Leptonic Decays and B→D$^{(*)}\tau\nu$

Toru Iijima
Steven Robertson

November 21, 2011
5th PBF Workshop at KEK

First, Apology for long delay !

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But,

finally, we are ramping up !

(private) draft exists.

2σ signal ?

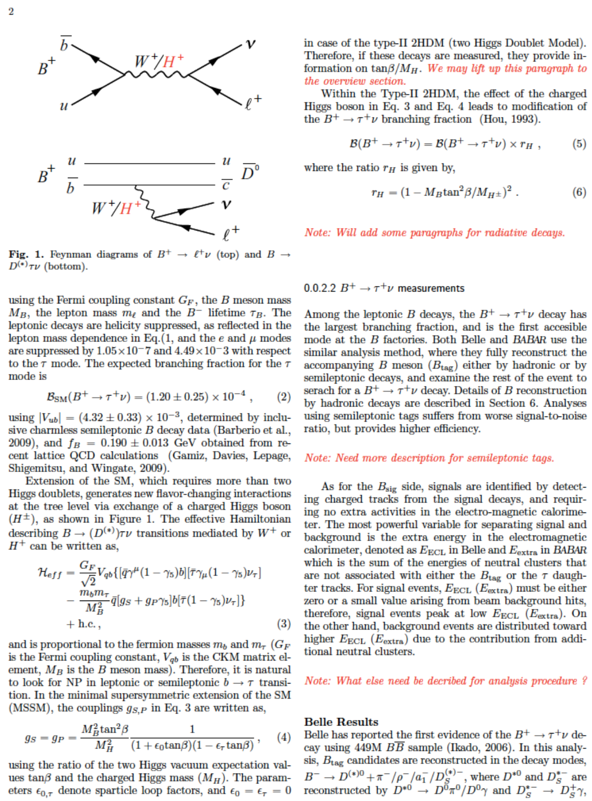

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Fig. 1. Feynman diagrams of $B^+ \to \ell^+\nu$ (top) and $B \to D^{(*)}\tau\nu$ (bottom).

using the Fermi coupling constant G_F, the B meson mass M_B, the lepton mass m_ℓ and the B^- lifetime τ_B. The leptonic decays are helicity suppressed, as reflected in the lepton mass dependence in Eq.(1, and the e and μ modes are suppressed by 1.05×10^-7 and 4.49×10^-3 with respect to the τ mode. The expected branching fraction for the τ mode is

$$\mathcal{B}_{\rm SM}(B^+ \to \tau^+\nu) = (1.20 \pm 0.25) \times 10^{-4} , \quad (2)$$

using $|V_{ub}| = (4.32 \pm 0.33) \times 10^{-3}$, determined by inclusive charmless semileptonic B decay data (Barberio et al., 2009), and $f_B = 0.190 \pm 0.013$ GeV obtained from recent lattice QCD calculations (Gamiz, Davies, Lepage, Shigemitsu, and Wingate, 2009).

Extension of the SM, which requires more than two Higgs doublets, generates new flavor-changing interactions at the tree level via exchange of a charged Higgs boson ($H^\pm$), as shown in Figure 1. The effective Hamiltonian describing $B \to (D^{(*)})\tau\nu$ transitions mediated by W^+ or H^+ can be written as,

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{qb}\{[\bar{q}\gamma^\mu(1-\gamma_5)b][\bar{\tau}\gamma_\mu(1-\gamma_5)\nu_\tau] - \frac{m_b m_\tau}{M_B^2}\bar{q}[g_S + g_P\gamma_5]b[\bar{\tau}(1-\gamma_5)\nu_\tau]\} + \text{h.c.} , \quad (3)$$

and is proportional to the fermion masses m_b and m_τ (G_F is the Fermi coupling constant, V_{qb} is the CKM matrix element, M_B is the B meson mass). Therefore, it is natural to look for NP in leptonic or semileptonic $b \to \tau$ transition. In the minimal supersymmetric extension of the SM (MSSM), the couplings $g_{S,P}$ in Eq. 3 are written as,

$$g_S = g_P = \frac{M_B^2 \tan^2\beta}{M_H^2} \frac{1}{(1+\epsilon_0\tan\beta)(1-\epsilon_\tau\tan\beta)} , \quad (4)$$

using the ratio of the two Higgs vacuum expectation values $\tan\beta$ and the charged Higgs mass (M_H). The parameters $\epsilon_{0,\tau}$ denote sparticle loop factors, and $\epsilon_0 = \epsilon_\tau = 0$ in case of the type-II 2HDM (two Higgs Doublet Model). Therefore, if these decays are measured, they provide information on $\tan\beta/M_H$. *We may lift up this paragraph to the overview section.*

Within the Type-II 2HDM, the effect of the charged Higgs boson in Eq. 3 and Eq. 4 leads to modification of the $B^+ \to \tau^+\nu$ branching fraction (Hou, 1993).

$$\mathcal{B}(B^+ \to \tau^+\nu) = \mathcal{B}(B^+ \to \tau^+\nu) \times r_H , \quad (5)$$

where the ratio r_H is given by,

$$r_H = (1 - M_B \tan^2\beta/M_{H^\pm})^2 . \quad (6)$$

Note: Will add some paragraphs for radiative decays.

0.0.2.2 $B^+ \to \tau^+\nu$ measurements

Among the leptonic B decays, the $B^+ \to \tau^+\nu$ decay has the largest branching fraction, and is the first accesible mode at the B factories. Both Belle and *BABAR* use the similar analysis method, where they fully reconstruct the accompanying B meson ($B_{\rm tag}$) either by hadronic or by semileptonic decays, and examine the rest of the event to serach for a $B^+ \to \tau^+\nu$ decay. Details of B reconstruction by hadronic decays are described in Section 6. Analyses using semileptonic tags suffers from worse signal-to-noise ratio, but provides higher efficiency.

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As for the $B_{\rm sig}$ side, signals are identified by detecting charged tracks from the signal decays, and requiring no extra activities in the electro-magnetic calorimeter. The most powerful variable for separating signal and background is the extra energy in the electromagnetic calorimeter, denoted as $E_{\rm ECL}$ in Belle and $E_{\rm extra}$ in *BABAR* which is the sum of the energies of neutral clusters that are not associated with either the $B_{\rm tag}$ or the τ daughter tracks. For signal events, $E_{\rm ECL}$ ($E_{\rm extra}$) must be either zero or a small value arising from beam background hits, therefore, signal events peak at low $E_{\rm ECL}$ ($E_{\rm extra}$). On the other hand, background events are distributed toward higher $E_{\rm ECL}$ ($E_{\rm extra}$) due to the contribution from additional neutral clusters.

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Belle Results

Belle has reported the first evidence of the $B^+ \to \tau^+\nu$ decay using 449M $B\overline{B}$ sample (Ikado, 2006). In this analysis, $B_{\rm tag}$ candidates are reconstructed in the decay modes, $B^- \to D^{(*)0} + \pi^-/\rho^-/a_1^-/D_S^{(*)-}$, where D^{*0} and D_S^{*-} are reconstructed by $D^{*0} \to D^0\pi^0/D^0\gamma$ and $D_S^{*-} \to D_S^+\gamma$,

Section Structure

		(private) draft
• 14.11.1 Overview of Leptonic Decays, and B-$\to D^{(*)} \tau \nu$	T.I	✓
• 14.11.2 $B^+ \to l^+ \nu$ ($l^+ = e, \mu, \tau$)		
– Theory of leptonic decays	T. I	✓
– $B^+ \to \tau^+ \nu$ measurements	T. I	✓ (*)
– $B^+ \to l^+ \nu$ measurements ($l^+ = e, \mu$)	S.R	✓
– $B^+ \to l^+ \nu \gamma$ measurements ($l^+ = e, \mu$)	S.R	✓
– Interpretation of results	T.I	✓
• 14.11.3 $B \to D^{(*)} \tau \nu$		
– Theory of $B \to D^{(*)} \tau \nu$	T.I	✓ (*)
– Experimental methodology and results	T.I/S.R	✓ (*)
– Interpretation of results	T.I/S.R	
• 14.11.4 Summary and future prospects		

(*): Theory/results for distribution measurements (BaBar) are not written yet.

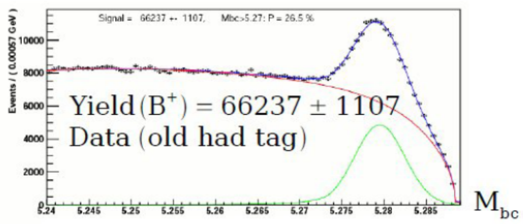
Available Results ($B\rightarrow \tau\nu$/ $l\nu$/ $l\nu\gamma$)

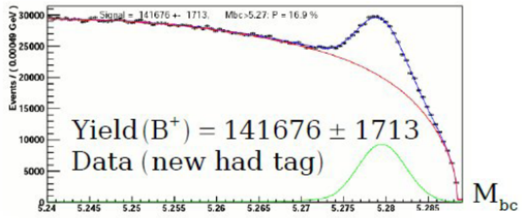
Mode	Exp	Tag	Ref.	
$B\rightarrow\tau\nu$	Belle	hadronic	PRL 97, 251802 (2006)	Update in progress
		SL	PRD-RC 82, 071101 (2010)	
	BaBar	hadronic	PRD-RC 77, 011107 (2008)	Update in progress
		SL	PRD-RC 81, 051101 (2010)	
$B\rightarrow l\nu$	Belle	untag	PLB 647, 67 (2007)	Update in progress
		hadronic		Analysis in progress
	BaBar	untag	PRD-RC 79, 091101 (2009)	
		hadronic	PRD-RC 77, 091104 (2008)	
		SL	PRD-RC 81, 051101 (2010)	
$B\rightarrow l\nu\gamma$	BaBar	hadronic	PRD-RC 80, 111105 (2009)	

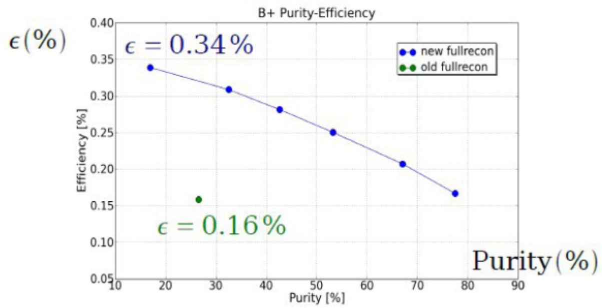
- Prospect for Belle results
  - $B\rightarrow\tau\nu$, $l\nu$ w/ hadronic tag → Target @ winter 2012
  - Others may not be in the scope of PBFB release in mid. 2012 (try our best, nevertheless).
- Also expect the final $B\rightarrow\tau\nu$ hadronic tag result from BaBar

Belle Prospect for B→ τ ν

- Results using the full data set (~770M $B\bar{B}$)
  - Present results: w/ 449M $B\bar{B}$ for hadronic tag
    w/ 657M BB for semileptonic tag
  - Reprocessed with improved tracking efficiency
- Improvement for the hadronic tag
  ⇨ effective luminosity improved by factor x2







Improved hadronic tag is being applied also for B→ lν, $D^{(*)}\tau\nu$.

Available Results ($B \rightarrow D^{(*)}\tau\nu$)

Mode	Exp	Tag	Ref.	
$B \rightarrow D^{(*)} \tau \nu$	Belle	inclusive	PRD-RC 82, 072005 (2010)	
		hadronic	arXiv: 0910.4301	Update in progress
	BaBar	hadronic	PRD 79, 092002 (2009), PRL 100, 021801 (2008)	Distribution also. Update in progress

- Belle analysis w/ (improved) hadronic tag + full data in progress, but don't know yet when results become ready…

Status/Plan

- Some email interactions between editors (w/cc to main editors) for the first draft (Nov. 13~).
- Will commit to SVN after writing unfinished sections a.s.a.p., and continue brushing up (by Dec/end.).
  - Draft tex files already moved to the SVN directory.
  - Bibliography files need be merged with the master bib files.
- Need consult theory editor for description of $B \to D \tau \nu$
  - BaBar authors show strong interest in direct contribution.
- Need replace some sections with updated results.

Belle $B \to \tau\nu$, $l\nu$ w/ hadronic tag.
BaBar $B \to \tau\nu$, $D^{(*)}\tau\nu$ w/ hadrnic tag.

Some Issues

$$\mathcal{B}(B^+ \to \ell^+\nu)_{\rm SM} = \frac{G_F^2 M_B m_\ell^2}{8\pi}\left(1-\frac{m_\ell^2}{M_B^2}\right)^2 \times f_B^2|V_{ub}|^2\tau_B\ ,$$

- Charged Higgs constraints from B→τν
  - SM Br. from $|V_{ub}|$ and f_B from CKM fit

$$\mathcal{B}_{\rm SM}(B^+ \to \tau^+\nu) = (1.20 \pm 0.25)\times 10^{-4}$$

$$\mathcal{B}(B \to \tau\nu)_{\rm CKMfit} = (0.786^{+0.179}_{-0.083})\times 10^{-4}$$

  - Need revisit the most updated experimental results, $|V_{ub}|$, f_B (CKMfit).
- Charged Higgs constraint from B→$D^{(*)}$ τ ν
  - Require some (non-trivial) works
    - Not so straight-forward
    - Revisit the most updated form factor parameters etc.
  - How to use distributions ?

Need work with theory editor.

- It is desirable to have new results for B→τν, $D^{(*)}$ τ ν
  - B→τν: prospect for update in Winter 2012.
  - Belle B→Dτν w/ hadronic tag: ???
    - The preliminary results in the present draft may have to be dropped.

Draft as of Nov.21

Contents

14.11 Leptonic decays, and $B \to D^{(*)}\tau\nu$

Editors:
Steve Robertson (BABAR)
Toru Iijima (Belle)

14.11.1 Overview of leptonic decays and $B \to D^{(*)}\tau^+\nu$

In this section, we review the measurements of purely leptonic decays, $B^+ \to \ell\nu (\ell = e, \mu, \tau)$, and semileptonic decays into τ final states, $B \to D^{(*)}\tau\nu$. Figure **??** shows Feynman diagrams of these tree level processes. Within the SM, they are sensitive to the magnitude of the CKM matrix elements $|V_{\rm cb}|$ and $|V_{\rm ub}|$. On the other hand, in extension of the SM, both are sensitive to a charged Higgs boson ($H^\pm$). In $B^+ \to \ell^+\nu$ decays, hadronic effects are encapsulated in the B decay constant f_B, while $B \to D^{(*)}\tau\nu$ decays have form factor uncertainties.

Experimentally, it is challenging to detect these decays because of small branching fractions and/or existence of neutrinos in the final state. Especially, tauonic decays, $B^+ \to \tau^+\nu$ and $B \to D^{(*)}\tau\nu$, involve more than two neutrinos in the final state, therefore, cannot be kinematically constrained. At B factories, one can fully reconstruct one of the B meson, referred to as the tag side ($B_{\rm tag}$), and compare properties of the remaining particle(s), referred to as the signal side ($B_{\rm sig}$), to those expected for signal and background. This method allows us to suppress strongly the combinatorial background. Disadvantage of the method is low efficiency in the full reconstruction at the level of $O(0.1)\%$. The high luminosity B factories have enabled us to measure these decays for the first time.

14.11.2 $B^+ \to \ell^+\nu$ ($\ell = e, \mu, \tau$)

14.11.2.1 Theory of leptonic decays

In the Standard Model (SM), the purely leptonic decay $B^+ \to \tau^+\nu$ proceeds via annihilation of $\overline{b}$ and u quarks to a W^+ boson (see Figure 1). The branching fraction is given by

$$\mathcal{B}(B^+ \to \ell^+\nu)_{\rm SM} = \frac{G_F^2 M_B m_\ell^2}{8\pi}\left(1 - \frac{m_\ell^2}{M_B^2}\right)^2 \times f_B^2 |V_{ub}|^2 \tau_B \,, \quad (1)$$

using the Fermi coupling constant G_F, the B meson mass M_B, the lepton mass m_ℓ and the B^- lifetime τ_B. The leptonic decays are helicity suppressed, as reflected in the

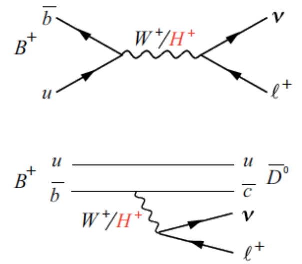


Fig. 1. Feynman diagrams of $B^+ \to \ell^+\nu$ (top) and $B \to D^{(*)}\tau\nu$ (bottom).

lepton mass dependence in Eq.(1, and the e and μ modes are suppressed by $1.05\times10{-7}$ and $4.49\times10{-3}$ with respect to the τ mode. The expected branching fraction for the τ mode is

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and is proportional to the fermion masses m_b and m_τ (G_F is the Fermi coupling constant, V_{qb} is the CKM matrix element, M_B is the B meson mass). Therefore, it is natural to look for NP in leptonic or semileptonic $b \to \tau$ transition. In the minimal supersymmetric extension of the SM (MSSM), the couplings $g_{S,P}$ in Eq. 3 are written as,

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Within the Type-II 2HDM, the effect of the charged Higgs boson in Eq. 3 and Eq. 4 leads to modification of the $B^+ \to \tau^+\nu$ branching fraction (Hou, 1993).

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where the ratio r_H is given by,

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Although the radiative mode $B^+ \to \ell^+\nu_\ell\gamma$ is additionally suppressed by the fine structure constant $\alpha_{\rm em}$, the presence of the photon can remove the helicity suppression of the purely leptonic modes by affecting the coupling of the spin-0 B meson to the spin-1 $W^\pm$ boson, possibly through an intermediate off-shell state **?**. The branching fraction of $B^+ \to \ell^+\nu_\ell\gamma$ is predicted in the SM to be of order 10^{-6} independent of the lepton type, making it potentially accessible at the B factories. Since the branching fractions for the e and μ modes exceed those of the non-radiative modes, they potentially provide an additional method to access $|V_{ub}|$ (or f_B) as well as a possible background for the non-radiative mode searches.

The decay rate depends on the $B \to \gamma$ form factor, but can be approximated as **?**

$$\mathcal{B}(B^+ \to \ell^+\nu_\ell\gamma) \approx \frac{\alpha_{\rm em} G_F^2 |V_{ub}|^2}{288\pi^2} f_B^2 m_B^5 \tau_B \left(\frac{Q_u}{\lambda_B} - \frac{Q_b}{m_b}\right)^2 \quad (7)$$

, where Q_i is the quark charge and λ_B is the first inverse moment of the B-meson wave function. This last parameter plays an important role in QCD factorization **?**. It also enters into calculations of the $B \to \pi$ form factor at zero momentum transfer and the branching fractions of two-body hadronic B-meson decays such as $B \to \pi\pi$, the benchmark channel for measuring the CKM angle α **?**. However, λ_B has large theoretical uncertainty, making $B^+ \to \ell^+\nu_\ell\gamma$ a crucial decay for obtaining a clean measurement of λ_B.

14.11.2.2 $B^+ \to \tau^+\nu$ measurements

Among the leptonic B decays, the $B^+ \to \tau^+\nu$ decay has the largest branching fraction, and is the first accessible mode at the B factories. Both Belle and *BABAR* use the similar analysis method, where they fully reconstruct the accompanying B meson ($B_{\rm tag}$) either by hadronic or by semileptonic decays, and examine the rest of the event to serach for a $B^+ \to \tau^+\nu$ decay. Details of B reconstruction by hadronic decays are described in Section 6. Analyses using semileptonic tags suffers from worse signal-to-noise ratio, but provides higher efficiency.

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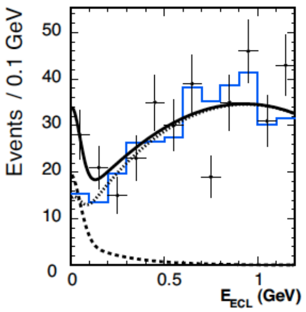
Belle has reported also a result with the semileptonic tagging method, using 657 M $B\overline{B}$ event sample (Hara, 2010). In this analysis, $B_{\rm tag}$ candidates were reconstructed by $B^- \to D^{*0}\ell^-\overline{\nu}$ and $B^- \to D^0\ell^-\overline{\nu}$ decays, where ℓ is electron (e) or muon (μ). D^0 mesons were reconstructed in the $K^-\pi^+$, $K^-\pi^+\pi^0$ and $K^-\pi^+\pi^-\pi^+$ modes. For the $B_{\rm sig}$ side, we use τ^+ decays to only one charged particle and neutrinos, i.e., $\tau^+ \to \ell^+\nu_\ell\overline{\nu}_\tau$ and $\tau^+ \to \pi^+\overline{\nu}_\tau$. Figure 2 show the $E_{\rm ECL}$ distribution overlaid with the fit results for sum of the all and each τ decay modes. We see a clear excess of signal events in the region near zero and obtain a signal yield of $N_S = 143^{+36}_{-35}$, corresponding to significance of 3.6σ including systematics. The deduced branching fraction is $\mathcal{B}(B \to \tau\nu) = (1.54^{+0.38}_{-0.37}({\rm stat.})^{+0.29}_{-0.31}({\rm syst.}))\times 10^{-4}$.

Note: The present Belle hadronic tag result is based on a version of $B_{\rm tag}$ reconstruction older than what is described in Section 6.



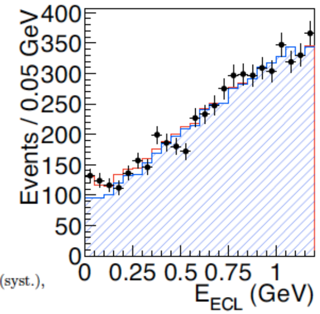


Fig. 2. Distribution of residual energy $E_{\rm ECL}$ reported by Belle using hadronic tags (top) and semileptonic tags (bottom). $B^+ \to \tau^+\bar{\nu}$ signals are seen near $E_{\rm ECL} = 0$.

Note: These Belle results will be replaced with the final results using the full data set and using the improved $B_{\rm tag}$ reconstruction.

***BABAR* Results**

Note: BABAR results will be put here.

Table 1 summarizes the branching fractions reported by Belle and *BABAR* using the hadronic and semileptonic tags. The naive average branching fraction is calculated to be,

$$\mathcal{B}(B \to \tau\nu)_{\rm AVG} = (1.73 \pm 0.35) \times 10^{-4}. \qquad (8)$$

Table 1. Summary of the results for $B^+ \to \tau^+\nu_\tau$. $N_{B\overline{B}}$: number of $B\overline{B}$ pairs in the data sample, $\mathcal{B}$: branching fraction (the first error is statistical, and the second systematic). *we may add more information: number of signals, significance, efficiency etc. BABAR semileptonic result may have been changed a little in the final publication, and need recalculation of the average.*

Exp.	Tag	$N_{B\overline{B}}$ (10^6)	B (10^{-4})	Ref.
Belle	hadronic	449	$1.79^{+0.56+0.46}_{-0.49-0.51}$	Ikado (2006)
Belle	semileptonic	657	$1.65^{+0.38+0.35}_{-0.37-0.37}$	Hara (2010)
BABAR	hadronic	383	$1.8^{+0.9}_{-0.8} \pm 0.4 \pm 0.2$	Aubert (2008)
BABAR	semileptonic	459	$1.8 \pm 0.8 \pm 0.1$	Aubert (2010)
Average			1.73 ± 0.35	

14.11.2.3 $B^+ \to \ell^+\nu$ ($\ell = e, \mu$)

Although the $B^+ \to e^+\nu$ and $B^+ \to \mu^+\nu$ branching fractions are substantially suppressed compared to the τ mode, these modes are still of considerable interest at the B factories. While the electron mode, within the SM, is well beyond reach, the μ mode has a predicted branching fraction of $\sim 5 \times 10^{-7}$ which is potentially detectable by *BABAR* and Belle. It is also notable that the relative enhancement (or suppression) of the leptonic branching fractions due to the existence of a charged Higgs boson is independent of the final state lepton mass (see Equation 6). Consequently, equally precise determinations of experimental branching fractions in any of the three leptonic modes would yield identical new physics constraints. Additionally, because the $B^+ \to \mu^+\nu$ final state contains only a single neutrino and a high momentum μ, there exist sufficient constraints that the search can be performed without the need for exclusive B tag reconstruction, with substantially higher signal efficiency than $B^+ \to \tau^+\nu$. To date, these inclusive searches have resulted in branching fraction limits which are within about a factor of two of the SM expectation, and which are limited by background statistics. *BABAR* has also performed a search using hadronic B tag reconstruction ?. While the signal efficiency in this case is reduced by a factor of $\sim 10^2$ relative to the inclusive search, reconstruction of the B tag provides information on the signal B four vector with the consequence that the μ momentum can be precisely determined in the B rest frame, resulting in a substantial reduction in backgrounds. With the data statistics available at *BABAR* and Belle the inclusive approach results in a significantly more stringent branching fraction limit than the tagged analysis, however it is notable that both methods current yield similar sensitivities for a 5σ signal observation due to the large statistical uncertainty in the background in the inclusive method. It is anticipated that both methods will provide complementary and precise $B^+ \to \mu^+\nu$ branching fraction measurements with the data statistics available at future super B factories.

Note: need comparison of BABAR and Belle $B^+ \to \ell^+\nu$ measurements.

14.11.2.4 $B^+ \to \ell^+\nu\gamma$ measurements

The only search for $\mathcal{B}(B^+ \to \ell^+\nu_\ell\gamma)$ which has been published by *BABAR* or Belle uses a hadronic-tag analysis ?. CLEO had previously published results based on an untagged search in 1997 ?, however BELLE ? and *BABAR* ? both performed studies using a similar "inclusive" method, both of which resulted in preliminary limits which have thus far not been published. Although the hadronic-tag technique results in a low signal efficiency (0.3% for signal modes), it compensates by providing a highly pure sample of B mesons with comparatively little non-$B\overline{B}$ (continuum) background, which proved problematic for the inclusive analyses. In addition, by reconstructing the $B_{\rm tag}$ using only detectable hadronic decay modes, the missing four-vector of the signal neutrino is fully determined. Thus, the hadronic tag analysis was able to avoid the model-dependent kinematic constraints in the signal selection that complicated the interpretations of the earlier analyses. *Note: How do we handle the fact that BABAR and Belle have not published their inclusive results?*

After reconstructing a $B_{\rm tag}$, the continuum background was suppressed using a multivariate selector of event shape variables since lighter $q\overline{q}$ pairs tend to produce a more jet-like shape with a strongly-preferred direction, usually at small angles to the beam axis. Only events with exactly one signal-side track pass the signal selection. This track must satisfy either electron or muon particle ID (PID), with Bremsstrahlung candidate clusters used to correct the four-vector of the electron. The highest energy photon within the rest of the particles was then selected as the signal photon, whose energy spectrum is expected to peak at $\sim$ 1 GeV. Remaining clusters were added to the extra energy ($E_{\rm extra}$) variable, on which was placed a loose requirement of < 0.8 GeV. To assure that the signal candidates were consistent with a three-body decay, the lepton momentum and the total missing momentum in the event were required to be essentially back-to-back in the frame recoiling from the photon. The most discriminating variable is the neutrino candidate's invariant mass, given as $m_\nu^2 \equiv |p_{\Upsilon(4S)} - p_{B_{\rm tag}} - p_\ell - p_\gamma|^2$ where p_i is the four-momentum of particle i. Fig. 3 shows that the signal peaks at zero, while the the background rises with m_ν^2.

The largest background for this analysis stemmed from $B^+ \to X_u^0\ell^+\nu_\ell$ events, where X_u is a neutral meson containing a u-quark. Thus, events in which the signal photon

Table 2. $B^+ \to \ell^+\nu$ branching fraction measurements by BABAR and Belle.

Exp.	Method/Tag	$N_{B\overline{B}}$ (10^6)	N_bg	N_obs	N^{SM}_{exp}	B (10^{-4})	Ref.
Belle	inclusive	449					Ikado (2006)
BABAR	inclusive						Aubert (2008)
BABAR	inclusive						Aubert (2008)
BABAR	hadronic						Aubert (2008)
BABAR	semileptonic						Aubert (2008)

candidate can be combined with another cluster to form an invariant mass consistent with the π^0 or η mass, or combined with a π^0 candidate to form an ω, were rejected. However, $B^+ \to X_u^0 \ell^+\nu_\ell$ events can mimic the signal decay kinematics, especially if only one high-energy photon daughter from the X_u decay is present in the signal-side clusters, or if the two photons from a $B^+ \to \pi^0\ell^+\nu_\ell$ decay are merged into a single EMC cluster containing the full energy of the π^0. This latter background was suppressed by limiting the lateral moment of the cluster energy deposit.

This analysis used a cut-and-count method and determined the background estimate in two parts: events which peak in the $m_{\rm ES}$ signal region which were estimated from various dedicated $B^+ \to X_u^0\ell^+\nu_\ell$ MC samples; and non-peaking events which were extrapolated directly from the data events in the $m_{\rm ES}$ sideband region, thus reducing the dependence on MC simulations. The largest background uncertainties stemmed, respectively, from the branching fractions and form factors of the various $B^+ \to X_u^0\ell^+\nu_\ell$ decays, and from the sideband data statistics.

The hadronic tag BABAR measurement used a data sample of 465 million $B\overline{B}$ pairs. A measurement of $\mathcal{B}(B^+ \to \ell^+\nu_\ell\gamma) = (6.5^{+7.6}_{-4.7}{}^{+2.8}_{-0.8}) \times 10^{-6}$ was obtained at 2.1σ, as well an upper limit of $\mathcal{B}(B^+ \to \ell^+\nu_\ell\gamma) < 15.6 \times 10^{-6}$ at 90% CL which approaches the SM expected value and is the most stringent reported limit to date. Because no requirements are applied to the lepton or photon kinematics, this analysis was the world's first measurement that is independent of the $B \to \gamma$ form factor models and valid over the full kinematic range.

However, using certain theoretical techniques, the extraction of λ_B can be improved by including a minimum energy requirement on the signal photon ?. A requirement that the signal photon candidate energy is > 1 GeV results in a partial branching fraction of $\Delta\mathcal{B}(B^+ \to \ell^+\nu_\ell\gamma) < 14 \times 10^{-6}$ at 90% CL. Tighter branching fraction limits that are dependent on the signal model were also determined by introducing a kinematic requirement on the angles between the three daughter particles of the signal decay. In the model that the two $B \to \gamma$ form-factors, f_V and f_A, are equal, $\mathcal{B}(B^+ \to \ell^+\nu_\ell\gamma) < 3.0 \times 10^{-6}$. In the less common $f_A = 0$ model, $\mathcal{B}(B^+ \to \ell^+\nu_\ell\gamma) < 18 \times 10^{-6}$.

Although we have yet to observe a significant $B^+ \to \ell^+\nu_\ell\gamma$ signal, the sensitivity of the these decays are such that the next generation B factories should have enough statistics to reach the SM predictions of order 10^{-6}.

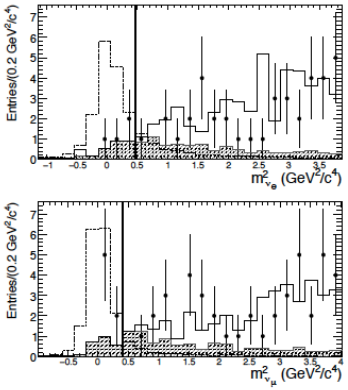


Fig. 3. m_ν^2 distribution after all selection criteria are applied, in electron (top) and muon (bottom) modes for the $m_{\rm ES}$-peaking (shaded) plus non-peaking (solid) contributions in the full background MC sample, signal MC normalized to $\mathcal{B} = 40 \times 10^{-6}$ (dashed), and data (points). Events to the left of the vertical lines are selected.

14.11.2.5 Interpretation of results

The $B^+ \to \tau^+\nu$ branching fraction $\mathcal{B}(B \to \tau\nu)_{\rm AVG} = (1.73 \pm 0.35) \times 10^{-4}$ (Eq. 8), is consistent with the above SM prediction, $\mathcal{B}(B \to \tau\nu)_{\rm SM} = (1.20 \pm 0.25) \times 10^{-4}$ (Eq. 2), calculated by the formula Eq. 1 and inputs of $|V_{\rm ub}|$ from semileptonic decay data and f_B from recent lattice QCD calculations. The ratio r_H, as defined in Eq. 6, is found to be $r_H = 0.95 \pm 0.32$. Based on this result and Eq. 5, the charged Higgs can be constrained in the $(\tan\beta, m_H)$ plane, as shown in Figure 4.

It should be noted however that there appears tension in this comparison, if the SM value is taken from the CKM fit. From the CKM fit, the $B^+ \to \tau^+\nu$ branching fraction is predeicted to be,

$$\mathcal{B}(B \to \tau\nu)_{\rm CKMfit} = (0.786^{+0.179}_{-0.083}) \times 10^{-4}\,. \qquad (9)$$

In this case, the average branching fraction $\mathcal{B}(B \to \tau\nu)_{AVG}$ is 2.4 σ higher than the prediction.

Note: The charged Higgs constraint is based on $\mathcal{B}_{\rm SM}$ rather than $\mathcal{B}_{\rm CKMfit}$. If we use the latter, constraint is changed significantly.

Note: Comparison to LHC should be made here or later in the grand summary of this section.

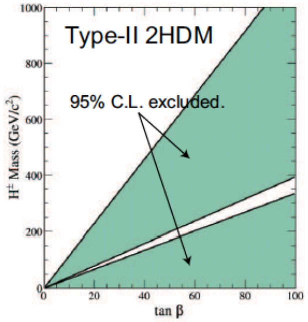


Fig. 4. Constraint on charged Higgs in the $(\tan\beta, m_H)$ plane in the type-II two Higgs doublet models. Hatched regions are excluded by $B \to \tau\nu$ at 95% confidence level.

14.11.3 $B \to D(*)\tau^+\nu$

14.11.3.1 Theory of $B \to D(*)\tau^+\nu$

The semileptonic B decay to the τ channel, $B \to D^{(*)}\tau\nu_\tau$, is also sensitive to the charged Higgs. In the SM, the branching fractions are predicted to be (0.69 ± 0.04)% and (1.41 ± 0.07)% for $\overline{B}^0 \to D^+\tau^-\overline{\nu_\tau}$ and $\overline{B}^0 \to D^{*+}\tau^-\overline{\nu_\tau}$ respectively (Chen and Geng, 2006). On the other hand, if a charged Higgs boson ($H^\pm$) exists, the branching fraction can be modified significantly. The $B \to D\tau\nu_\tau$ decay has similar sensitivity to $H^\pm$ as the $B \to \tau\nu_\tau$ decay, but with different theoretical systematics; the former suffers from uncertainty in the form factor, while the latter requires knowledge of the B decay constant f_B. Moreover, the three-body $B \to D\tau\nu_\tau$ allows us to study τ polarization or resulting decay distributions, which discriminate between $H^\pm$ and $W^\pm$ exchange. Therefore, they provide complementary approaches to searching for $H^\pm$ signatures in B decays.

Effects of the charged Higgs to $B \to D\tau\nu$ decays are discussed in a number of theoretical liratures (Grzadkowski and Hou, 1992; Itoh, Komine, and Okada, 2005; Kiers and Soni, 1997; Nierste, Trine, and Westhoff, 2008; Tanaka, 1995). The analysis of $B \to D\tau\nu$ requires the knowledge of the vector and scalar $B \to D$ form factors. The vector form factor can be dedcued from the semileptonic decay into the light leptons $B \to D\ell\nu_\ell (\ell = e, \mu)$. The charged Higgs can be constrained based on the ratio, $R(D) = \mathcal{B}(B \to D\tau\nu)/\mathcal{B}(B \to D\ell\nu)$, which is related to the scalar couping constant g_S in Eq. 4. The normalization to $\mathcal{B}(B \to D\ell\nu)$ reduces both experimental and theoretical systematic errors, where the latter arises mainly from the vector form factor uncertainty.

Note: Also need description on charged Higgs constraint using distribution (polarization). Theory description for $B \to D\tau\nu$ is not trivial, and we'd better consul theorists how to describe them in a not-too-long way.

14.11.3.2 Experimental methodology and results

Similarly to $B^+ \to \tau^+\nu$, the $B \to D^{(*)}\tau\nu$ decay has more than two neutrinos in the final state, and cannot be kinematically constrained. Therefore, tagging methods are applied in analyses for measuring $B \to D^{(*)}\tau\nu$. Both Belle and BABAR have reported results using hadronic decay tags. The Belle analysis uses also another method, referred to as "inclusive tags", where $B_{\rm tag}$'s are reconstructed by calculating the four-vector sum of the tracks inclusively without reconstructing the intermediate mesons.

Belle results

The Belle collaboration reported the first observation (5.2 σ) of an exclusive $b \to c\tau\nu_\tau$ decay in the $B^0 \to D^*\tau^+\nu$ channel using the inclusive tag method in the data sample of 535M $B\overline{B}$ pairs (Matyja, 2007). The $\tau^+ \to e^+\nu_e\overline{\nu}_\tau$ and $\tau^+ \to \pi^+\overline{\nu}_\tau$ decays were used to reconstruct τ lepton candidates. The deduced branching fraction was $\mathcal{B}(\overline{B}^0 \to D^{*+}\tau^-\overline{\nu}_\tau) = (2.02^{+0.40}_{-0.37} \pm 0.37) \times 10^{-2}$.

More recently, a new analysis for $B \to D^{(*)}\tau\nu_\tau$ has been performed using 657 M $B\overline{B}$ pairs (Bozek, 2010). The signal and combinatorial background yields are extracted from an extended unbinned maximum likelihood fit to the $M_{\rm tag}$ (beam-energy constraint mass of the $B_{\rm tag}$ and P_{D^0} (momentum of D^0 from $B_{\rm sig}$ measured in the $\Upsilon(4S)$ frame) variables. The $\tau^+ \to e^+\nu_e\overline{\nu}_\tau$, $\tau^+ \to \mu^+\nu_\mu\overline{\nu}_\tau$ and $\tau^+ \to \pi^+\overline{\nu}_\tau$ are used to reconstruct the τ^+ lepton candidates. In total, 13 different decay channels, 8 for D^{*0} and 5 for D^0 are considered. The fits are performed simultaneously to all data subsets. In each of the sub-channels, the data was described as the sum of four components; signal, cross-feed between $\overline{D}^{*0}\tau^+\nu_\tau$ and $\overline{D}^0\tau^+\nu_\tau$, combinatorial and peaking backgrounds. Figure 5 shows the $M_{\rm tag}$ and P_{D^0} distributions and fit by the four components. The extracted signal yields (significance) are 446^{+58}_{-56} (8.1 σ) for $B^+ \to \overline{D}^{*0}\tau^+\nu_\tau$ and 146^{+42}_{-41} (3.5 σ) for $B^+ \to \overline{D}^0\tau^+\nu_\tau$. This is the first evidence of the $B^+ \to \overline{D}^0\tau^+\nu_\tau$ decay. The branching fractions are found to be $\mathcal{B}(B^+ \to \overline{D}^{*0}\tau^+\nu_\tau) = (2.12^{+0.28}_{-0.27} \pm 0.29) \times 10^{-2}$ and $\mathcal{B}(B^+ \to \overline{D}^0\tau^+\nu_\tau) = (0.77^{+0.22}_{-0.22} \pm 0.12) \times 10^{-2}$.

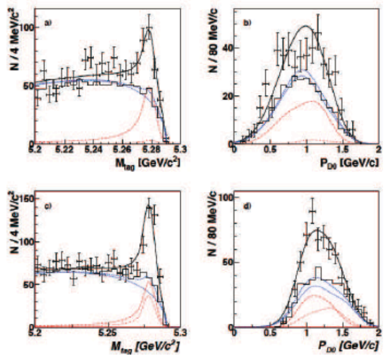


Fig. 5. The fit projection to $M_{\rm tag}$ and P_{D^0} for $M_{\rm tag} > 5.26$ GeV/c^2 (a, b) for $D^{*0}\tau^-\bar{\nu}_\tau$ and (c, d) for $D^0\tau^-\bar{\nu}_\tau$

The Belle collaboration also present results using hadronic tags based on 657M $B\overline{B}$ (Adachi, 2009). The $B \to D\tau\nu_\tau$ and $B \to D^*\tau\nu_\tau$ signals are extracted using unbinned extended maximum likelihood fits to the two-dimensional ($m^2_{\rm miss}, E_{\rm ECL}$) distributions obtained after the selection of the signal decays. Signals are characterized by relatively large missing mass $m_{\rm miss}$ and $E_{\rm ECL}$ near zero. The B^+ and B^0 tag samples are fitted separately, since the cross talk between the two tags are found to be small. Then for each B^0 and B^+ tag, a fit is performed simultaneously to the two distributions for the $D\tau\nu_\tau$ and $D^*\tau\nu_\tau$ modes. The fit components are two signal modes; $B \to D\tau\nu_\tau$ and $B \to D^*\tau\nu_\tau$, the backgrounds from $B \to D\ell\nu_\ell$, $B \to D^*\ell\nu_\ell$, and other processes. Figure 6 shows the $m_{\rm miss}$ and $E_{\rm ECL}$ distributions and fit of the four components for the $B^+ \to \overline{D}{}^0\tau^+\nu_\tau$ and $B^0 \to D^-\tau^+\nu_\tau$ decays.

Note: In the near future, the Belle collaboration will be able to provide results with the full data set (∼ 770M $B\overline{B}$). Especially, significant improvement is expected for the hadronic tag, by increasing the data size (× ∼ 1.7) and also by introducing a newly developed full reconstruction code with higher tagging efficiency (× ∼ 2).

BABAR results

BABAR results (including distributions) will be presented.

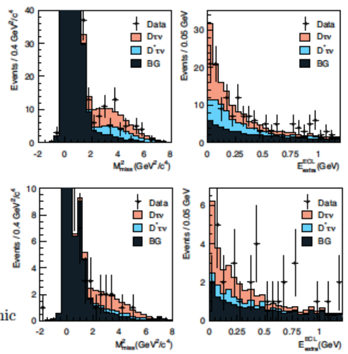


Fig. 6. The fit projection to $M^2_{\rm miss}$ and $E_{\rm ECL}$ (top) for $B^- \to D^0\tau^-\bar{\nu}_\tau$ and (bottom) for $\overline{B}{}^0 \to D^+\tau^-\bar{\nu}_\tau$.

14.11.4 Interpretation of results

Table 3 summarizes the results of the $B \to D^{(*)}\tau\nu$ branching fraction measurements by Belle and *BABAR*The naive average of the ratio $R(D)$ is found to be $R(D) = 0.40 \pm 0.08$, and is consistent with the SM value $R(D)_{\rm SM} = 0.302 \pm 0.015$.

Note: Work on creating new plot for the charged higgs constraint.

Note: What about constraint using also distribution ?

14.11.5 Summary and future prospects

Note: Will be added later

Table 3. Summary of the results for $B \to D^{(*)}\tau\nu$. $N_{B\overline{B}}$: number of $B\overline{B}$ pairs in the data sample used for the analysis, $\mathcal{B}$: branching fraction (the first error is statistical, the second systematic, and the third due to the branching fraction uncertainty in the normlization mode), Σ: significance of the signal including systematic, $R(D^{(*)})$: the ratio $\mathcal{B}(B \to D^{(*)}\tau\nu)/\mathcal{B}(B \to D^{(*)}\ell\nu)$.

Exp.	Tag	$N_{B\overline{B}}$ (10^6)	$\mathcal{B}$ (10^{-4})	Σ	$R(D^{(*)})$ (%)	Ref.
$\overline{B}{}^0 \to D^{*+}\tau^-\bar{\nu}_\tau$						
Belle	inclusive	535	$2.02^{+0.40}_{-0.37} \pm 0.37$	3.5		Matyja (2007)
Belle	hadronic	657	$2.56^{+0.75+0.31}_{-0.66-0.22} \pm 0.10$	4.7	48^{+14+6}_{-12-4}	Adachi (2009)
BABAR	hadronic	232	$1.11 \pm 0.51 \pm 0.04 \pm 0.04$	2.7	$20.7 \pm 9.5 \pm 0.8$	Aubert (2009)
$B^- \to D^{*0}\tau^-\bar{\nu}_\tau$						
Belle	inclusive	657	$2.12^{+0.28}_{-0.27} \pm 0.29$	8.1		Bozek (2010)
Belle	hadronic	657	$3.04^{+0.69+0.40}_{-0.66-0.47} \pm 0.22$	3.9	47^{+11+6}_{-10-7}	Adachi (2009)
BABAR	hadronic	232	$2.25 \pm 0.48 \pm 0.22 \pm 0.17$	5.3	$34.6 \pm 7.3 \pm 3.4$	Aubert (2009)
$\overline{B}{}^0 \to D^+\tau^-\bar{\nu}_\tau$						
Belle	hadronic	657	$1.01^{+0.46+0.13}_{-0.41-0.11} \pm 0.10$	2.6	48^{+22+6}_{-19-5}	Adachi (2009)
BABAR	hadronic	232	$1.04 \pm 0.35 \pm 0.15 \pm 0.10$	3.3	$48.9 \pm 16.5 \pm 6.9$	Aubert (2009)
$B^- \to D^0\tau^-\bar{\nu}_\tau$						
Belle	inclusive	657	$0.77 \pm 0.22 \pm 0.12$	3.5		Bozek (2010)
Belle	hadronic	657	$1.51^{+0.41+0.24}_{-0.39-0.19} \pm 0.15$	3.8	70^{+19+11}_{-18-9}	Adachi (2009)
BABAR	hadronic	232	$0.67 \pm 0.37 \pm 0.11 \pm 0.07$	1.8	$31.4 \pm 17.0 \pm 4.9$	Aubert (2009)

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