

Heavy Flavours II

FAPPS 2011 at Les Houches
Emi KOU (LAL/IN2P3)

CP “*invariance*” of K system



How can we make two CP (+ and -) states from K^0 and \bar{K}^0 ?



$$\begin{aligned}\mathcal{CP}|K^0\rangle &= |\bar{K}^0\rangle & K^0 &= \bar{s}d \\ \mathcal{CP}|\bar{K}^0\rangle &= |K^0\rangle & \bar{K}^0 &= \bar{d}s\end{aligned}$$

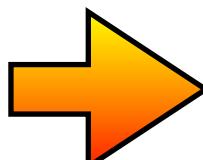
Gell-Mann and Pais (1955)

ANSWER

If the K is a mixed state of K^0 and \bar{K}^0 in nature...

$$|K_1\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle)$$

$$|K_2\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle)$$



$$\begin{aligned}\mathcal{CP}|K_1\rangle &= +\frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle) \\ &= |K_1\rangle\end{aligned}$$

CP EVEN

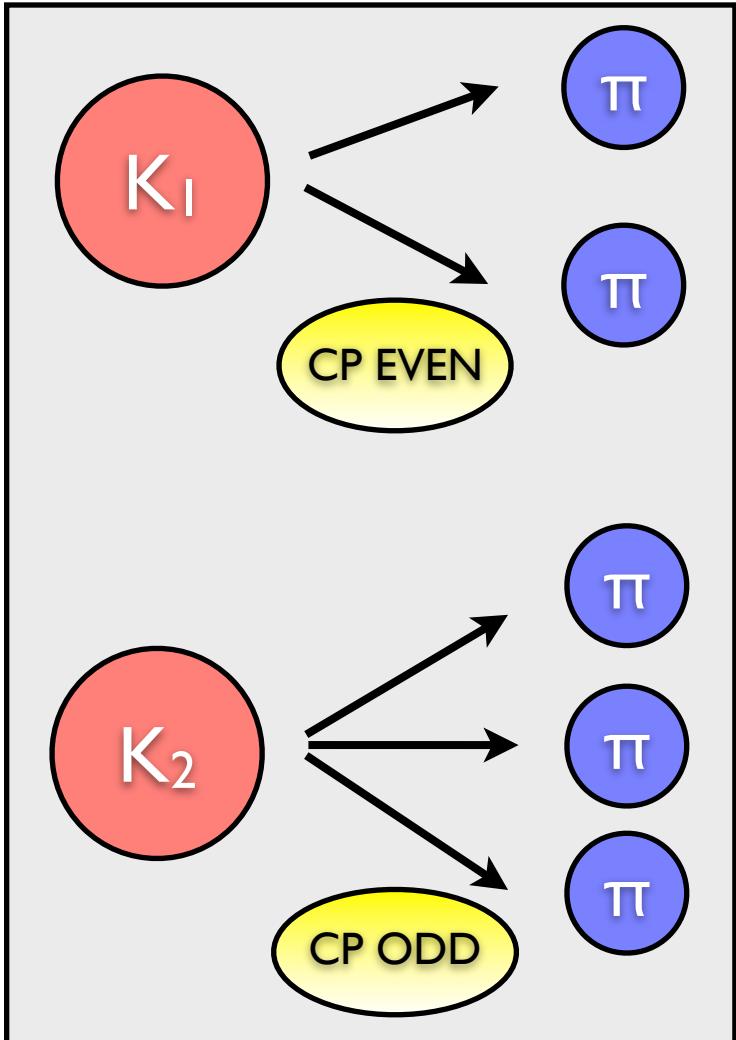
$$\begin{aligned}\mathcal{CP}|K_2\rangle &= -\frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle) \\ &= -|K_2\rangle\end{aligned}$$

CP ODD

CP “*invariance*” of K system

Distinguishing K_1 and K_2

By the decay channel



By the life-time

$$M_K = 498 \text{ MeV}$$

$$M_\pi = 140 \text{ MeV}$$

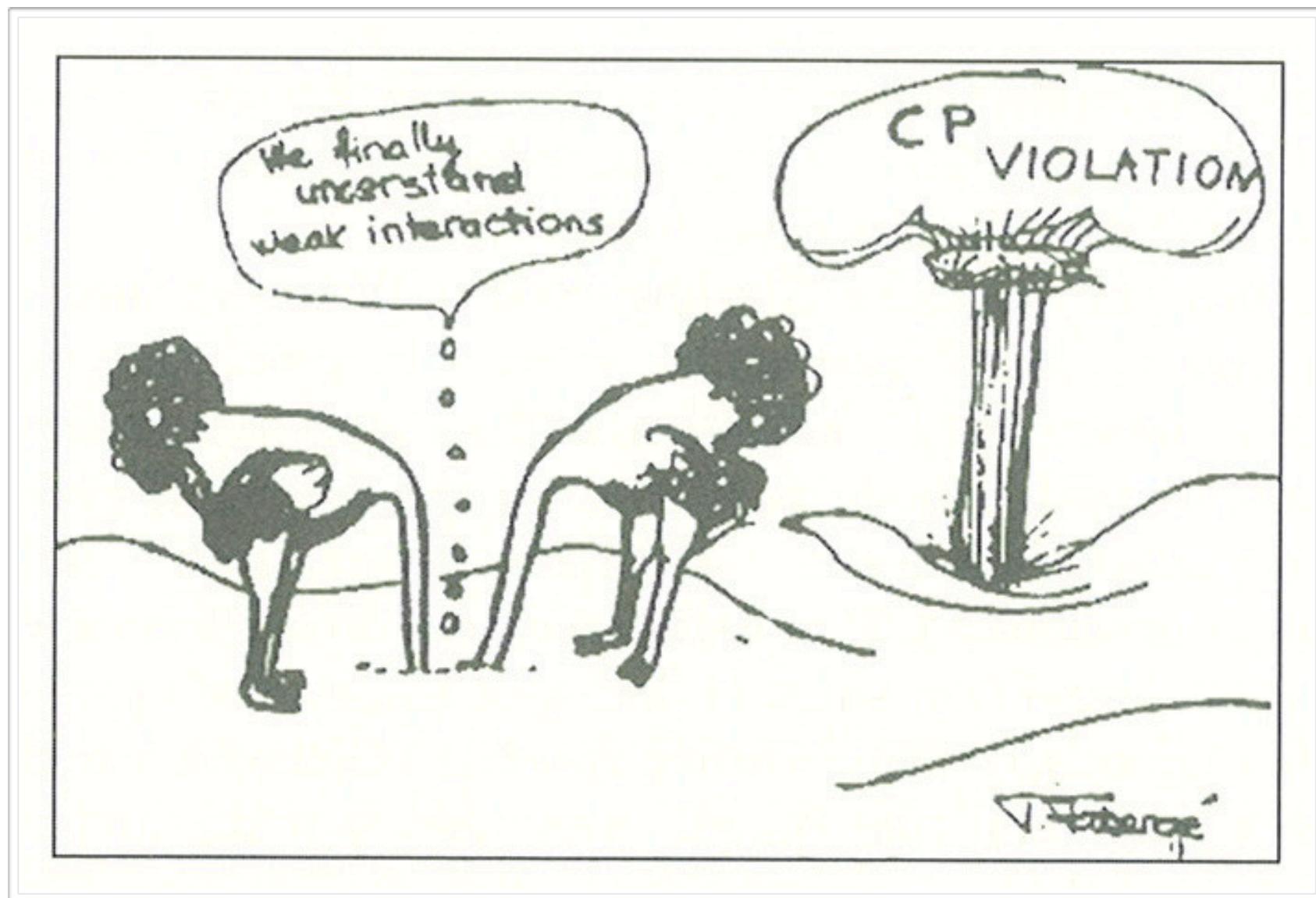
Phase space for 2π is about 600
larger than for 3π

$$\tau(K_1) \simeq 0.90 \times 10^{-10} s$$

$$\tau(K_2) \simeq 5.1 \times 10^{-8} s$$

Accidental phase space suppression:
short-lived K is K_1 and long-lived one is K_2

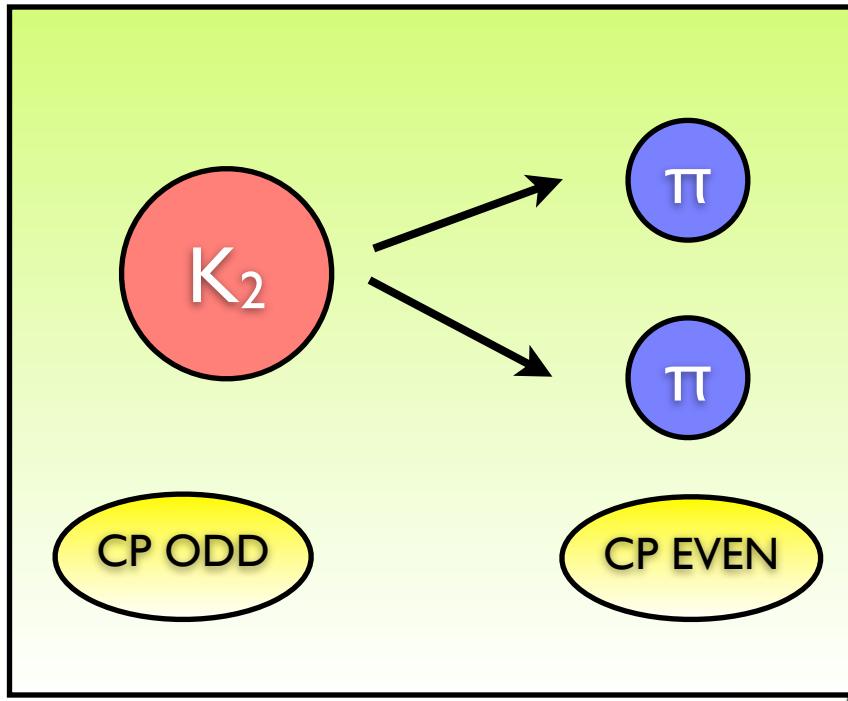
CP non-invariance of K system



Cabibbo (1966)

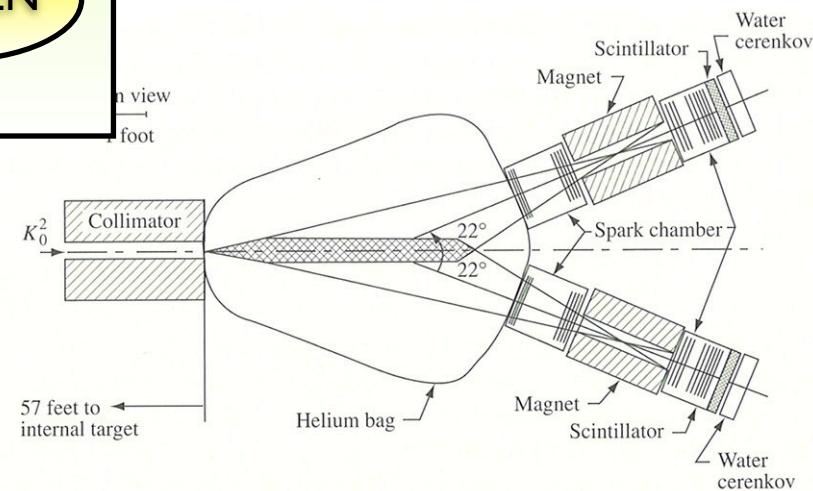
CP non-invariance of K system

First observation of the CP violation



**Cronin, Fitch
Christenson, Turlay
(1964)**

Long-lived K_2
decaying to 2π !!!

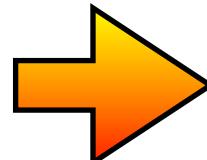


CP non-invariance of K system

We thought...

$$|K_1\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle)$$

$$|K_2\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle)$$



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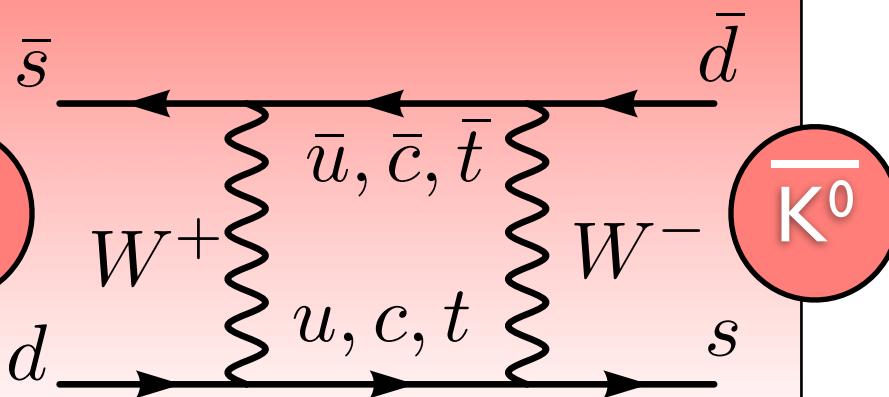
$$\begin{aligned}\mathcal{CP}|K_2\rangle &= -\frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle) \\ &= -|K_2\rangle\end{aligned}$$

CP ODD

But, actually...

K^0 and \bar{K}^0 can mix through box diagram.
Thus, they are not mass eigenstate.

K^0 and \bar{K}^0 can mix!



$$|K_S\rangle = \frac{1}{\sqrt{2}}(\textcolor{red}{p}|K^0\rangle + \textcolor{blue}{q}|\bar{K}^0\rangle)$$

$$|K_L\rangle = \frac{1}{\sqrt{2}}(\textcolor{red}{p}|K^0\rangle - \textcolor{blue}{q}|\bar{K}^0\rangle)$$

If $q/p \neq 1$,
the mass eigenstate $K_{S/L}$
are not CP eigen state
CP violation!!



Oscillation with Weak interaction

Now we *diagonalize this matrix*

$$\begin{aligned}\mathcal{H} &= \mathbf{M} - \frac{i}{2}\boldsymbol{\Gamma} \\ &= \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{21} - \frac{i}{2}\Gamma_{21} & M_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix}\end{aligned}$$

Using *CPT invariance ($M_{11}=M_{22}, \Gamma_{11}=\Gamma_{22}$) and \mathbf{M} and $\boldsymbol{\Gamma}$ being Hermitian, we find the mass eigenstate P_1 and P_2*

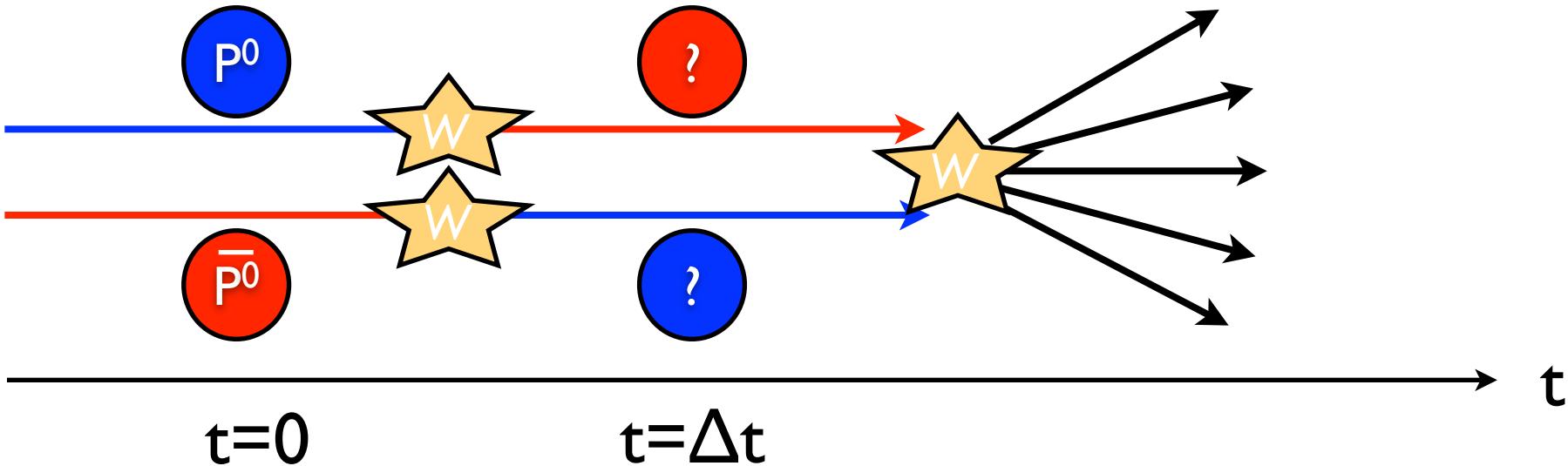
$$\begin{aligned}|P_1\rangle &= p|P^0\rangle + q|\bar{P}^0\rangle ; & \frac{q}{p} &= \pm \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}} \\ |P_2\rangle &= p|P^0\rangle - q|\bar{P}^0\rangle\end{aligned}$$

$$\begin{aligned}|P^0(t)\rangle &= f_+(t)|P^0\rangle + \frac{q}{p}f_-(t)|\bar{P}^0\rangle ; & f_\pm(t) &= \frac{1}{2}e^{-i(M_1 - i\Gamma_1/2)t} [1 \pm e^{-i(\Delta M + i\Delta\Gamma/2)t}] \\ |\bar{P}^0(t)\rangle &= f_+(t)|P^0\rangle + \frac{p}{q}f_-(t)|\bar{P}^0\rangle\end{aligned}$$

Time evolution formula



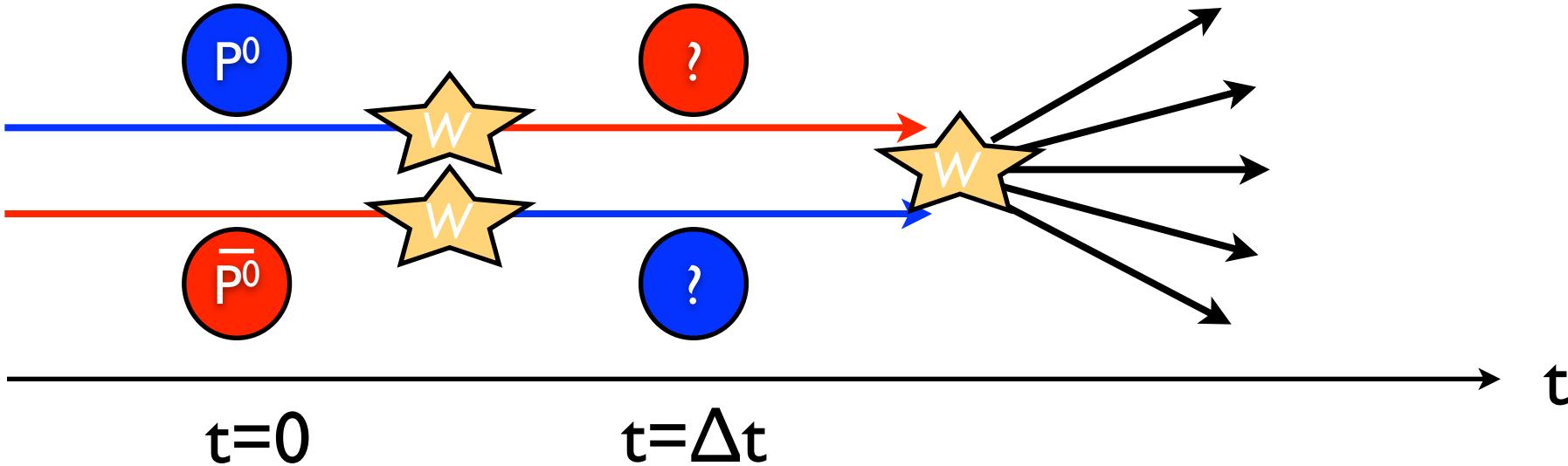
Decay width with Weak interaction



$$A(f) = \langle f | \mathcal{H}^{\Delta F=1} | P^0 \rangle \quad \overline{A}(f) = \langle f | \mathcal{H}^{\Delta F=1} | \bar{P}^0 \rangle$$



Decay width with Weak interaction



$$A(f) = \langle f | \mathcal{H}^{\Delta F=1} | P^0 \rangle \quad \bar{A}(f) = \langle f | \mathcal{H}^{\Delta F=1} | \bar{P}^0 \rangle$$

$$\Gamma(P^0(t) \rightarrow f) \propto e^{-\Gamma_1 t} |A(f)|^2 \left[K_+(t) + K_-(t) \left| \frac{q}{p} \right|^2 \left| \frac{\bar{A}(f)}{A(f)} \right|^2 + 2Re \left[L^*(t) \left(\frac{q}{p} \right) \bar{\left(\frac{\bar{A}(f)}{A(f)} \right)} \right] \right]$$
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$$K_{\pm}(t) = 1 + e^{\Delta\Gamma t} \pm 2e^{\frac{1}{2}\Delta\Gamma t} \cos \Delta M t, \quad L^*(t) = 1 - e^{\Delta\Gamma t} + 2ie^{\frac{1}{2}\Delta\Gamma t} \sin \Delta M t$$

Time-dependent CP asymmetry in B decay

Time-dependent CP asymmetry is defined as (f is some CP eigenstate):

$$\frac{\Gamma(B^0(t) \rightarrow f) - \Gamma(\bar{B}^0(t) \rightarrow f)}{\Gamma(B^0(t) \rightarrow f) + \Gamma(\bar{B}^0(t) \rightarrow f)}$$

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In B meson system, we can use $\Delta\Gamma \ll \Delta M$, which simplifies our formula:

$$\frac{\Gamma(B^0(t) \rightarrow f) - \Gamma(\bar{B}^0(t) \rightarrow f)}{\Gamma(B^0(t) \rightarrow f) + \Gamma(\bar{B}^0(t) \rightarrow f)} = \xi_f \sin(\Delta M_B t) S_f + \cos(\Delta M_B t) C_f$$
$$S_f = \frac{2 \operatorname{Im} \left(\frac{q}{p} \frac{A(\bar{B}^0 \rightarrow f)}{A(B^0 \rightarrow f)} \right)}{1 + \left| \frac{A(\bar{B}^0 \rightarrow f)}{A(B^0 \rightarrow f)} \right|^2}, \quad C_f = \frac{1 - \left| \frac{A(\bar{B}^0 \rightarrow f)}{A(B^0 \rightarrow f)} \right|^2}{1 + \left| \frac{A(\bar{B}^0 \rightarrow f)}{A(B^0 \rightarrow f)} \right|^2}$$

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Asymmetry is nonzero only when there is a complex phase in the theory!!!

Plan

- 1st lecture: Introduction to flavour physics
 - ★ Weak interaction processes (charges, neutral processes, GIM mechanism)
 - ★ Discovery of CP violation in the K system
 - ★ Measuring oscillation in the B system
- 2nd lecture: Describing oscillations within SM
 - ★ Kobayashi-Maskawa mechanism for CP violation
 - ★ Testing the unitarity of the CKM matrix

Plan

- 3rd lecture: Searching new physics with flavour physics
 - ★ Some examples in the past
 - ★ Some examples in the future

Where is the complex number in SM??!

Theoretically, there are only a few possible couplings which can be complex in SM!



Where is the complex number in SM??!

Theoretically, there are only a few possible couplings which can be complex in SM!

It took nearly 10 years to find the solution for this complex coupling...

Kobayashi, Maskawa
(1973)

Parameter counting of the unitary matrix to go to diagonalize the Yukawa coupling

Unitarity condition

$$UU^\dagger = 1 \longrightarrow 2n^2 - n^2 = n^2$$



Phase convention

$$n^2 - (2n - 1) = (n - 1)^2$$

Where is the complex number in SM?!

Theoretically, there are only a few possible

For two generation, only 1 rotation remains while for three generation, 3 rotations plus 1 phase remains (prediction of the 3rd generation).

go to diagonalize the Yukawa coupling

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Phase convention

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3 mixings and 1 phase

phase. The rotation is defined as follows:

$$\omega(\theta_{12}, 0) = \begin{pmatrix} \cos \theta_{12} & \sin \theta_{12} & 0 \\ -\sin \theta_{12} & \cos \theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (1)$$

$$\omega(\theta_{13}, \delta_1) = \begin{pmatrix} \cos \theta_{13} & 0 & \sin \theta_{13} e^{-i\delta_1} \\ 0 & 1 & 0 \\ -\sin \theta_{13} e^{-i\delta_1} & 0 & \cos \theta_{13} \end{pmatrix} \quad (2)$$

$$\omega(\theta_{23}, 0) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_{23} & \sin \theta_{23} \\ 0 & -\sin \theta_{23} & \cos \theta_{23} \end{pmatrix} \quad (3)$$

Then, the standard CKM matrix is obtained by choosing to multiply these matrices in the following order:

$$V_{\text{CKM}}^{3 \times 3} = \omega(\theta_{23}, 0)\omega(\theta_{13}, \delta_1)\omega(\theta_{12}, 0). \quad (4)$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix},$$

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We need experimental verifications that all 9 complex elements can be explained by the 4 input parameters.

A new parameterization

phase. The rotation is defined as follows:

$$\omega(\theta_{12}, 0) = \begin{pmatrix} \cos \theta_{12} & \sin \theta_{12} & 0 \\ -\sin \theta_{12} & \cos \theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (1)$$

$$= \begin{pmatrix} \cos \theta_{12} & 0 & \sin \theta_{12} e^{-i\delta_1} \\ 0 & 1 & 0 \\ 0 & \sin \theta_{12} e^{-i\delta_1} & \cos \theta_{12} \end{pmatrix}$$

We re-parametrize in terms of λ, A, ρ and η :

$$\sin \theta_{12} = \lambda, \sin \theta_{13} = A(\rho - i\eta)\lambda^3, \sin \theta_{23} = A\lambda^2$$

Realizing the hierarchy in the matrix, we

Then,
order:

expand in terms of $\lambda \sim 0.22$:

$$\sin \theta_{12} = \mathcal{O}(\lambda), \sin \theta_{23} = \mathcal{O}(\lambda^2), \sin \theta_{13} = \mathcal{O}(\lambda^3)$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix},$$

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Wolfenstein's parameterization

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

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New parameters:
 λ, A, ρ and η

Wolfenstein's parameterization

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Phases appear at
|3, 3| elements

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Expansion in
order λ^3

$$= \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$= \begin{pmatrix} 1 - \lambda^2/2 - \lambda^4/8 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 + (-1/8 - A^2/2)\lambda^4 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 + A\lambda^4(1/2 - \rho - i\eta) & 1 \end{pmatrix} + \mathcal{O}(\lambda^5)$$

Expansion in
order λ^4

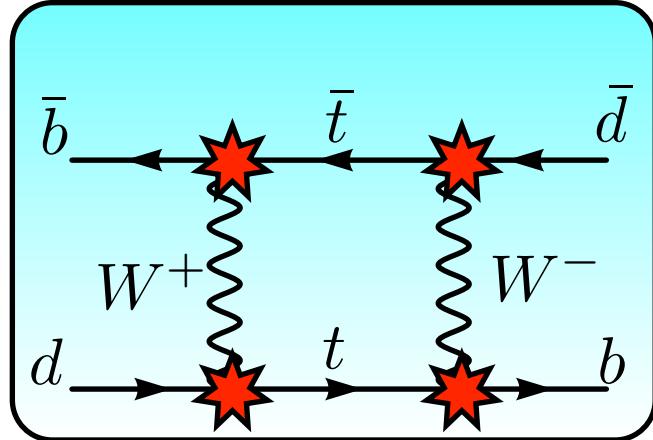


Computing q/p

In the B system, we have $M_{12} \gg \Gamma_{12}$, thus

$$\frac{q}{p} = \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}} \simeq \sqrt{\frac{M_{12}^*}{M_{12}}} \equiv e^{i\phi}$$

Loop function
dominant=top quark



$$M_{12} = \frac{G_F^2 m_W^2}{16\pi^2} (V_{tb} V_{td}^*)^2 S_0 \left(\frac{m_t^2}{m_W^2} \right) \times \eta_{\text{QCD}} \frac{\langle B^0 | (\bar{d}b)_{V-A} (\bar{d}b)_{V_A} | \bar{B}^0 \rangle}{m_B}$$

Strong interaction part

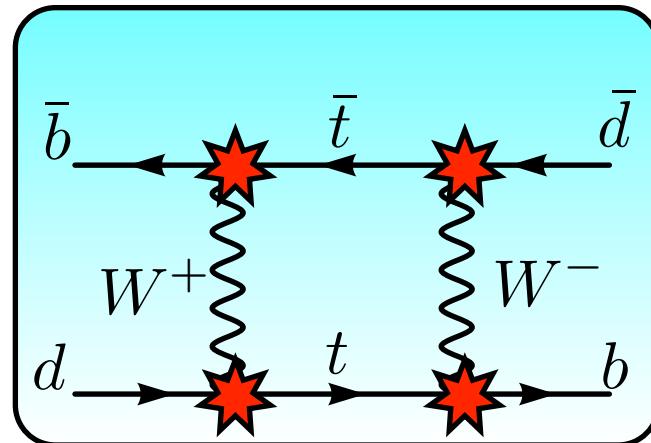


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$$V_{td} = A\lambda^3(1 - \rho - i\eta)$$

$$V_{tb} = 1$$



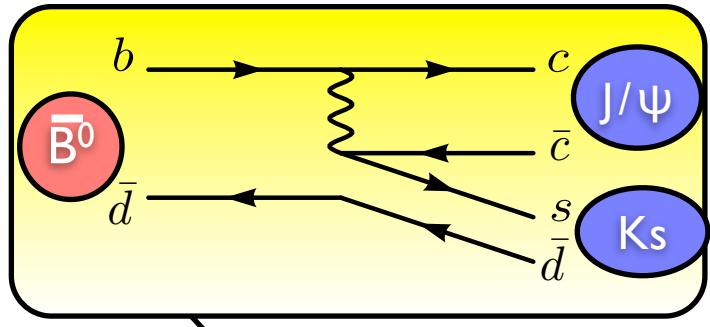
$$\frac{q}{p} = e^{-2i \arg(V_{tb}^* V_{td})}$$

Strong interaction part

DONE!



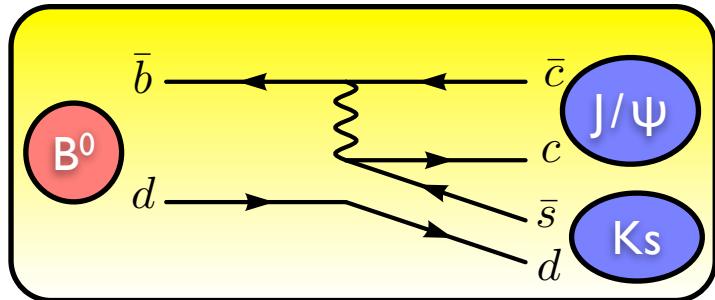
Computing $A(\bar{B}^0 \rightarrow J/\psi K_S)/A(B^0 \rightarrow J/\psi K_S)$



$$A(\bar{B}^0 \rightarrow J/\psi K_S)$$

$$= \frac{4G_F}{\sqrt{2}} V_{cb} V_{cs}^* C \langle J/\psi K_S | \bar{s}_L \gamma_\mu b_L \bar{c}_L \gamma^\mu c_L | \bar{B}^0 \rangle$$

Strong interaction part

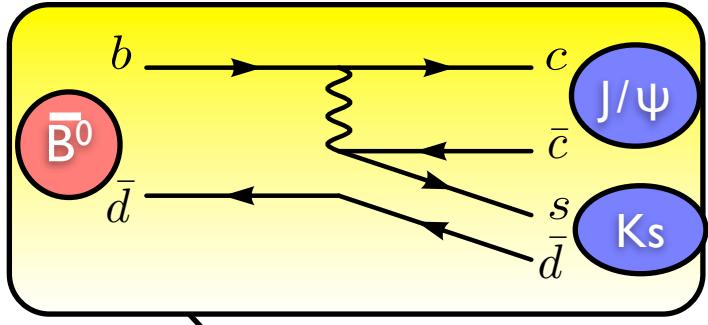


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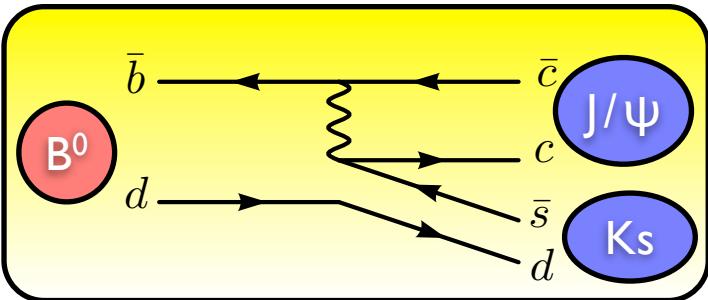
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Strong interaction part



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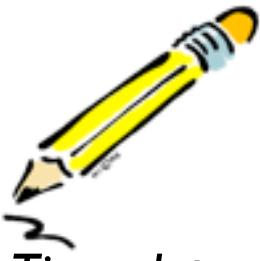
$$V_{cb} = A\lambda^2$$

$$V_{cs} = 1 - \lambda^2/2$$



$$\frac{A(\bar{B}^0 \rightarrow J/\psi K_S)}{A(B^0 \rightarrow J/\psi K_S)} = 1$$

DONE!



CP asymmetry in $B \rightarrow J/\psi K_S$

Time-dependent CP asymmetry is defined as (f is some CP eigenstate):

$$\frac{\Gamma(B^0(t) \rightarrow f) - \Gamma(\bar{B}^0(t) \rightarrow f)}{\Gamma(B^0(t) \rightarrow f) + \Gamma(\bar{B}^0(t) \rightarrow f)}$$

In B meson system, we can use $\Delta\Gamma \ll \Delta M$, which simplifies our formula:

$$\frac{\Gamma(B^0(t) \rightarrow f) - \Gamma(\bar{B}^0(t) \rightarrow f)}{\Gamma(B^0(t) \rightarrow f) + \Gamma(\bar{B}^0(t) \rightarrow f)} = \xi_f \sin(\Delta M_B t) S_f + \cos(\Delta M_B t) C_f$$
$$S_f = \frac{2 \operatorname{Im} \left(\frac{q}{p} \frac{A(\bar{B}^0 \rightarrow f)}{A(B^0 \rightarrow f)} \right)}{1 + \left| \frac{A(\bar{B}^0 \rightarrow f)}{A(B^0 \rightarrow f)} \right|^2}, \quad C_f = \frac{1 - \left| \frac{A(\bar{B}^0 \rightarrow f)}{A(B^0 \rightarrow f)} \right|^2}{1 + \left| \frac{A(\bar{B}^0 \rightarrow f)}{A(B^0 \rightarrow f)} \right|^2}$$

$$\frac{q}{p} = e^{-2i \arg(V_{tb}^* V_{td})}$$

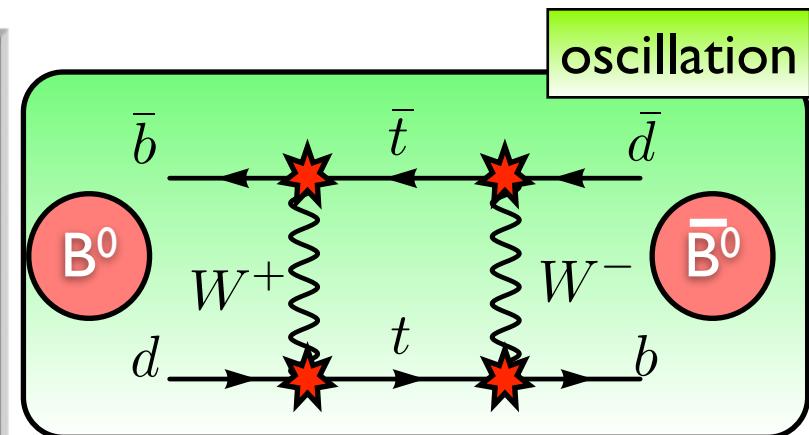
$$\frac{A(\bar{B}^0 \rightarrow J/\psi K_S)}{A(B^0 \rightarrow J/\psi K_S)} = 1$$



CP asymmetry in $B \rightarrow J/\psi K_S$

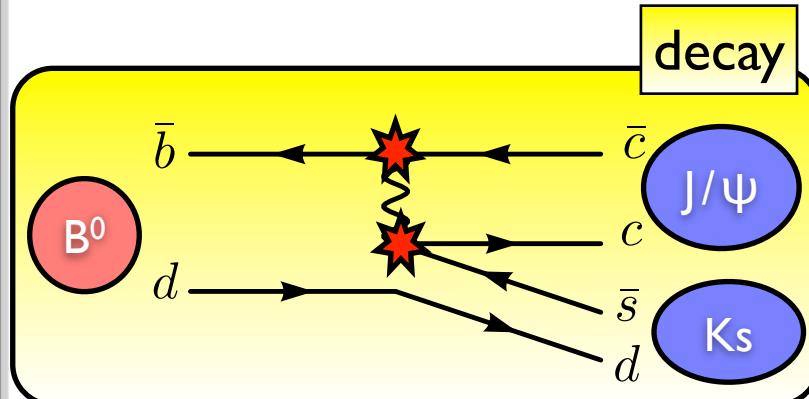
$$A_{J/\psi K_S}(t) = \frac{\Gamma(\bar{B}^0(t) \rightarrow J/\psi K_S) - \Gamma(B^0(t) \rightarrow J/\psi K_S)}{\Gamma(\bar{B}^0(t) \rightarrow J/\psi K_S) + \Gamma(B^0(t) \rightarrow J/\psi K_S)} = S_{J/\psi K_s} \sin \Delta M_B t$$

$$S_{J/\psi K_s} = \text{Im} \left[\underbrace{\frac{M_{12}}{M_{12}^*} \frac{A(\bar{B} \rightarrow J/\psi K_S)}{A(B \rightarrow J/\psi K_S)}}_{\text{oscill. } \text{decay}} \right]$$



$$= \text{Im} \left[\underbrace{\frac{V_{tb} V_{td}^*}{V_{tb}^* V_{td}}}_{\text{oscill.}} \underbrace{\frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}}}_{\text{decay}} \right]$$

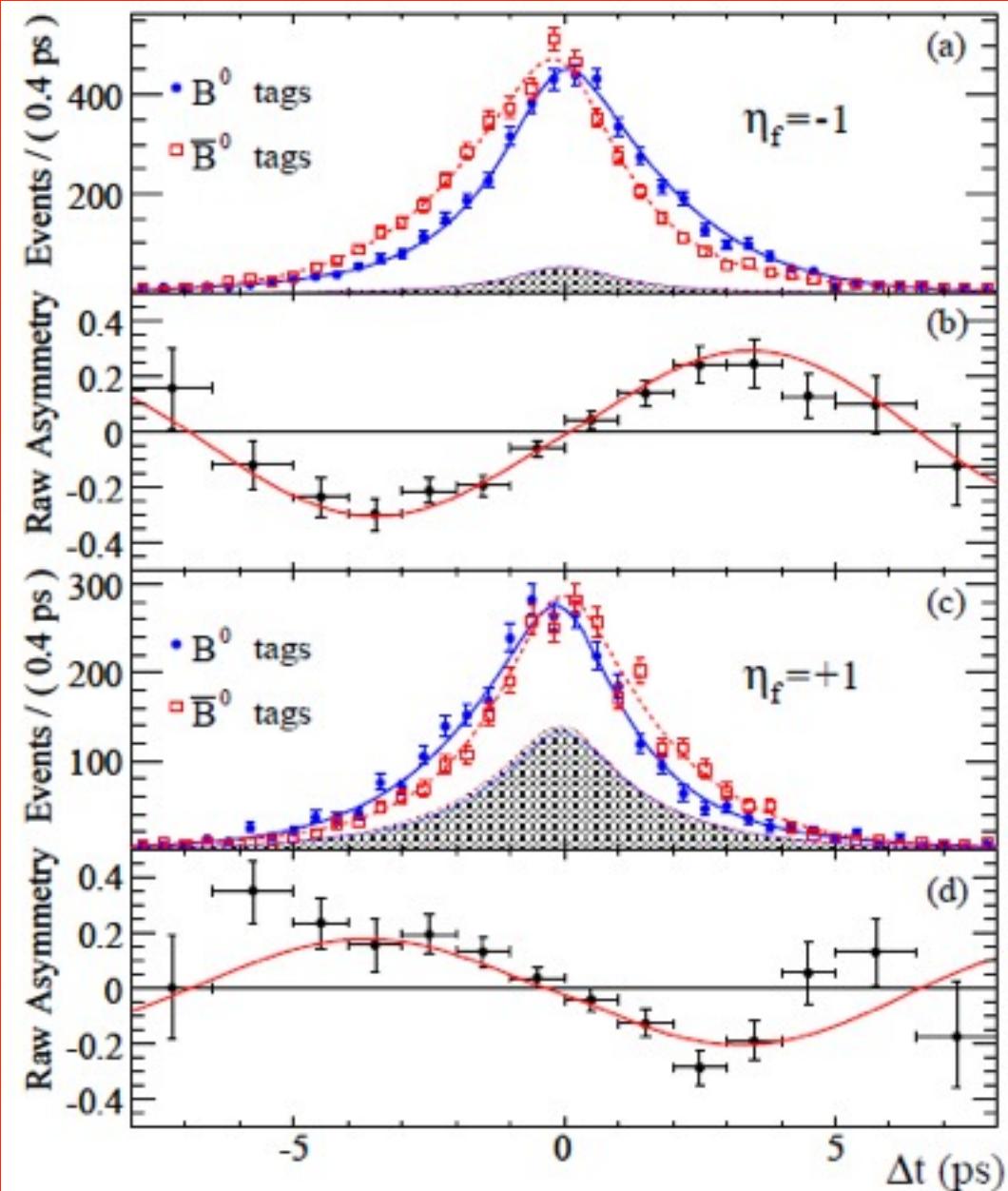
$$= \sin 2\phi_1(\beta)$$



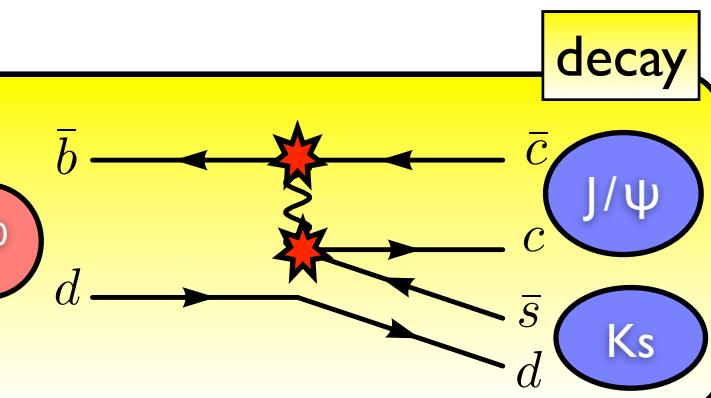
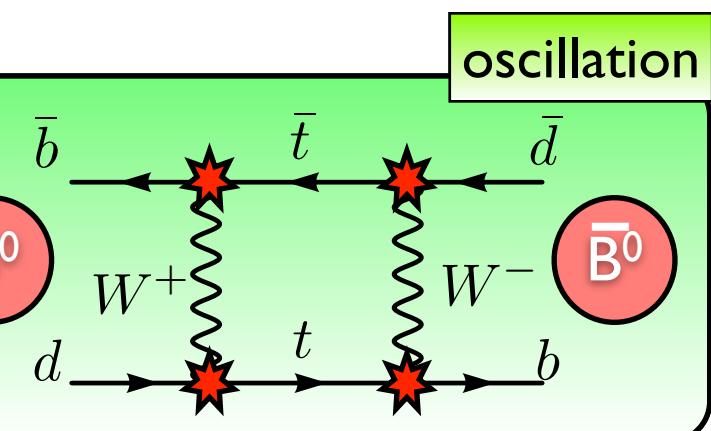
CP asymmetry in $B \rightarrow J/\psi K_s$



$A_{J/\psi K_s}$



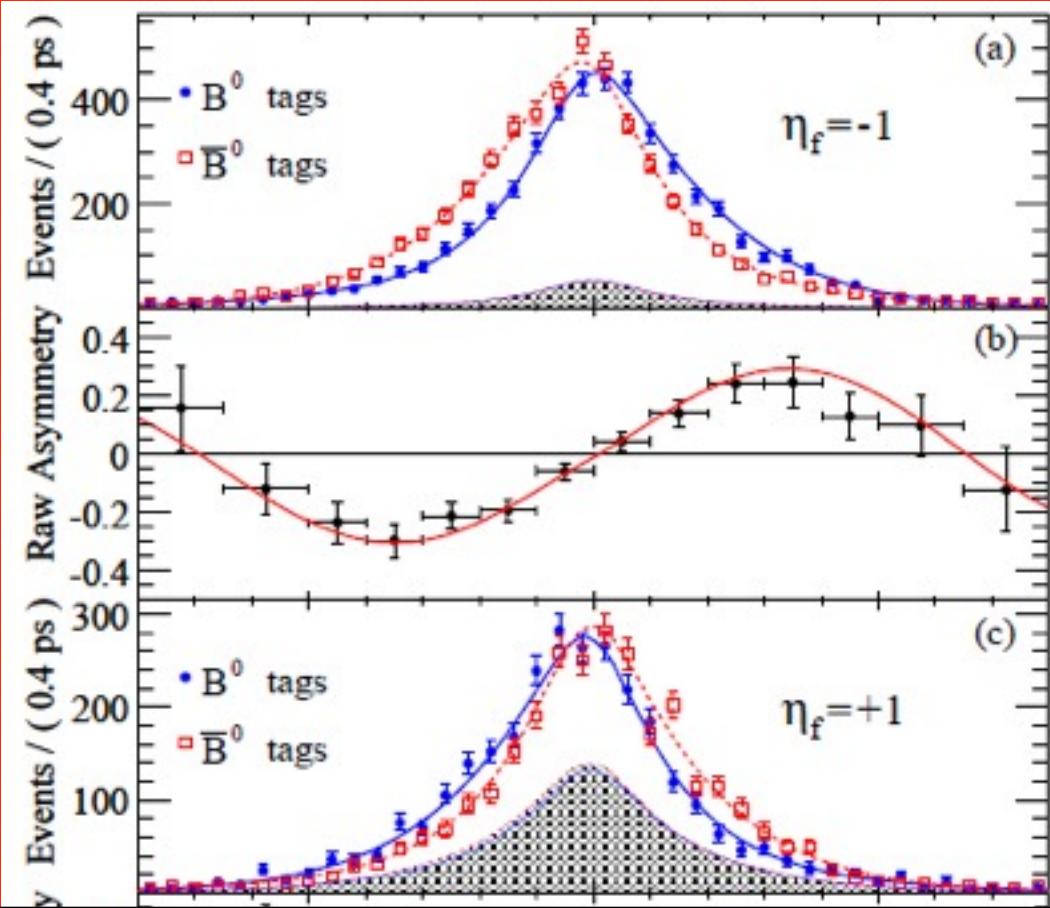
$$= S_{J/\psi K_s} \sin \Delta M_B t$$



CP asymmetry in $B \rightarrow J/\psi K_s$

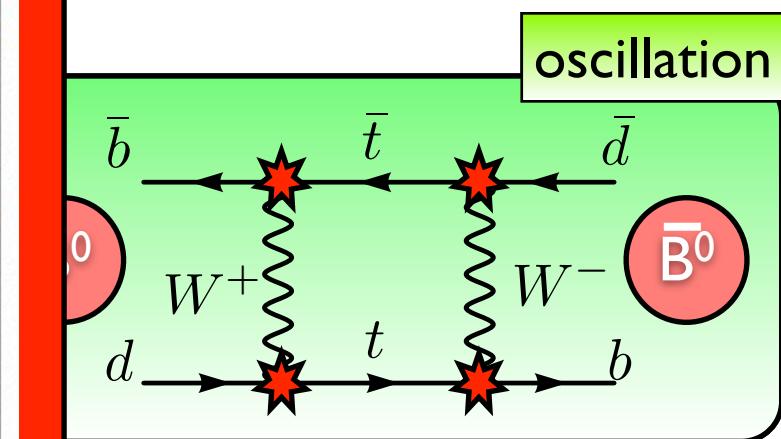


$A_{J/\psi K_s}$



$$= S_{J/\psi K_s} \sin \Delta M_B t$$

$S_{J/\psi K_s}$



Including various $b \rightarrow cc\bar{c}s$ the
measurements, we find

$$\Phi_I = (21.1 \pm 0.9)^\circ$$

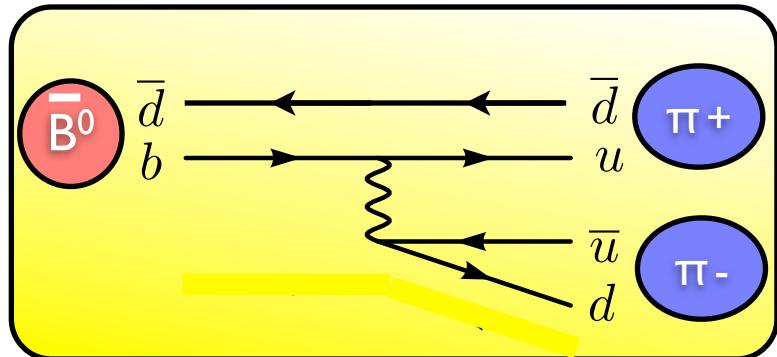
Ψ

K_s

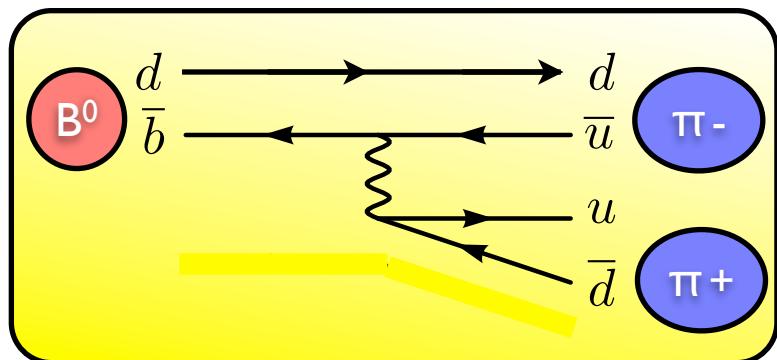


Computing $A(\bar{B} \rightarrow \pi^+ \pi^-)/A(B \rightarrow \pi^+ \pi^-)$

Assuming tree-dominant...



$$A(\bar{B}^0 \rightarrow \pi^+ \pi^-) = \frac{4G_F}{\sqrt{2}} V_{ub} V_{ud}^* C \langle \pi^+ \pi^- | \bar{u}_L \gamma_\mu b_L \bar{d}_L \gamma^\mu u_L | \bar{B}^0 \rangle$$



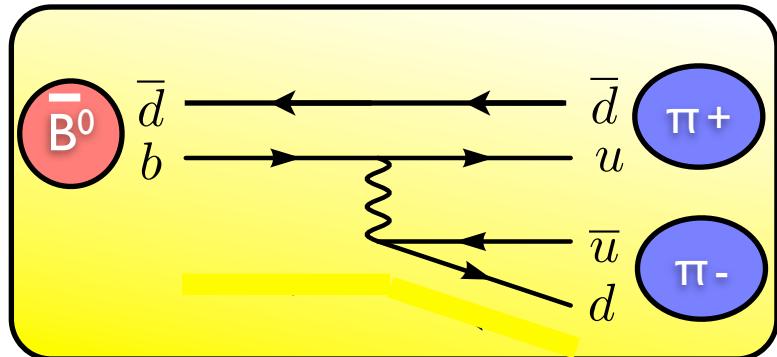
$$A(B^0 \rightarrow \pi^+ \pi^-) = \frac{4G_F}{\sqrt{2}} V_{ub}^* V_{ud} C \langle \pi^+ \pi^- | (\bar{u}_L \gamma_\mu b_L \bar{d}_L \gamma^\mu u_L)^\dagger | B^0 \rangle$$

Strong interaction part

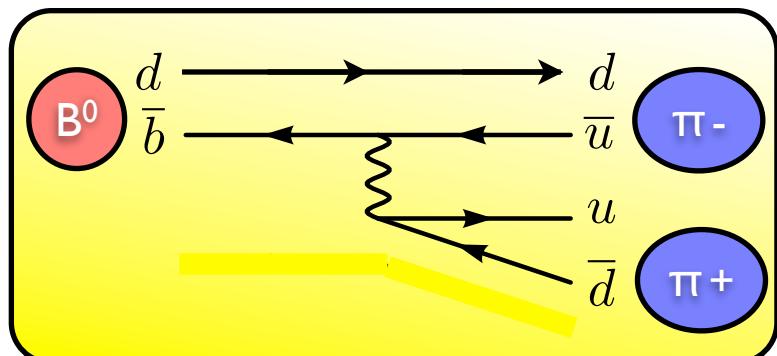


Computing $A(\bar{B}^0 \rightarrow \pi^+ \pi^-)/A(B^0 \rightarrow \pi^+ \pi^-)$

Assuming tree-dominant...



$$A(\bar{B}^0 \rightarrow \pi^+ \pi^-) = \frac{4G_F}{\sqrt{2}} V_{ub} V_{ud}^* C \langle \pi^+ \pi^- | \bar{u}_L \gamma_\mu b_L \bar{d}_L \gamma^\mu u_L | \bar{B}^0 \rangle$$



$$A(B^0 \rightarrow \pi^+ \pi^-) = \frac{4G_F}{\sqrt{2}} V_{ub}^* V_{ud} C \langle \pi^+ \pi^- | (\bar{u}_L \gamma_\mu b_L \bar{d}_L \gamma^\mu u_L)^\dagger | B^0 \rangle$$

Strong interaction part

$$V_{ub} = A \lambda^3 (\rho - i\eta)$$

$$V_{ud} = 1 - \lambda^2/2$$



$$\frac{A(\bar{B}^0 \rightarrow \pi^+ \pi^-)}{A(B^0 \rightarrow \pi^+ \pi^-)} = e^{2i \arg(V_{ub} V_{ud}^*)}$$

DONE!



CP asymmetry in $B \rightarrow \pi^+ \pi^-$

Time-dependent CP asymmetry is defined as (f is some CP eigenstate):

$$\frac{\Gamma(B^0(t) \rightarrow f) - \Gamma(\bar{B}^0(t) \rightarrow f)}{\Gamma(B^0(t) \rightarrow f) + \Gamma(\bar{B}^0(t) \rightarrow f)}$$

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$$S_f = \frac{2 \operatorname{Im} \left(\frac{q}{p} \frac{A(\bar{B}^0 \rightarrow f)}{A(B^0 \rightarrow f)} \right)}{1 + \left| \frac{A(\bar{B}^0 \rightarrow f)}{A(B^0 \rightarrow f)} \right|^2}, \quad C_f = \frac{1 - \left| \frac{A(\bar{B}^0 \rightarrow f)}{A(B^0 \rightarrow f)} \right|^2}{1 + \left| \frac{A(\bar{B}^0 \rightarrow f)}{A(B^0 \rightarrow f)} \right|^2}$$

$$\frac{q}{p} = e^{-2i \arg(V_{tb}^* V_{td})}$$

$$\frac{A(\bar{B}^0 \rightarrow \pi^+ \pi^-)}{A(B^0 \rightarrow \pi^+ \pi^-)} = e^{2i \arg(V_{ub} V_{ud}^*)}$$



CP asymmetry in $B \rightarrow \pi^+ \pi^-$

$$A_{\pi^+ \pi^-}(t) = \frac{\Gamma(\bar{B}^0(t) \rightarrow \pi^+ \pi^-) - \Gamma(B^0(t) \rightarrow \pi^+ \pi^-)}{\Gamma(\bar{B}^0(t) \rightarrow \pi^+ \pi^-) + \Gamma(B^0(t) \rightarrow \pi^+ \pi^-)} = S_{\pi^+ \pi^-} \sin \Delta M_B t$$

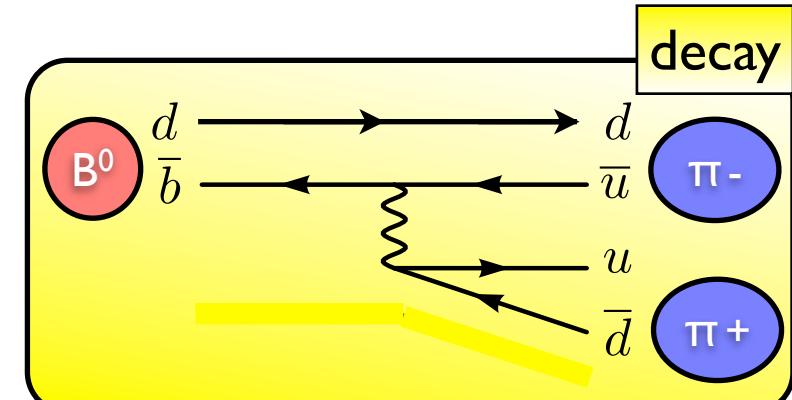
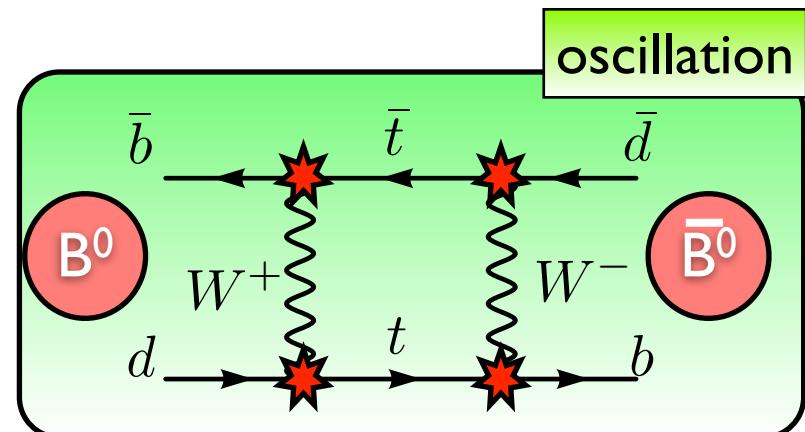
$$S_{\pi^+ \pi^-} = \text{Im} \left[\frac{M_{12}}{M_{12}^*} \frac{A(\bar{B} \rightarrow \pi^+ \pi^-)}{A(B \rightarrow \pi^+ \pi^-)} \right]$$

$\underbrace{M_{12}}_{\text{oscill.}}$ $\underbrace{\frac{A(\bar{B} \rightarrow \pi^+ \pi^-)}{A(B \rightarrow \pi^+ \pi^-)}}_{\text{decay}}$

$$= \text{Im} \left[\frac{V_{tb} V_{td}^*}{V_{tb}^* V_{td}} \frac{V_{ub} V_{ud}^*}{V_{ub}^* V_{ud}} \right]$$

$\underbrace{V_{tb} V_{td}^*}_{\text{oscill.}}$ $\underbrace{\frac{V_{ub} V_{ud}^*}{V_{ub}^* V_{ud}}}_{\text{decay}}$

$$= \sin 2\phi_2(\alpha)$$



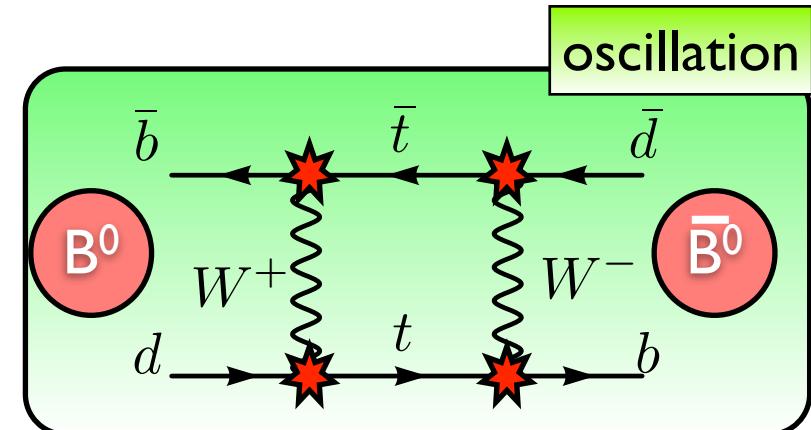
* Assuming tree-dominant...



CP asymmetry in $B \rightarrow \pi^+ \pi^-$

$$A_{\pi^+ \pi^-}(t) = \frac{\Gamma(\bar{B}^0(t) \rightarrow \pi^+ \pi^-) - \Gamma(B^0(t) \rightarrow \pi^+ \pi^-)}{\Gamma(\bar{B}^0(t) \rightarrow \pi^+ \pi^-) + \Gamma(B^0(t) \rightarrow \pi^+ \pi^-)} = S_{\pi^+ \pi^-} \sin \Delta M_B t$$

**Penguin pollution
prevents a precise
measurement of $\Phi_2(\alpha)$**



Including the measurements with

$$\pi\pi, \rho\rho, \pi\rho,
 $\alpha = (89.0^{+4.4}_{-4.2})^\circ$$$

* Assuming tree-dominant...

Test of Unitarity of CKM

$$V_{CKM}^\dagger V_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$V_{CKM} V_{CKM}^\dagger = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Test of Unitarity

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$
$$= \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

Unitarity: 9 complex numbers can be replaced by the 4 real number parameters



We must test *at which extent* this is satisfied!

Unitarity triangles

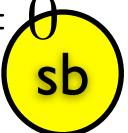
$$\underbrace{V_{ud}V_{us}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{cd}V_{cs}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{td}V_{ts}^*}_{\mathcal{O}(\lambda^5)} = 0$$



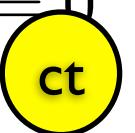
$$\underbrace{V_{ud}V_{cd}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{us}V_{cs}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{ub}V_{cb}^*}_{\mathcal{O}(\lambda^5)} = 0$$



$$\underbrace{V_{us}V_{ub}^*}_{\mathcal{O}(\lambda^4)} + \underbrace{V_{cs}V_{cb}^*}_{\mathcal{O}(\lambda^2)} + \underbrace{V_{ts}V_{tb}^*}_{\mathcal{O}(\lambda^2)} = 0$$



$$\underbrace{V_{td}V_{cd}^*}_{\mathcal{O}(\lambda^4)} + \underbrace{V_{ts}V_{cs}^*}_{\mathcal{O}(\lambda^2)} + \underbrace{V_{tb}V_{cb}^*}_{\mathcal{O}(\lambda^2)} = 0$$



$$\underbrace{V_{ud}^*V_{ub}}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{cd}^*V_{sb}}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{td}^*V_{tb}}_{\mathcal{O}(\lambda^3)} = 0$$



$$\underbrace{V_{ud}V_{ub}^*}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{cd}V_{cb}^*}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{td}V_{tb}^*}_{\mathcal{O}(\lambda^3)} = 0$$



Unitarity triangles

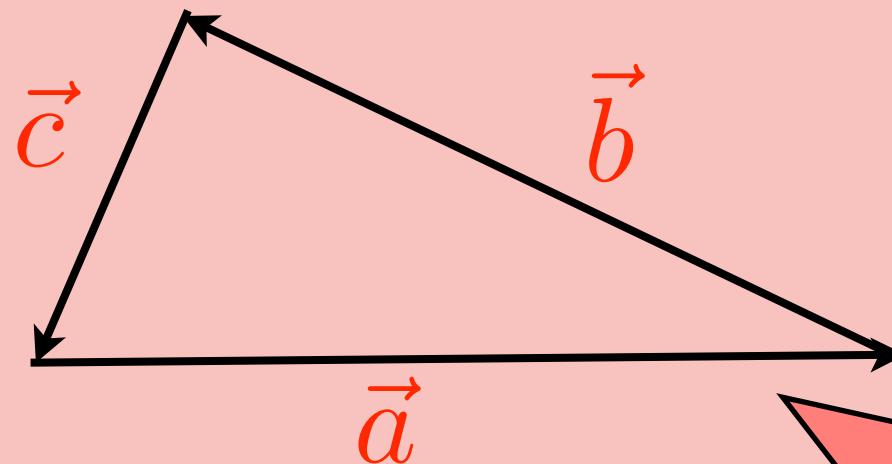
$$\underbrace{V_{ud}V_{us}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{cd}V_{cs}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{td}V_{ts}^*}_{\mathcal{O}(\lambda)} = 0$$

$$\underbrace{V_{ud}V_{cd}^*}_{\mathcal{O}(\lambda^5)} + \underbrace{V_{us}V_{cs}^*}_{\mathcal{O}(\lambda^5)} + \underbrace{V_{ub}V_{cb}^*}_{\mathcal{O}(\lambda^5)} = 0$$

$$\underbrace{V_{us}V_{ub}^*}_{\mathcal{O}(\lambda^4)} + \underbrace{V_{cd}V_{cb}^*}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{ts}V_{tb}^*}_{\mathcal{O}(\lambda^3)} = 0$$

$$\underbrace{V_{ud}^*V_{ub}}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{cd}^*V_{cb}}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{ts}^*V_{tb}}_{\mathcal{O}(\lambda^3)} = 0$$

$$\underbrace{V_{1i}^*V_{1k}}_{\vec{a}} + \underbrace{V_{2i}^*V_{1k}}_{\vec{b}} + \underbrace{V_{3i}^*V_{1k}}_{\vec{c}} = 0$$



Unitarity Triangle

uc

ct

Unitarity triangles

$$\underbrace{V_{ud}V_{us}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{cd}V_{cs}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{td}V_{ts}^*}_{\mathcal{O}(\lambda^5)} = 0$$

A blue unitarity triangle with three sides labeled $\mathcal{O}(\lambda)$, $\mathcal{O}(\lambda)$, and $\mathcal{O}(\lambda^5)$. The hypotenuse is a long blue arrow pointing from the bottom-left vertex to the top-right vertex. The two legs are shorter blue arrows forming the base and height of the triangle.

$$\underbrace{V_{ud}V_{cd}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{us}V_{cs}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{ub}V_{cb}^*}_{\mathcal{O}(\lambda^5)} = 0$$

A blue unitarity triangle with three sides labeled $\mathcal{O}(\lambda)$, $\mathcal{O}(\lambda)$, and $\mathcal{O}(\lambda^5)$. The hypotenuse is a long blue arrow pointing from the bottom-left vertex to the top-right vertex. The two legs are shorter blue arrows forming the base and height of the triangle.

$$\underbrace{V_{us}V_{ub}^*}_{\mathcal{O}(\lambda^4)} + \underbrace{V_{cs}V_{cb}^*}_{\mathcal{O}(\lambda^2)} + \underbrace{V_{ts}V_{tb}^*}_{\mathcal{O}(\lambda^2)} = 0$$

A green unitarity triangle with three sides labeled $\mathcal{O}(\lambda^4)$, $\mathcal{O}(\lambda^2)$, and $\mathcal{O}(\lambda^2)$. The hypotenuse is a long green arrow pointing from the bottom-left vertex to the top-right vertex. The two legs are shorter green arrows forming the base and height of the triangle.

$$\underbrace{V_{td}V_{cd}^*}_{\mathcal{O}(\lambda^4)} + \underbrace{V_{ts}V_{cs}^*}_{\mathcal{O}(\lambda^2)} + \underbrace{V_{tb}V_{cb}^*}_{\mathcal{O}(\lambda^2)} = 0$$

A green unitarity triangle with three sides labeled $\mathcal{O}(\lambda^4)$, $\mathcal{O}(\lambda^2)$, and $\mathcal{O}(\lambda^2)$. The hypotenuse is a long green arrow pointing from the bottom-left vertex to the top-right vertex. The two legs are shorter green arrows forming the base and height of the triangle.

$$\underbrace{V_{ud}^*V_{ub}}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{cd}^*V_{cb}}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{td}^*V_{tb}}_{\mathcal{O}(\lambda^3)} = 0$$

A red unitarity triangle with three sides labeled $\mathcal{O}(\lambda^3)$, $\mathcal{O}(\lambda^3)$, and $\mathcal{O}(\lambda^3)$. The hypotenuse is a long red arrow pointing from the bottom-left vertex to the top-right vertex. The two legs are shorter red arrows forming the base and height of the triangle.

$$\underbrace{V_{td}V_{ud}^*}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{ts}V_{us}^*}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{tb}V_{ub}^*}_{\mathcal{O}(\lambda^3)} = 0$$

A red unitarity triangle with three sides labeled $\mathcal{O}(\lambda^3)$, $\mathcal{O}(\lambda^3)$, and $\mathcal{O}(\lambda^3)$. The hypotenuse is a long red arrow pointing from the bottom-left vertex to the top-right vertex. The two legs are shorter red arrows forming the base and height of the triangle.

Unitarity triangles

$$\underbrace{V_{ud}V_{us}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{cd}V_{cs}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{td}V_{ts}^*}_{\mathcal{O}(\lambda^5)} = 0$$

K physics

A blue unitarity triangle with vertices at the origin, a point on the horizontal axis, and a point on the hypotenuse. Arrows indicate a clockwise cycle.

$$\underbrace{V_{ud}V_{cd}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{us}V_{cs}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{ub}V_{cb}^*}_{\mathcal{O}(\lambda^5)} = 0$$

D physics

A blue unitarity triangle with vertices at the origin, a point on the horizontal axis, and a point on the hypotenuse. Arrows indicate a clockwise cycle.

$$\underbrace{V_{us}V_{ub}^*}_{\mathcal{O}(\lambda^4)} + \underbrace{V_{cs}V_{cb}^*}_{\mathcal{O}(\lambda^2)} + \underbrace{V_{ts}V_{tb}^*}_{\mathcal{O}(\lambda^2)} = 0$$

B_s physics

A green unitarity triangle with vertices at the origin, a point on the horizontal axis, and a point on the hypotenuse. Arrows indicate a clockwise cycle.

$$\underbrace{V_{td}V_{cd}^*}_{\mathcal{O}(\lambda^4)} + \underbrace{V_{ts}V_{cs}^*}_{\mathcal{O}(\lambda^2)} + \underbrace{V_{tb}V_{cb}^*}_{\mathcal{O}(\lambda^2)} = 0$$

ct

A green unitarity triangle with vertices at the origin, a point on the horizontal axis, and a point on the hypotenuse. Arrows indicate a clockwise cycle.

$$\underbrace{V_{ud}^*V_{ub}}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{cd}^*V_{cb}}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{td}^*V_{tb}}_{\mathcal{O}(\lambda^3)} = 0$$

B_d physics

A red unitarity triangle with vertices at the origin, a point on the horizontal axis, and a point on the hypotenuse. Arrows indicate a clockwise cycle.

$$\underbrace{V_{td}V_{ud}^*}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{ts}V_{us}^*}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{tb}V_{ub}^*}_{\mathcal{O}(\lambda^3)} = 0$$

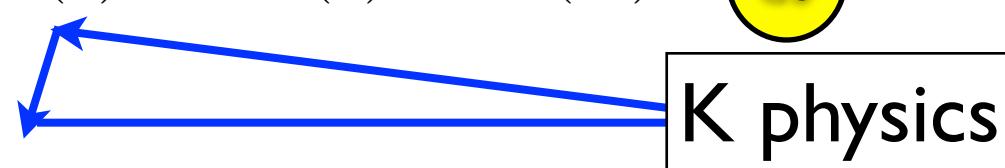
ut

A red unitarity triangle with vertices at the origin, a point on the horizontal axis, and a point on the hypotenuse. Arrows indicate a clockwise cycle.

Unitarity triangles

$$\underbrace{V_{ud}V_{us}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{cd}V_{cs}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{td}V_{ts}^*}_{\mathcal{O}(\lambda^5)} = 0$$

ds



$$\underbrace{V_{ud}V_{cd}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{us}V_{cs}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{ub}V_{cb}^*}_{\mathcal{O}(\lambda^5)} = 0$$

uc



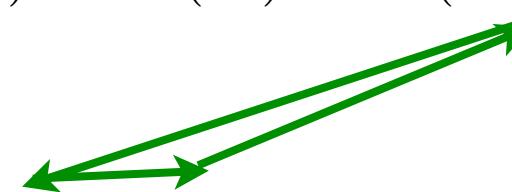
$$\underbrace{V_{us}V_{ub}^*}_{\mathcal{O}(\lambda^4)} + \underbrace{V_{cs}V_{cb}^*}_{\mathcal{O}(\lambda^2)} + \underbrace{V_{ts}V_{tb}^*}_{\mathcal{O}(\lambda^2)} = 0$$

sb



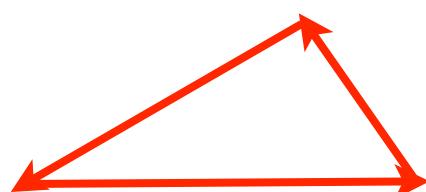
$$\underbrace{V_{td}V_{cd}^*}_{\mathcal{O}(\lambda^4)} + \underbrace{V_{ts}V_{cs}^*}_{\mathcal{O}(\lambda^2)} + \underbrace{V_{tb}V_{cb}^*}_{\mathcal{O}(\lambda^2)} = 0$$

ct



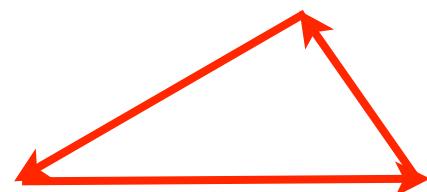
The Unitarity Triangle!

db



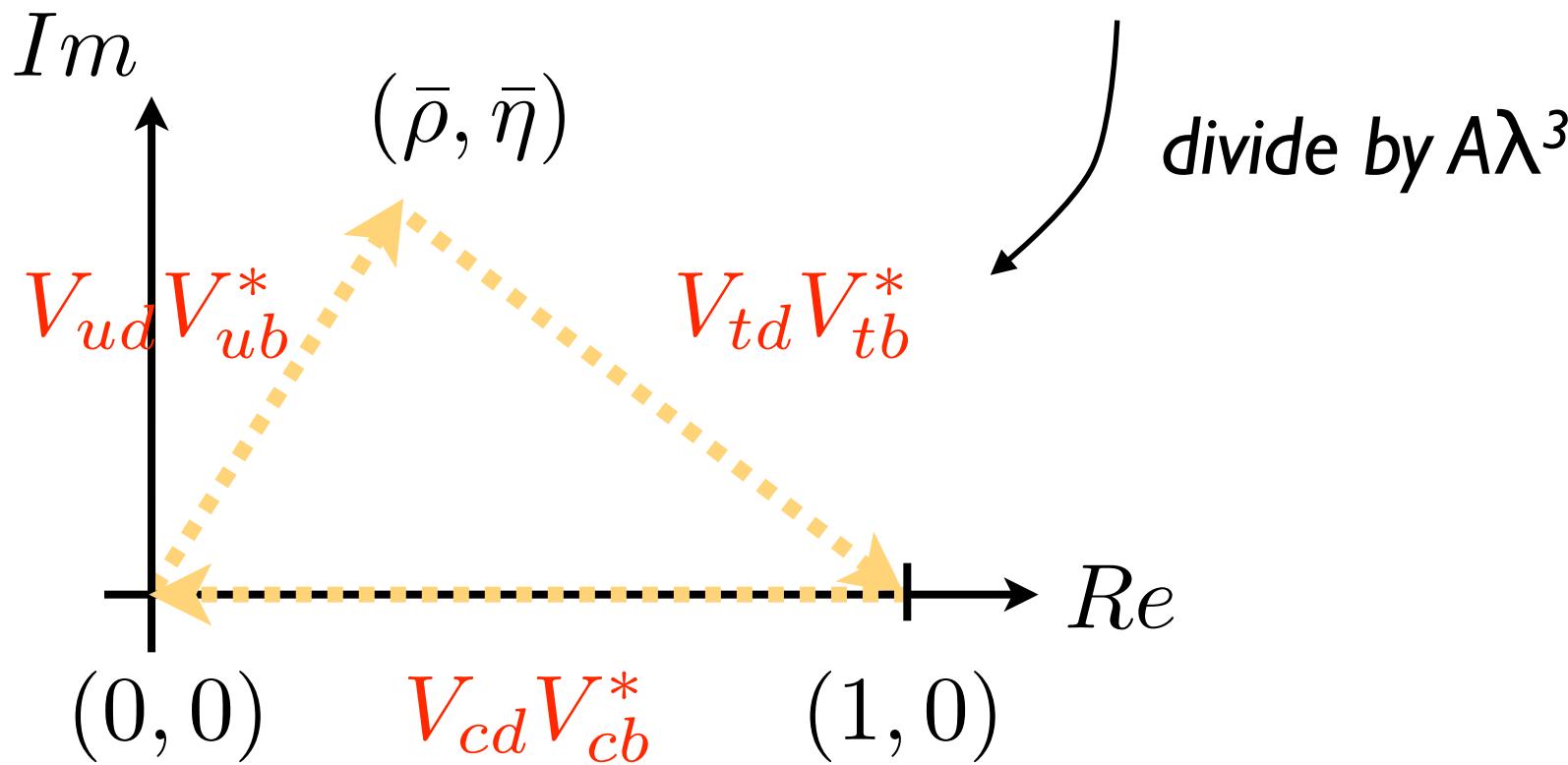
$$\underbrace{V_{td}V_{ud}^*}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{ts}V_{us}^*}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{tb}V_{ub}^*}_{\mathcal{O}(\lambda^3)} = 0$$

ut



The Unitarity Triangle

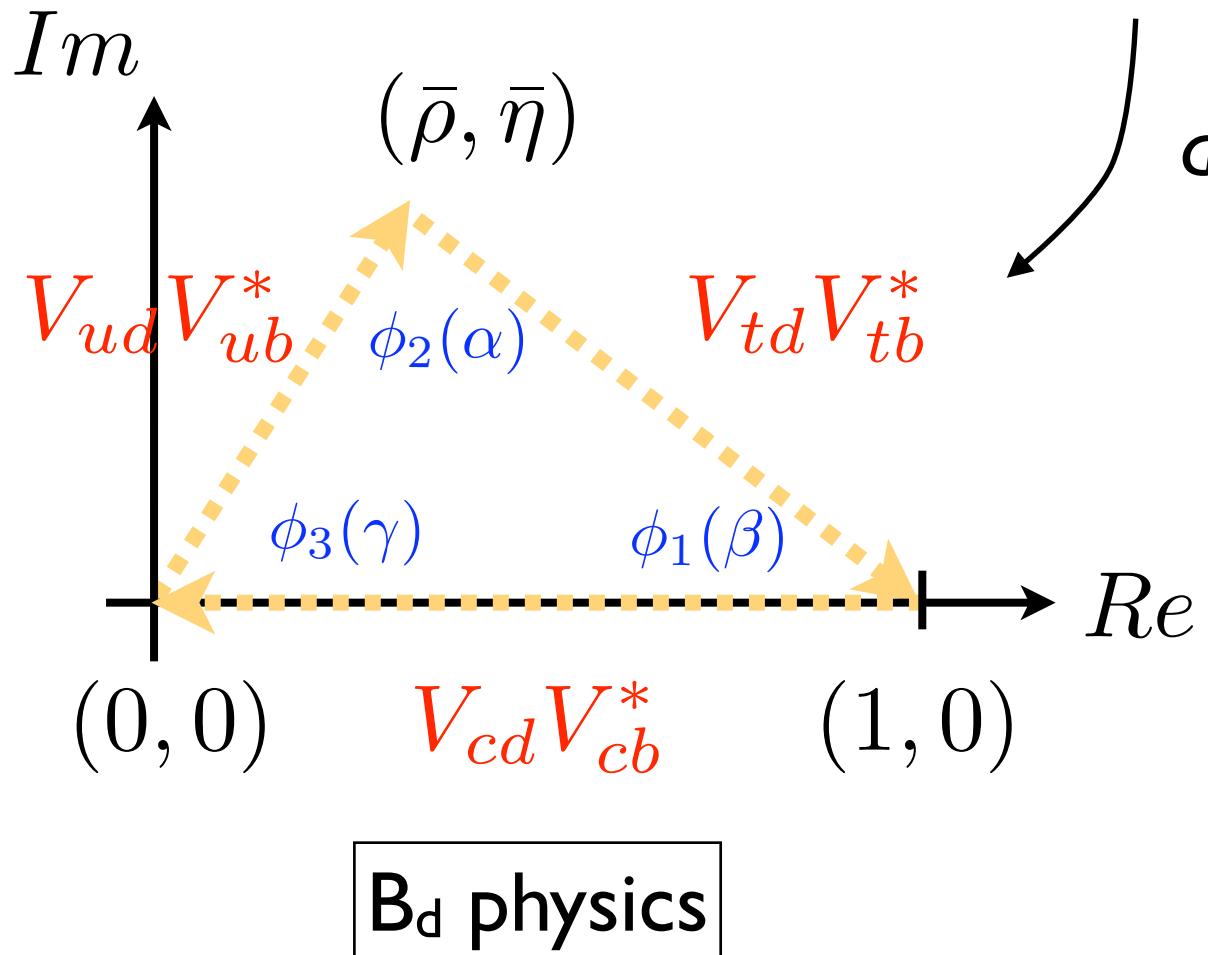
$$\underbrace{V_{ud}V_{ub}^*}_{A\lambda^3(\rho+i\eta)} + \underbrace{V_{cd}V_{cb}^*}_{-A\lambda^3} + \underbrace{V_{td}V_{tb}^*}_{A\lambda^3(1-\rho-i\eta)} = 0$$



B_d physics

The Unitarity Triangle

$$\underbrace{V_{ud}V_{ub}^*}_{A\lambda^3(\rho+i\eta)} + \underbrace{V_{cd}V_{cb}^*}_{-A\lambda^3} + \underbrace{V_{td}V_{tb}^*}_{A\lambda^3(1-\rho-i\eta)} = 0$$

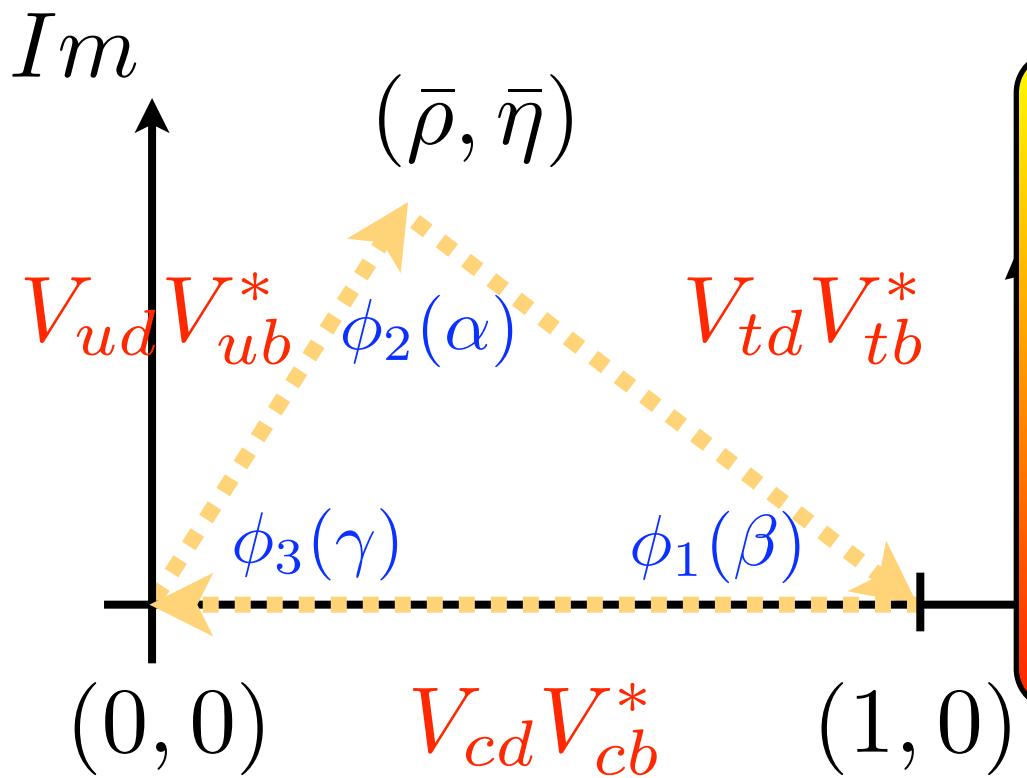


$$\begin{aligned} \arg\left(\frac{V_{tb}^* V_{td}}{V_{cb}^* V_{cs}}\right) &\equiv -\phi_1 \\ \arg\left(\frac{V_{tb}^* V_{td}}{V_{ub}^* V_{ud}}\right) &\equiv -\phi_2 \\ \arg\left(\frac{V_{cb}^* V_{cs}}{V_{ub}^* V_{ud}}\right) &\equiv -\phi_3 \end{aligned}$$

divide by $A\lambda^3$

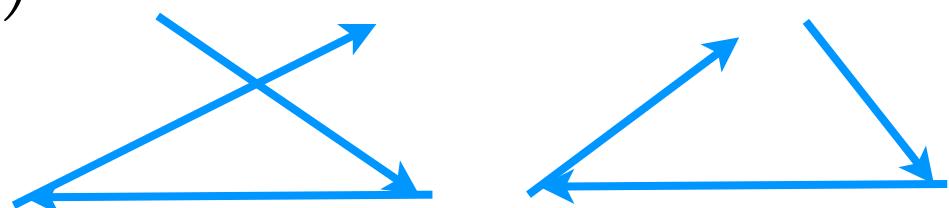
The Unitarity Triangle

$$\underbrace{V_{ud}V_{ub}^*}_{A\lambda^3(\rho+i\eta)} + \underbrace{V_{cd}V_{cb}^*}_{-A\lambda^3} + \underbrace{V_{td}V_{tb}^*}_{A\lambda^3(1-\rho-i\eta)} = 0$$

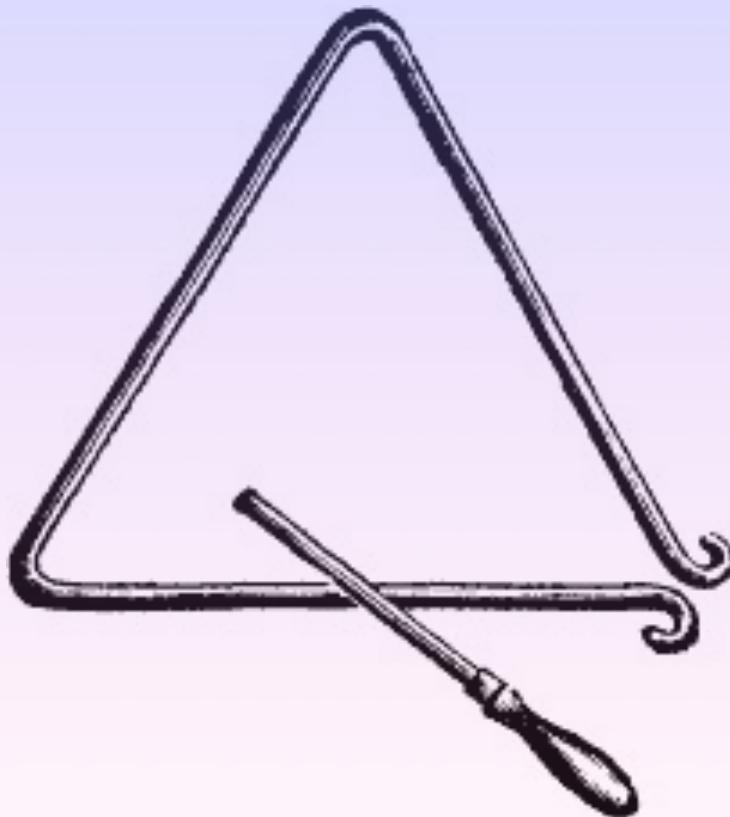


B_d physics

Unitarity test is to verify if the triangle closes at the apex from independent measurements for three sides and three angles!!

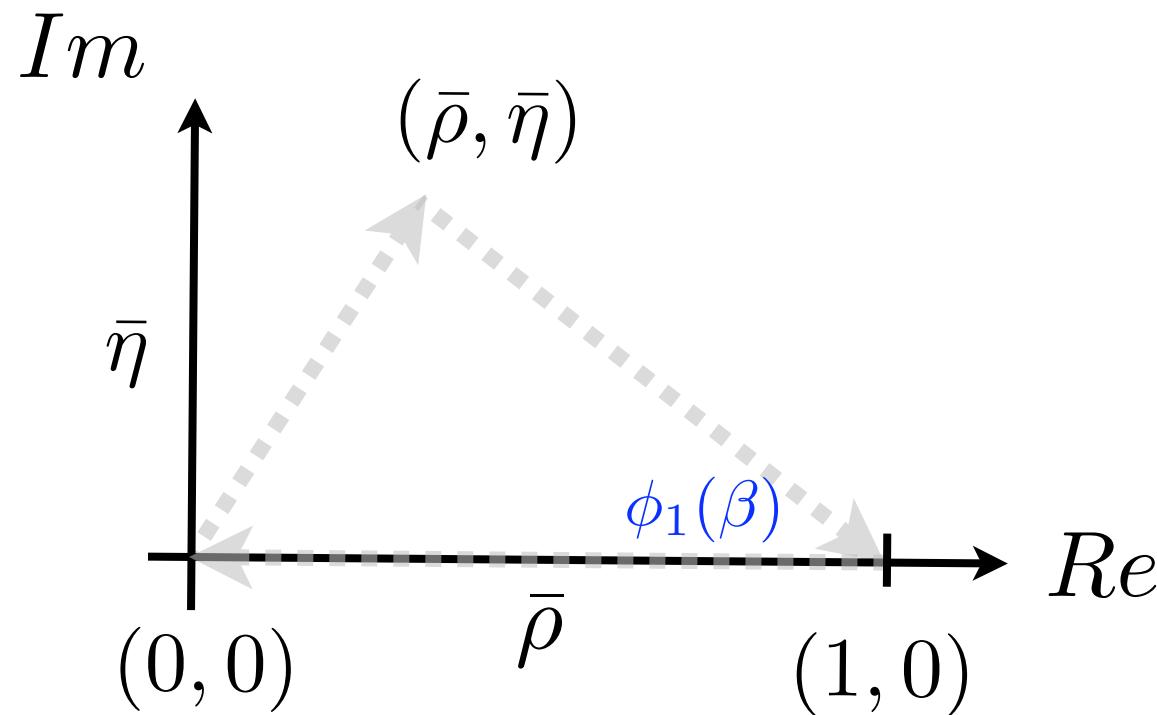


Determination of the CKM matrix



Determination of the CKM matrix: $\sin 2\Phi_1$ (β) (phase)

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

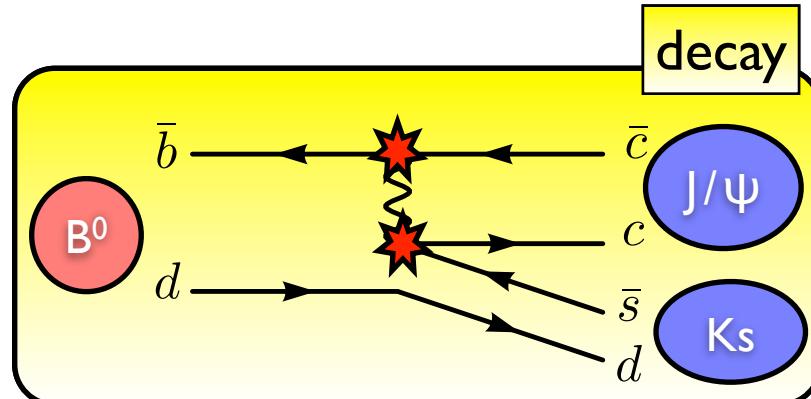
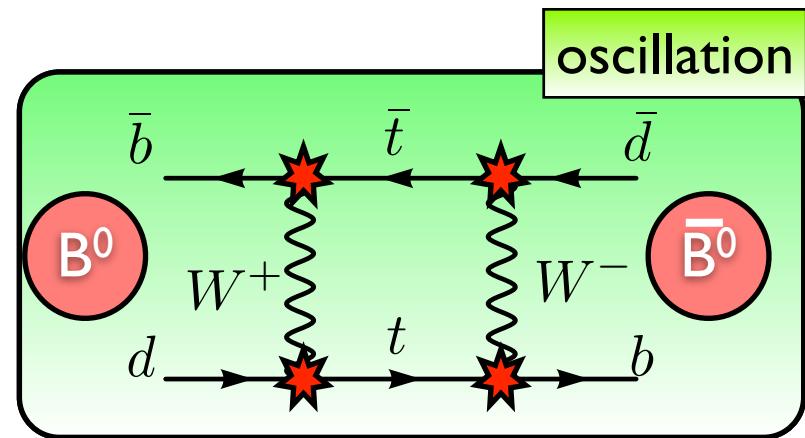




CP asymmetry in $B \rightarrow J/\psi K_S$

$$A_{J/\psi K_S}(t) = \frac{\Gamma(\bar{B}^0(t) \rightarrow J/\psi K_S) - \Gamma(B^0(t) \rightarrow J/\psi K_S)}{\Gamma(\bar{B}^0(t) \rightarrow J/\psi K_S) + \Gamma(B^0(t) \rightarrow J/\psi K_S)} = S_{J/\psi K_s} \sin \Delta M_B t$$

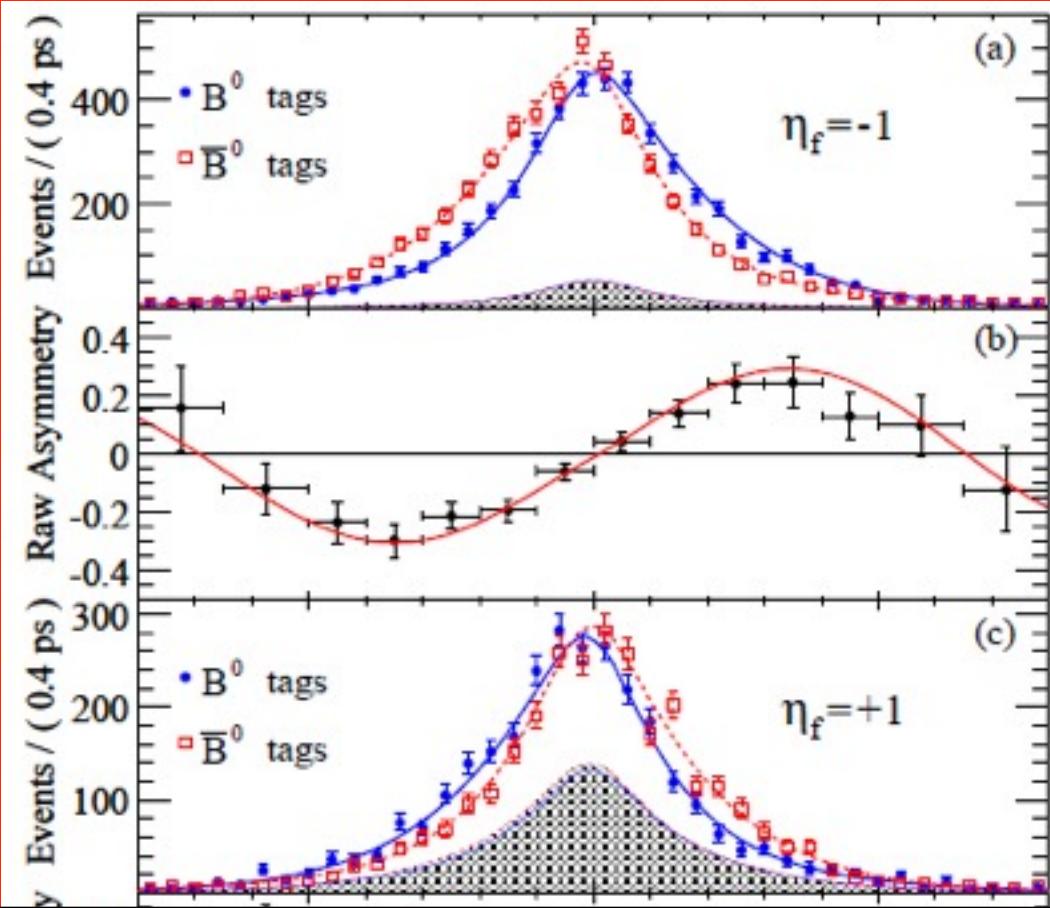
$$\begin{aligned} S_{J/\psi K_s} &= \text{Im} \left[\underbrace{\frac{M_{12}}{M_{12}^*}}_{\text{oscill.}} \underbrace{\frac{A(\bar{B} \rightarrow J/\psi K_S)}{A(B \rightarrow J/\psi K_S)}}_{\text{decay}} \right] \\ &= \text{Im} \left[\underbrace{\frac{V_{tb} V_{td}^*}{V_{tb}^* V_{td}}}_{\text{oscill.}} \underbrace{\frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}}}_{\text{decay}} \right] \\ &= \sin 2\phi_1 \end{aligned}$$



CP asymmetry in $B \rightarrow J/\psi K_s$

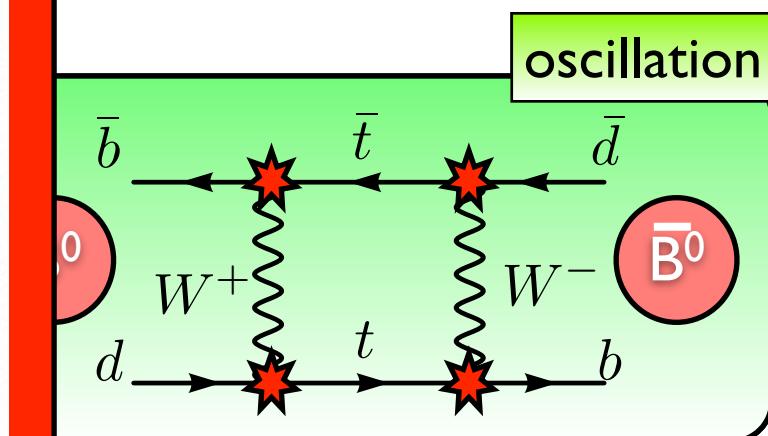


$A_{J/\psi K_s}$



$$= S_{J/\psi K_s} \sin \Delta M_B t$$

$S_{J/\psi K_s}$



decay

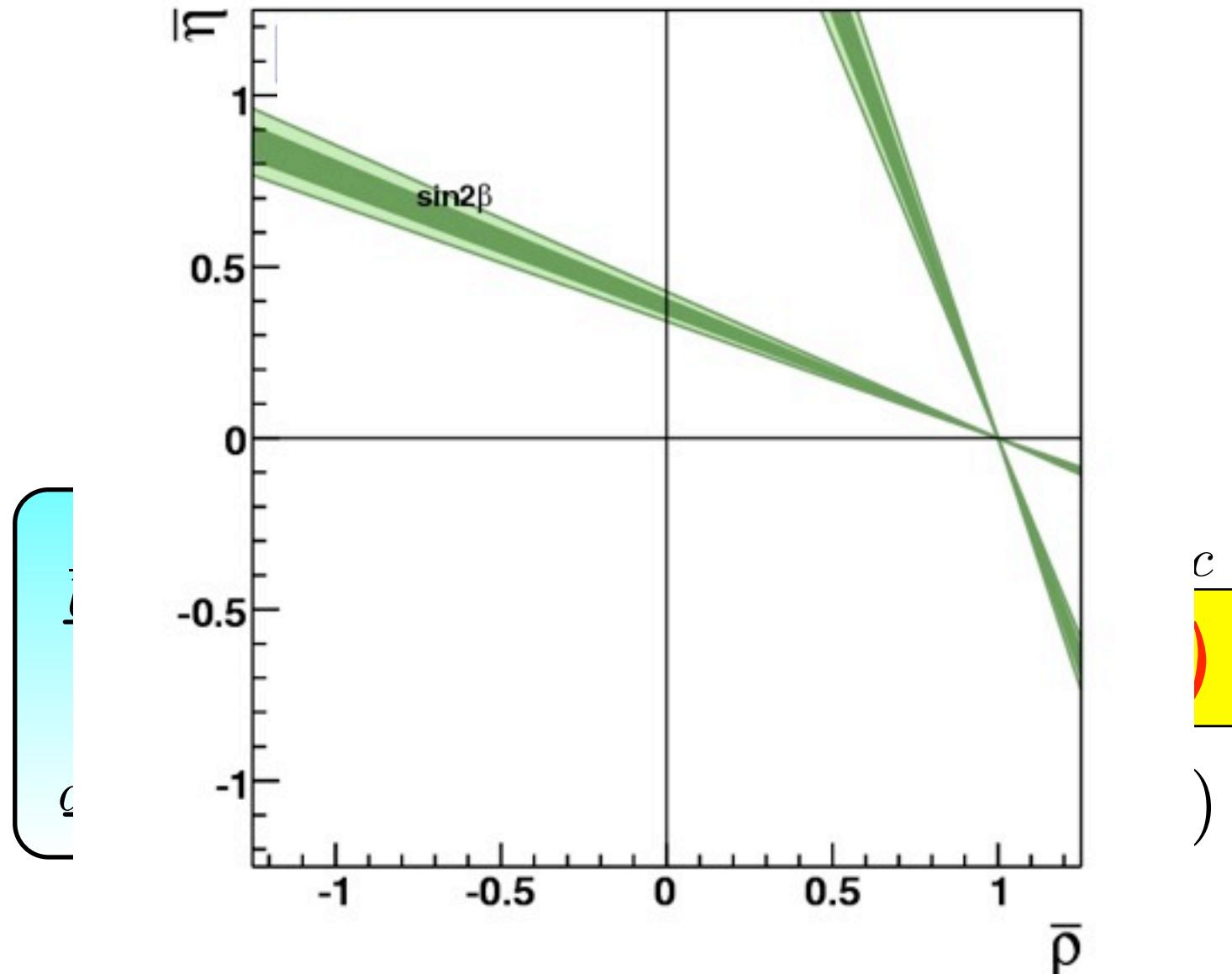
Including various $b \rightarrow cc\bar{c}s$ the
measurements, we find

$$\Phi_I = (21.1 \pm 0.9)^\circ$$

Ψ

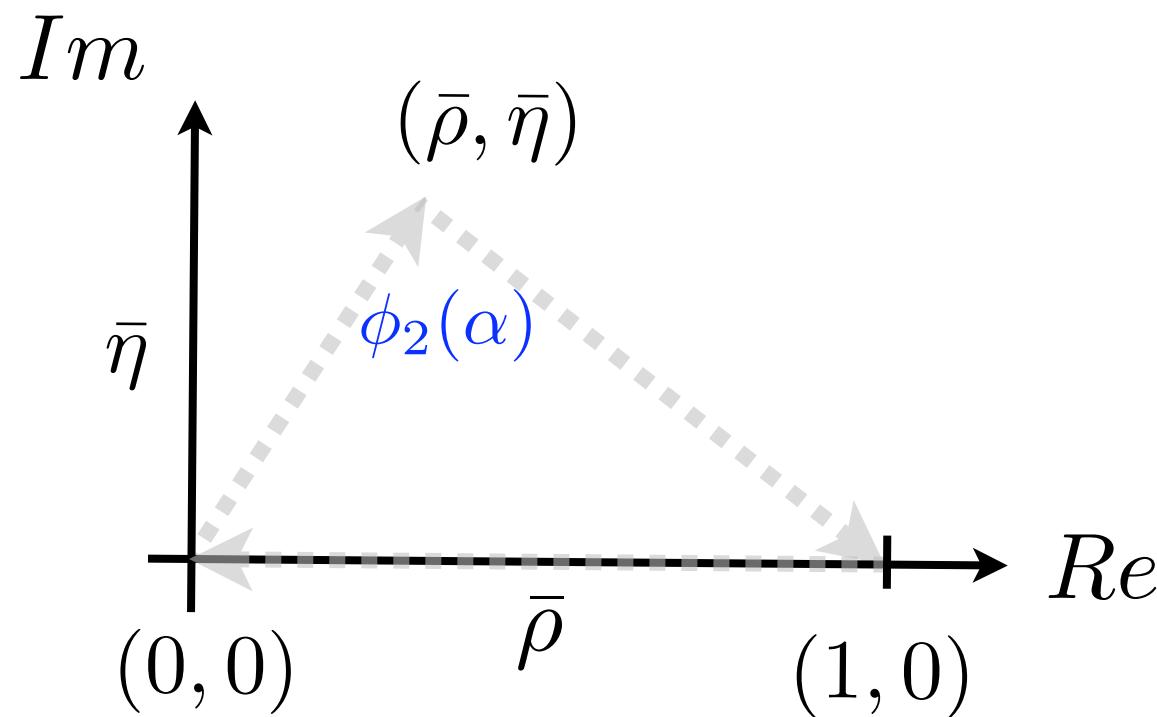
K_s

Determination of the CKM matrix:



Determination of the CKM matrix: $\sin 2\Phi_2 (\alpha)$ (phase)

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$





CP asymmetry in $B \rightarrow \pi^+ \pi^-$

$$A_{\pi^+\pi^-}(t) = \frac{\Gamma(\bar{B}^0(t) \rightarrow \pi^+\pi^-) - \Gamma(B^0(t) \rightarrow \pi^+\pi^-)}{\Gamma(\bar{B}^0(t) \rightarrow \pi^+\pi^-) + \Gamma(B^0(t) \rightarrow \pi^+\pi^-)} = S_{\pi^+\pi^-} \sin \Delta M_B t$$

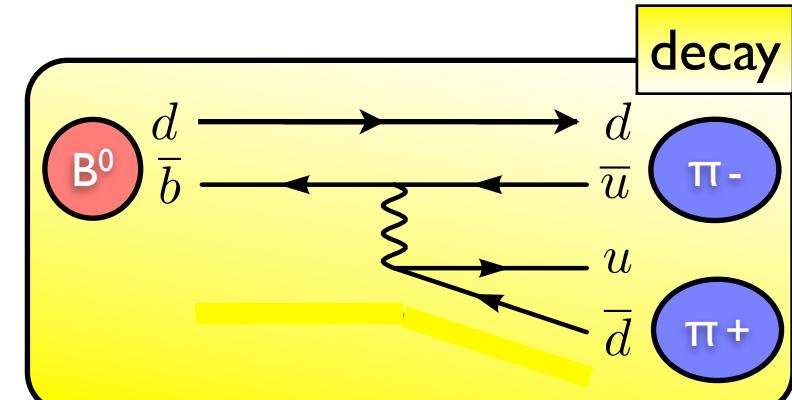
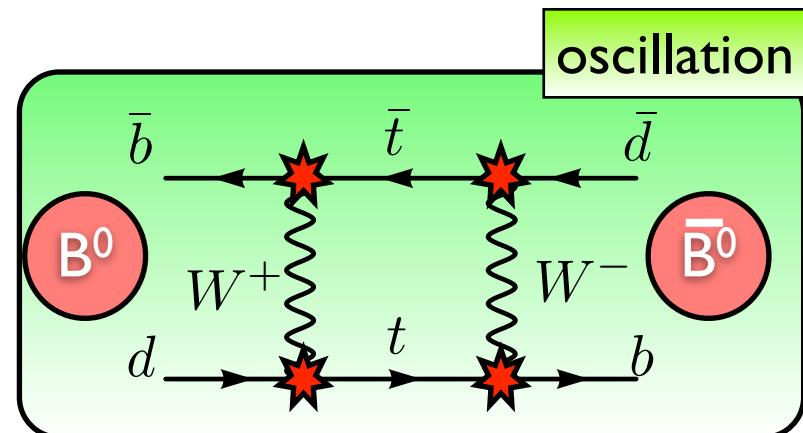
$$S_{\pi^+\pi^-} = \text{Im} \left[\frac{M_{12}}{M_{12}^*} \frac{A(\bar{B} \rightarrow \pi^+\pi^-)}{A(B \rightarrow \pi^+\pi^-)} \right]$$

$\underbrace{M_{12}}_{\text{oscill.}}$ $\underbrace{\frac{A(\bar{B} \rightarrow \pi^+\pi^-)}{A(B \rightarrow \pi^+\pi^-)}}_{\text{decay}}$

$$= \text{Im} \left[\frac{V_{tb} V_{td}^*}{V_{tb}^* V_{td}} \frac{V_{ub} V_{ud}^*}{V_{ub}^* V_{ud}} \right]$$

$\underbrace{V_{tb} V_{td}^*}_{\text{oscill.}}$ $\underbrace{\frac{V_{ub} V_{ud}^*}{V_{ub}^* V_{ud}}}_{\text{decay}}$

$$= \sin 2\phi_2(\alpha)$$



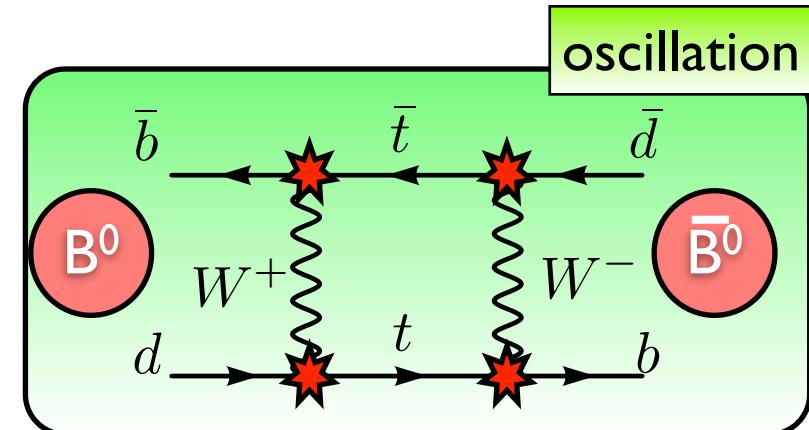
* Assuming tree-dominant...



CP asymmetry in $B \rightarrow \pi^+ \pi^-$

$$A_{\pi^+ \pi^-}(t) = \frac{\Gamma(\bar{B}^0(t) \rightarrow \pi^+ \pi^-) - \Gamma(B^0(t) \rightarrow \pi^+ \pi^-)}{\Gamma(\bar{B}^0(t) \rightarrow \pi^+ \pi^-) + \Gamma(B^0(t) \rightarrow \pi^+ \pi^-)} = S_{\pi^+ \pi^-} \sin \Delta M_B t$$

**Penguin pollution
prevents a precise
measurement of $\Phi_2(\alpha)$**



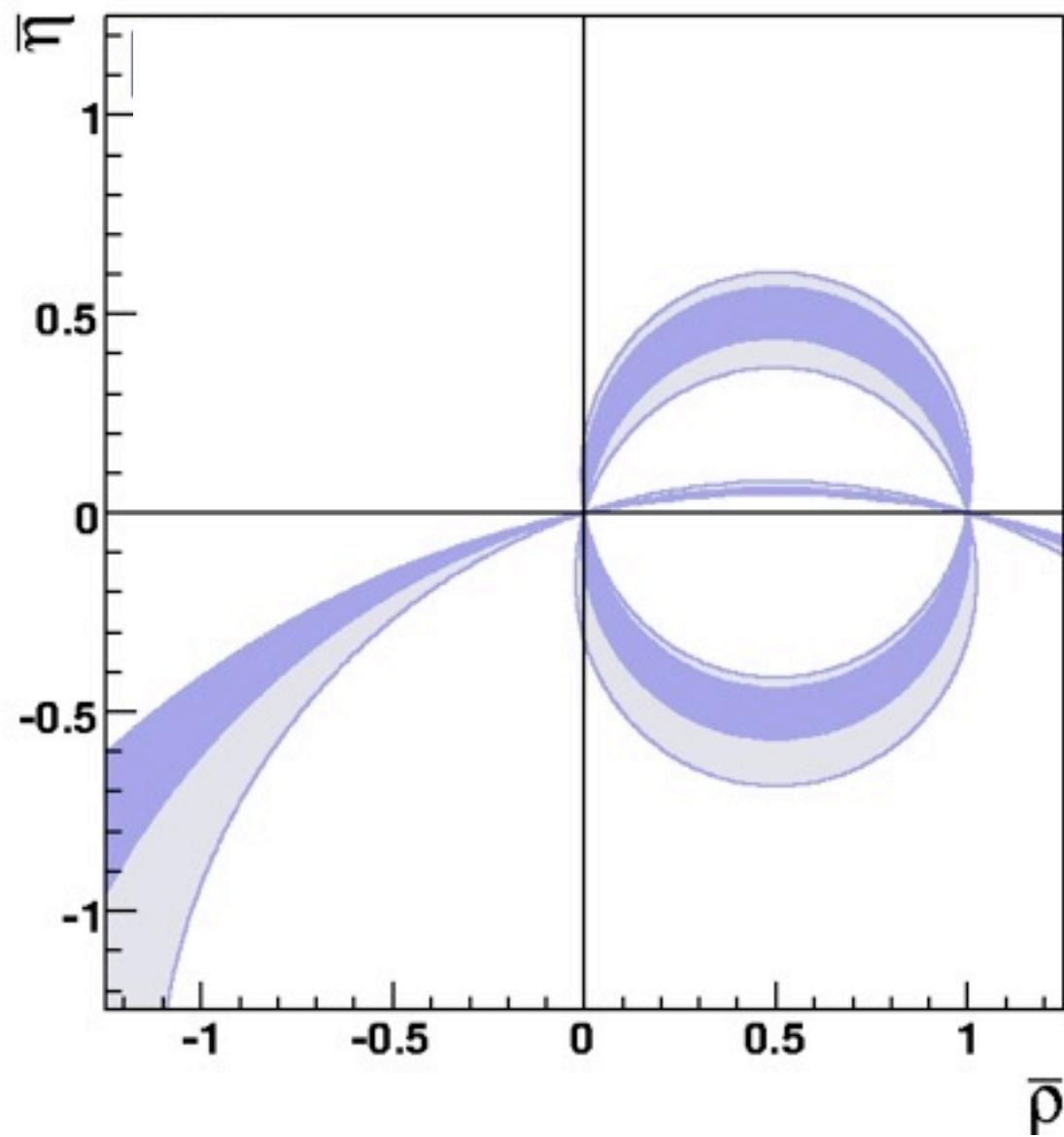
Including the measurements with

$$\pi\pi, \rho\rho, \pi\rho,$$

$$\alpha = (89.0^{+4.4}_{-4.2})^\circ$$

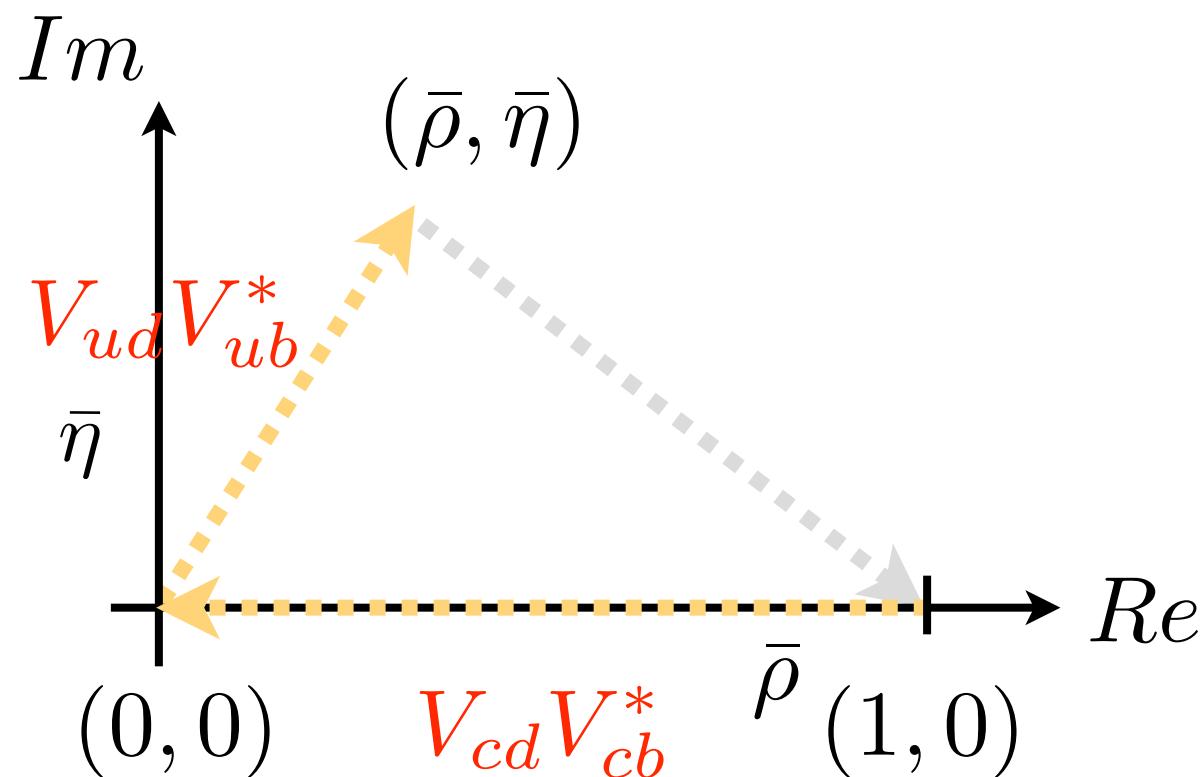
* Assuming tree-dominant...

Determination of the CKM matrix:



Determination of the CKM matrix: $|V_{ub}|$ and $|V_{cb}|$

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

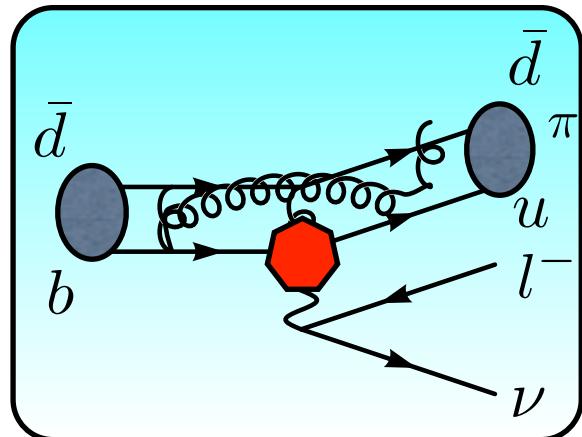


Determination of the CKM matrix: $|V_{ub}|$

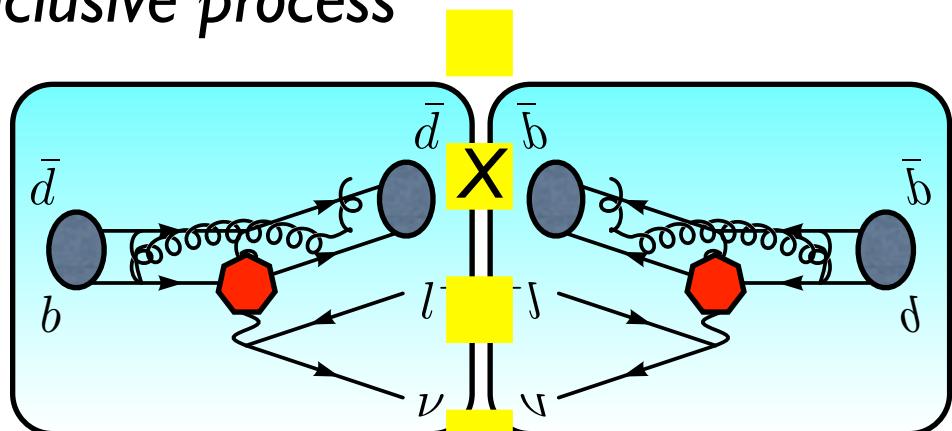
$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

** Similar in $|V_{cb}|$*

Exclusive process



Inclusive process



Optical theorem

$$\mathcal{A}(B \rightarrow \pi l \nu) \propto |V_{ub}| F^{B \rightarrow \pi}(q^2)$$

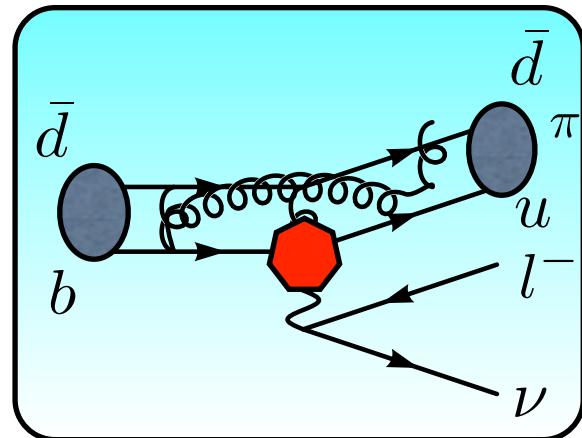
$$\sum |\mathcal{A}(B \rightarrow X_u l \nu)|^2 \propto |V_{ub}|^2 f(q^2, \mu_\pi, \dots)$$

Determination of the CKM matrix: $|V_{ub}|$

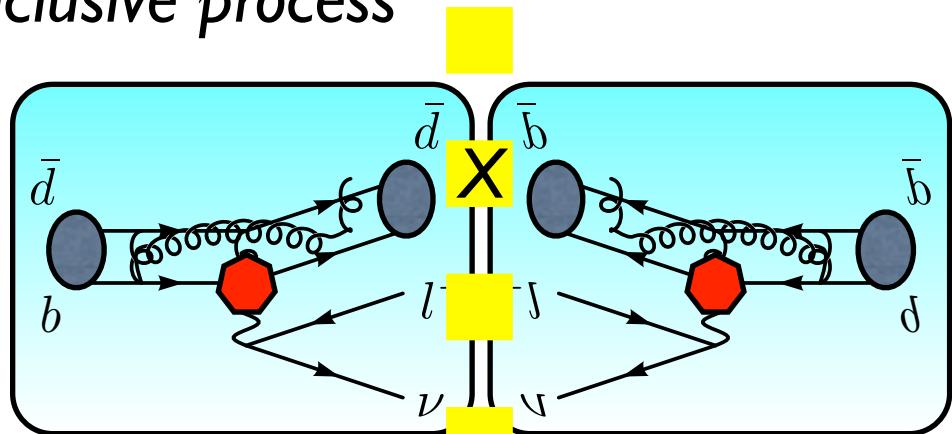
$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

** Similar in $|V_{cb}|$*

Exclusive process



Inclusive process



Optical theorem

$$\mathcal{A}(B \rightarrow \pi l \nu) \propto |V_{ub}| F^{B \rightarrow \pi}(q^2)$$

$$\sum |\mathcal{A}(B \rightarrow X_u l \nu)|^2 \propto |V_{ub}|^2 f(q^2, \mu_\pi, \dots)$$

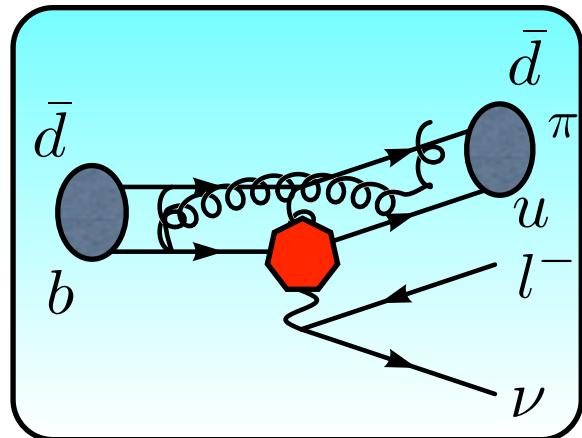
Hadronic uncertainties!

Determination of the CKM matrix: $|V_{ub}|$

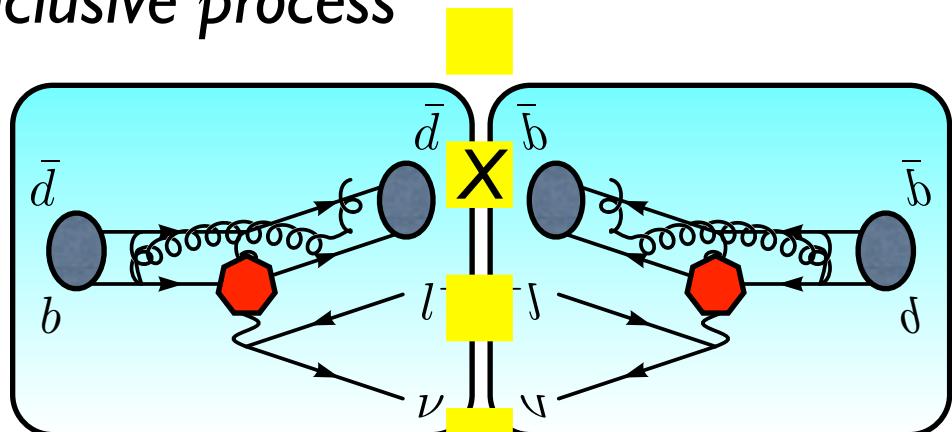
$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

** Similar in $|V_{cb}|$*

Exclusive process



Inclusive process



Optical theorem

$$\mathcal{A}(B \rightarrow \pi l \nu) \propto |V_{ub}| F^{B \rightarrow \pi}(q^2)$$

$|V_{ub}| = (3.89 \pm 0.44) \cdot 10^{-3}$

$$\sum |\mathcal{A}(B \rightarrow X_u l \nu)|^2 \propto |V_{ub}|^2 f(q^2, \mu_\pi, \dots)$$

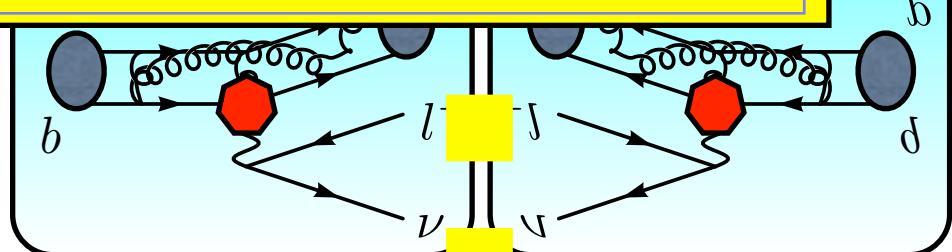
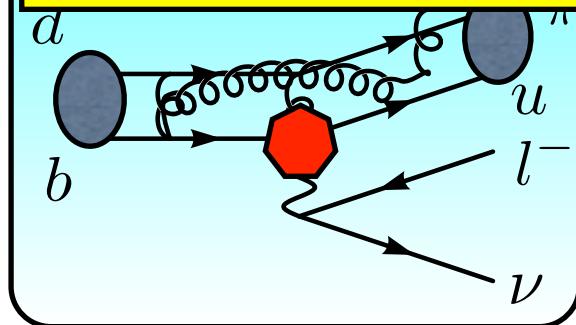
$|V_{ub}| = (4.27 \pm 0.38) \cdot 10^{-3}$

Determination of the CKM matrix: $|V_{ub}|$

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \end{pmatrix} \quad * \text{Similar in } |V_{cb}|$$

Further improvements in hadronic uncertainties:

- Many efforts in Lattice QCD
- Heavy quark effective theory (expansion using $m_{c,b} \gg \Lambda_{\text{QCD}}$)



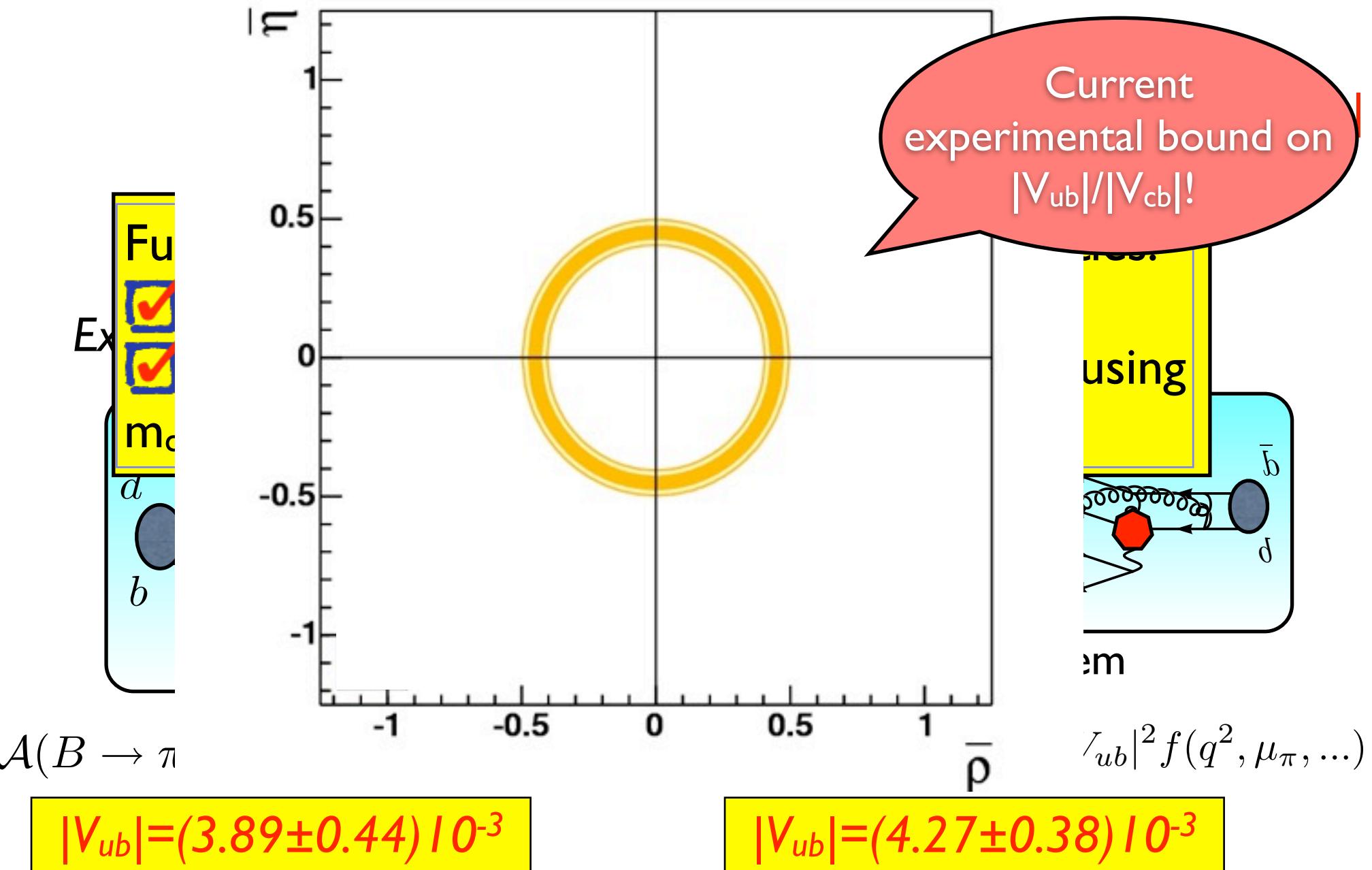
$$\mathcal{A}(B \rightarrow \pi l \nu) \propto |V_{ub}| F^{B \rightarrow \pi}(q^2)$$

$$|V_{ub}| = (3.89 \pm 0.44) \cdot 10^{-3}$$

$$\sum |\mathcal{A}(B \rightarrow X_u l \nu)|^2 \propto |V_{ub}|^2 f(q^2, \mu_\pi, \dots)$$

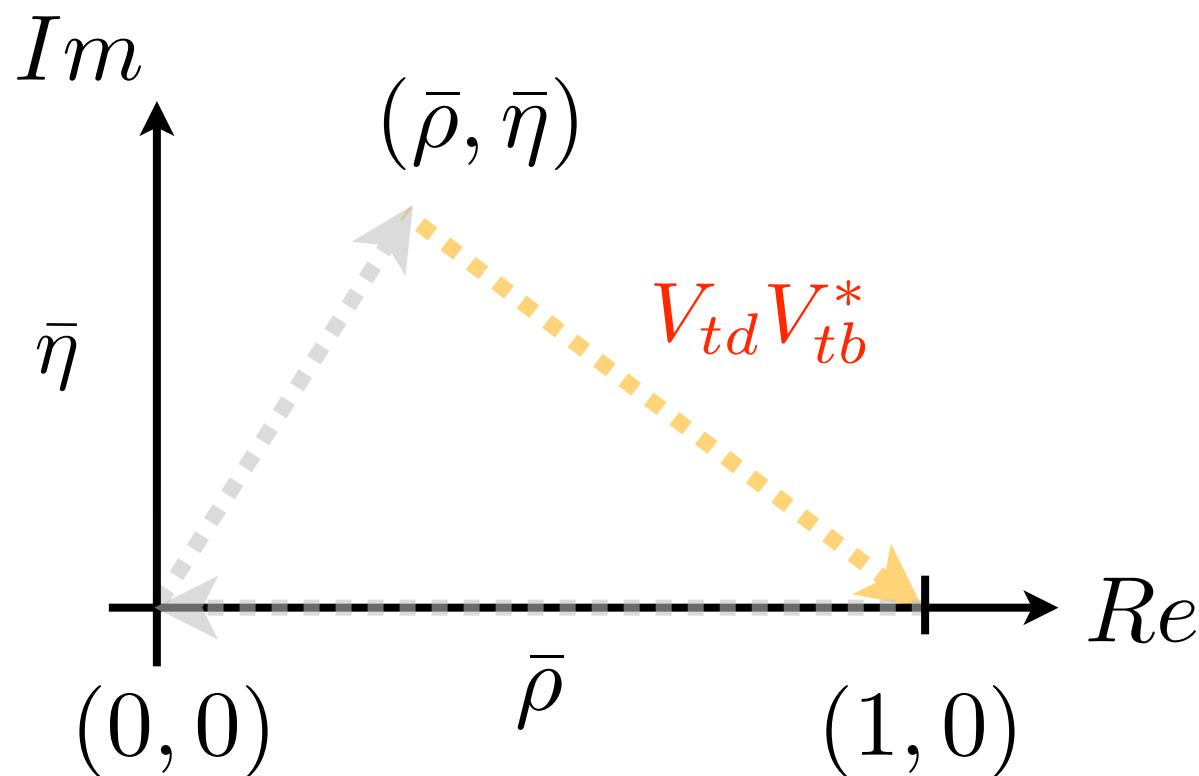
$$|V_{ub}| = (4.27 \pm 0.38) \cdot 10^{-3}$$

Determination of the CKM matrix:



Determination of the CKM matrix: $|V_{td} V_{tb}^*|$

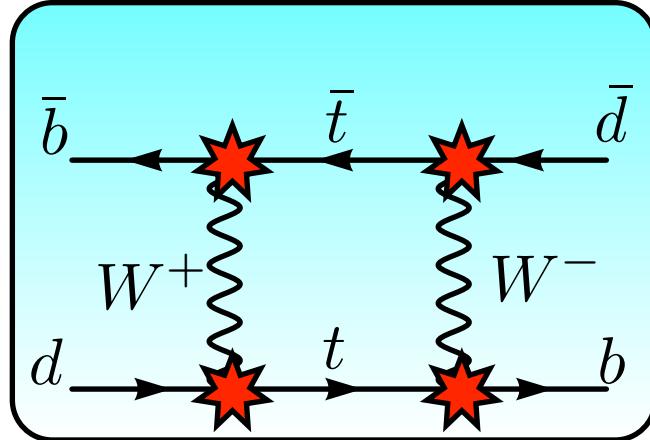
$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



Determination of the CKM matrix:

$$|V_{td} V_{tb}^*|$$

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

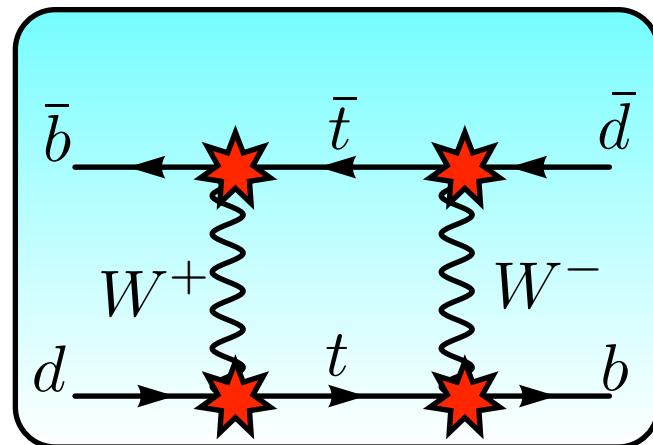


$\Delta M_d \Rightarrow |V_{td} V_{tb}^*|$

Determination of the CKM matrix:

$$|V_{td} V_{tb}^*|$$

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



$$\Delta M_d (\propto |M_{12}|)$$

$$= \frac{G_F^2 m_W^2}{16\pi^2} |V_{tb} V_{td}^*|^2 S_0 \left(\frac{m_t^2}{m_W^2} \right) \eta_{\text{QCD}}$$

$$\times \frac{1}{m_B} \langle B^0 | (\bar{d}b)_{V-A} (\bar{d}b)_{V-A} | \overline{B}^0 \rangle$$

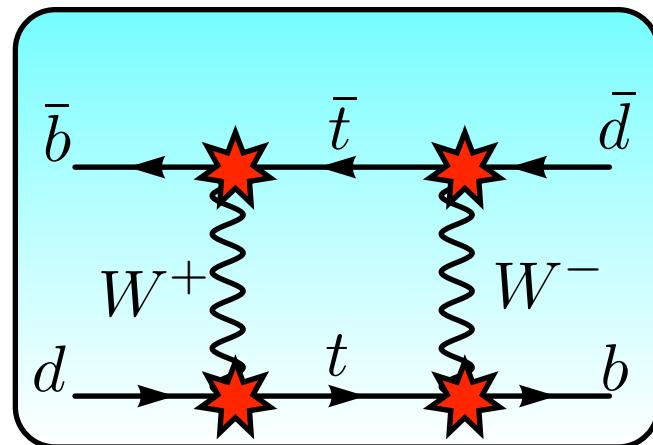
$\Delta M_d \Rightarrow |V_{td} V_{tb}^*|$

Determination of the CKM matrix:

$$|V_{td} V_{tb}^*|$$

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Perturbative QCD correction



$$\Delta M_d \rightarrow |V_{td} V_{tb}^*|$$

$$\Delta M_d (\propto |M_{12}|)$$

$$= \frac{G_F^2 m_W^2}{16\pi^2} |V_{tb} V_{td}^*|^2 S_0 \left(\frac{m_t^2}{m_W^2} \right) \eta_{\text{QCD}}$$

$$\times \frac{1}{m_B} \langle \bar{B}^0 | (\bar{d}b)_{V-A} (\bar{d}b)_{V-A} | \bar{B}^0 \rangle$$

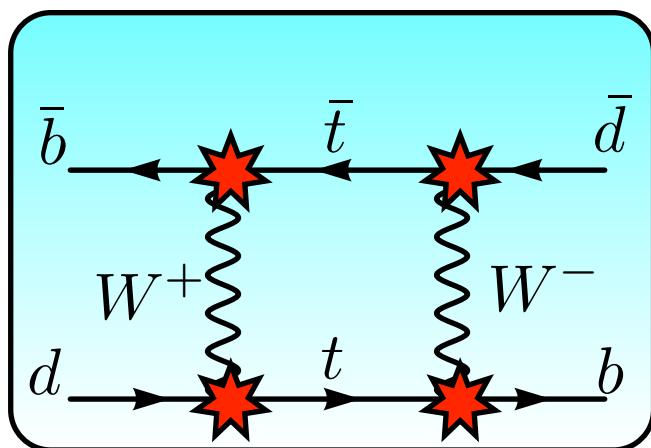
$$\equiv \frac{8}{3} B_B f_B^2 m_B^2$$

Main source of the hadronic uncertainties
in determining $|V_{tb}|$:
Lattice QCD computation very important!

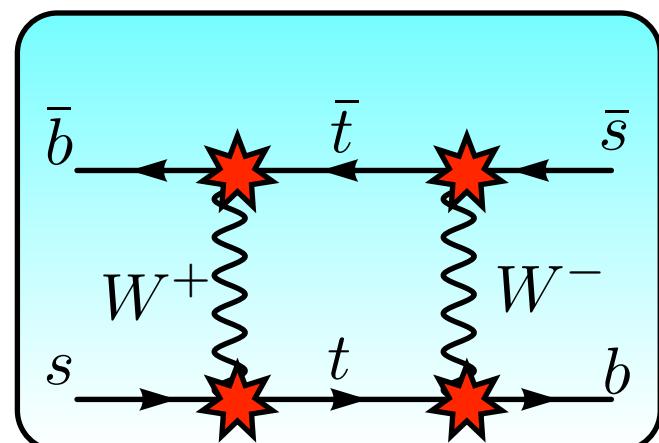
Determination of the CKM matrix: $|V_{td}V_{tb}^*|$ and $|V_{ts}V_{tb}^*|$

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Some hadronic uncertainties cancel in the ratio between ΔM_d and ΔM_s

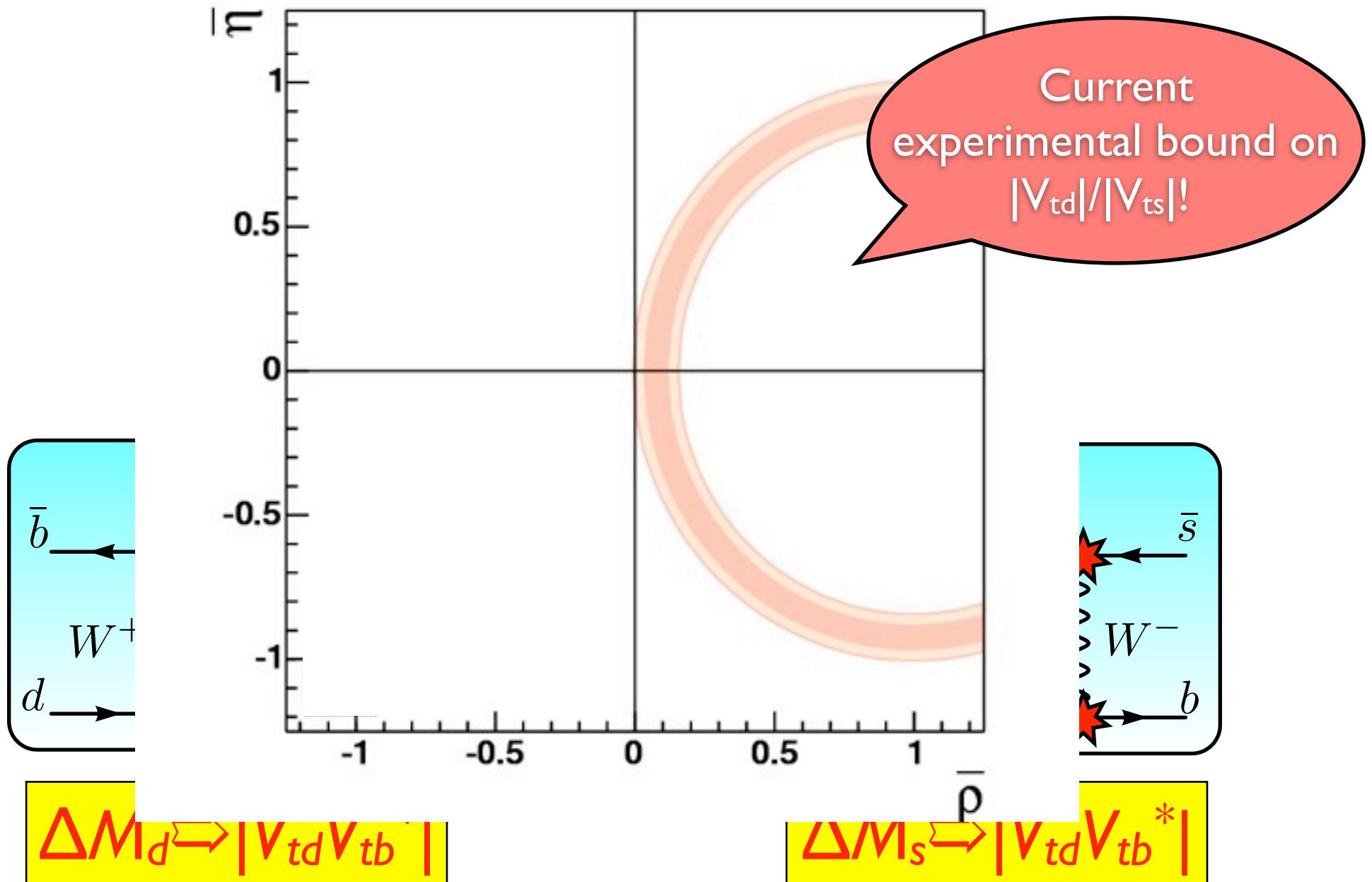


$\Delta M_d \Rightarrow |V_{td}V_{tb}^*|$

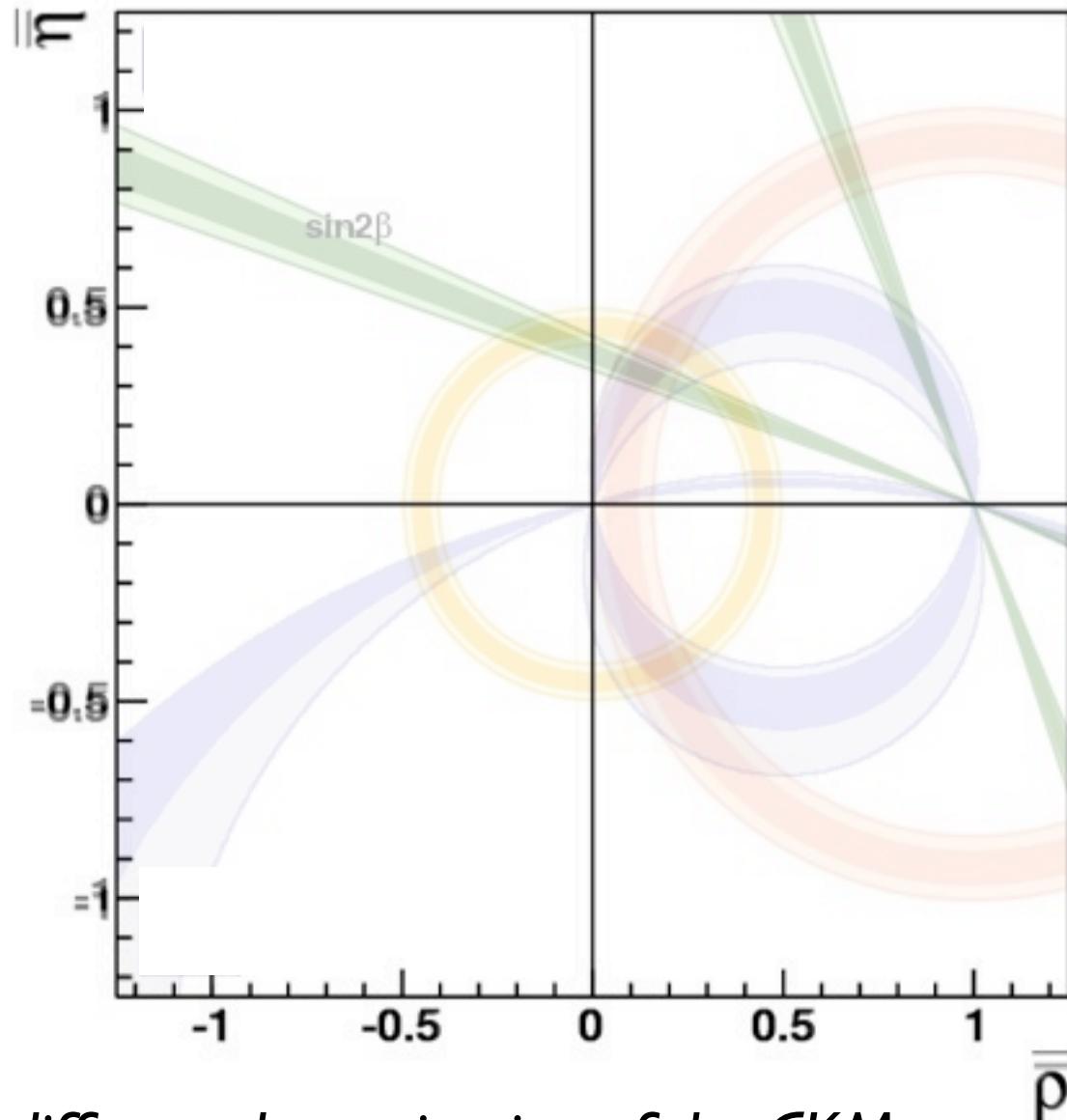


$\Delta M_s \Rightarrow |V_{ts}V_{tb}^*|$

Determination of the CKM matrix:

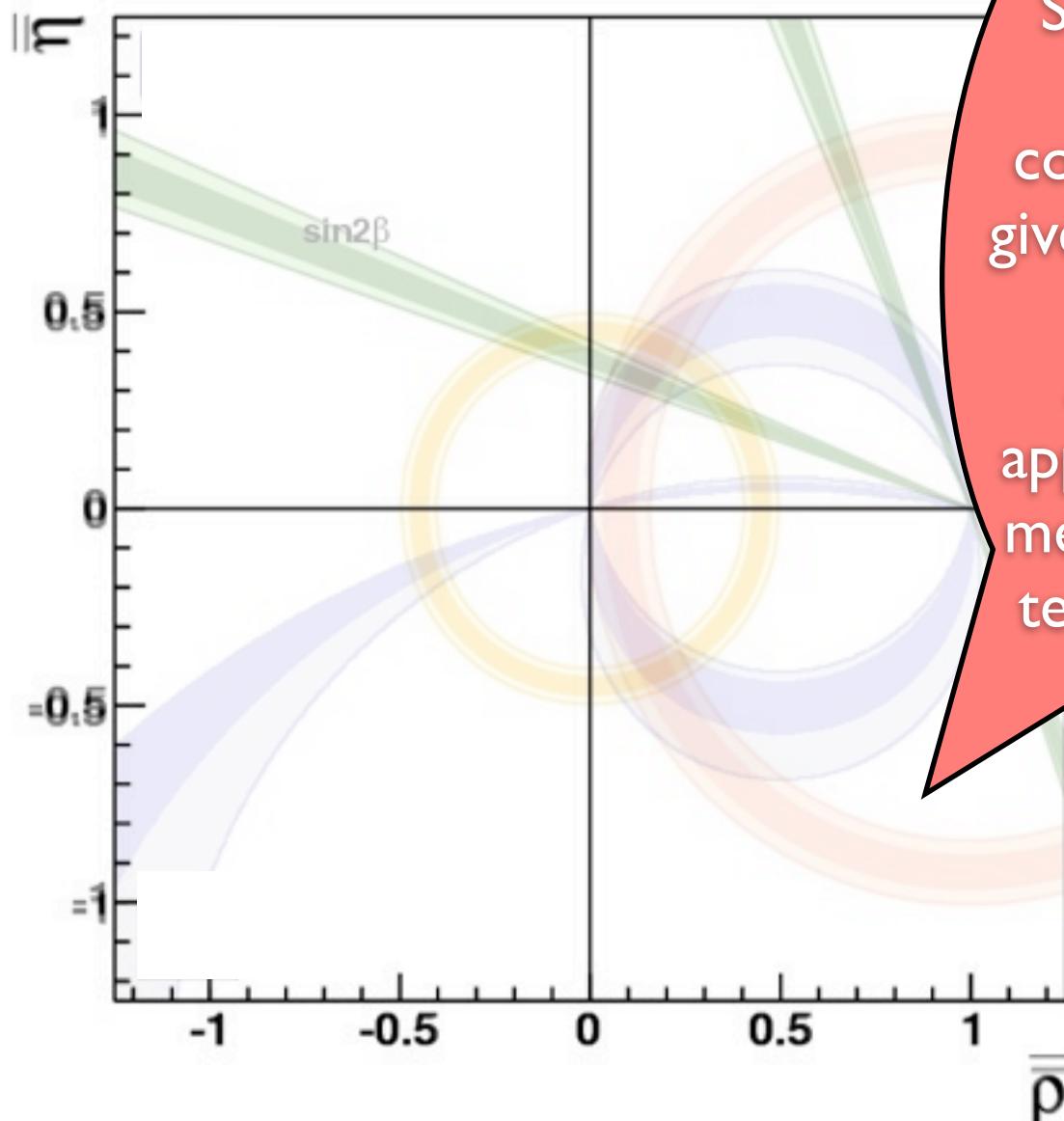


Combining the constraints...

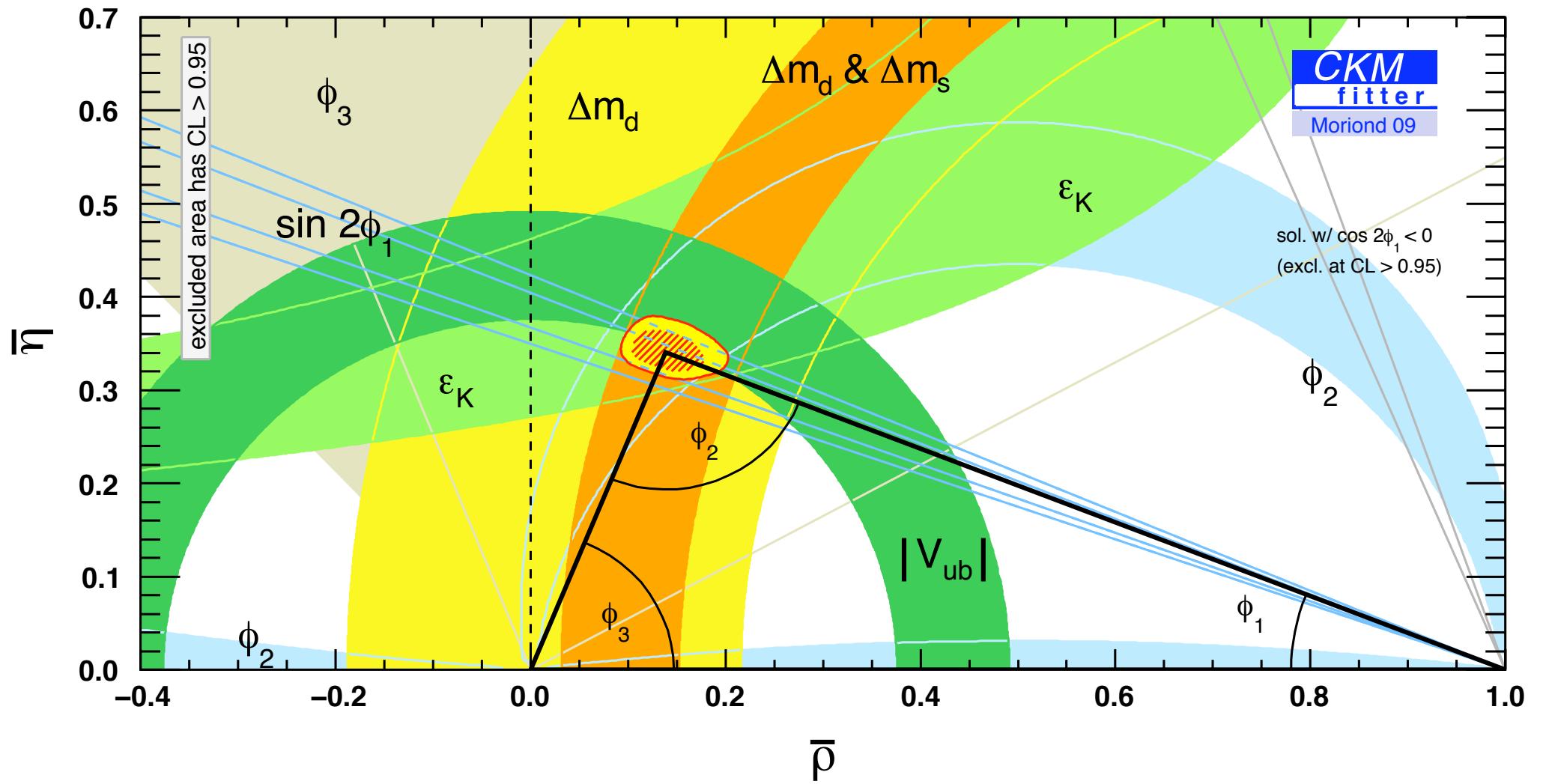


Indeed the different determination of the CKM parameters are “relatively” consistent to each other...

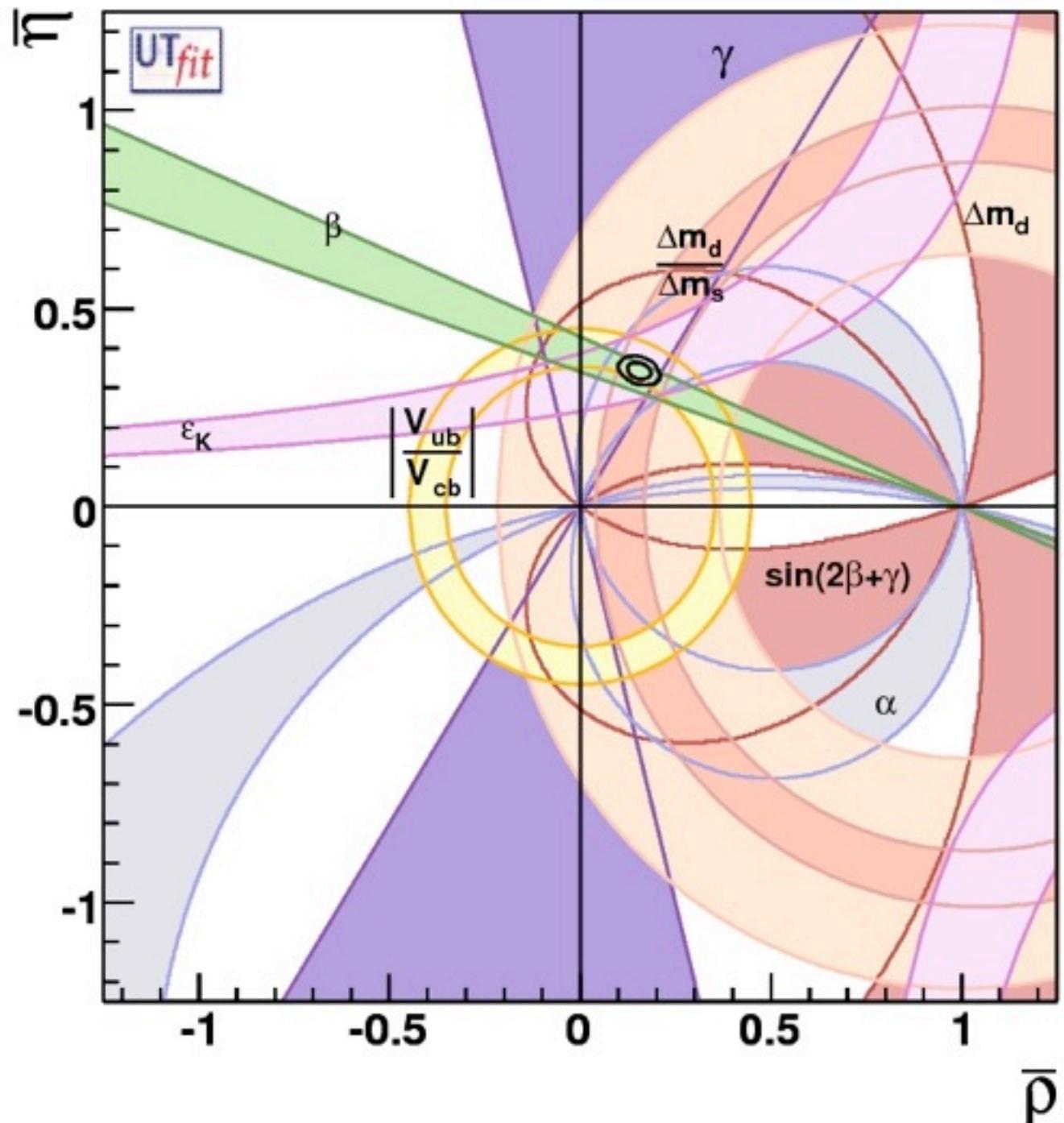
Combining the constraints

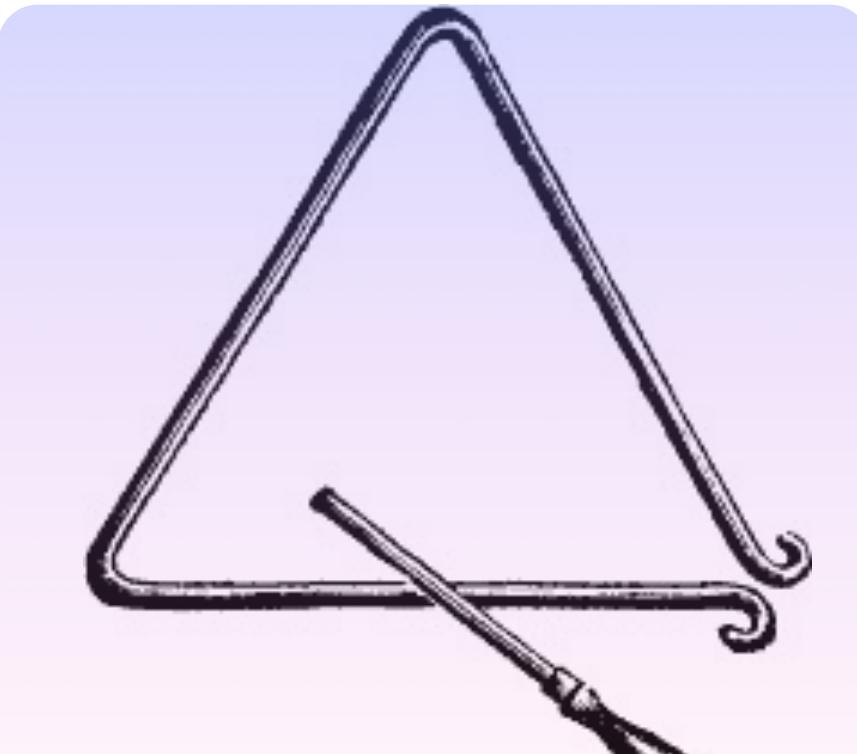


Warning:
Simply overlapping
the different
constraints does not
give a right confidence
level.
An application of
appropriate statistical
methods is crucial to
test the unitarity!!



<http://ckmfitter.in2p3.fr/>





We can say that the main part of the CP violation comes from the complex phase in the CKM matrix. However, there is still a possibility that the unitarity is not exact for a certain extent. Our challenges for more precise experimental data as well as improvements in the theoretical predictions continue!!!

Determination of CKM matrix

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Determination of the CKM matrix: Cabibbo angle

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$
$$= \begin{pmatrix} \cos \theta_c & \sin \theta_c & V_{ub} \\ -\sin \theta_c & \cos \theta_c & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

You remember the
Cabibbo angle.

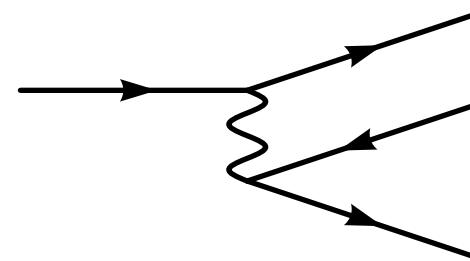
Determination of the CKM matrix: Cabibbo angle

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$V_{ud} = 0.9746 \underbrace{(4)}_{\tau_n} \underbrace{(18)}_{G_A/G_V} \underbrace{(2)}_{\text{RCorr.}}$$

| Nucleus | ft (sec) | V_{ud} |
|---------------|------------|---------------------|
| ^{10}C | 3039.5(47) | 0.97370(80)(14)(19) |
| ^{14}O | 3042.5(27) | 0.97411(51)(14)(19) |
| ^{26}Al | 3037.0(11) | 0.97400(24)(14)(19) |
| ^{34}Cl | 3050.0(11) | 0.97417(34)(14)(19) |
| ^{38}K | 3051.1(10) | 0.97413(39)(14)(19) |
| ^{42}Sc | 3046.4(14) | 0.97423(44)(14)(19) |
| ^{46}V | 3049.6(16) | 0.97386(49)(14)(19) |
| ^{50}Mn | 3044.4(12) | 0.97487(45)(14)(19) |
| ^{54}Co | 3047.6(15) | 0.97490(54)(14)(19) |
| Weighted Ave. | | 0.97418(13)(14)(19) |

Hadronic
uncertainties!



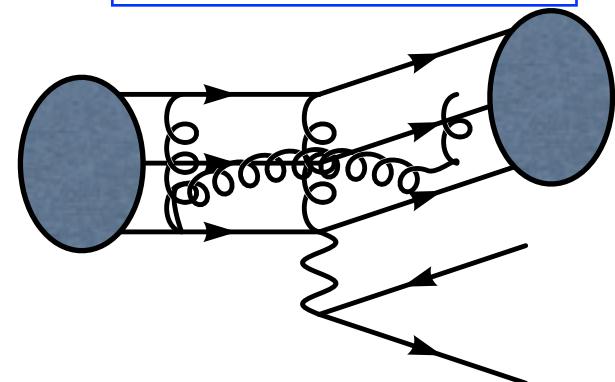
Determination of the CKM matrix: Cabibbo angle

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

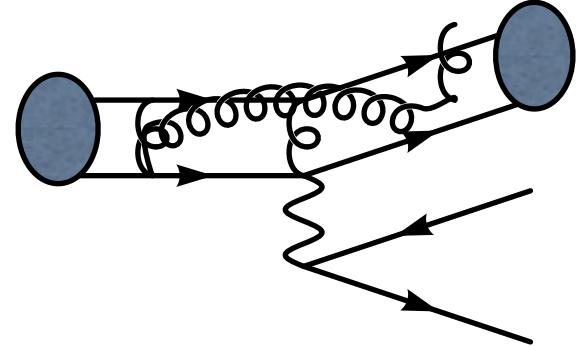
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| Nucleus | ft (sec) | V_{ud} |
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| ^{10}C | 3039.5(47) | 0.97370(80)(14)(19) |
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| Weighted Ave. | | 0.97418(13)(14)(19) |

Hadronic
uncertainties!



Determination of the CKM matrix: Cabibbo angle

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$


$$f_+(0)|V_{us}| = 0.21668(45)$$

Hadronic
uncertainties!

Chiral Perturbation
Theory

$$f_+(0) = 0.961 \pm 0.08 \text{ (2%)}$$

Lattice

$$f_+(0) = 0.9609(51)$$

| Decay Mode | $ V_{us} f_+(0)$ |
|---------------|-----------------------|
| $K^\pm e3$ | 0.21746 ± 0.00085 |
| $K^\pm \mu 3$ | 0.21810 ± 0.00114 |
| $K_L e3$ | 0.21638 ± 0.00055 |
| $K_L \mu 3$ | 0.21678 ± 0.00067 |
| $K_S e3$ | 0.21554 ± 0.00142 |
| Average | 0.21668 ± 0.00045 |

Determination of the CKM matrix: Cabibbo angle

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \end{pmatrix}$$

V_{ud} and V_{us} :

All in all, it is close to $V_{ud} = \cos\theta_c$, $V_{us} = \sin\theta_c$.
BUT many efforts continue!

Hadron
uncertainties:

Chiral Perturbation
Theory

$$f_+(0) = 0.961 \pm 0.08 \text{ (2%)}$$

Lattice

$$f_+(0) = 0.9609(51)$$

| | |
|-------------|-----------------------|
| $K^+\mu 3$ | 0.21746 ± 0.00085 |
| $K_L e 3$ | 0.21810 ± 0.00114 |
| $K_L \mu 3$ | 0.21638 ± 0.00055 |
| $K_S e 3$ | 0.21678 ± 0.00067 |
| Average | 0.21554 ± 0.00142 |
| | 0.21668 ± 0.00045 |

Determination of the CKM matrix: Charm meson decays

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Hadronic uncertainties for heavy mesons are more difficult to control.

- No chiral perturbation
- Lattice possible (but much more computing power needed)
- Heavy quark effective theory (expansion using $m_{c,b} \gg \Lambda_{\text{QCD}}$)

$$\begin{aligned} V_{cd} &= 0.230 \pm 0.011 \\ V_{cs} &= 1.04 \pm 0.06 \\ V_{cb} &= (41.2 \pm 1.1) \times 10^{-3} \\ V_{ub} &= (3.93 \pm 0.36) \times 10^{-3} \end{aligned}$$

Determination of the CKM matrix: Charm meson decays

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \end{pmatrix}$$

The unitarity is more or less fine. For the most of decay channels, hadronic uncertainties are larger than the experimental ones...

Theoretical challenges!!!

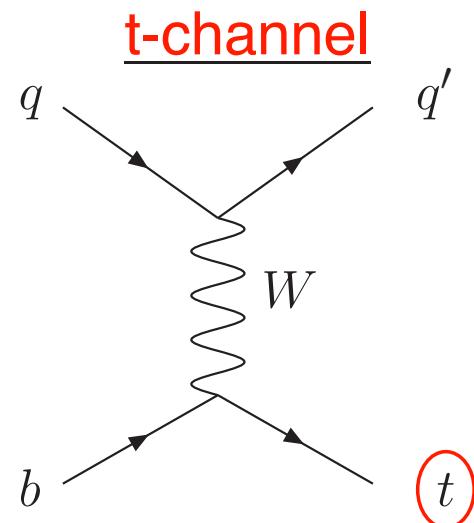
- No charm quarks
- Lattice possible (but requires more computer power needed)
- Heavy quark effective theory (expansion using $m_{c,b} \gg \Lambda_{\text{QCD}}$)

$$\begin{aligned} V_{cb} &= (41.2 \pm 1.1) \times 10^{-3} \\ V_{ub} &= (3.93 \pm 0.36) \times 10^{-3} \end{aligned}$$

Determination of the CKM matrix: $|V_{tb}|$ from single top production

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & \textcircled{V_{tb}} \end{pmatrix}$$

The tree level determination becomes only possible by the top physics.



| | |
|------|-----------------------|
| Tev. | $\sim 0.9 \text{ pb}$ |
| LHC | $\sim 240 \text{ pb}$ |

Why we need single top production to measure V_{tb} ?

Tait, Yuan

hep-ph/0007298

Top quarks may be produced in pairs at a hadron collider via the strong interaction, through processes such as $q\bar{q} \rightarrow t\bar{t}$ and $gg \rightarrow t\bar{t}$. Thus, the rate and kinematic distributions of top quarks produced in this way are a measure of the top's interactions with the gluons. The top decay proceeds via the weak interaction, and as we shall see does provide interesting information about the chiral structure of the W - t - b interaction [7, 9]. However, decays are experimentally relatively insensitive to the magnitude of the interaction by which they are mediated. For example, in the case of top, there is one SM decay mode, $t \rightarrow b W^+$. If this vertex were somehow modified by new physics to have a different magnitude, it would affect the top's intrinsic width. However, at a hadron collider the width cannot be measured because the experimental resolutions are much larger than the width itself [10]. Similarly, while observing exotic top decays would certainly be interesting and would suggest what type of new vertices describe the observed decays, it would not determine the magnitude of these new interactions. Even a study of branching fractions compared to the SM decay mode may be misleading, because one must have already measured the W - t - b interaction strength itself through some other means.

These drawbacks lead one to study weak production mechanisms of the top quark, which have cross sections directly proportional to the top's weak couplings. The Z - t - t coupling will presumably contribute to $t\bar{t}$ production, though distinguishing this from the

Determination of the CKM matrix: $\sin 2\beta_s$ (phase of V_{ts})

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Expansion in
order λ^3

$$= \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$= \begin{pmatrix} 1 - \lambda^2/2 - \lambda^4/8 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 + (-1/8 - A^2/2)\lambda^4 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 + A\lambda^4(1/2 - \rho - i\eta) & 1 \end{pmatrix} + \mathcal{O}(\lambda^5)$$

Expansion in
order λ^4

Determination of the CKM matrix: $\sin 2\beta_s$ (phase of V_{ts})

- While the final state $J/\psi\phi$ is not a CP eigenstate, its **polarisation decomposed states** are.
- The $B_s \rightarrow J/\psi\phi$ process is quite similar to $B_d \rightarrow J/\psi K_S$, except for the fact that **the width difference $\Delta\Gamma_s$ cannot be neglected**.

By summing over all the polarisation final states, we obtain the **master formula** for the time dependent CP asymmetry:

$$\begin{aligned}\frac{\Gamma(\bar{B}_s^0 \rightarrow J/\psi\phi) - \Gamma(B_s^0 \rightarrow J/\psi\phi)}{\Gamma(\bar{B}_s^0 \rightarrow J/\psi\phi) + \Gamma(B_s^0 \rightarrow J/\psi\phi)} &= \frac{D \operatorname{Im} \left[\frac{q}{p} \bar{\rho}_{\text{odd}} \right] + \operatorname{Im} \left[\frac{q}{p} \bar{\rho}_{\text{even}} \right]}{D F_{\text{odd}}(t) + F_{\text{even}}(t)} \sin \Delta M_{st} \\ &= \boxed{\sin 2\beta_s} \sin \Delta M_{st}\end{aligned}$$

$$F_{\text{odd,even}}(t) = \cosh \left(\frac{\Delta\Gamma_s}{2} t \right) + \operatorname{Re} \left[\frac{q}{p} \bar{\rho}_{\text{odd,even}} \right] \sinh \left(\frac{\Delta\Gamma_s}{2} t \right)$$

where $\bar{\rho} = \operatorname{Amp}(\bar{B}_s^0 \rightarrow J/\psi\phi) / \operatorname{Amp}(B_s^0 \rightarrow J/\psi\phi)$ and a hadronic quantity $D \equiv \frac{|A_\perp|^2}{|A_\parallel|^2 + |A_0|^2}$ is estimated as 0.3 ± 0.2 where the amplitudes $A_{0,\parallel,\perp}$ characteristic for the final state ($A_{0,\parallel}$ for CP-even and A_\perp for CP-odd).