Lectures on Neutrino Physics

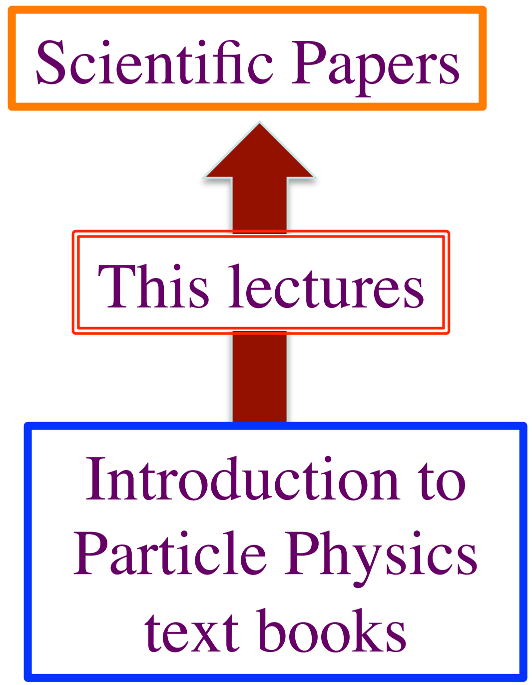
Fumihiko Suekane
Research Center for Neutrino Science
Tohoku University
suekane@awa.tohoku.ac.jp
http://www.awa.tohoku.ac.jp/~suekane

France Asia Particle Physics School
@Ecole de Physique Les Houchess
18-20/Oct./2011

* This lecture is intended to give intuitive understanding of neutrino physics for students and young physicists of other field.

* I will try to make this lecture to be a bridge between general text books and scientific papers.

* 3 lectures are very short to mention about all the varieties of neutrino physics and only limited but important topics are mentioned.



Contents

* History
* Neutrinos in the Standard Model
* Neutrino Oscillations (Main)
* Double Beta decays
* Prospects

Day-1

Day-2

Day-3

What is known for neutrinos

Only a few things are known about neutrinos.

.... There is much room to study.

PDG2010

Neutrino Properties

See the note on "Neutrino properties listings" in the Particle Listings.

Mass $m < 2$ eV (tritium decay)

Mean life/mass, $\tau/m > 300$ s/eV, CL = 90% (reactor)

Mean life/mass, $\tau/m > 7 \times 10^9$ s/eV (solar)

Mean life/mass, $\tau/m > 15.4$ s/eV, CL = 90% (accelerator)

Magnetic moment $\mu < 0.54 \times 10^{-10}\ \mu_B$, CL = 90% (solar)

Number of Neutrino Types

Number $N = 2.984 \pm 0.008$ (Standard Model fits to LEP data)

Number $N = 2.92 \pm 0.05$ (S = 1.2) (Direct measurement of invisible Z width)

Neutrino Mixing

The following values are obtained through data analyses based on the 3-neutrino mixing scheme described in the review "Neutrino Mass, Mixing, and Oscillations" by K. Nakamura and S.T. Petcov in this *Review*.

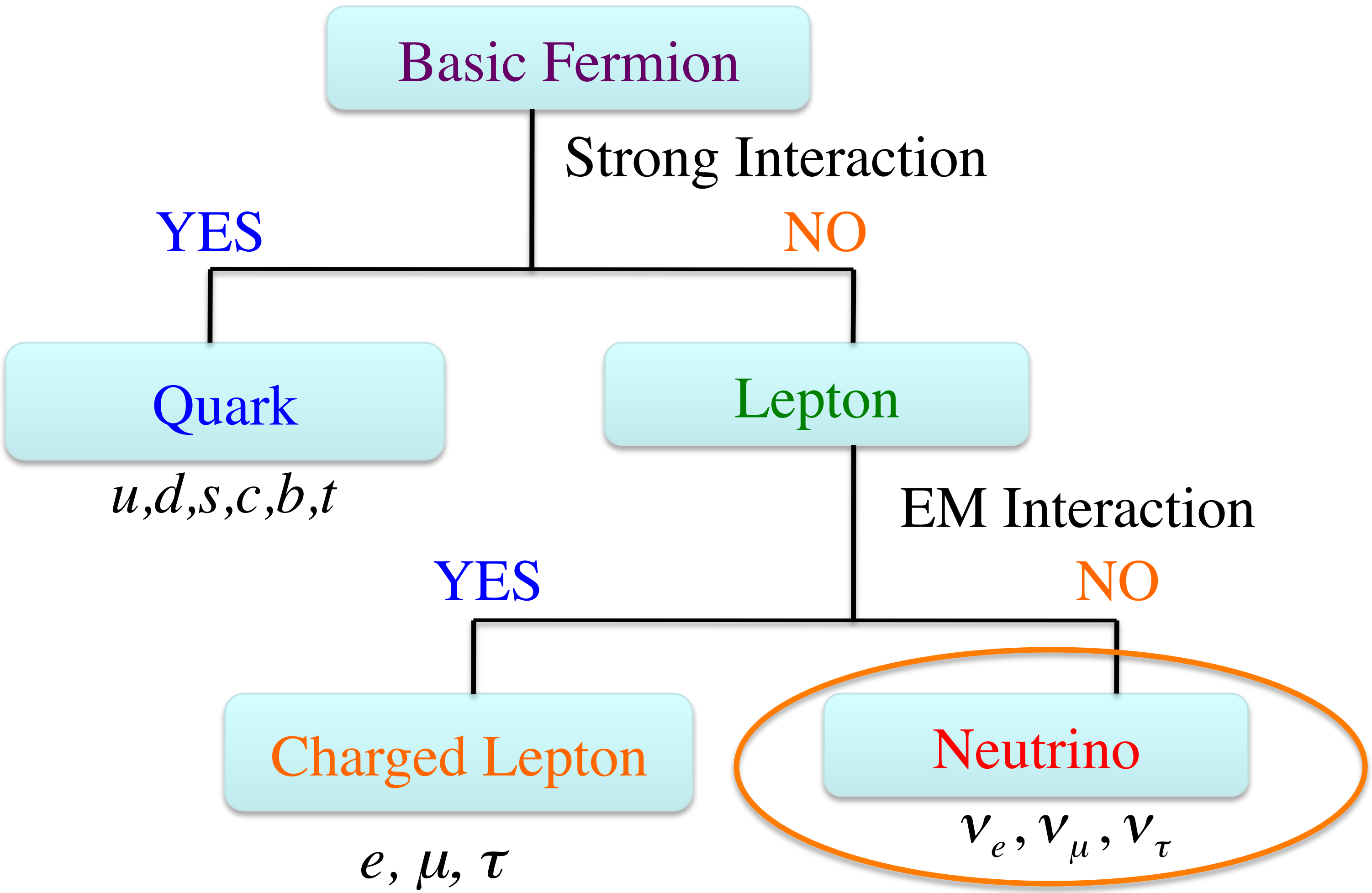
$\sin^2(2\theta_{12}) = 0.87 \pm 0.03$

$\Delta m^2_{21} = (7.59 \pm 0.20) \times 10^{-5}$ eV^2

$\sin^2(2\theta_{23}) > 0.92$ [i]

$\Delta m^2_{32} = (2.43 \pm 0.13) \times 10^{-3}$ eV^2 [j]

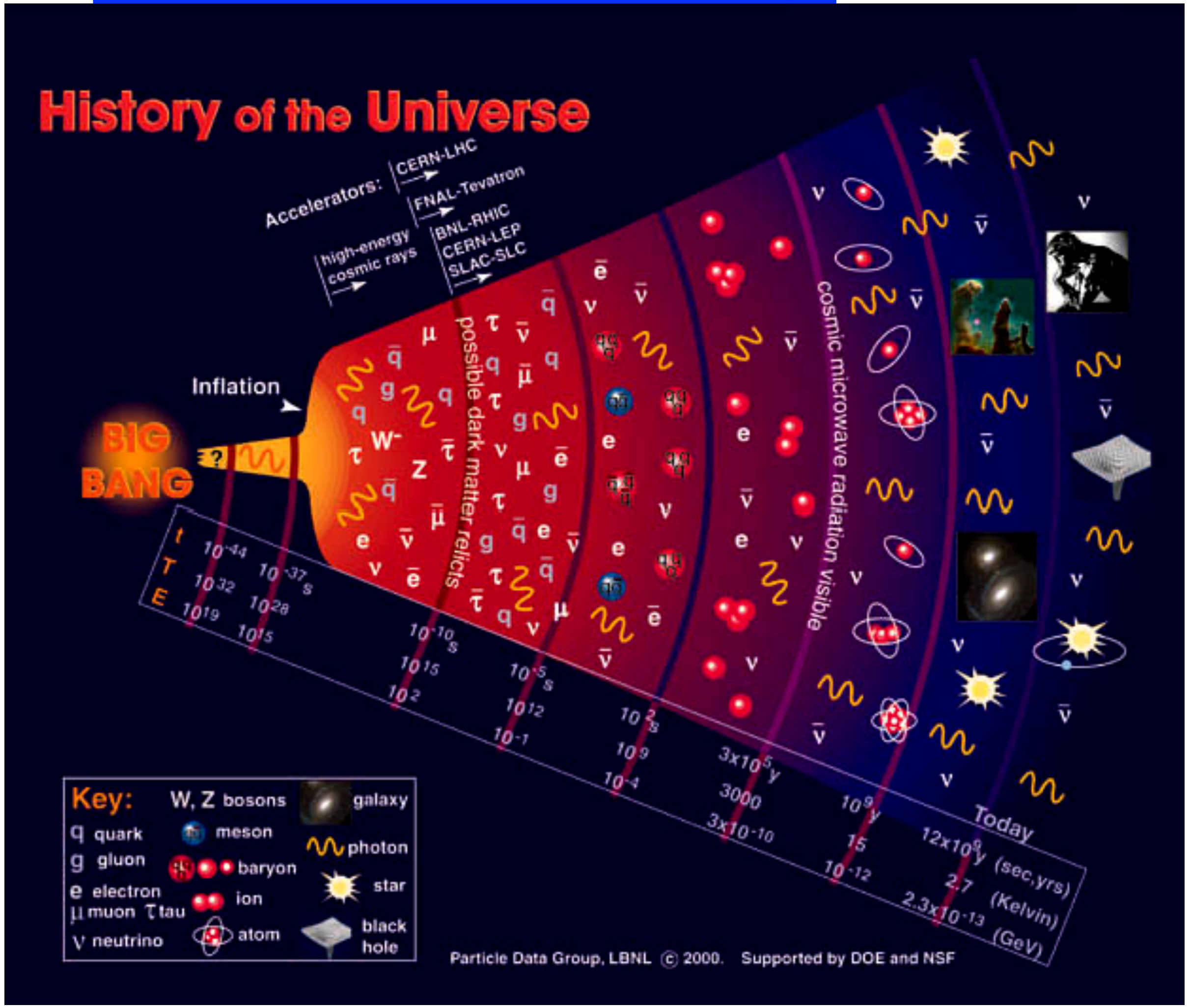
$\sin^2(2\theta_{13}) < 0.15$, CL = 90%

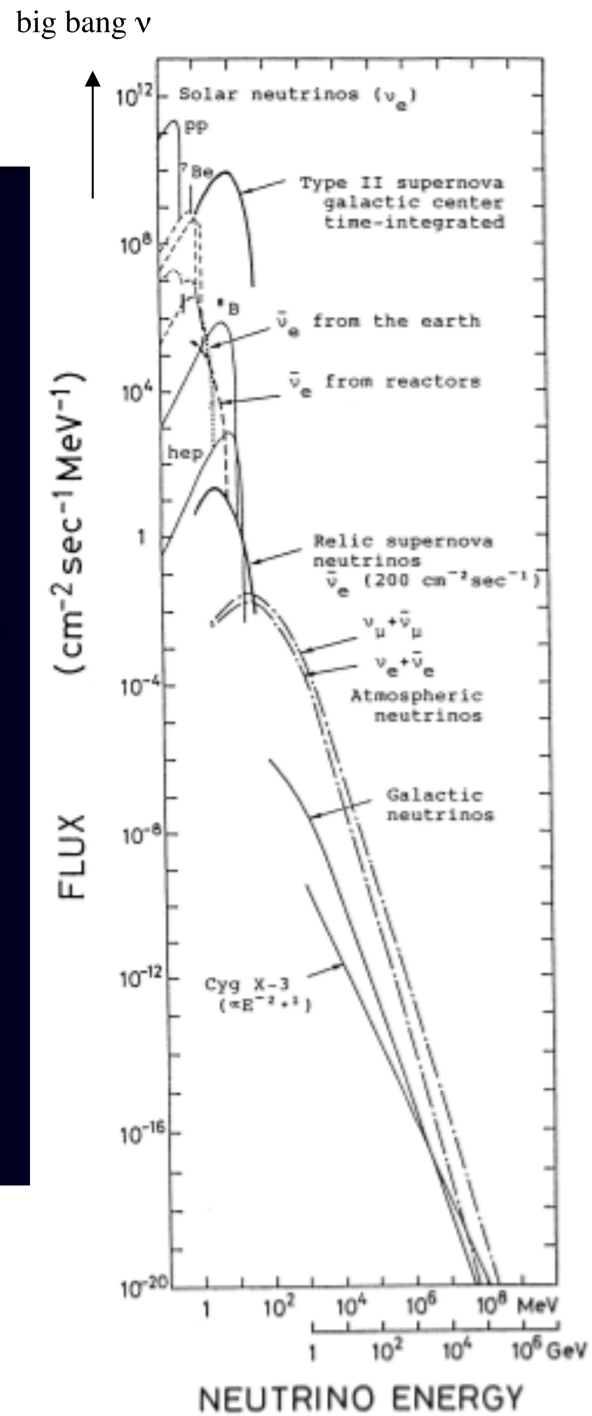


We call Fermions which do not perform strong nor EM interaction,

Neutrinos

Neutrinos in Nature





$e^+e^- \rightarrow \nu\bar{\nu}$ in early universe.

Most abundant next to photons (~100ν/cc)

ν Timeline

(years are approximate)

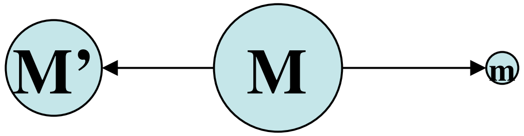
1899 Discovery of β-decay	[Rutherford]
1914 β-ray has continuous energy spectrum	[Chadwick]
1930 Neutrino hypothesis	[Pauli]
1956 1st Evidence of neutrino @ reactor	[Reines & Cowan]
1961 Discovery of ν_μ	[Shwartz, Ledermann, Steinberger]
1969~ Deficit of solar neutrino	[Davis]
1977 Discovery of τ lepton (indirect evidence of ν_τ)	[Perl]
1985 Proposal of MSW effect	[Mikheyev, Smirnov, Wolfenstein]
1987 Detection of neutrinos from SN1987A	[Koshiba]
1989 N_ν=3 by Z^0 shape	[LEP]
1995 Nobel prize to Reines	
(1996, 1997Claim of ν_μ->ν_e oscillation	[LSND])
1998 1st evidence of neutrino oscillation by atmospheric ν	[SuperKamiokande]
2000 Direct evidence of ν_τ	[DONUT]
(2001 Claim of neutrinoless $\beta\beta$ decay	[Klapdor])
2002 Nobel prize to Davis & Koshiba	
2002 Flavor transition	[SNO]
Reactor Neutrino Deficit	[KamLAND]
2004 ν_μ disappearance @ Accelerator	[K2K]
2010 $\nu_\mu \rightarrow \nu_\tau$	[OPERA]
2011 Indication of ν_e appearance @ Accelerator	[T2K]

1^{st} Indication of Neutrino

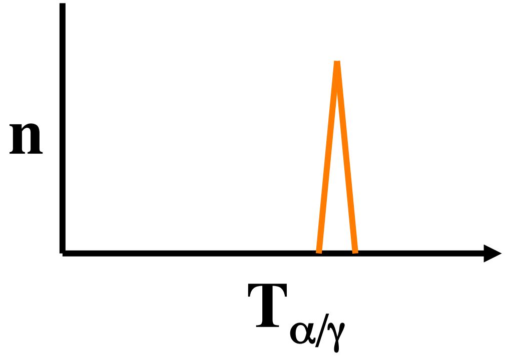
~1914, an anomaly found

(The 1st anomaly in neutrino which lead great discovery.)

γ & α decays ➔ The energy of the decay particle is unique

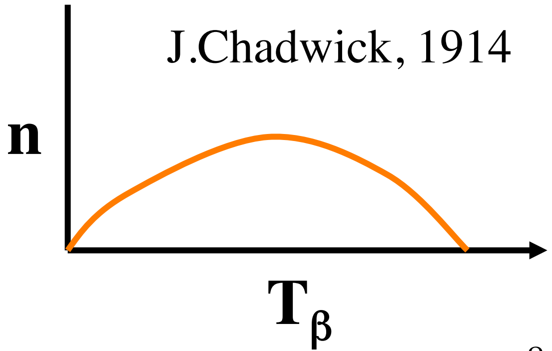


$$T_\alpha = \frac{(M-m)^2 - M'^2}{2M}$$



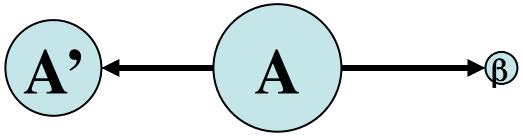
However, for β-decays,
it is continuous.

Why??



Neutrino Hypothesis

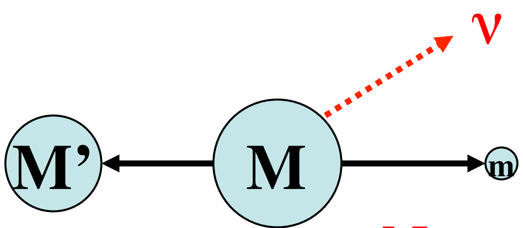
* Energy conservation low is broken　(N.Bohr, 1932)



$$E_\beta \neq E_A - E_{A'}$$

* β-decay is a 3 body reaction (W.Pauli, 1930)

$$A \rightarrow B + \beta^- + \nu$$



$$E_\beta = E_A - E_{A'} - E_\nu < E_A - E_{A'}$$

Neutrino Hypothesis

4/Dec./1930
Letter from Pauli
to participants of
a conference.

Dear Radioactive Ladies and Gentlemen,

As the bearer of these lines, to whom I graciously ask you to listen, will explain to you in more detail, how because of the "wrong" statistics of the N and Li^6 nuclei and the continuous beta spectrum, I have hit upon a deseperate remedy to save the "exchange theorem" of statistics and the law of conservation of energy. Namely, the possibility that there could exist in the nuclei electrically neutral particles, that I wish to call neutrons, which have spin 1/2 and obey the exclusion principle and which further differ from light quanta in that they do not travel with the velocity of light. The mass of the neutrons should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 proton masses> The continuous beta spectrum would then become understandable by the assumption that in beta decay a neutron is emitted in addition to the electron such that the sum of the energies of the neutron and the electron is constant...

I agree that my remedy could seem incredible because one should have seen those neutrons very earlier if they really exist. But only the one who dare can win and the difficult situation, due to the continuous structure of the beta spectrum, is lighted by a remark of my honoured predecessor, Mr Debye, who told me recently in Bruxelles: "Oh, It's well better not to think to this at all, like new taxes". From now on, every solution to the issue must be discussed. Thus, dear radioactive people, look and judge. Unfortunately, I cannot appear in Tubingen personally since I am indispensable here in Zurich because of a ball on the night of 6/7 December. With my best regards to you, and also to Mr Back.

Your humble servant
. W. Pauli

Expected properties of ν from β-decays

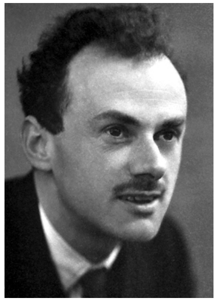
(1) Q=0 ← charge conservation

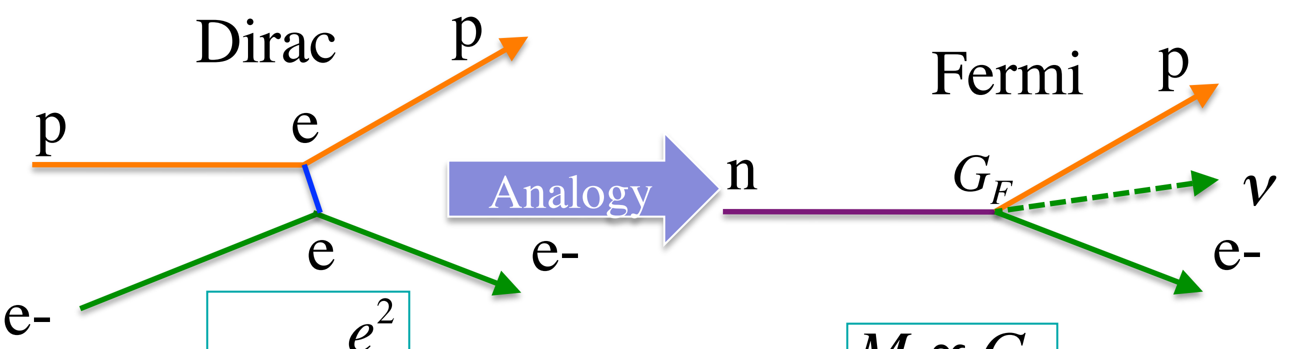
(2) s=1/2 ← spin conservation

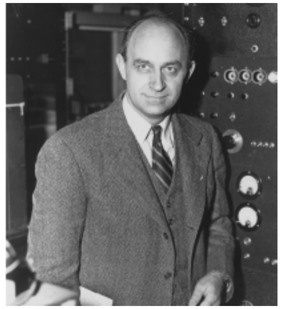
(3) mass is small if exists ← maximum energy of β-rays.

(4) Interact very weakly ← lifetime of β-decays.

How ν-N cross section was estimated in early days. Fermi's model







$M \propto \frac{e^2}{q^2}$ $\qquad$ $M \propto G_F$

Various $\beta^{\pm}$-decays & Electron caputure ➔ $G_F \sim 10^{-11}/\mathrm{MeV}^2$

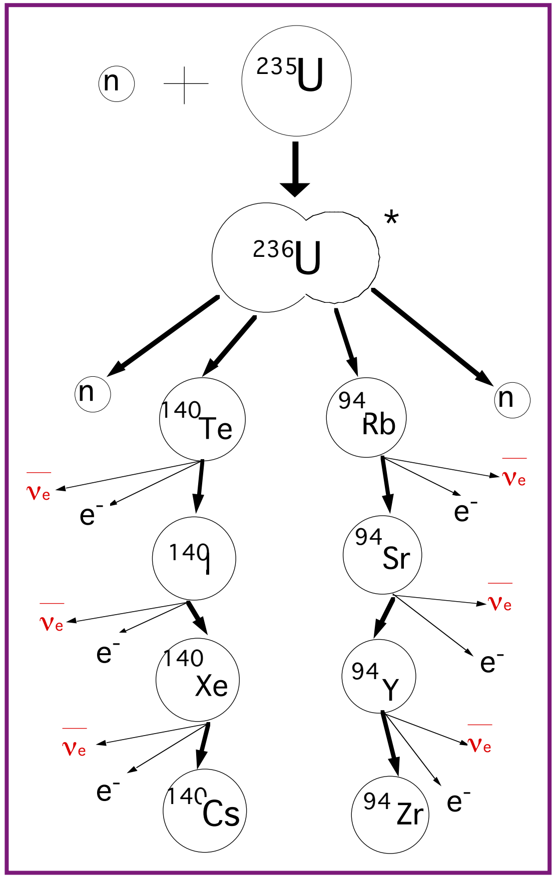
Then, $\sigma(\bar{\nu} + p \to e^+ + n) = \frac{G_F^2}{\pi} p_{CM}^2 \sim \boxed{10^{-20}[b]!!}$

"I did something a physicist should never do. I predicted something which will never be observed experimentally..". (W.Pauli)

"There is no practically possible way of observing the neutrino" (Bethe & Peierls, 1934)

Then 30 years had passed ....

Discovery of ν



$E_\nu \sim$ MeV

Very strong ν sources are necessary,

← Chain reactions of nuclear fissions.

Energy release: 200MeV/fission

β-decays: ~ 6 ν/fission

$1.9\text{x}10^{11}\nu$/J

Reactor or Nuclear Explosion

An early idea to detect ν (not realized)

Reines & Cowan

Nuclear Explosion

Vacuum shaft

Neutrino Detector

Free fall to prevent the shock wave.

by Reines & Cowan

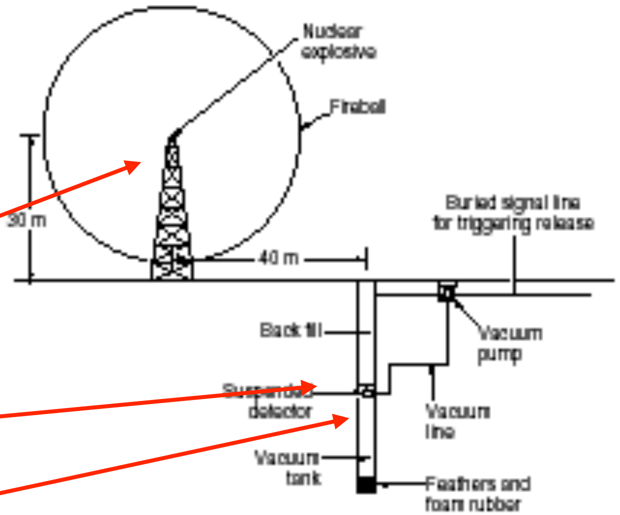


Figure 1. Detecting Neutrinos from a Nuclear Explosion
Antineutrinos from the fireball of a nuclear device would impinge on a liquid scintillation detector suspended in the hole dug below ground at a distance of about 40 meters from the 30-meter-high tower. In the original scheme of Reines and Cowan, the antineutrinos would induce inverse beta decay, and the detector would record the positrons produced in that process.

=> Physicists make use of everything available

While preparing the experiment, they realized nuclear reactor is more relevant to perform experiment.

Then they moved to a Savannah River Reactor

200L Cd loaded water tanks

1400L liquid scintillator tanks

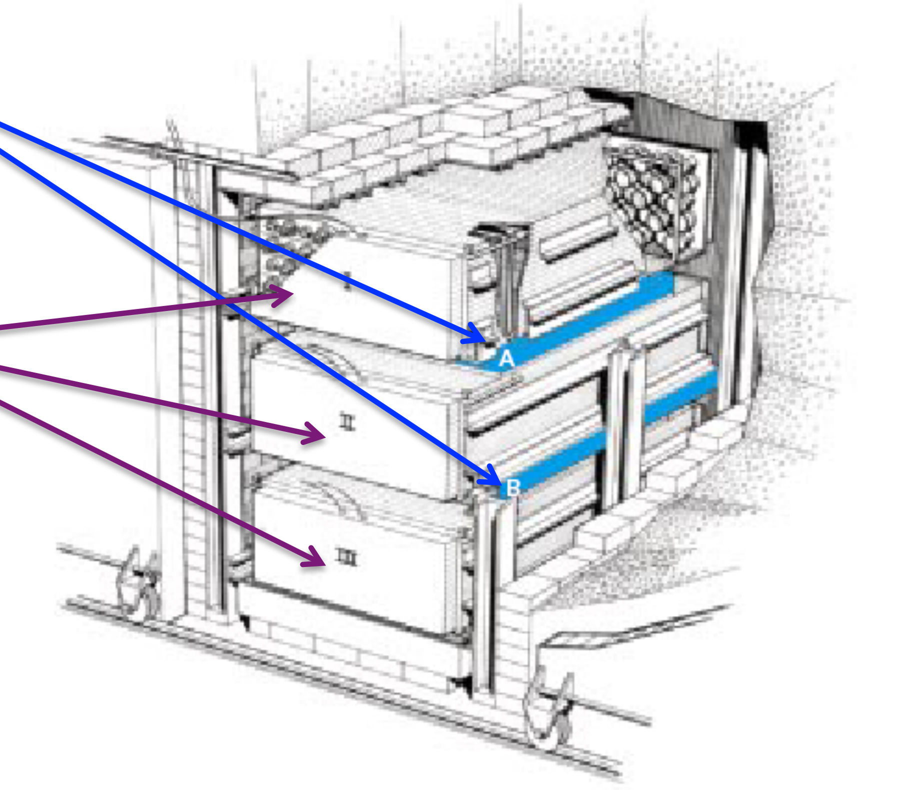
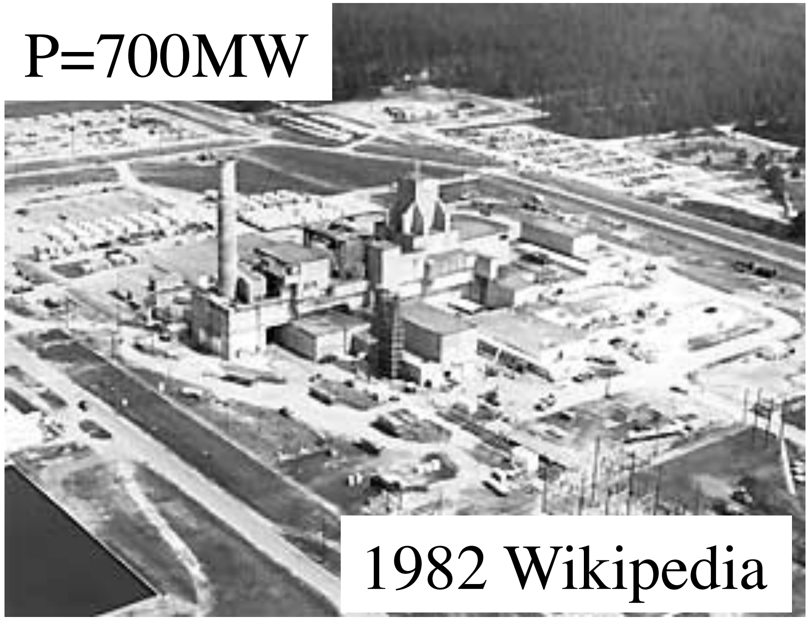


Figure 4. The Savannah River Neutrino Detector—A New Design
The neutrino detector is illustrated here inside its lead shield. Each of two large, flat plastic tanks (pictured in light blue and labeled A and B) was filled with 200 liters of water. The protons in the water provided the target for inverse beta decay; cadmium chloride dissolved in the water provided the cadmium nuclei that would capture the neutrons. The target tanks were sandwiched between three scintillation detectors (I, II, and III). Each detector contained 1,400 liters of liquid scintillator that was viewed by 110 photomultiplier tubes. Without its shield, the assembled detector weighed about 10 tons.

P=700MW



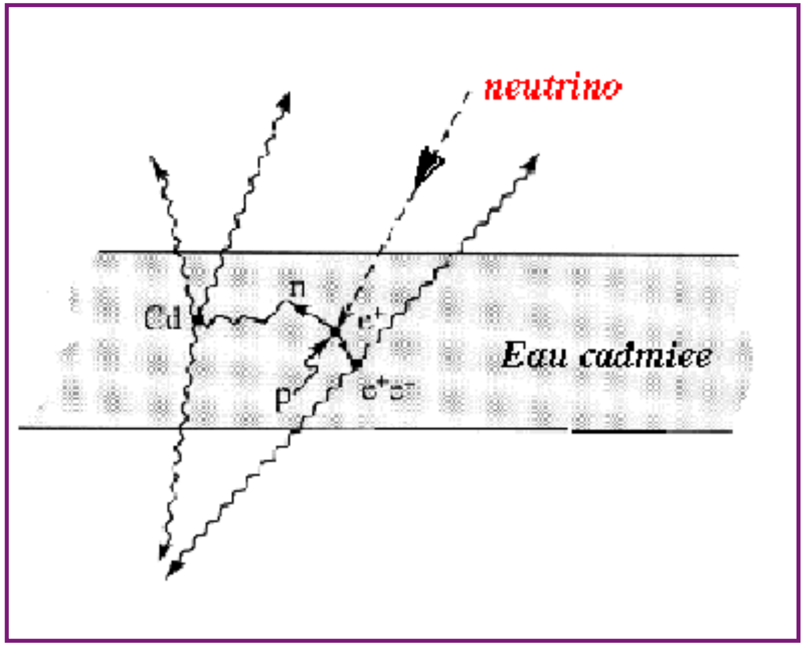
1982 Wikipedia

Principle of ν detection

* ν flux:

@15m from Savannah Liver P reactor core. (P=700MW)

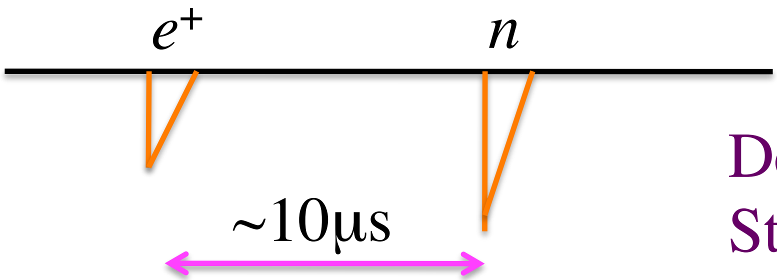
flux~$5\times10^{12}\nu/cm^2/s$



* Detection Principle:

$$\bar{\nu}_e + p \rightarrow e^+ + n \quad \Rightarrow$$

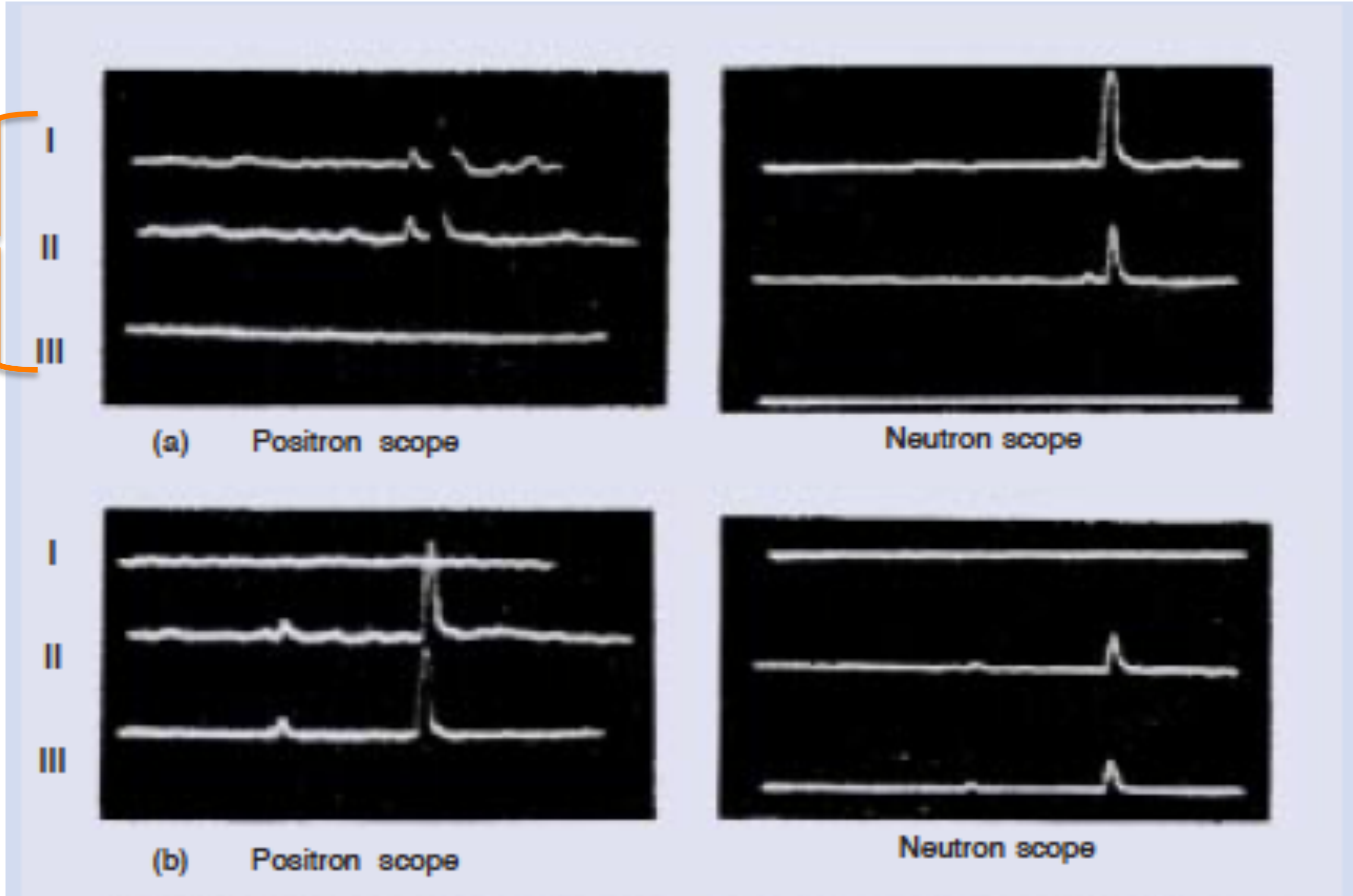
$$\begin{cases} e^+ + e^- \rightarrow 2\gamma(0.5MeV) \\ n + Cd \rightarrow Cd^* \rightarrow Cd + n\gamma(9MeV) \end{cases}$$



Delayed Coincidence Technique
Still used in modern experiments

2 examples of delayed coincidence

LS tank ID



http://library.lanl.gov/cgi-bin/getfile?00326606.pdf#search='delayed%20coincidence%20cadmium%20neutrino'

An episode

At the same time of Reines& Cowan, R.Davis and L.Alvarez performed neutrino experiment at a Savannah river reactor, too. Their detection principle was,

$$\nu + Cl \rightarrow e^- + Ar$$

However, they failed to detect positive result.
But this actually means the reactor neutrino (anti neutrino) dose not cause the reaction

$$\bar{\nu} + Cl \nrightarrow e^- + Ar$$

and <u>neutrino and anti-neutrino are different particles (concept of that time)</u>
Later on, Davis also won Novel prize by detecting solar neutrinos with the same technique.

Lessons : Negative result can be an important signature.
: Hanging on is important for success.

Discovery of μ-neutrino

Lederman, Schwarts, Steinberger

* Neutrino Source:
@ Brookhaven AGS

$$p(15GeV) + Be \to \pi + X$$

$$\pi \to \mu + \nu$$

π decays with 21m decay space.

$$\frac{\Gamma(\pi \to e + \nu)}{\Gamma(\pi \to \mu + \nu)} \approx 10^{-4}$$

99.99% of neutrinos are associated with muon production

Detection of neutrino

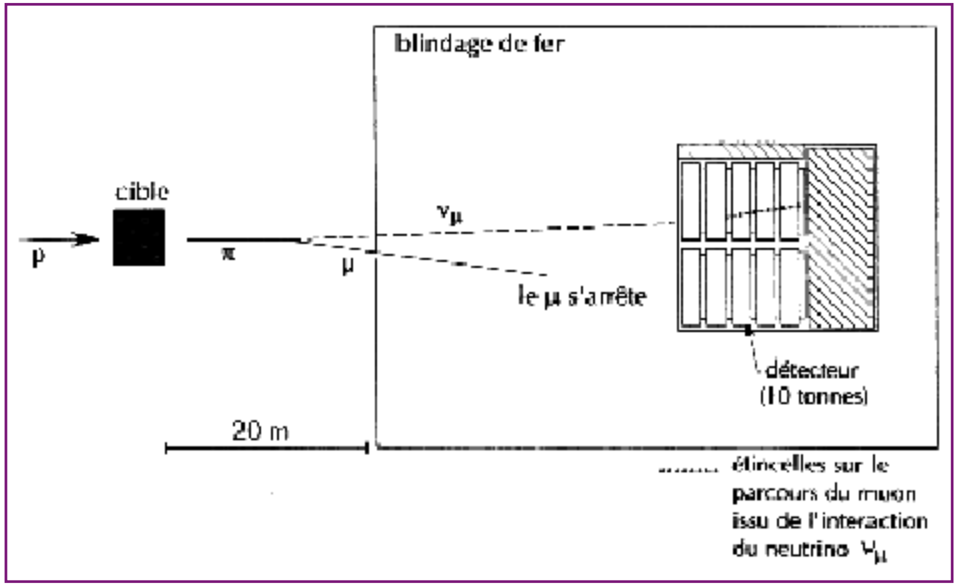
* Target: 90 x 2.5cmt Al slab

Looked for $\nu + Al \rightarrow \begin{cases} \mu + X \\ e + X \end{cases}$

μ signal => a single track
e signal => EM shower

They observed
34 single track μ events
22 μ+X
6 backgrounds (not like e)

spark chambers



➔ The neutrinos from β-decay and π decay are different particle

τ neutrino

(2000 DONUT group)

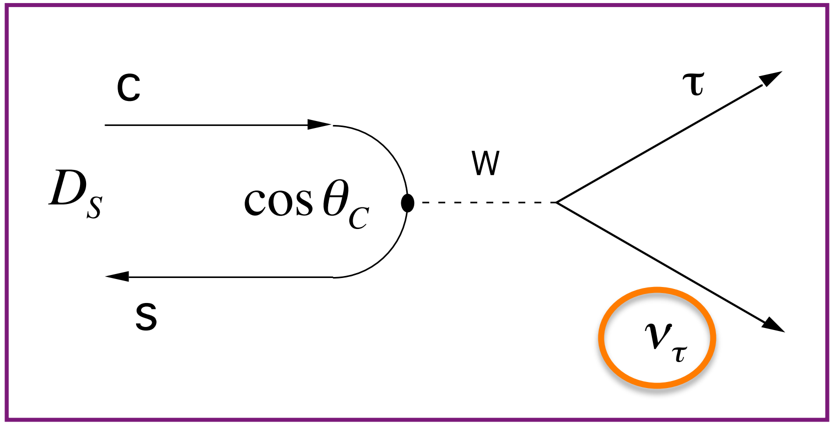
* Production of τ neutrino

FNAL TEVATRON

$$p(E=800GeV)+W \rightarrow D_S^{\pm}(m=1.97GeV)+X$$

Then

$$D_S^{\pm} \rightarrow \tau^{\pm}(m=1.78GeV)+\nu_\tau \quad (Br \sim 4\%)$$



$$\begin{cases} m_{D_s} > m_\tau \\ \text{Cabibbo favor} \end{cases}$$

$$E_\nu \sim 70GeV, \quad \frac{\phi_{\nu_\tau}}{\phi_{\nu_{e,\mu}}} \sim 0.05$$

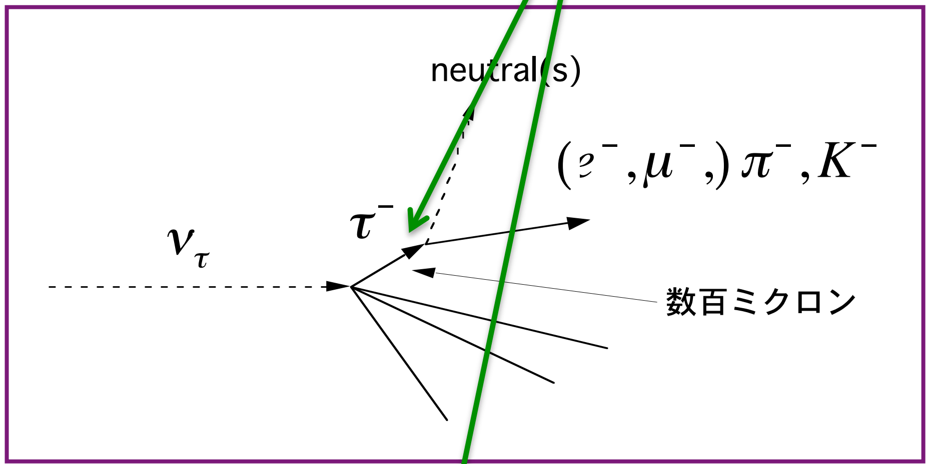
Detection principle

$$\nu_\tau + A \rightarrow \tau + X$$

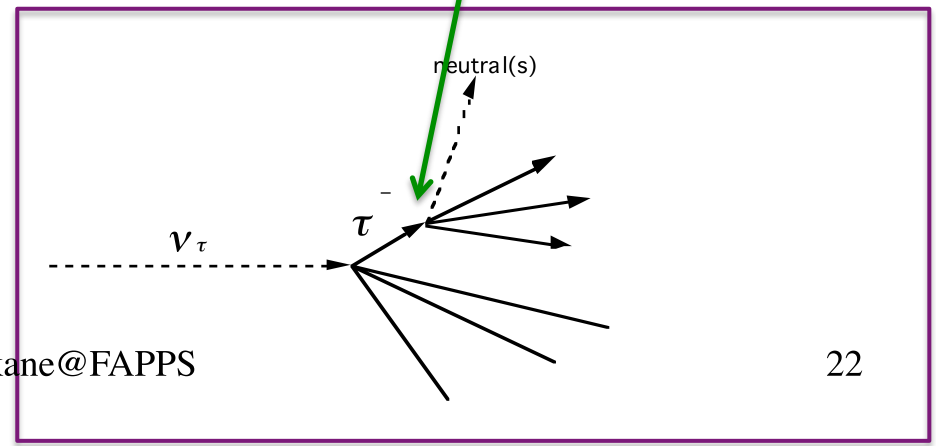
τ decays after ct~87um,

Look for the "kink"

85% for 1 prong mode



15% for 3 prongs mode.



the Detector

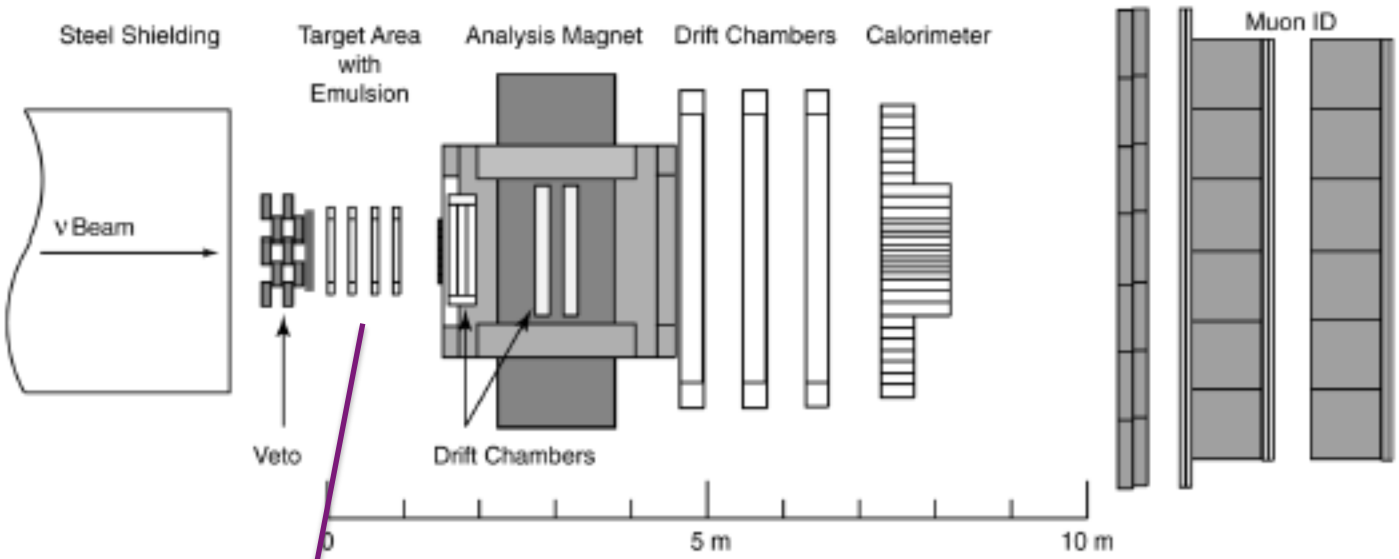


FIG. 4. Schematic plan view of the spectrometer. The neutrinos are incident from the left, emerging from the passive shield. The design is relatively compact to optimize identification of electrons and muons.

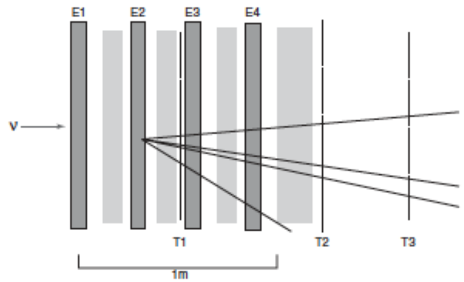
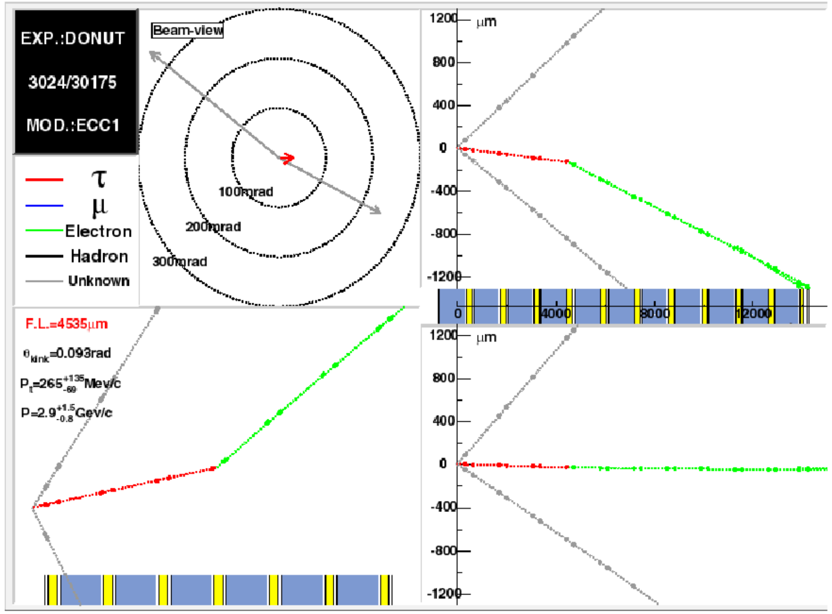


FIG. 3. Schematic plan view of the target region. The emulsion modules are indicated with E labels, the trigger hodoscopes with T labels. The lighter gray areas are occupied by scintillating fiber planes, 44 in total. The paths of charged particles in a typical interaction are superimposed.

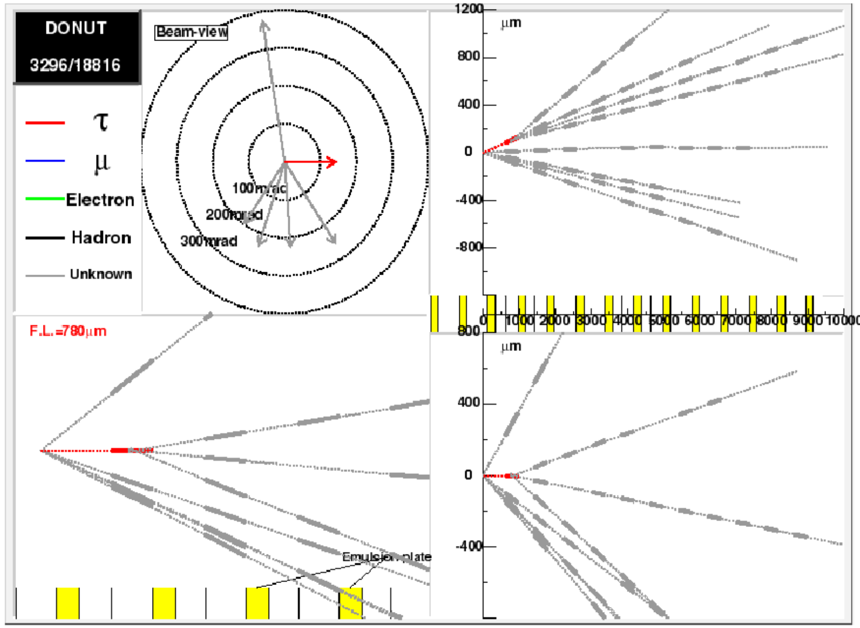
Nuclear Emulsion
(~thick camera film)

position resolution <1μm

Results (2000)



1prong



3 prongs

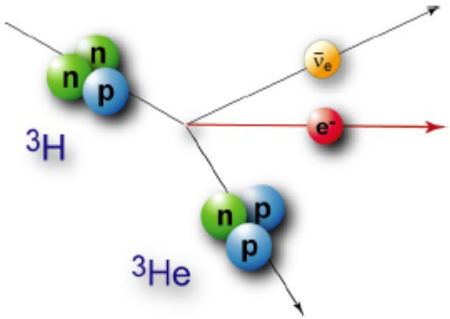
4 ± 0.44 ν_τ was observed in 1000 neutrino events. (9σ significance)

"We did R&D for τ-neutrino detection around 1980 but once gave up because it seemed too difficult to success". K.Niwa

Search for neutrino mass

β-decay: absolute ν-mass
model independent, kinematics
status: $m_\nu < 2.3$ eV
potential: $m_\nu \approx 200$ meV
e.g.: KATRIN, MARE-II

0νββ-decay: eff. Majorana mass
model-dependent (CP-phases)
status: $m_{\beta\beta} < 0.35$ eV (evidence?)
potential: $m_{\beta\beta} \approx 20\text{-}50$ meV
e.g.: GERDA, CUORE, EXO, SNO+, Majorana, Nemo 3, COBRA, KamLAND-Zen

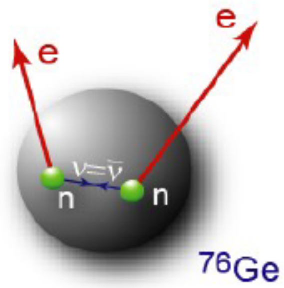


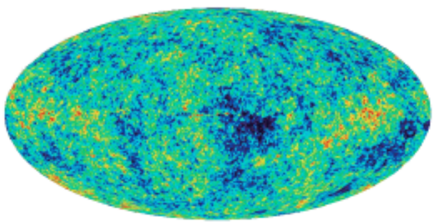
neutrino mass measurements

m_ν

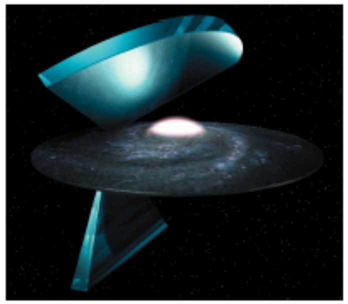
$m_{\beta\beta}$

Σm_i





cosmology: ν hot dark matter Ω_ν
model dependent, analysis of LSS data
status: $\Sigma m_\nu < 0.44$ eV (Hannestad et al., JCAP08(2010)001)
potential: $\Sigma m_\nu \approx 20\text{-}50$ meV
e.g.: WMAP, SDSS, LSST, Planck



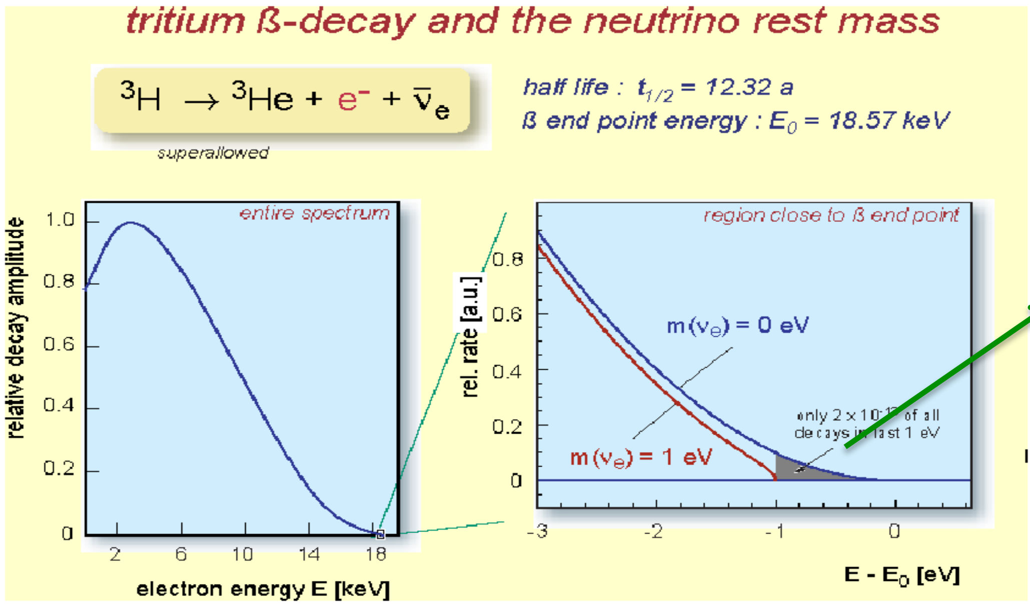
Direct neutrino mass detection

electron neutrino

Principle $A \rightarrow B + e^- + \overline{\nu}_e$

$$N(p_e)dp_e \propto p_e^2 (E_0 - E_e)^2 \sqrt{1 - \left(\frac{m_\nu}{E_0 - E_e}\right)^2} dp_e$$

$$E_e^{MAX} = E_0 \rightarrow E_0 - m_\nu$$



Distortion of energy spectrum at the end point

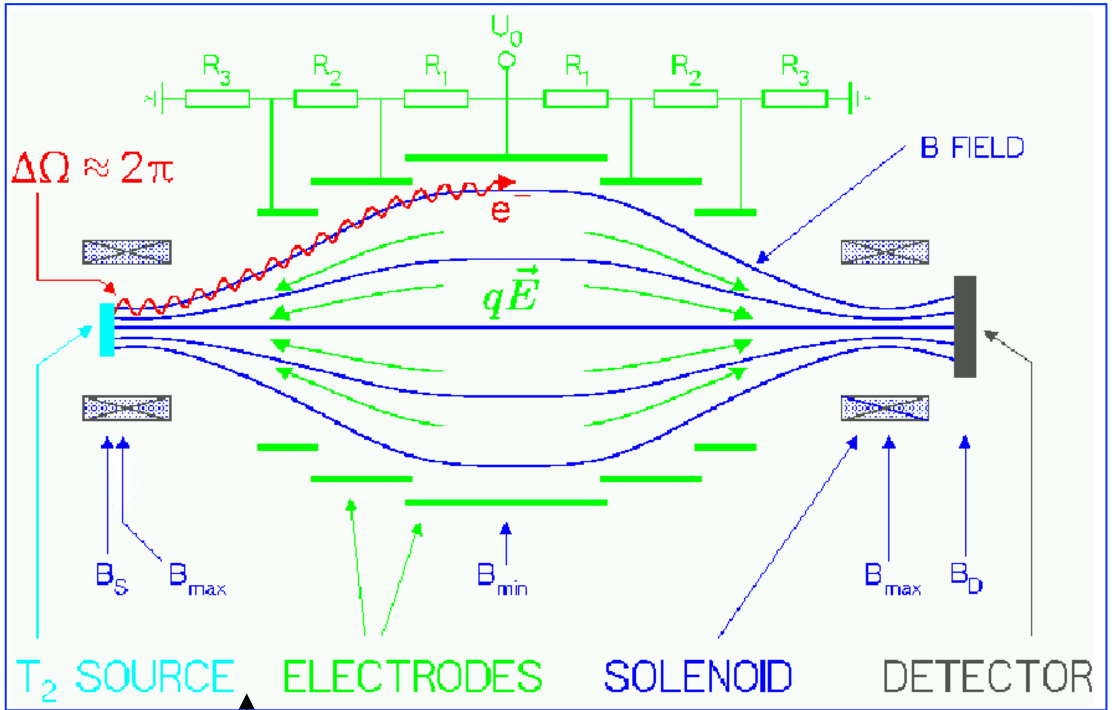
Source: Tritium

$$^{3}H \rightarrow {}^{3}He + e^{-} + \overline{\nu}_{e}$$

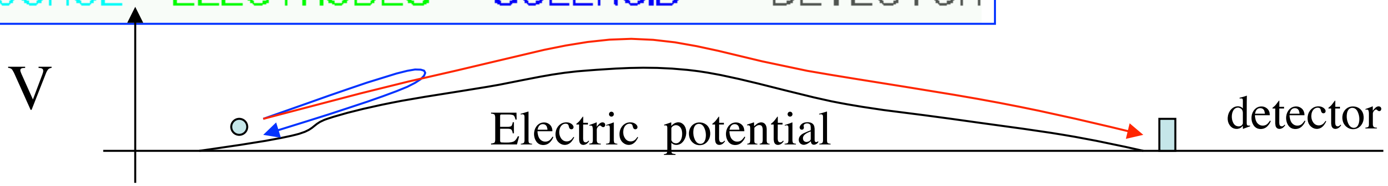
$$\left(Q = 18.6 KeV, \tau_{1/2} = 12.3y\right)$$

E_0=small ➔ good m_ν sensitivity
Lifetime ➔ reasonably short & long
Z = small ➔ small correction

} ideal isotope to seek for m_ν



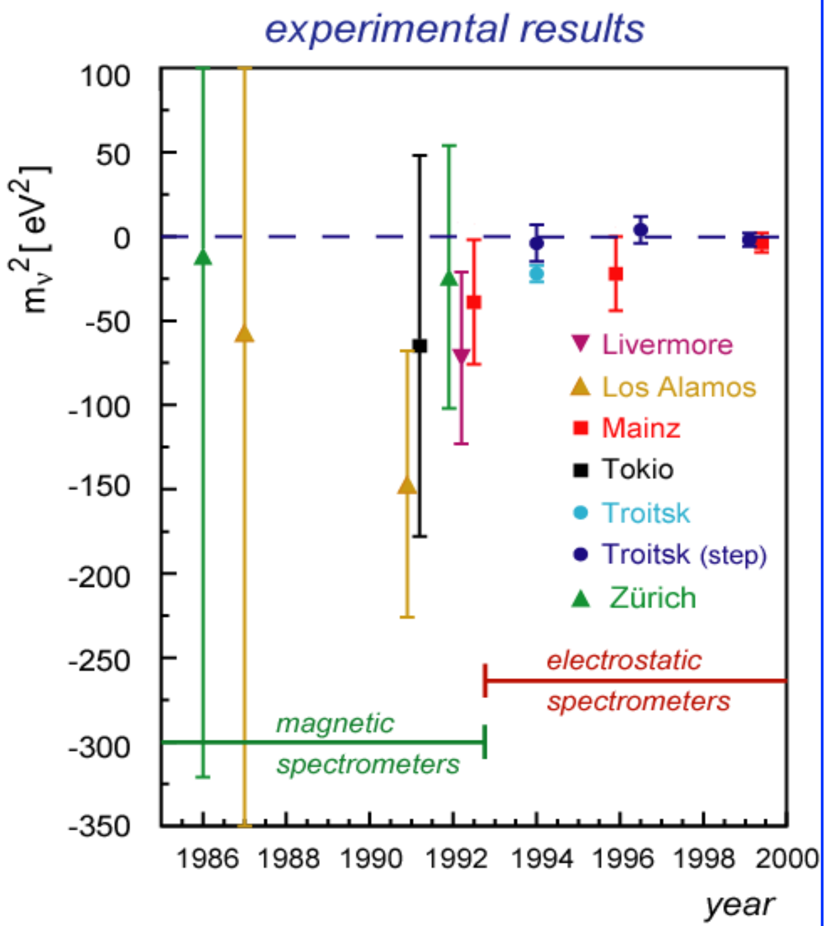
* Strong Source
* Large Acceptance
* Energy measurement
  by Electric potential



K. Eitel (Neutrino04)

$m_{\nu e}$ Results

	m_ν
ITEP *T_2 in complex molecule* *magn. spectrometer (Tret'yakov)*	17-40 eV
Los Alamos *gaseous T_2 - source* *magn. spectrometer (Tret'yakov)*	< 9.3 eV
Tokio *T - source* *magn. spectrometer (Tret'yakov)*	< 13.1 eV
Livermore *gaseous T_2 - source* *magn. spectrometer (Tret'yakov)*	< 7.0 eV
Zürich *T_2 - source impl. on carrier* *magn. spectrometer (Tret'yakov)*	< 11.7 eV
Troitsk (1994-today) *gaseous T_2 - source* *electrostat. spectrometer*	< 2.2 eV
Mainz (1994-today) *frozen T_2 - source* *electrostat. spectrometer*	< 2.2 eV



From current to future experiments

Mainz:

$m_\nu^2 = -1.2(-0.7) \pm 2.2 \pm 2.1\ eV^2$

$m_\nu < 2.2(2.3)$ eV (95%CL)

C. Weinheimer, Nucl. Phys. B (Proc. Suppl.) 118 (2003) 279
C. Kraus, EPS HEP2003 (neighbour excitations self-consistent)

Troitsk:

$m_\nu^2 = -2.3 \pm 2.5 \pm 2.0\ eV^2$

$m_\nu < 2.1$ eV (95%CL)

V. Lobashev, private communication
(allowing for a step function near endpoint)

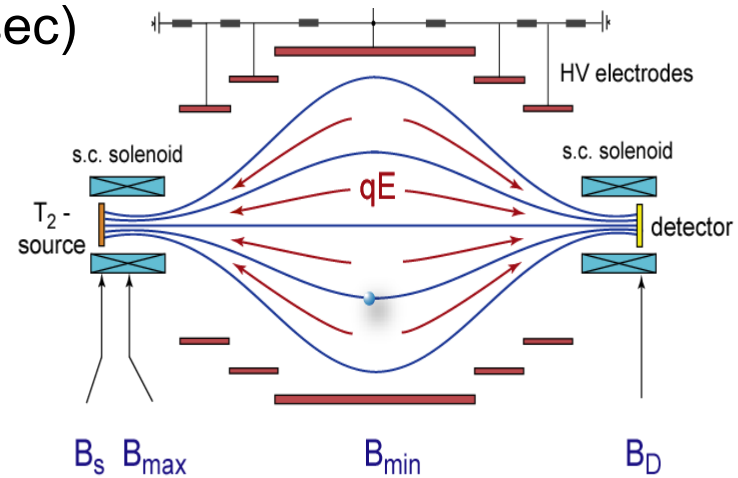
aim: improvement of m_ν by one order of magnitude (2eV → 0.2eV)
→ improvement of uncertainty on m_ν^2 by 100 ($4eV^2$ → $0.04eV^2$)

statistics:

- stronger Tritium source (>>10^{10} β´s/sec)
- longer measurement
  (~100 days → ~1000 days)

energy resolution:

- $\Delta E/E = B_{min}/B_{max}$
  → spectrometer with ΔE=1eV
  → Ø 10m UHV vessel

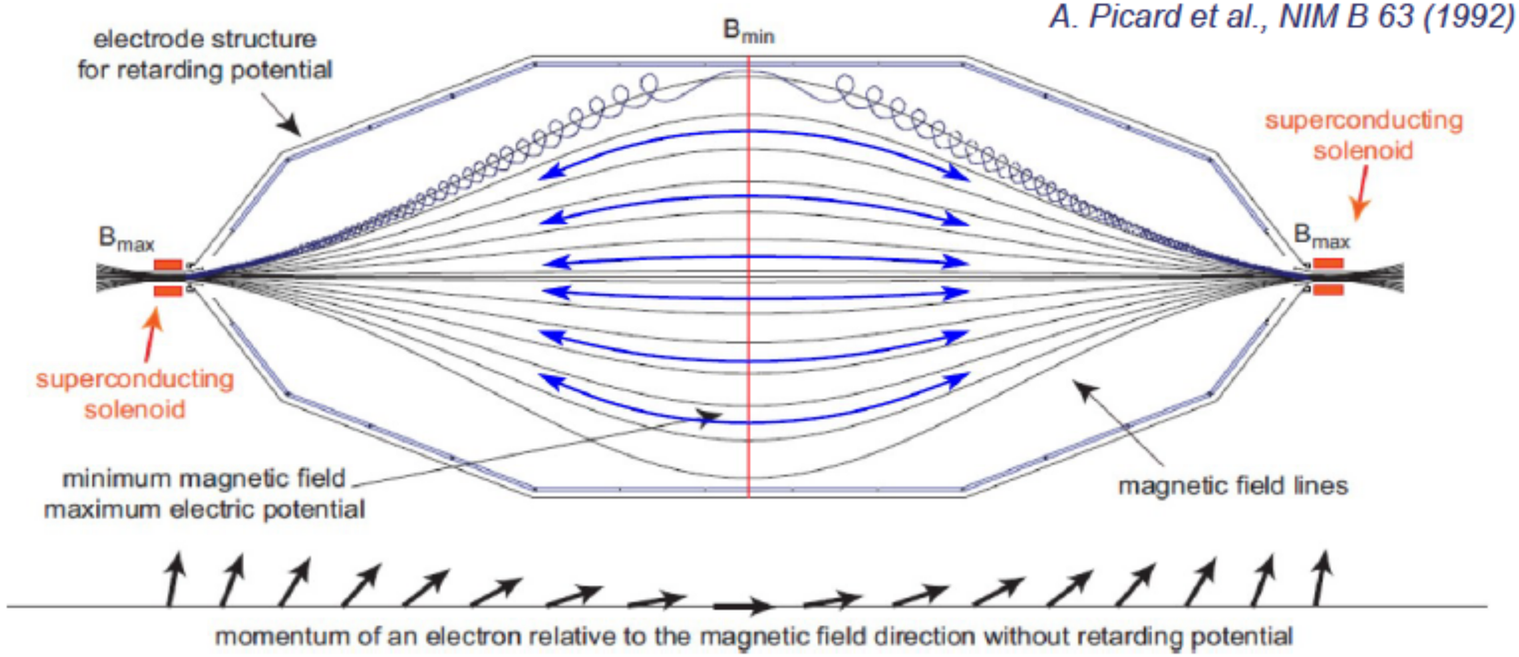


KATRIN: MAC-E filter concept

Westfälische Wilhelms-Universität Münster

Magnetic Adiabatic Collimation with Electrostatic Filter

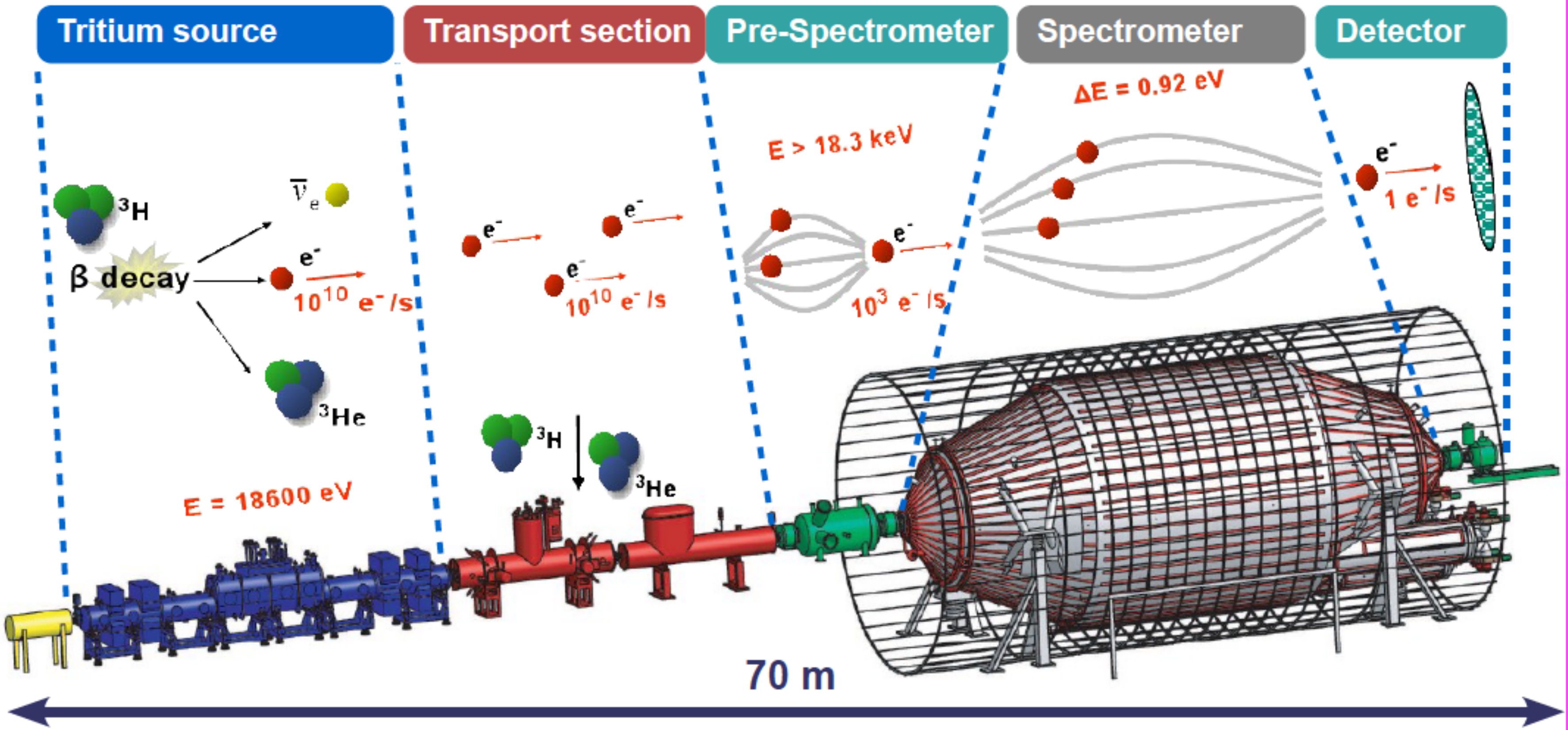
A. Picard et al., NIM B 63 (1992)



- adiabatic transport → $\mu = E_\perp / B = \text{const.}$
- B drops by $2 \cdot 10^4$ from solenoid to analyzing plane → $E_\perp \rightarrow E_{\parallel}$
- only electrons with $E_{\parallel} > eU_0$ can pass the retardation potential
- Energy resolution $\Delta E = E_{\perp,\text{max, start}} \cdot B_{min} / B_{max} < 1$ eV

Volker Hannen, TAUP 2011 conference, 7.9.2011

KATRIN experiment at KIT

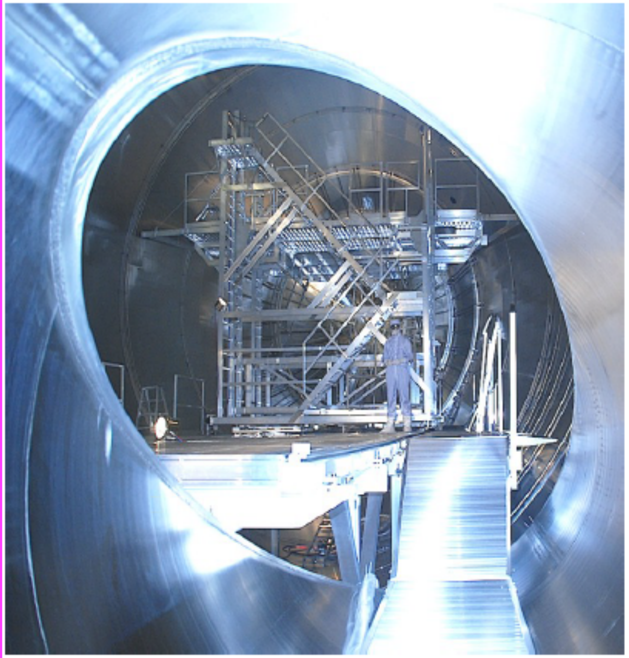


A famous picture

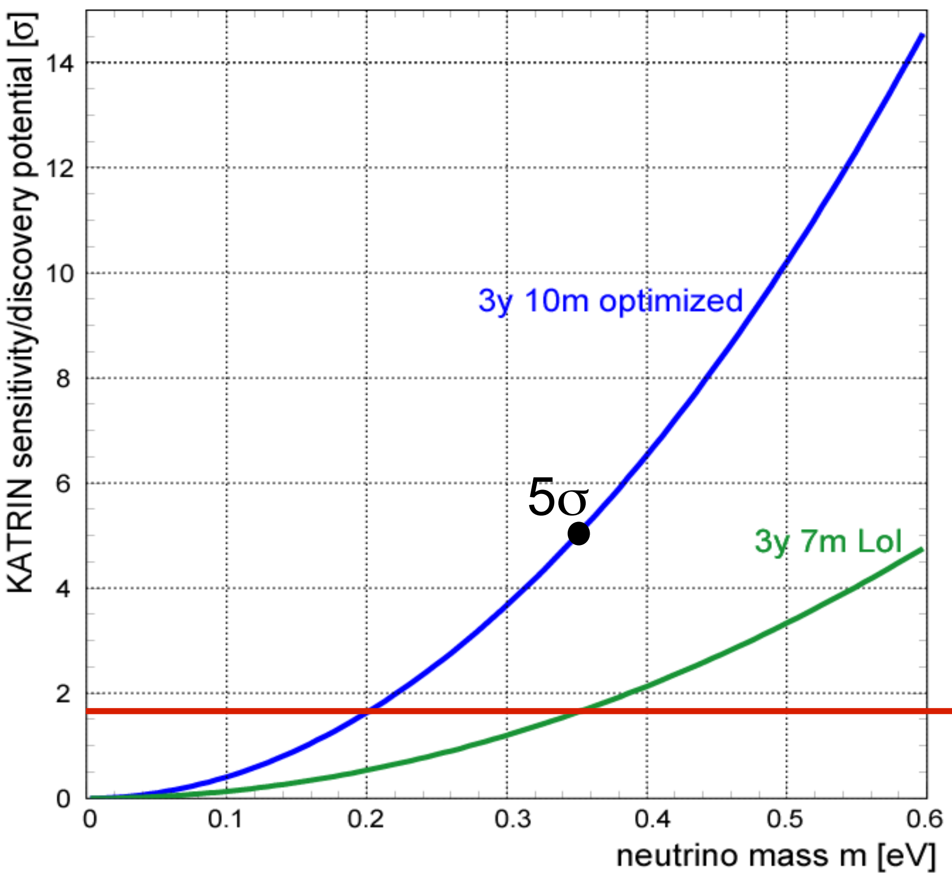
Some KATRIN highlights

Main spectr. inner wire electrode

- Purpose: - electrostatic shielding of background electrons
  - shaping of eletric fields
  - removal of trapped particles
- 224 of 248 double layer wire modules installed
- last 24 modules to be installed after preparation of the pump port region in fall 2011
- main spectrometer commissioning spring 2012



KATRIN sensitivity & discovery potential



expectation:

after 3 full beam years

$\sigma_{syst} \sim \sigma_{stat}$

$m_\nu = 0.35\text{eV } (5\sigma)$

$m_\nu = 0.3\text{eV } (3\sigma)$

discovery potential

$m_\nu < 0.2\text{eV } (90\%\text{CL})$

sensitivity

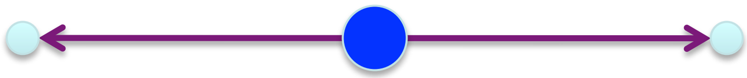
ν_μ mass limit

PHYSICAL REVIEW D VOLUME 53, NUMBER 11 1 JUNE 1996

Upper limit of the muon-neutrino mass and charged-pion mass from momentum analysis of a surface muon beam

K. Assamagan,[4,*] Ch. Brönnimann,[1,2] M. Daum,[1] H. Forrer,[1,3,†] R. Frosch,[1] P. Gheno,[1] R. Horisberger,[1] M. Janousch,[1,2,‡] P.-R. Kettle,[1] Th. Spirig,[1,2,§] and C. Wigger[1,2]

$$\pi(stop) \rightarrow \mu + \nu_\mu$$



$$m_\pi = \sqrt{m_\mu^2 + p_\mu^2} + \sqrt{m_\nu^2 + p_\mu^2} \rightarrow \boxed{m_\nu = 0.34\, m_\pi \sqrt{1 - \frac{p_\mu}{p_\mu^0}}}$$

A precise spectrometer

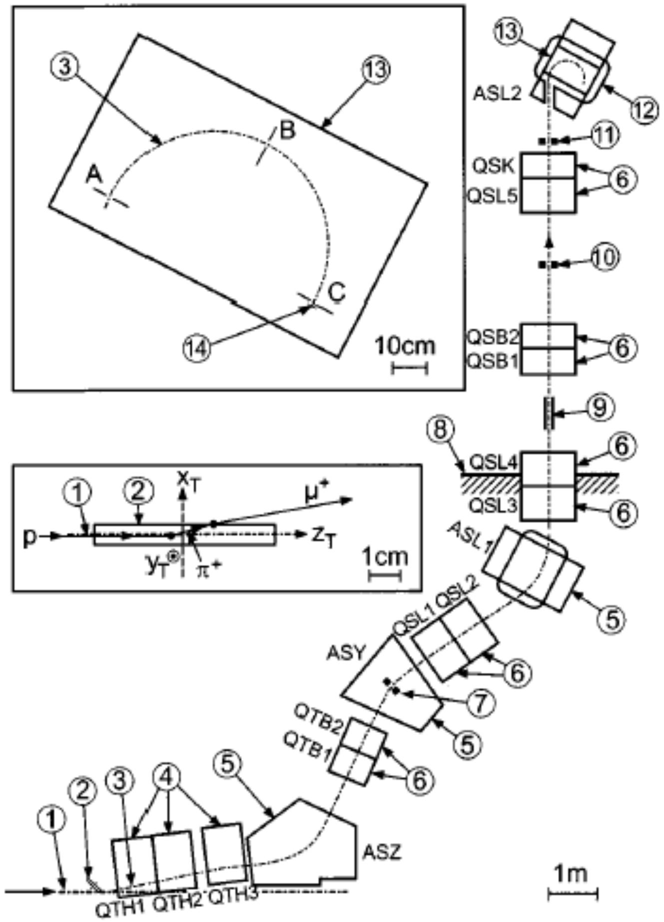


FIG. 1. Experimental setup. (1) Central trajectory of 590 MeV proton beam; (2) graphite target; (3) central trajectory of muon beam; (4) half-quadrupole magnets; (5) dipole magnets; (6) quadrupole magnets; (7) collimator defining the beam momentum acceptance; (8) concrete shielding of proton channel; (9) crossed-field particle separator; (10) lead collimator; (11) remotely movable collimator system (normally open); (12) magnetic spectrometer; (13) pole of spectrometer; (14) muon detectors (silicon microstrip and single surface-barrier detectors); A,B,C: copper collimators.

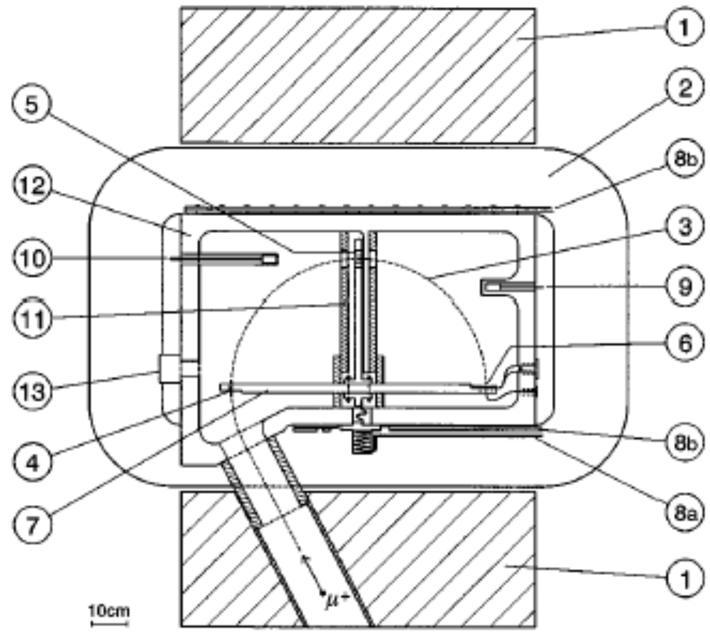


FIG. 3. The muon spectrometer: (1) magnet yoke; (2) magnet coils; (3) central muon trajectory; (4)–(6) copper collimators A,B,C; (7) titanium support; (8a) and (8b) cooling water pipes; (9) and (10) NMR probes; (11) lead shielding; (12) vacuum chamber; (13) port for vacuum pump.

@PSI

K.Assamagan et al. PRD53,6065(1996)

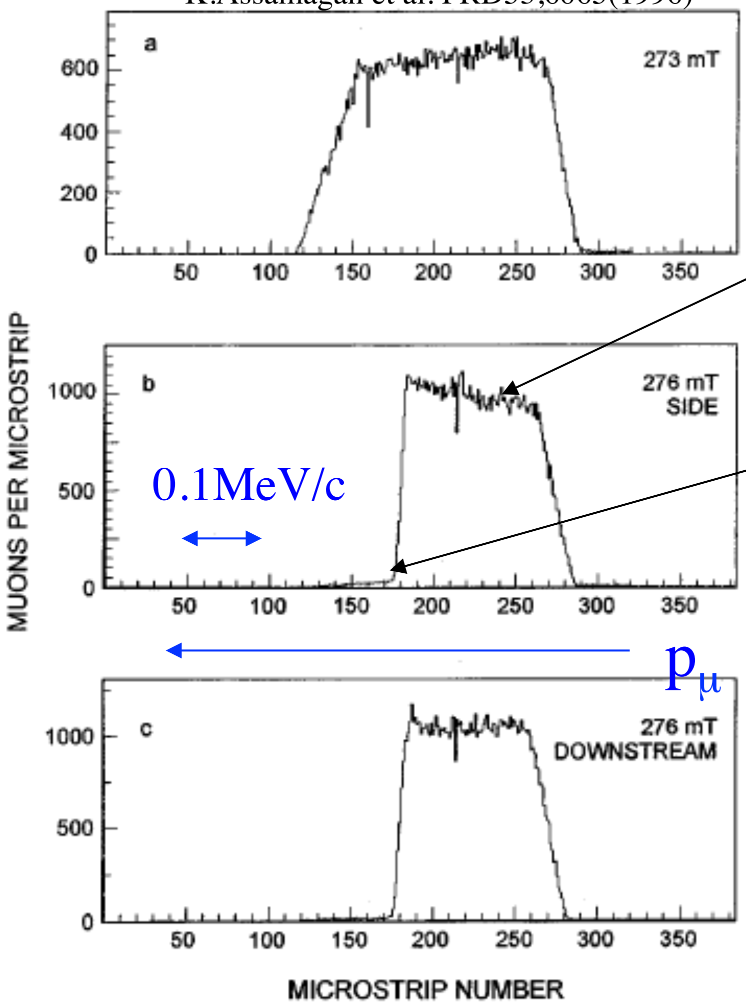


FIG. 6. Distribution of muons in the microstrip detector for three typical runs. (a) Central muon-beam momentum 29.45 MeV/c, spectrometer field 273.0 mT; (b) and (c) central muon-beam momentum 29.75 MeV/c, spectrometer field 276.0 mT. One microstrip width (0.05 mm) corresponds to $\Delta p_{\mu^+} \approx 0.0021 \mathrm{MeV}/c$. The muon momentum increases to the left. For details, see Sec. III B.

energy loss in the target

$$p_\mu^{\max} = (29.79200 \pm 0.00011) MeV/c$$

$$\boxed{m_{\nu_\mu} < 0.19\, MeV (90\% CL)}$$

(PDG average)

ν_μ mass limit

$$m_\nu = 0.34 m_\pi \sqrt{1 - \frac{p_\mu}{p_\mu^0}}$$

$$\delta m_{\nu_\mu} \sim \sqrt{m_\pi^2 - m_\mu^2}\left(\sqrt{\frac{\delta m_\pi}{m_\pi}} + \sqrt{\frac{1-(m_\mu/m_\pi)^2}{1+(m_\mu/m_\pi)^2}}\sqrt{\frac{\delta p_\mu}{p_\mu}}\right)$$

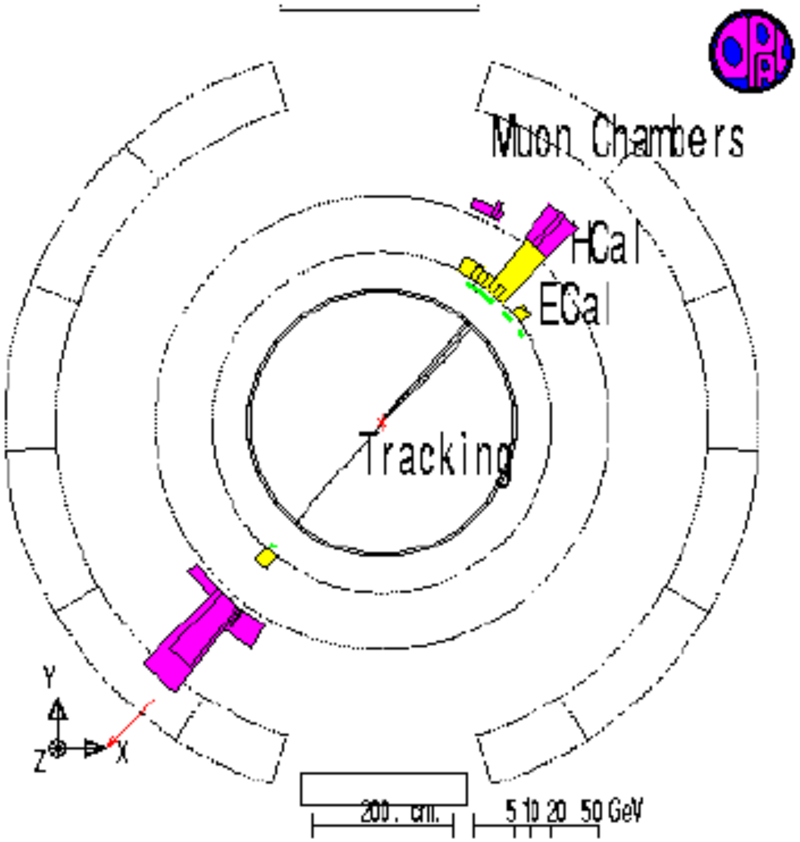
$$\underbrace{\sim 0.15 MeV}_{\boxed{\delta m_\pi}} \oplus \underbrace{49 MeV \sqrt{\frac{\delta p_\mu}{p_\mu}}}_{\boxed{\sim 0.1 MeV}}$$

δm_π limits the precision

τ neutrino mass limit

$$e^+e^- \to \tau^+\tau^-$$

$$\tau \to \nu_\tau + X \quad (LEP, CLEO)$$

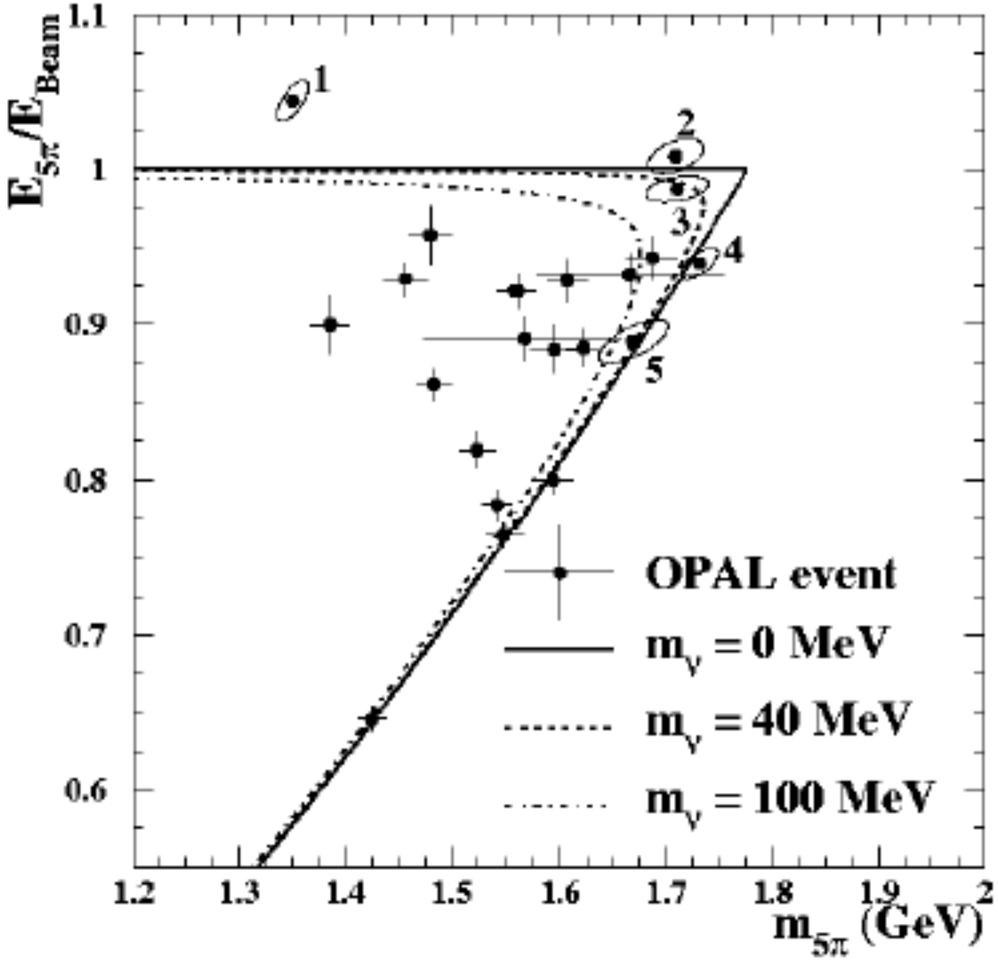


$$\tau \rightarrow \nu_\tau + X \quad (LEP, CLEO)$$

distribution of m_X & E_x,

$$\frac{d\Gamma}{dm_X dE_X} = f(m_X, E_X; m_\nu)$$

➔ obtain most likely m_ν



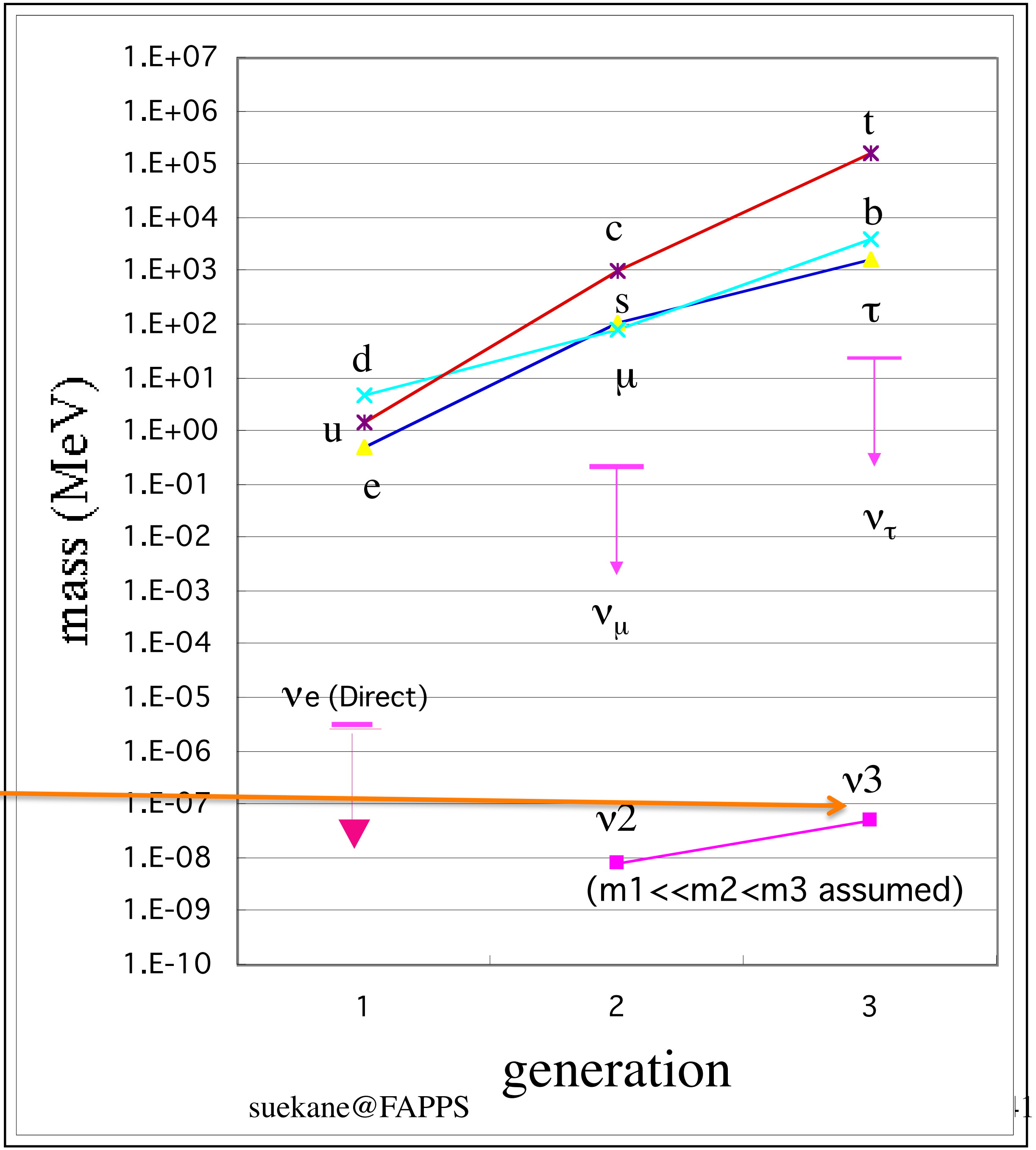
Better precision for smaller Q-value, but low statistics,

$\tau \rightarrow 5\pi + \nu_\tau$ are used.

PDG average ; $\boxed{m_{\nu_\tau} < 18.2 MeV (95\% CL)}$

Quark-Lepton masses

ν mass is very small



Lower limit of heaviest neutrino mass is ~50meV

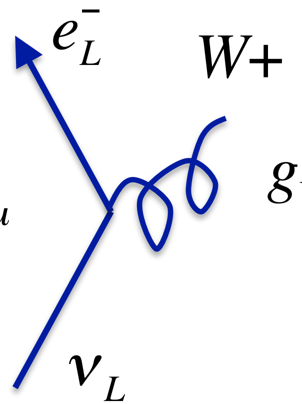
← ν oscillation

Neutrinos in the Standard Model

* Q=0,
* No color
* m=0,
* s=1/2
* $\overline{\nu} \neq \nu$
* only ν_L exists

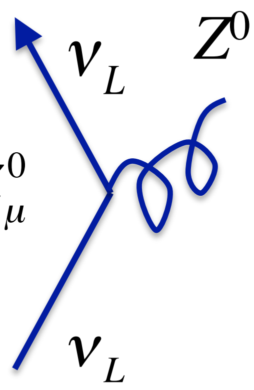
(or ν_L may exist but it does not interact at all)

$$-ig_W\left[\overline{e}_R\gamma^\mu\nu_L\right]W_\mu$$



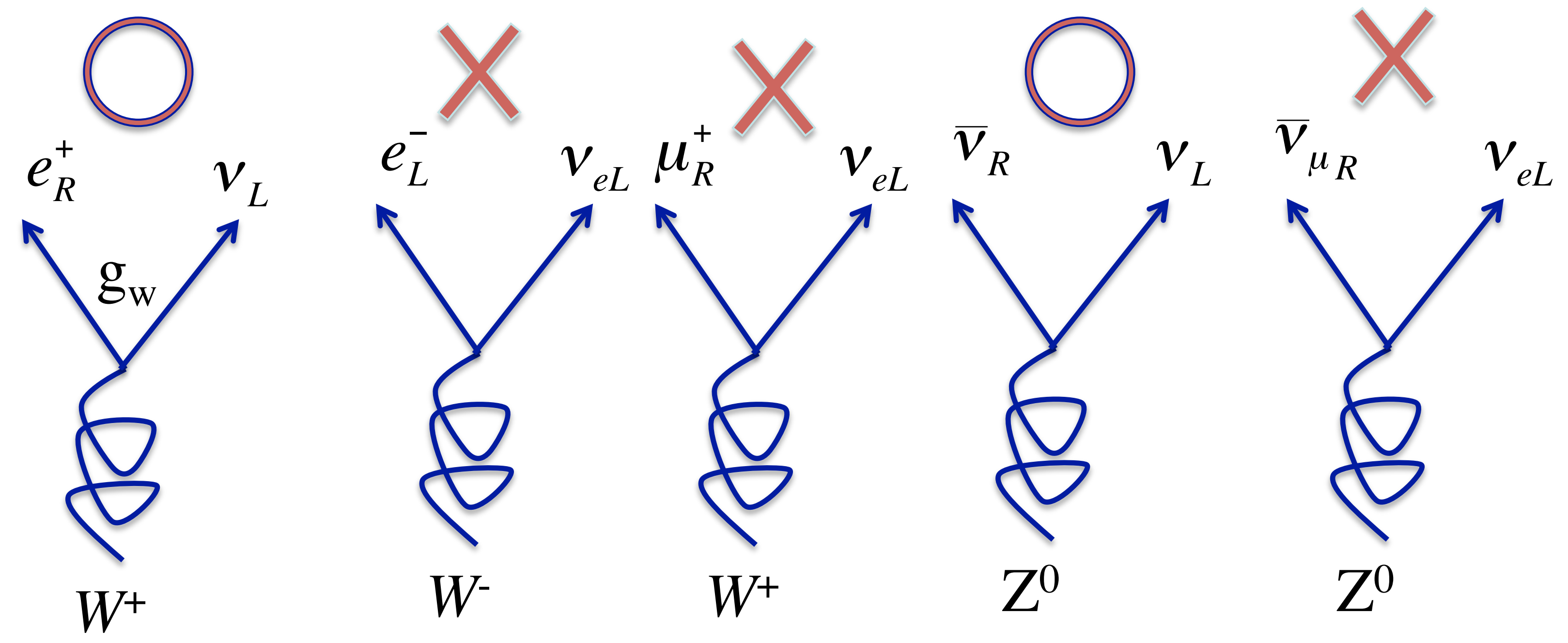
$$g_W = \frac{e}{\sqrt{2}\sin\theta_W} \sim 1.4e$$

$$-ig_Z\left[\overline{\nu}_R\gamma^\mu\nu_L\right]Z_\mu^0$$



$$g_Z = \frac{e}{\sin 2\theta_W} \sim 1.2e$$

$\sin^2\theta_W \sim 0.23$ (Weinberg angle)



Lepton number violation

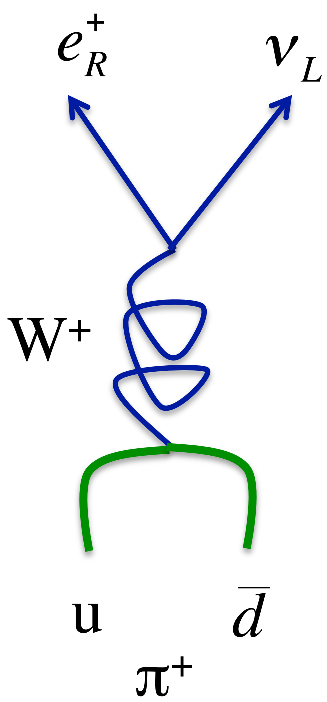
Flavor violation

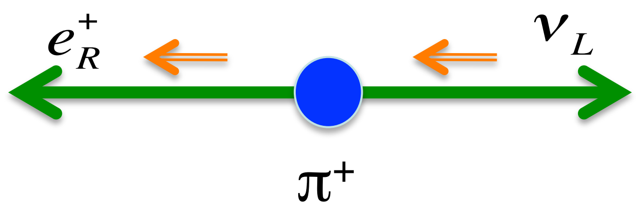
Flavor violation

'Helicity' Suppression of π-decay

Experimental fact: $$\frac{\Gamma_{\pi\to e\nu}}{\Gamma_{\pi\to\mu\nu}} = 1.2\times 10^{-4}$$

How it is explained?





You may say W couple only LH ν and RH e.
So that J(eν)=1, while pion spin=0
=> violates spin conservation

However, this decay exists if very small.
And for $\pi\to\mu+\nu$ decay, the spin conservation seems to strongly violated.

Helicity and Chirality

Sometimes Helicity and Chirality are used in confuse.
Here they are defined and their relations are discussed.

Dirac equation in free space is,

$$\left(i\gamma^{\mu}\partial_{\mu}-m\right)\psi(x)=0$$

General solution is,

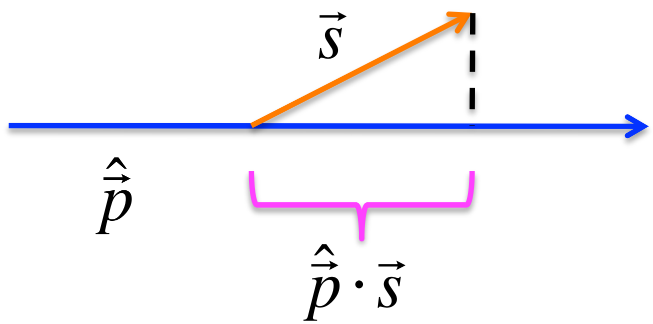
$$\psi(x)=\begin{pmatrix} u \\ \vec{\eta}\cdot\vec{\sigma}u \end{pmatrix} e^{i(\vec{p}\vec{x}-Et)}+\begin{pmatrix} -\vec{\eta}\cdot\vec{\sigma}v \\ v \end{pmatrix} e^{i(\vec{p}\vec{x}+Et)}$$

$$\left[\vec{\eta}\equiv\frac{\vec{p}}{E+m},\quad E\equiv+\sqrt{\vec{p}^{\,2}+m^{2}},\quad u=\begin{pmatrix} u_1 \\ u_2 \end{pmatrix},\quad v=\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}\right]$$

Now we take initial condition as positive energy and $\vec{p}=(0,0,p)$

$$\psi(0)=\begin{pmatrix} u \\ \eta\sigma_z u \end{pmatrix}$$

Helicity is the spin component to the direction of the movement.



If the movement is along the z-direction, helicity components are,

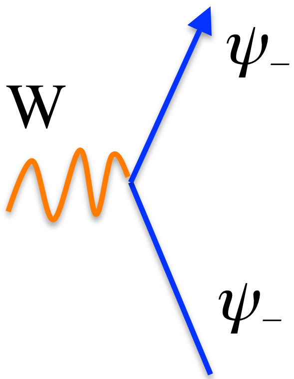
$$\begin{cases} \psi_R \equiv \frac{1}{2}\left(1+\hat{\vec{p}}\cdot\vec{\sigma}\right)\psi = \frac{1}{2}\left(1+\sigma_z\right)\begin{pmatrix} u \\ \eta u \end{pmatrix} = u_1\begin{pmatrix} \chi_1 \\ \eta\chi_1 \end{pmatrix} \\ \psi_L \equiv \frac{1}{2}\left(1-\hat{\vec{p}}\cdot\vec{\sigma}\right)\psi = \frac{1}{2}\left(1-\sigma_z\right)\begin{pmatrix} u \\ -\eta u \end{pmatrix} = u_2\begin{pmatrix} \chi_2 \\ -\eta\chi_2 \end{pmatrix} \end{cases}$$

These helicity states show actual spin direction.

Here after we call Right(Left) handed Helicity =RH (LH)

What W couples to: Chirality

W couples to negative Chirality (NC) particle and positive Chirality (PC) state anti-particle.



Chirality components of ψ is defined by,

$$\psi_\pm \equiv \frac{1}{2}(1 \pm \gamma_5)\psi = \frac{1}{2}\begin{pmatrix} 1 & \pm 1 \\ \pm 1 & 1 \end{pmatrix}\begin{pmatrix} u \\ \eta\sigma_z u \end{pmatrix} = \frac{1 \pm \eta\sigma_z}{2}\begin{pmatrix} u \\ \pm u \end{pmatrix}$$

For m → 0, η→1 and

$$\psi_\pm = \frac{1 \pm \eta\sigma_z}{2}\begin{pmatrix} u \\ \pm u \end{pmatrix} \xrightarrow{m=0} \frac{1 \pm \sigma_z}{2}\begin{pmatrix} u \\ \pm u \end{pmatrix} = \psi_{R/L}$$

For high energy, the helicity and chirality are same and sometimes they are confused.

For low energy, NC has RH component.

In $\pi^- \rightarrow e^- \nu$ decay, e$^-$ is NC state and ν is PC state.

Then the The RH component of electron in the π decay is,

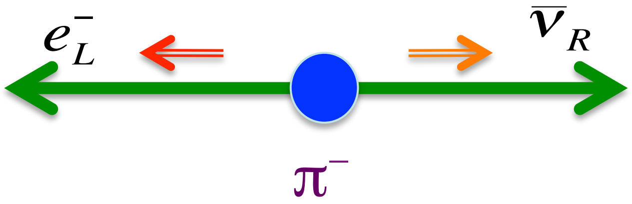
$$\psi_{R-} = \frac{1}{2}\left(1+\hat{\vec{p}}\cdot\vec{\sigma}\right)\psi_- = \frac{(1-\eta)}{4}\left(1+\sigma_z\right)\begin{pmatrix} u \\ -u \end{pmatrix}$$

So that the probability which is RH is,

$$\left|\psi_{R-}\right|^2 = \frac{(1-\eta)^2}{8}\left[u^\dagger\left(1+\sigma_z\right)u\right] = \frac{\gamma}{\gamma+1}\frac{(1-\beta)}{2}\left|u_1\right|^2 \xrightarrow{m/E<<1} \left(\frac{m^2}{4E^2}\right)\left|u_1\right|^2$$

This means the electron has right handed component with probability

$$P \propto \left(m_e^2 / 4E_e^2\right) \sim 1.3\times 10^{-5}$$



This conserve spin

For muon case, m_μ/E_μ~1 and the suppression is not strong.

Taking into account the phase space the theoretical prediction is $\frac{\Gamma_{\pi\to e\nu}}{\Gamma_{\pi\to\mu\nu}} = \left(\frac{m_e}{m_\mu}\right)^2 \left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right) = 1.28\times10^{-4}$

while observation is

π^+ DECAY MODES		Fraction (Γ_i/Γ)	Confidence
$\mu^+\nu_\mu$	[b]	(99.98770±0.00004) %	
$\mu^+\nu_\mu\gamma$	[c]	(2.00 ±0.25) × 10^{-4}	
$e^+\nu_e$	[b]	(1.230 ±0.004) × 10^{-4}	

→ $1.23\text{x}10^{-4}$

Likewise for K decay, $\frac{\Gamma_{K\to e\nu}}{\Gamma_{K\to\mu\nu}} = \left(\frac{m_e}{m_\mu}\right)^2 \left(\frac{m_K^2 - m_e^2}{m_K^2 - m_\mu^2}\right) = 2.4\times10^{-5}$

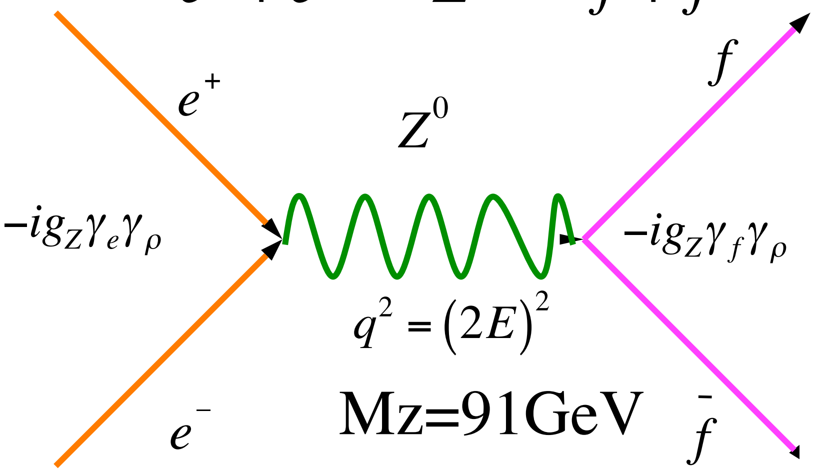
while observation is

K^+ DECAY MODES	Fraction (Γ_i/Γ)	Confide
Leptonic and semileptonic modes		
$K^+ \to e^+\nu_e$	(1.584±0.020) × 10^{-5}	
$K^+ \to \mu^+\nu_\mu$	(63.55 ±0.11) %	

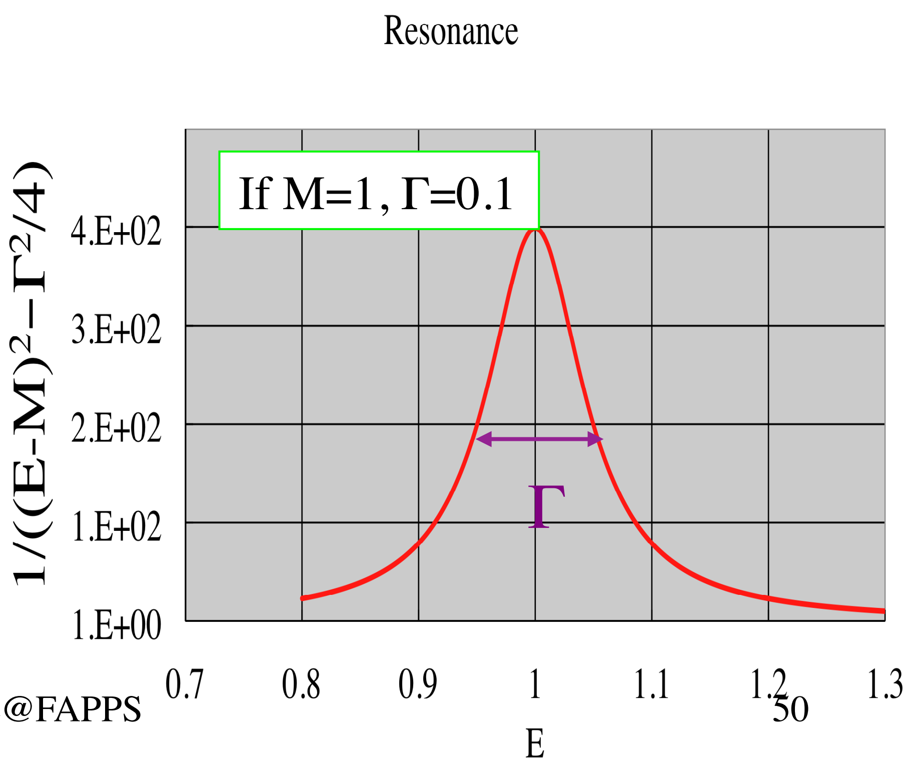
→ $2.5\text{x}10^{-5}$

neutrino flavor counting using Z^0

$$e^+ + e^- \to Z^0 \to f + \bar{f}$$



$$\sigma \propto \frac{g_e^2 g_f^2}{4M_Z^2\left(\left(\sqrt{q^2}-M_Z\right)^2+\Gamma_Z^2/4\right)}$$



The width of the Z^0: Γ is inverse of the lifetime of Z^0: τ

The lifetime is proportional to inverse sum of decay width.

$$\frac{1}{\Gamma_Z} = \tau = \frac{1}{\Gamma_{Z\to u\bar{u}} + \Gamma_{Z\to d\bar{d}} + \cdots + \Gamma_{Z\to \nu_e\bar{\nu}_e}}$$

If the number of neutrino flavors is n_ν,

$$\Gamma_Z = 6\Gamma_U + 9\Gamma_D + 3\Gamma_L + n_\nu\Gamma_\nu$$

$$\Gamma_\nu \propto 1, \quad \Gamma_L \propto 1 - 4x_w + 8x_w^2, \quad \Gamma_U \propto 1 - \frac{8}{3}x_w + \frac{32}{9}x_w^2, \quad \Gamma_D \propto 1 - \frac{4}{3}x_w + \frac{8}{9}x_w^2$$

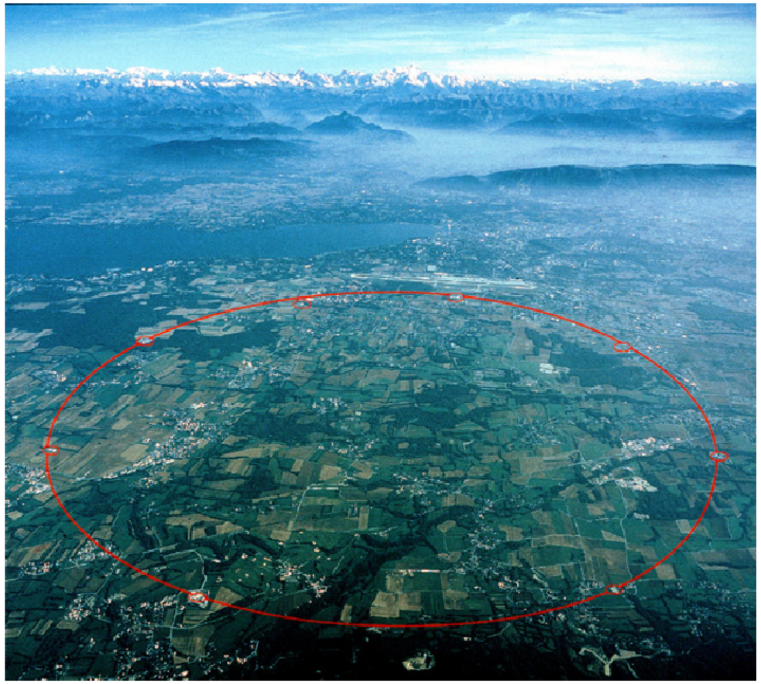
$$x_w = \sin\theta_w \sim 0.23$$

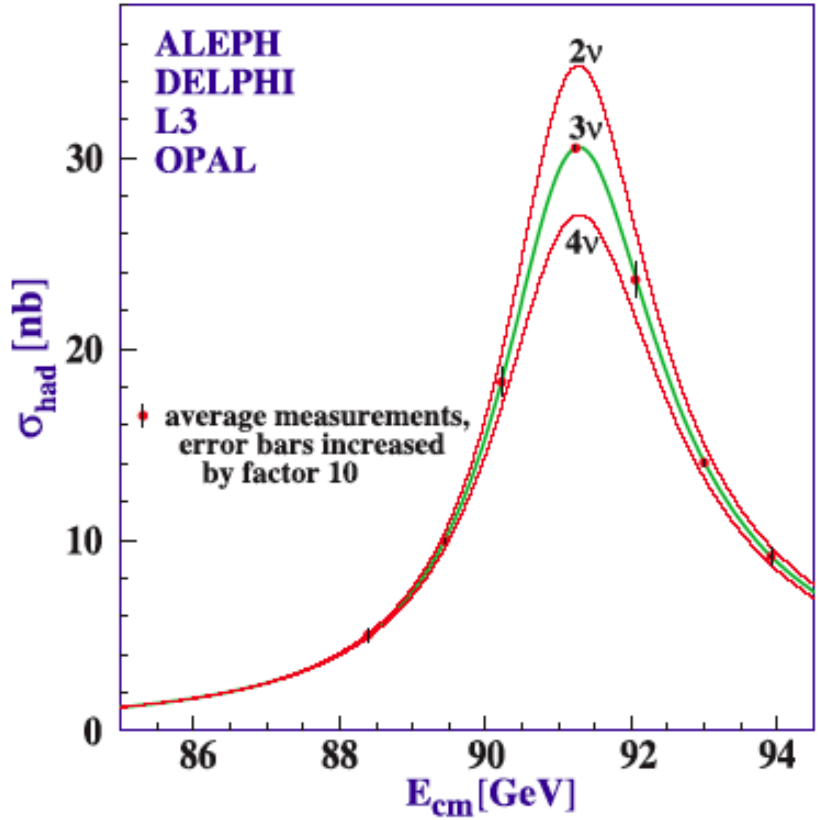
measurement

$$n_\nu = \frac{\Gamma_Z - 6\Gamma_U - 9\Gamma_D - 3\Gamma_L}{\Gamma_\nu}$$

known

CERN's aerial view in 1994. (Copyright CERN)
Vue aerienne du CERN avec le trac du tunnel LEP et du futur LHC (1994). (Copyright CERN)





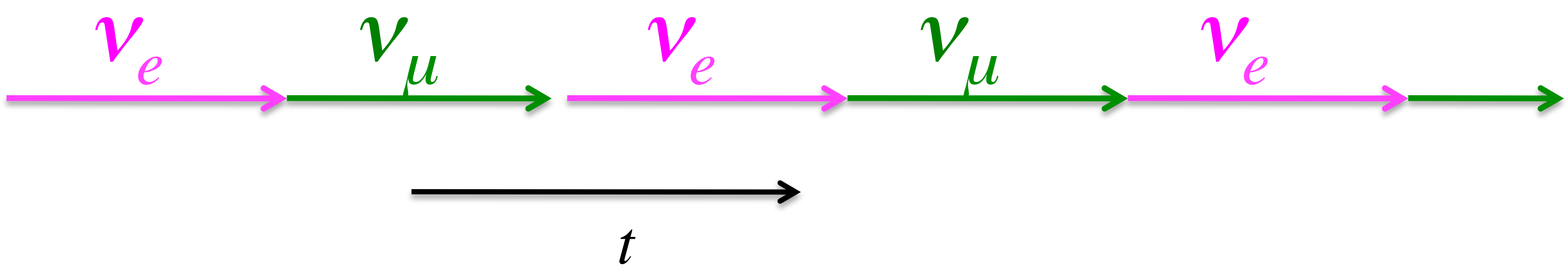
4 experiments at LEP (ALEPH, DELPHI, OPAL, L3) showed

$$\boxed{n_\nu = 2.984 \pm 0.008}$$

If 4th neutrino exists $m_4 > 45\mathrm{GeV}$ or
it does not couple to Z^0, called sterile neutrino.

ν Oscillation: An Introduction

Neutrino oscillation is phenomena in which flavor of neutrino oscillatory changes as time passed by.



If we start from ν_e, the probability to find ν_μ at time t is expressed as:

$$P_{\nu_e \to \nu_\mu}(t) = \sin^2 2\theta \sin^2 \frac{m_2^2 - m_1^2}{4E} t$$

Where m_2 and m_1 are masses of energy-eigenstate neutrinos,
θ is called mixing angle.

ν oscillation is the 1st firm evidence beyond the standard model and its studies are important to understand the nature.

Phenomena of spin-1/2 for ν Oscillation

The formalism of the ν oscillation is very similar to that of spin-1/2 under magnetic field.
So let's review the spin motion as introduction.

The spin motion under magnetic field is described by the Pauli equation:

$$i\dot{\psi} = \mu\vec{B}\cdot\vec{\sigma}\psi$$

Where $\vec{B} = \left(B_x, B_y, B_z\right)$ is the magnetic field and μ is magnetic dipole moment of the particle.

This equation was 1stly introduced empirically by Pauli, and later on obtained by taking non relativistic limit of the Dirac equation with electro-magnetic interaction.

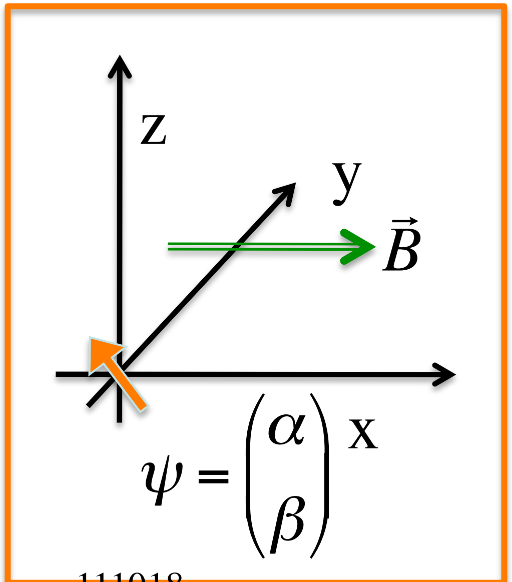
Phenomena of spin-1/2 for ν Oscillation

$$i\dot{\psi} = \mu\vec{B}\vec{\sigma}\psi$$

The wave function is a mixture of spin up and down states

$$\psi(t) = \alpha(t)|\Uparrow\rangle + \beta(t)|\Downarrow\rangle \equiv \begin{pmatrix}\alpha(t)\\ \beta(t)\end{pmatrix}$$

Here, we think the case that magnetic field is along the x axis.



$$\vec{B} = (B,0,0)$$

Then the Pauli equation becomes

$$\begin{pmatrix}\dot{\alpha}\\ \dot{\beta}\end{pmatrix} = -i\mu B\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}\alpha\\ \beta\end{pmatrix}$$

Phenomena of spin-1/2 for ν Oscillation

$$\begin{cases} \dot{\alpha} = -i\mu B\beta \\ \dot{\beta} = -i\mu B\alpha \end{cases}$$

by taking the delivertive of the 1st equation,
and replacing $\dot{\beta}$ by the 2nd equation,

$$\ddot{\alpha} = -(\mu B)^2 \alpha$$

This is the harmonic oscillator and we know the general solution;

$$\begin{cases} \alpha(t) = pe^{-i\mu Bt} + qe^{i\mu Bt} \\ \beta(t) = pe^{-i\mu Bt} - qe^{i\mu Bt} \end{cases}$$

Where p & q are integral constants to be determined by initial condition.

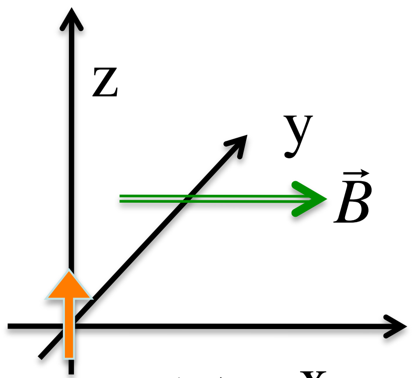
Phenomena of spin-1/2 for ν Oscillation

Then the general spin state is,

$$\psi(t) = \left(pe^{-i\mu Bt} + qe^{i\mu Bt}\right)|\Uparrow\rangle + \left(pe^{-i\mu Bt} - qe^{i\mu Bt}\right)|\Downarrow\rangle$$

Now we assume that at t=0, the spin pointed upward.

$$\psi(0) = |\Uparrow\rangle$$



$$\psi(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Then we can determine p and q for this case,

$$\psi(0) = (p+q)|\Uparrow\rangle + (p-q)|\Downarrow\rangle \equiv |\Uparrow\rangle$$

$$\rightarrow \boxed{p = q = \frac{1}{2}}$$

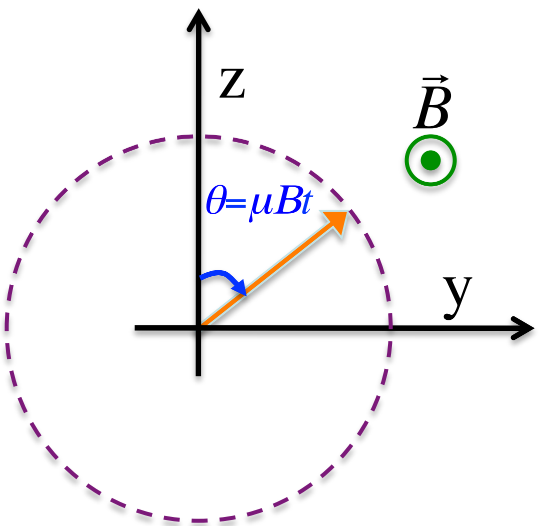
Phenomena of spin-1/2 for ν Oscillation

Then we get specific wave function;

$$\psi(t) = \cos(\mu Bt)|\Uparrow\rangle - i\sin(\mu Bt)|\Downarrow\rangle$$

This means at later time, $|\Downarrow\rangle$ state is generated with oscillating probability

$$\boxed{P_{\Uparrow\to\Downarrow}(t) = \sin^2(\mu Bt)}$$



$$\psi(\theta = \mu Bt, \varphi = 0)$$

If we recall that the wave function of the spin, which is in z-y plane and polar angle is θ is

$$\psi(\theta) = \cos\theta|\Uparrow\rangle - i\sin\theta|\Downarrow\rangle$$

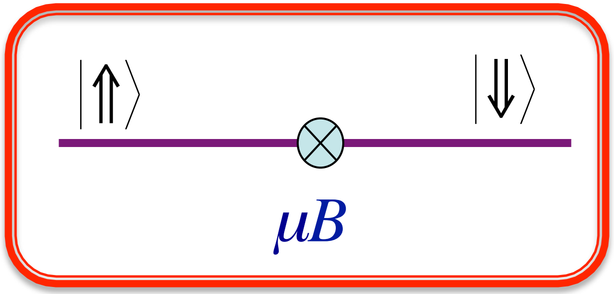
Physically it corresponds to the precession of the spin, caused by the torque by B and μ.

Phenomena of spin-1/2 for ν Oscillation

$$\begin{pmatrix} \dot{\alpha} \\ \dot{\beta} \end{pmatrix} = -i\mu B \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

Quantum Mechanically, this is understood as the effect that magnetic field causes transition between $|\Uparrow\rangle \leftrightarrow |\Downarrow\rangle$ with amplitude μB.

We will draw this kind of effect schematically as follows.



The neutrino oscillation can be understood as exactly same manner.

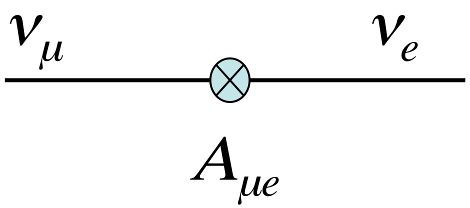
A simple case of ν Oscillation

We assume 2 neutrino system; ν_e & ν_μ

The general state is;

$$\psi_\nu(t) = \alpha(t)|\nu_e\rangle + \beta(t)|\nu_\mu\rangle$$

We assume <u>something</u> makes transition: $\nu_e \Leftrightarrow \nu_\mu$



Then there are correspondences to the spin case

$$\boxed{\begin{cases} |\nu_e\rangle \Leftrightarrow |\Uparrow\rangle \\ |\nu_\mu\rangle \Leftrightarrow |\Downarrow\rangle \\ A_{\mu e} \Leftrightarrow \mu B \end{cases}}$$

If the initial state is pure ν_e state, like beta decay, then

$$P_{\nu_e \to \nu_\mu}(t) = \sin^2 A_{\mu e} t$$

This is the very basic of neutrino oscillation.

A simple case of ν Oscillation

However, we often see the neutrino oscillation probability as

$$P_{\nu_\mu}(t) = \underline{\sin^2 2\phi} \sin^2 \frac{m_2^2 - m_1^2}{4E} L$$

Where does this come from?
What is the analogy of spin motion?

Still spin-1/2 for ν Oscillation

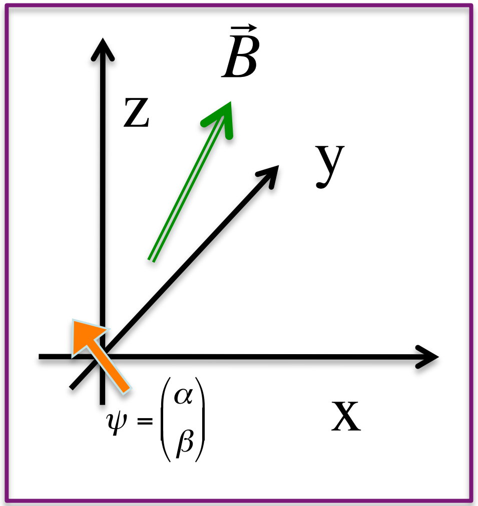
In actual case, mass term has to be included in the Pauli equation

$$i\dot{\psi} = \left(m + \mu\vec{B}\vec{\sigma}\right)\psi$$

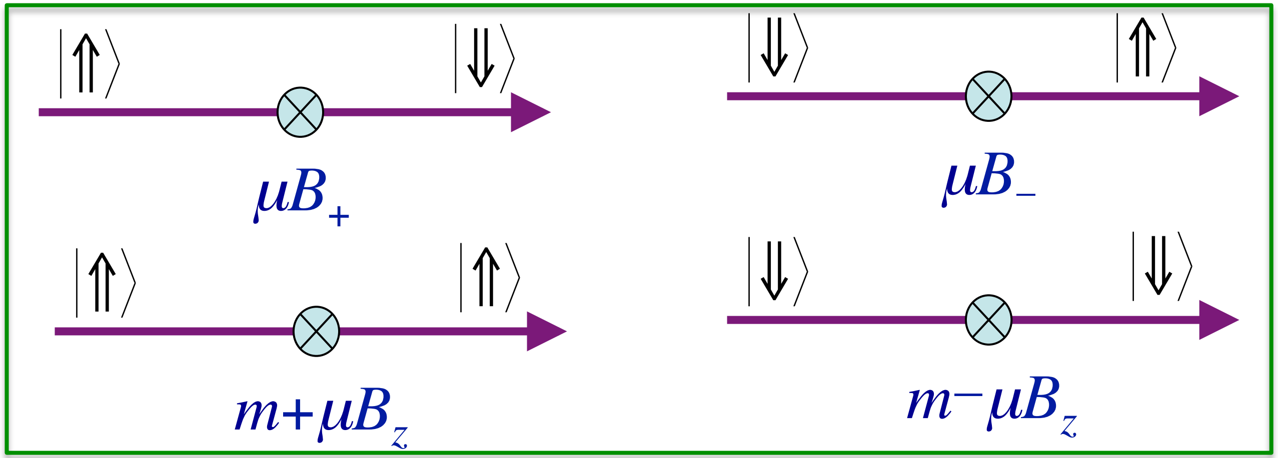
and the most general equation with arbitrary magnetic field is,

$$\begin{pmatrix}\dot{\alpha} \\ \dot{\beta}\end{pmatrix} = -i\begin{pmatrix} m+\mu B_z & \mu B_- \\ \mu B_+ & m-\mu B_z \end{pmatrix}\begin{pmatrix}\alpha \\ \beta\end{pmatrix}; \quad B_\pm \equiv B_x \pm iB_y$$

Spin transition amplitudes are



=



Still spin-1/2 for ν Oscillation

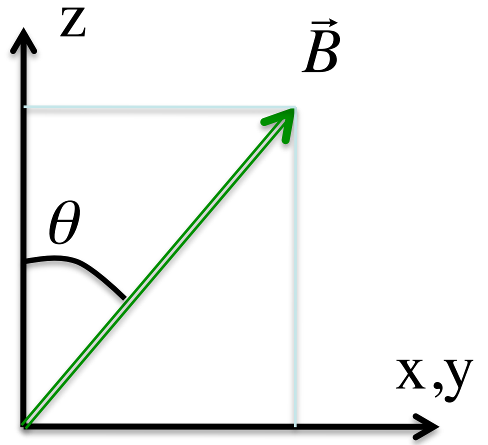
$$\begin{pmatrix} \dot{\alpha} \\ \dot{\beta} \end{pmatrix} = -i \begin{pmatrix} m + \mu B_z & \mu B_- \\ \mu B_+ & m - \mu B_z \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

This equation has the general form

$$\begin{pmatrix} \dot{\alpha} \\ \dot{\beta} \end{pmatrix} = -i \begin{pmatrix} P & Q^* \\ Q & R \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}, \quad P, R \in \mathrm{Re}\,al$$

Relation between Polar angle θ of the magnetic field and transition amplitudes is,

$$\tan\theta \equiv \frac{B_\perp}{B_z} = \boxed{\frac{2|Q|}{R-P}}$$



Still spin-1/2 for ν Oscillation

$$\begin{pmatrix} \dot{\alpha} \\ \dot{\beta} \end{pmatrix} = -i \begin{pmatrix} P & Q^* \\ Q & R \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}, \quad P, R \in \mathrm{Re}\,al$$

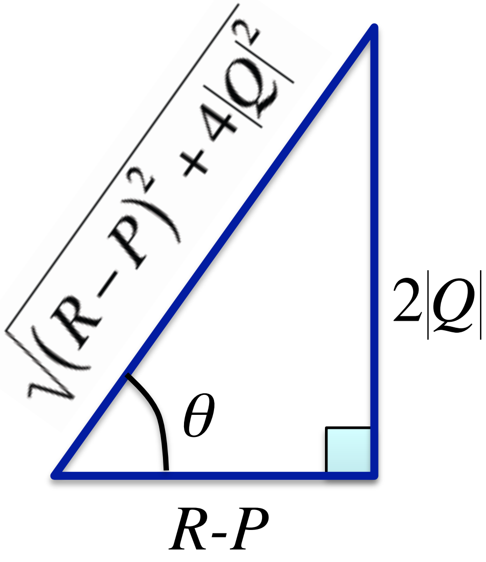
Then the general solution can be expressed using θ as,

$$\begin{cases} \alpha(t) = C_1 \cos(\theta/2) e^{-iE_+ t} - C_2 \cos(\theta/2) e^{-iE_- t} \\ \beta(t) = C_1 \sin(\theta/2) e^{-iE_+ t} + C_2 \cos(\theta/2) e^{-iE_- t} \end{cases}$$

where,

$$\begin{cases} E_+ = \frac{1}{2}\left((P+R) + \sqrt{(P-R)^2 + 4|Q|^2}\right) \\ E_- = \frac{1}{2}\left((P+R) - \sqrt{(P-R)^2 + 4|Q|^2}\right) \end{cases}$$

(Note: by definition, $E_+ > E_-$)

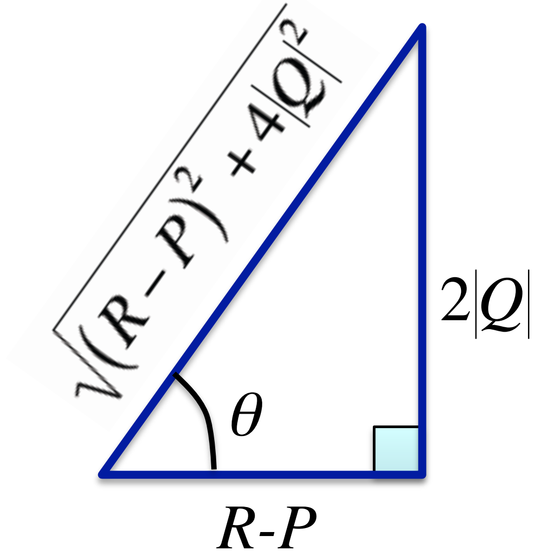


$$\tan\theta \equiv \frac{2|Q|}{R-P}$$

Still spin-1/2 for ν Oscillation

The general wave function is,

$$\boxed{\begin{aligned}\psi(t) = &\left(C_1\cos(\theta/2)e^{-iE_+t} - C_2\sin(\theta/2)e^{-iE_-t}\right)|\Uparrow\rangle \\ &+\left(C_1\sin(\theta/2)e^{-iE_+t} + C_2\cos(\theta/2)e^{-iE_-t}\right)|\Downarrow\rangle\end{aligned}}$$



Again we start with

$$\psi(0) = |\Uparrow\rangle$$

Then, the integral constants are determined.

$$\tan\theta \equiv \frac{2|Q|}{R-P}$$

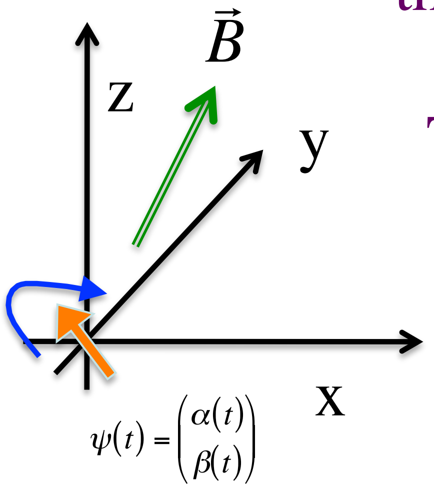
$$\begin{cases} C_1\cos(\theta/2) - C_2\sin(\theta/2) = 1 \\ C_1\sin(\theta/2) + C_2\cos(\theta/2) = 0 \end{cases} \Rightarrow \begin{cases} C_1 = \cos(\theta/2) \\ C_2 = -\sin(\theta/2) \end{cases}$$

Still spin-1/2 for ν Oscillation

In this case, the specific wave function is

$$\psi(t) = \left(\cos^2(\theta/2)e^{-iE_+t} + \sin^2(\theta/2)e^{-iE_-t}\right)|\Uparrow\rangle + \frac{1}{2}\sin\theta\left(e^{-iE_+t} - e^{-iE_-t}\right)|\Downarrow\rangle$$

This state corresponds to spin precession within the plane perpendicular to the magnetic field.



The time dependent probability of spin-down state is

$$P_{\Uparrow\to\Downarrow}(t) = \left|\frac{\sin\theta}{2}\left(e^{-iE_+t} - e^{-iE_-t}\right)\right|^2 = \sin^2\theta\sin^2\mu Bt$$

corresponds to the angle between the precession plane and z axis.

Difference of the energies in the energy eigenstate

Still spin-1/2 for ν Oscillation

Look for Energy eigenstate,

Remember the general state

$$\psi_\nu(t) = \left(C_1 \cos(\theta/2) e^{-iE_+t} - C_2 \sin(\theta/2) e^{-iE_-t}\right)|\Uparrow\rangle + \left(C_1 \sin(\theta/2) e^{-iE_+t} + C_2 \cos(\theta/2) e^{-iE_-t}\right)|\Downarrow\rangle$$

If we choose, $C_1=1$, $C_2=0$,

$$\psi_+(t) = \left(\cos(\theta/2)|\Uparrow\rangle + \sin(\theta/2)|\Downarrow\rangle\right)e^{-iE_+t}$$

This means

$$|+\rangle \equiv \cos(\theta/2)|\Uparrow\rangle + \sin(\theta/2)|\Downarrow\rangle$$

is energy eigenstate with energy E_+.

Similarly, if we choose, $C_1=0$, $C_2=1$,

$$\psi_-(t) = \left(-\sin(\theta/2)|\Uparrow\rangle + \cos(\theta/2)|\Downarrow\rangle\right)e^{-iE_-t} \equiv |-\rangle e^{-iE_-t}$$

Still spin-1/2 for ν Oscillation

The spin states $|\Uparrow\rangle$, $|\Downarrow\rangle$ itselvs are NOT energy eigenstate and do not have definite energy.
(If you try to measure the energy of $|\Uparrow\rangle$ state, you will see 2 energies.)

But the mixed state,

$$\begin{pmatrix} |+\rangle \\ |-\rangle \end{pmatrix} = \begin{pmatrix} \cos(\theta/2) & \sin(\theta/2) \\ -\sin(\theta/2) & \cos(\theta/2) \end{pmatrix} \begin{pmatrix} |\Uparrow\rangle \\ |\Downarrow\rangle \end{pmatrix}$$

ARE energy eigenstate and have definite energy.
$\theta/2$ is called mixing angle between energy eigenstate and spin state.
=The mixing angle corresponds to 1/2 of the polar angle of $\vec{B}$.