overline{f}|O_1^\dagger|\overline{B}\rangle + \lambda_2^* \langle \overline{f}|O_2^\dagger|\overline{B}\rangle \qquad (27)$$

The matrix elements of $\mathcal{O}_{1/2}^{(\dagger)}$ involve only strong interactions which we assume to be CP invariant. Hence we have

$$\langle \overline{f}|O_1^\dagger|\overline{B}\rangle = \langle f|O_1|B\rangle \quad \text{and} \quad \langle \overline{f}|O_2^\dagger|\overline{B}\rangle = \langle f|O_2|B\rangle \quad (28)$$

Thus the CP Asymmetry becomes

$$\mathcal{A}_{\mathrm{CP}}(B \to f) = \frac{\Gamma(B \to f) - \Gamma(\overline{B} \to \overline{f})}{\Gamma(B \to f) + \Gamma(\overline{B} \to \overline{f})} \qquad (29)$$
$$\propto 2\,\mathrm{Im}[\lambda_1 \lambda_2^*]\,\mathrm{Im}[\langle f|O_1|B\rangle \langle f|O_2|B\rangle^*]$$

Remaining To Do‘s

- Few minor comment still need to be included
- References need to be checked and validated
- Interplay with Chapter 7 (Mixing and Time Dependence) needs to be checked
- Fix a few TeXnicalities.