Beyond Standard Model

(BSM)

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Contents

- Lecture I : Effective Field Theory (EFT) Approach
  - Why BSM ?
  - Naive Dimensional Analysis
  - (SM as an) EFT
- Lecture II : BSM w/o Considering Hierarchy Problem
  - Additional Matters: 4th Generation, Additional Scalar (with DM)
  - New Gauge Interactions: Extra U(1), (LR model)
  - Extra Dim (UED)
- Lecture III : BSM Considering Hierarchy Problem
  - SUSY (GUT)
  - Technicolor
  - Large Extra Dim (ADD) and Warped spacetime (RS)

Great Success and Some Drawbacks of the SM

Contents

What is the SM of particle physics ?

Standard model (SM): Matter

Fundamental Constituent of Matter : Spin 1/2 Fermions

- Leptons do not feel strong interactions
- Quarks and gluons do ! (Masses in GeV)

$$\begin{pmatrix} \nu_e \ (\sim 0) \\ e \ (0.511\mathrm{MeV}) \end{pmatrix}_L , \quad \begin{pmatrix} \nu_\mu \ (\sim 0) \\ \mu \ (0.106) \end{pmatrix}_L , \quad \begin{pmatrix} \nu_\tau \ (\sim 0) \\ \tau \ (1.777) \end{pmatrix}_L ,$$

$$\begin{pmatrix} u^\alpha \ (\sim 0.003) \\ d^\alpha (\sim 0.005) \end{pmatrix}_L , \quad \begin{pmatrix} c^\alpha (\sim 1.50) \\ s^\alpha (\sim 0.12) \end{pmatrix}_L , \quad \begin{pmatrix} t^\alpha (\sim 175) \\ b^\alpha (\sim 5) \end{pmatrix}_L$$

$e_R,\ \mu_R,\ \tau_R,\ u^\alpha_R,\ c^\alpha_R,\ t^\alpha_R,\ d^\alpha_R,\ s^\alpha_R,\ b^\alpha_R$ and

$(N_{eR},\ N_{\mu R},\ N_{\tau R}?)$ or Something else ?

SM : Forces (Interactions)

Interactions and Their Force Quanta

- Gravity (spin-2 massless graviton G)
  $\rightarrow$ stars, galxies,..
- Electromagnetic Interactions (spin-1 photon γ)
  $\rightarrow$ atoms and molecules
- Weak Interaction
  (spin-1 massive vector bosons $W^{\pm}, Z^0$)
  $\rightarrow$ Radioactivity (e.g., $n \rightarrow p e^- \bar{\nu}$)
- Strong Interaction (spin-1 massless gluons g)
  $\rightarrow$ quarks and gluons, nucleons, nucleus
- Mathematically, all interactions except gravity are described by $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge theory
- And force that breaks EW symmetry (Higgs ?)

SM Lagrangian

- Standard Model (Glashow-Weinberg-Salam) based on $SU(3)_c \times SU(2)_L \times U(1)_Y$
- The Renormalizable SM lagrangian :

$$\mathcal{L}_{\rm SM}^{\rm Ren} = \mathcal{L}_{\rm kin}(f, A) + \mathcal{L}_{\rm kin}(A_\mu) + \mathcal{L}_{\rm kin-pot}(H) + \mathcal{L}_{\rm Yukawa}(f, \bar{f}, H)$$

- where

$$\begin{aligned}
\mathcal{L}_{\rm kin}(f, A) &= \bar{f} i\gamma \cdot D f \\
\mathcal{L}_{\rm kin}(A_\mu) &= -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu^a} \\
\mathcal{L}_{\rm kin-pot}(H) &= (DH)^\dagger (DH) - V(|\phi|) \\
\mathcal{L}_{\rm Yukawa}(f, \bar{f}, H) &= \bar{f}_{iR} H f_{jL} + \ldots
\end{aligned}$$

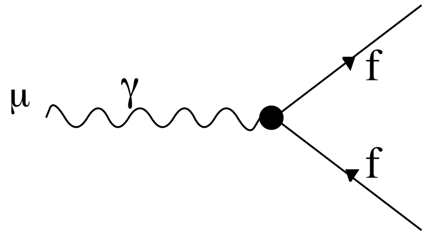
SM Lagrangian-II

- The first two are very well verified upto $E \sim 200$ GeV (or down to $\sim 10^{-3}$ fm or 10^{-16} cm)
- The last two have to be studied more in the future, at Tevatron, LHC and ILC, ...
- SM : An effective theory upto $E \sim$ a few hundred GeV

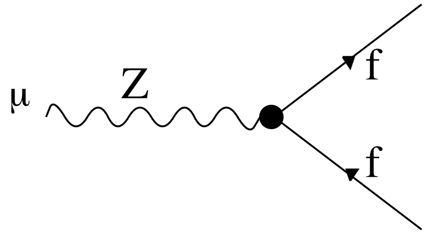
$$\mathcal{L}^{\text{Eff}}_{\text{SM}} = \mathcal{L}^{\text{Ren}}_{\text{SM}} + \sum_{r=1}^{\infty} \frac{g_r}{\Lambda^r}\, \mathcal{O}_{(r+4)}$$

- Excellent agreement of the SM predictions with the almost all the data indicates that the new physics scale $\lambda > O(10 - 100)$ TeV, depending on the channels you study
- Baryon and Lepton numbers are accidental symmetries of the SM
  $\rightarrow B$ and L violating scales are very high (see later)

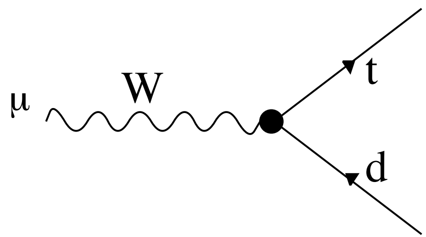
Feynman rules for gauge interactions



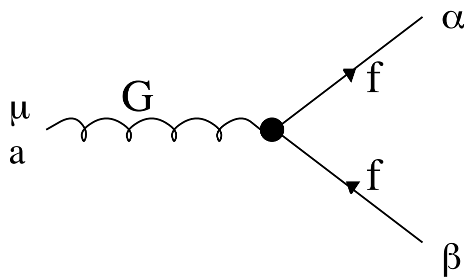
$$-ie\gamma_\mu Q_f$$



$$i\frac{g_2}{2\cos\theta_W}\gamma_\mu\left(v_f - a_f\gamma_5\right)$$



$$i\frac{g_2}{2\sqrt{2}}\gamma_\mu\left(1-\gamma_5\right)V_{td}$$



$$-ig_s\gamma_\mu\left(T^a\right)_{\alpha\beta}$$

Flavor physics and CP violation in the quark sector

Flavor and CP violation in SM

- Weak eigenstates are mixtures of mass eigenstates
- Cabibbo - Kobayashi – Maskawa (CKM) matrix describes flavor mixing and CP violation in the charged weak current interaction
- Unitarity : $VV^\dagger = 1$

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

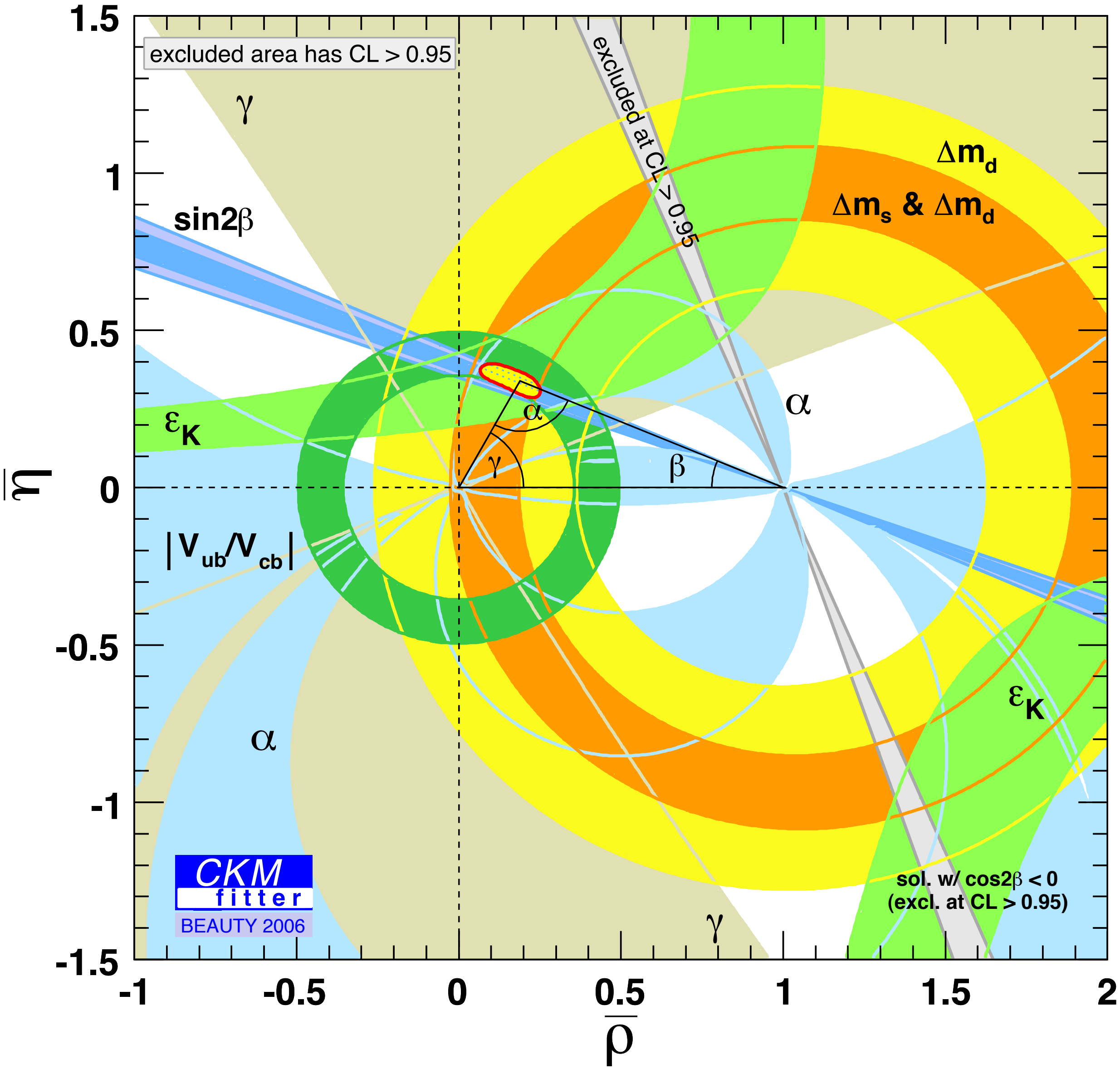
CKMology

- Wolfenstein parametrization:

$$V_{CKM} \simeq \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 - iA^2\lambda^4\eta & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

- Current date : $\lambda = 0.22, \quad A = (0.826 \pm 0.083)$
- Small mixing and Hierarchical structure
  $\lambda \sim 0.2, \lambda^2 \sim 0.04, \lambda^3 \sim 0.008$
- Why are quark masses and mixings so hierarchical ?
- η: the unique source of CPV in K and B meson systems cf. Neutrino sector has completely different behavior !

Constraints in the $\rho - \eta$ plane



Appraise for (C)KM paradigm

- Many different independent (both tree and loop) processes single out a region for the apex of the UT, (ρ, η)
- This is highly nontrivial, because this would be not the case if the top was lighter of heavier than the current value
- Any new physics around EW scale may have additional Flavor and CP violation, which are now strongly constrained by the CKMology
- Even the $b \to s$ transition is now strongly constrained by the recent measurement of the mudulus of $B_s - \overline{B_s}$ mixing, ΔM_s by D0 & CDF @ Tevatron

Electroweak Precision tests (EWPT)

- $(g-2)_\mu$: Muon anomalous magnetic dipole moment
- Tests of (P)QCD
- Correlation between m_W, m_t and m_H
- Where (What) is the Higgs after all ?

Where (What) is the Higgs after all ?

Higgs mechanism in the SM

- Gauge Interaction has Universality, which is very well confirmed by many exp.'ts
- Still only known long range force is E & M and Gravity
- Other gauge bosons should get masses or confined
- Higgs boson has not been found yet: $m_H^{\text{exp}} > 114$ GeV
- Higgs Mechanism in EW vs. Confinement in QCD
- Recall the Landau–Ginzburg Theory for Superconductivity
  London Equation $\rightarrow$ Meissner Effect $\rightarrow$ Massive Photon inside SC

Bound on m_H within the SM

- Upper bound from tree level unitarity :
  - $W_L W_L$ elastic scattering violates perturbative unitarity w/o Higgs boson (B.W.Lee, C. Quigg and H.B. Thacker)
  - Either Higgs boson or new resonances (as ρ, K^*... in hadron physics) to restore unitarity
  - Another way out : Unitarity can be restored, if there are infinitely many massive gauge bosons, as predicted in higher dim. gauge theory without Higgs (Higgsless EWSB)
- Upper bound from triviality condition : $\lambda\phi^4$ theory is not asymptotically free
  $\rightarrow$ No Landau pole appears until the scale Λ where new physics comes in

Bound on m_H within the SM-II

- Lower bound from vacuum stability :

$$\frac{d\lambda}{dt} = \frac{3}{4\pi^2}\left[\lambda^2 + 3\lambda y_t^2 - 9y_t^4 + \text{small gauge and Yukawa terms}\right]$$

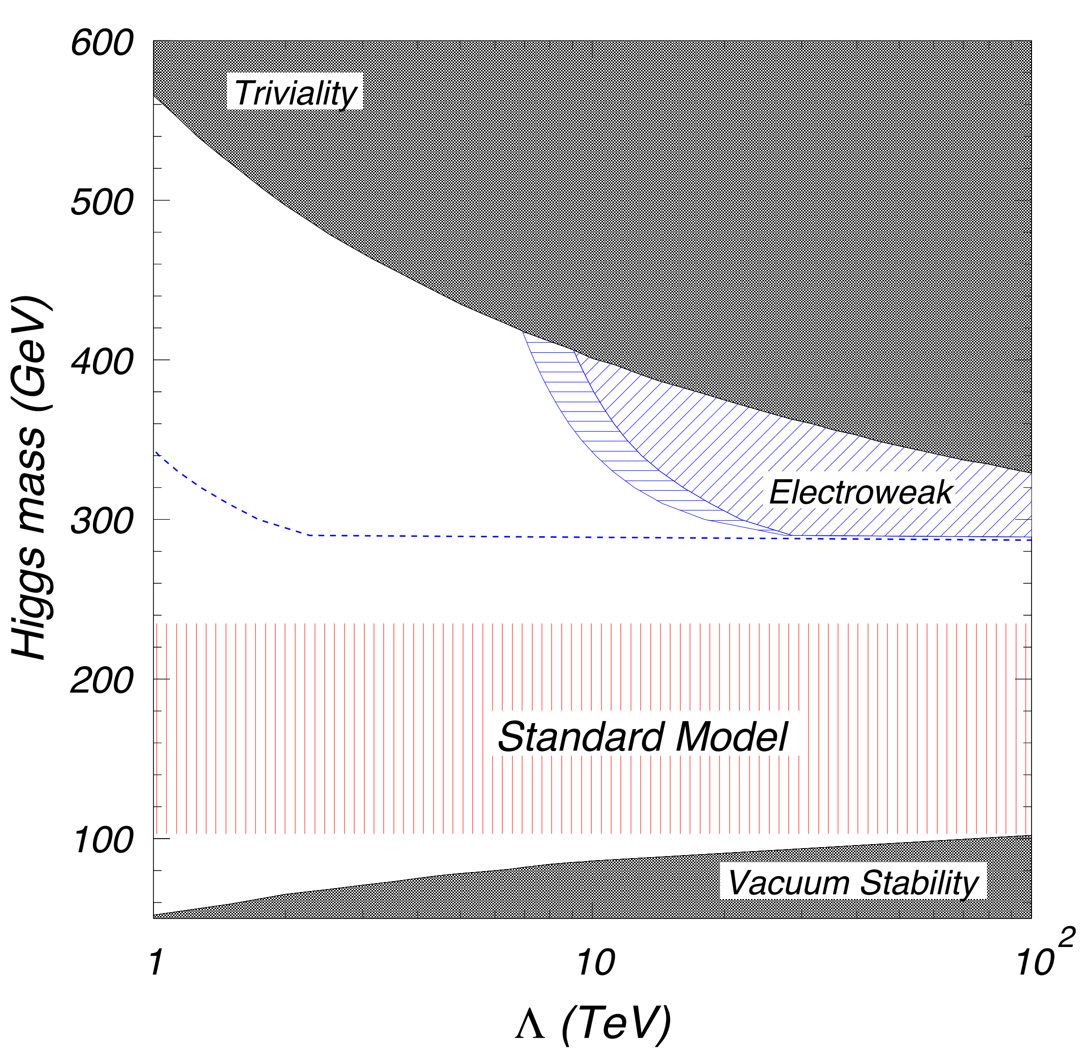
with $\lambda = \lambda_0 = m_H^2/2v^2$ and $y_t^0 = m_t/v$

- For small λ and fixed m_t, λ decreases with t and can be negative $\rightarrow$ condition for $\lambda(\Lambda) > 0$:

$$m_H(\text{GeV}) > 129.5 + 2.1(m_t - 171.4) - 4.5\left(\frac{\alpha_s(m_Z) - 0.118}{0.006}\right)$$

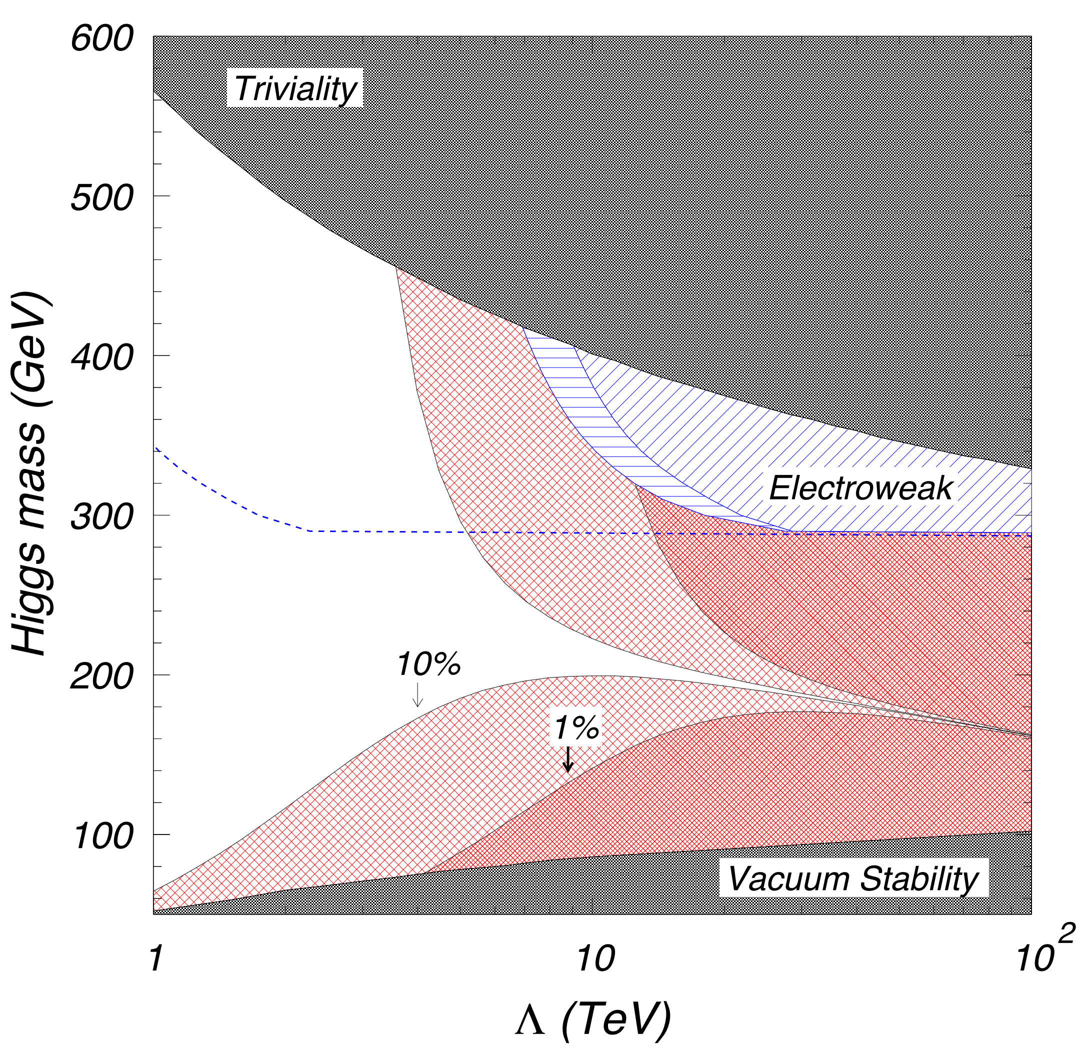
- $130 < m_H(\text{in GeV}) < 200$ for $m_t = 171$ GeV, if $\Lambda \sim M_{\text{GUT}}$ or M_{Pl}

m_H vs. Λ: Classical



Higgs effects on EWPT included

m_H vs. Λ: plus fine tuning



Do we have to care about fine tuning ?

SM Higgs: Summary

- Combining with the EWPT, the light fundamental Higgs is well supported with $m_H \lesssim 199$ GeV 95 % CL
- Such a light SM Higgs is well within the reach of LHC, and one can definitely find out Higgs boson, if the SM is the correct picture
- To test the Higgs mechanism by recontruting the Higgs potential, one needs to build the ILC or its relative, to accurately measure the tiple and the quartic couplings of Higgs boson
- Heavy Higgs or Higgsless models need conspiracy in order to be consistent with some parameters such as S or T

Beyond the SM Higgs ?

- The previous plot is not valid, lighter Higgs possible (as in MSSM)
- More fundamental Higgs bosons ($SU(2)_L$ singlet, doublet, triplet,...)
- No Higgs ? (Technicolor, Walking, and relatives)
- Composite Higgs ? (Little Higgs, Fat higgs, Top condensate, ...)
- What is realized in Nature ?
  $\rightarrow$ Very important and expensive question to be answered at LHC

Phenomenological drawbacks of the SM

- Neutrino masses and mixings
- Dark matter of our universe

Neutrino oscillations

- Neutrinos are hard to detect, and their masses are not precisely known
- Massless spin 1/2 particle in the renormalizable SM
- Mass limits from direct searches:
  - $m_\nu < 3$ eV from tritium β decay
  - $m_\nu < 0.19$ keV from $\pi \to \mu\nu_\mu$
  - $m_\nu < 18.2$ MeV from $\tau \to 5\pi + \nu_\tau$
- Indirect bound from cosmology : $\sum m_\nu < 2(11)$ eV from WMAP data analysis
- Why are they so small compared with other fermion masses ?
  $m_e = 0.511$ MeV

Charged lepton flavor violation (LFV) ?

- LFV in neutrino sector has been confirmed
- How about in the charged lepton sector ?
- Upper bounds on Br for some modes (2004 PDG) :

Mode	Br
$\mu \to e\gamma$	$< 1.2 \times 10^{-11}$
$\mu \to 3e$	$< 1.0 \times 10^{-12}$
$\tau \to e\gamma$	$< 2.7 \times 10^{-6}$
$\tau \to \mu\gamma$	$< 1.1 \times 10^{-6}$
$\tau \to 3\mu$	$< 1.9 \times 10^{-6}$
$\tau \to \mu\eta$	$< 9.6 \times 10^{-6}$

Charged LFV - II ?

- Why is it so small in the charged lepton sector, whereas it is large in the neutrino sector ?
  Answer: Not well understood yet
- Charged LFV can be enhanced in SUSY models or some physics beyond the SM
- Search for charged LFV's still going on :
  $\mu \to e\gamma$ (MEG)
  $\mu^- \ \mathrm{Ti} \to e^- \ \mathrm{Ti}$ (MECO)
  $\tau \to \mu\gamma, \ 3\mu, \ \mu\eta$, etc. ($B, \tau$ factories)
- Sensitive probe of physics beyond the SM

Some DM candidates

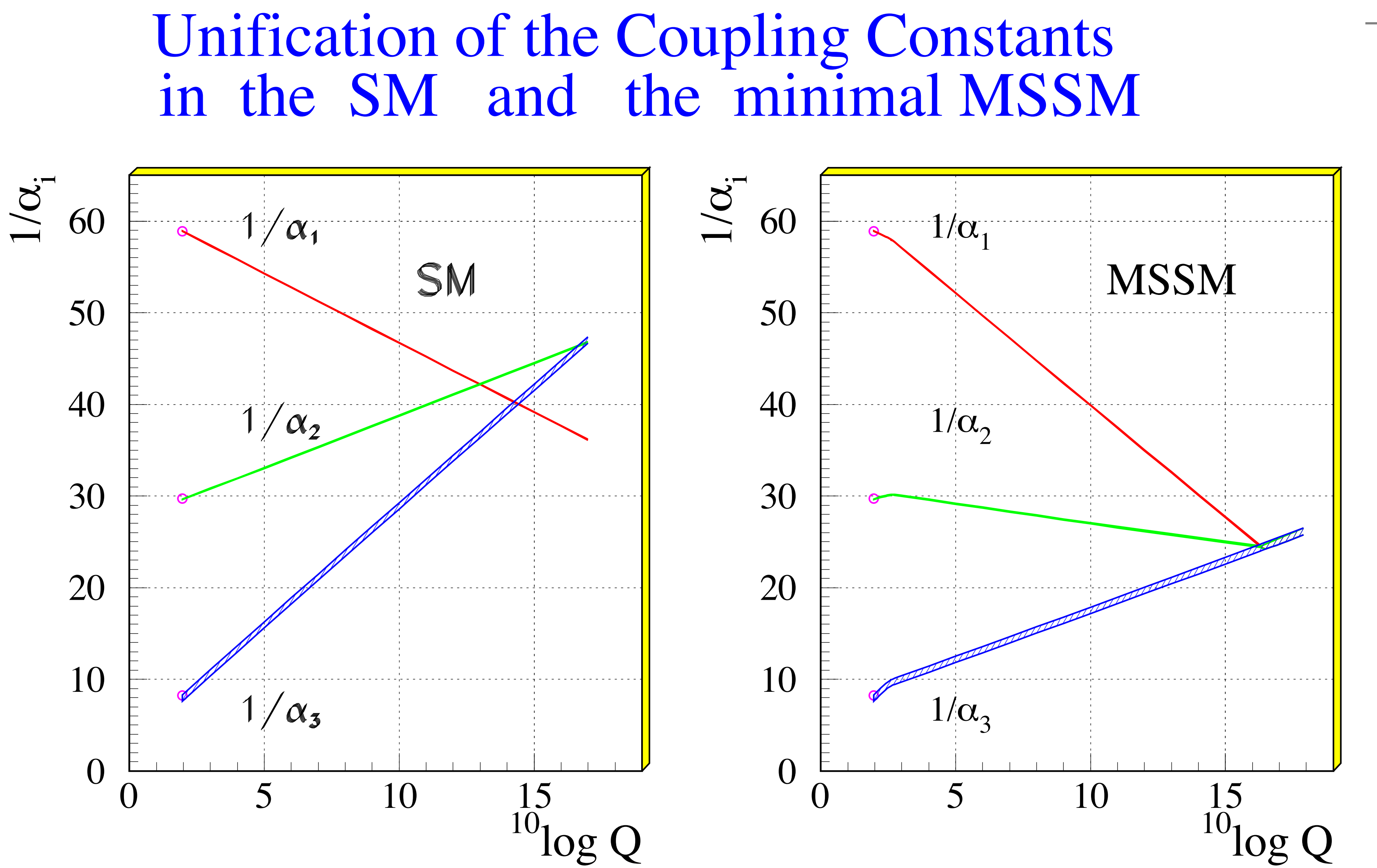
Particle	Solve another problem ?
Singlet scalar	No (Simplest extension)
Singlet fermion	No (The next simplest extension)
Axion	Y (Strong CP)
LSP (χ_1^0 or $\tilde{G}$)	Y (fine tuning & proton stablity)
Lightest KK	Y(?) (Hierarchy problem)
Axino	Y (SUSY version of strong CP)
Branon	Y (?) (Baneworld scenario)

Some of them can be studied at colliders (LHC/ILC), whereas some of them are not

Other theoretical/aesthetical drawbacks

- Gauge coupling (force) unification ?
- Some Why and fine tuning problems
(# of generations, Why rich structures in masses and mixings, Why now ?...; strong CP, gauge hierarchy problem, cosmological constant problem, ...)
- Quantum gravity

Running of 3 gauge couplings



NB: GCU can be achieved in other ways without SUSY (RS1 with SM in the bulk, more matters in TeV regions,....)

Grand Unification (GUT)

- Unification $\rightarrow$ Progress in theoretical physics
  Maxwell's E & M, QM and Special Relativity $\rightarrow$ QFT, ...
- Unanswered Questions within SM
  - Why $Q_p = -Q_e$ and $U(1)_Y$ quantum numbers ?
  - Why 3 different forces ? Are they UNIFIABLE ?
    $SU(3)_c \times SU(2)_L \times U(1)_Y \rightarrow G_{\mathrm{GUT}}$
  - Why proton is stable ?
    $\tau(p \rightarrow e^+\pi^0) > 1.6 \times 10^{33}$ years
  - Why 3 generations ?
  - Quantum Gravity ?
  - Many other questions ...

GUT and proton decay in $SU(5)$

- $5^* = (d_1^c, d_2^c, d_3^c, e^-, \nu_e)_L^T$

$$10 = \begin{pmatrix} 0 & u_3^c & -u_2^c & -u^1 & -d^1 \\ & 0 & u_1^c & -u^2 & -d^2 \\ & & 0 & -u^3 & -d^3 \\ & & & 0 & -e^+ \\ & & & & 0 \end{pmatrix}_L$$

$1 = N_L^c$

- SM particles fit into $5^* + 10 + 1$ of $SU(5)$

$\boxed{\text{Quark–Lepton Unification}}$

$SU(5)$ GUT: gauge bosons

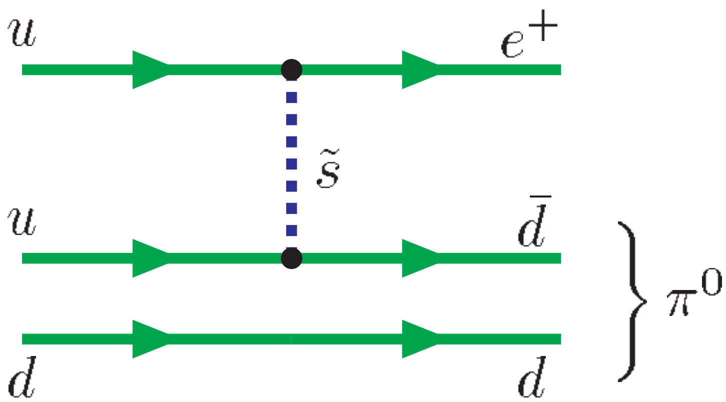
- 24 Gauge bosons in adjoint

$$
\begin{pmatrix}
G^1_1 - \frac{2B}{\sqrt{30}} & G^1_2 & G^1_3 & \bar{X}^1 & \bar{Y}^1 \\
G^2_1 & G^2_2 - \frac{2B}{\sqrt{30}} & G^2_3 & \bar{X}^2 & \bar{Y}^2 \\
G^3_1 & G^3_2 & G^3_3 - \frac{2B}{\sqrt{30}} & \bar{X}^3 & \bar{Y}^3 \\
X_1 & X_2 & X_3 & \frac{W^3}{\sqrt{2}} + \frac{3B}{\sqrt{30}} & W^+ \\
Y_1 & Y_2 & Y_3 & W^- & -\frac{W^3}{\sqrt{2}} + \frac{3B}{\sqrt{30}}
\end{pmatrix}
$$

- X, Y gauge bosons couple to quark + lepton (Leptoquarks) $\rightarrow$ Proton decays
  cf. Similar if $R-$parity is violated in the MSSM

$SU(5)$ GUT and proton decay

- Superheavy X, Y gauge boson excahnge:

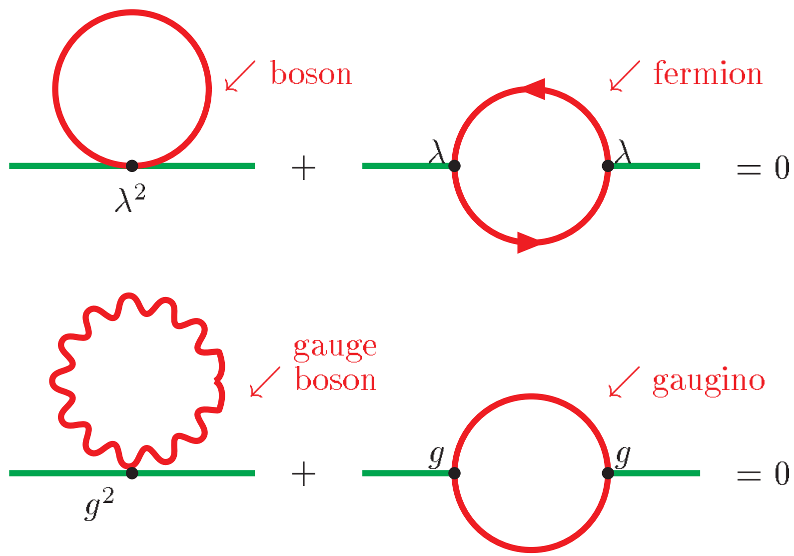


$$p \to e^+ \pi^0$$

$$\tau^{-1} \sim \frac{\alpha_{\mathrm{GUT}}^2 m_p^5}{M_X^4}$$

- NonSUSY $SU(5)$: $M_X \simeq 3 \times 10^{14}$ GeV $\to \tau \simeq 10^{30\pm1}$ years EXCLUDED
- SUSY SU(5) is OK with proton decay exp. and Gauge Coupling Unif.

Solving Gauge Hierarchy Problem



- Fermion Loop Contribution

$$\Delta m_H^2 = \frac{|\lambda_f|^2}{16\pi^2} \left[-2\Lambda_{UV}^2 + 6m_f^2 \ \ln(\Lambda_{UV}/m_f) + \ldots \right]$$

- Scalar Loop Contribution

$$\Delta m_H^2 = \frac{\lambda_S}{16\pi^2} \left[+\Lambda_{UV}^2 - 2m_S^2 \ \ln(\Lambda_{UV}/m_S) + \ldots \right]$$

Solving Gauge Hierarchy Problem-II

- $\Lambda_{UV} \sim M_{pl} \sim 10^{19}$ GeV vs. $m_H \sim 10^2$ GeV
$\rightarrow$ Technical Gauge Hierarchy Problem
- Dangerous Λ_{UV}^2 terms cancel, if $\lambda_S = |\lambda_f|^2$
- The result will be
$\Delta m_H^2 = m_{soft}^2 \left[\frac{\lambda}{16\pi^2} \ln(\Lambda_{UV}/m_{soft}) + ...\right]$
- m_{soft} cannot be too huge
- These two relations can be realized in **SUSY**
$*$ scalar quartic self couplings are related with Yukawa couplings
$*$ f and S have the same masses in SUSY limit

Why SUSY ?

- SUSY : FERMION $\leftrightarrow$ BOSON
- Maximal Symmetry of S-matrix in Rel. Local QFT with graded Lie algebra
  (Haag, Lopusansky and Sohnius)
- Can solve Technical Hierarchy Problem
- Better High Energy Behavior in SUSY QFT
- Low Energy Measurements of 3 Gauge Couplings + SUSY $\rightarrow$ SUSY GUT
- Cold dark matter if $R-$parity is conserved (Bonus)
- Essential in String Theories (quantum theory of gravity)
- Local SUSY (SUGRA) includes Gravity

Effetive Field Theory (EFT)

- Why EFT ?
- SM (Ren + Nonren) as an EFT
- EFT for Dark Matter Physics

Why EFT ? (weak coupling case)

- We don’t know what happens at energy higher than it is affordable
- High Energy physics can leave footprints in low energy regime, which can be adequately described by effective lagrangian with an infinite tower of local operators
- If new physics scale is much higher than experimental energy scale, the lowest dim nonrenormalizable operators will give the dominant corrections to the SM prdictions
  Fermi’s theory of weak interaction is a good example
- One can do meaningful phenomenology with a few number of unknown parameters
- Existing proof : the very most successful SM down to $r \lesssim 10^{-18}$ m
- In any case, we are living with EFT any way in real life

Why EFT ? (strong coupling case)

- In a strongly coupled theory such as QCD where nonperturbative aspects are very important, it is ususally very difficult to solve a problem
- Very often physical dof is different from fields in the lagrangian
(quarks and gluon vs. hadrons in QCD)
- Useful (often critical) to construct EFT based on the symmetries of the underlying strongly interacting theory, using the relevant dof only
- Most important to identify the relevant dof and relevant symmetries
- Many examples in QCD: chiral lagrangian [$+$ (axial) vector mesons, heavy hadrons], NRQCD for heavy quarkonium, HQET for heavy hadrons, SCET etc.

Naive Dimensional Analysis

- Natural Units in HEP:

$$c = \hbar = 1 \rightarrow [\vec{L} = \vec{r} \times \vec{p}] = 0$$

$$[L] = [T] = [\vec{p}]^{-1}$$

$$E = \sqrt{(pc)^2 + (mc^2)^2} \quad \longrightarrow \quad E = \sqrt{p^2 + m^2},$$

$$\text{QM Amp} \sim \int_{\text{path}} e^{iS/\hbar} \quad \longrightarrow \quad [\text{Action}] = 0 = [\int d^4x \mathcal{L}]$$

- $[E] = [p] = [M] = [L]^{-1} = [T]^{-1}$
- Everything will be in mass dimensions:

$$[\mathcal{L}] = 4, \;\; [\sigma(= \text{Area})] = -2, \;\; [\tau(= \Gamma^{-1})] = -1$$

- Both the decay rate ($\Gamma \equiv \tau^{-1}$) and the cross section (σ) are given by

  Fermi's Golden Rule

  with suitable flux facors

$$|\mathcal{M}|^2 \times \text{phase space} \left(\equiv \Pi_{i=1^n} \frac{d^3\vec{p}_i}{(2\pi)^3 2E_i}) \right) \times (2\pi)^4 \delta(\sum_i p_i - \sum_f p_f)$$

- Note that $[\Gamma] = +1$ and $[\sigma] = -2$
- It is often enough to do the dimensional analysis for Γ and σ, when there is only one important mass scale from the phase space integration
- A number of easy examples will be given in this lecture

Scalar fields

- Lagrangian for a real scalar field:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}\phi^2 - \mu\phi^3 - \frac{\lambda}{4}\phi^4 + \sum_{i=1}^{\infty}\frac{C_{4+i}}{\Lambda^i}\phi^{4+i}$$

- $[\partial] = +1, [\mathcal{L}] = 4 \rightarrow [\phi] = 1$
- $[m] = [\mu] = +1$ and $[\lambda] = [C_i] = 0$
- C_i terms are nonrenormalizable interaction terms ($\phi^{d>4}$: Irrelevant operators $\rightarrow$ Will discuss shortly)
- Field op ϕ create or annihilate a particle of mass m:

$$\phi \sim a(p)e^{-ip\cdot x} + a^\dagger(p)e^{+ip\cdot x}$$

- Complex scalar $\phi \sim a + b^\dagger$ with a and b relevant to particle and antiparticle

Fermion fields

- Lagrangian for fermion fields :

$$\mathcal{L} \;=\; \overline{\psi}(i\partial\cdot\gamma - m_\psi)\psi + \frac{C}{\Lambda^2}(\overline{\psi}\psi)^2 + ...$$

- $[\psi] = 3/2\ ,\ [m] = 1\ ,\ [C] = 0$
- C term: nonrenormalizable (irrelevant at low energy)
- Dirac field operator:

$$\begin{aligned} \psi &\sim bu + d^\dagger v \\ \overline{\psi} &\sim b^\dagger\overline{u} + d\overline{v} \end{aligned}$$

- Fermi's theory of weak interaction is the classic example

- Dimensional analysis for $\psi\overline{\psi}$ scattering

$$\mathcal{M}(\psi(p_1, s_1)\overline{\psi}(p_2, s_2) \to \psi(p_3, s_3)\overline{\psi}(p_4, s_4)) \sim \frac{1}{\Lambda^2}$$

$$\sigma \sim \left(\frac{1}{\Lambda^2}\right)^2 \times (phasespace) \sim \left(\frac{1}{\Lambda^2}\right)^2 \times s$$

- Mandelstam variables for $2 \to 2$ scattering:

$$s \equiv (p_1 + p_2)^2, t = (p_3 - p_1)^2, u = (p_4 - p_1)^2$$

$$s + t + u = \sum_{i=1}^{4} m_i^2$$

- Cross section becomes zero as $s \to 0$: Irrelevant

Unitarity Violation

- What happen at high energy ?

$$\sigma \to \infty \to$$

Violation of perturbative Unitarity near $\sqrt{s} \sim \Lambda/\sqrt{C}$
$\to$ New dof's will come into play for cure (e.g., $W^{\pm}$ or Z^0)

- This is the wonder of Nature with special relativity and quantum mechanics
- In the SM, the pointlike interaction is replaced by the $W^{\pm}, Z^0$ propagator, which cuts off the bad high energy behavior
- $\sigma \sim 1/s$ at very high energy scale $\sqrt{s} \gg m_{W,Z}$

Vector fields

- Lagrangian for abelian gauge field with a charged particle (QED):

$$
\begin{aligned}
\mathcal{L} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}(iD\cdot\gamma - m_\psi)\psi \\
F_{\mu\nu} &\equiv \partial_\mu A_\nu - \partial_\nu A_\mu \\
D_\mu\psi &\equiv (\partial_\mu + ieA_\mu)\,\psi
\end{aligned}
$$

- $[A_\mu] = 1, [F_{\mu\nu}] = 2, [e] = 0$
- Dimensionless coupling $e \to$ Renormalizable interaction (marginal operator, meaning that it is important at all energy scales)
- RG equation for e may run into a Landau pole, above which the coupling diverge $\to$ Either new theory before/around Landau pole, or low energy theory is free field theory

Renormalizable Opertors

- dim 0 : $I_{\rm op}$ (cosmological constant)
- dim 1 : S (scalar tadpole)
- dim 2 : S^2 , $A_\mu A^\mu$ (mass terms for bosons)
- dim 3 : $\overline{\psi}\psi$ (Fermion mass term) , S^3 (self interaction of singlet scalar)
- dim 4 : $S\overline{\psi}\psi$ (Yukawa interaction) , S^4 (Scalar self coupling) , A_μ^4 , $\partial_\mu A_\nu A^\mu A^\nu$ (self interactions of gauge fields)

NB: S, S^3 etc possible only for gauge singlet S

Some remarks on QFT

- QFT is the basic framework for particle physics, and is a marriage of QM and Special Relativity
- Spin-Statistics theorem
  - Bosons : totally symmetric wavefunction
  - Fermions : totally antisymmetric wavefunction
  - Intrinsic $P(B, F) = (+B, -F)$
- CPT is a symmetry of any local QFT
  $\to CP$ violation implies T (time-reversal) violation
- CPT theorem: $m_n = m_{\bar{n}}$ and $\tau_n = \tau_{\bar{n}}$, $\mu_n = \mu_{\bar{n}}$
- However, a partial width of n and $\bar{n}$ can be different $\to$ Direct CP Violation :

$$\Gamma(n \to f) \neq \Gamma(\bar{n} \to \bar{f})$$

- No renormalizable interactions possible for $s \geq 3/2$
  (Higher spin would be OK for composite particles)

Heavy Quarknia Quantum Numbers

- Bound State of spin-1/2 Q and $\bar{Q}$ with $^{2S+1}L_J$:

$$P = (-1)^{L+1}, \quad C = (-1)^{L+S} \to 0^{-+}, 1^{--}, 1^{++}, 1^{+-},$$

- Bound State of spin-0 Q and $\bar{Q}$ with $^{2S+1}L_J$ (with $S = 0$ and $L = J$):

$$P = (-1)^{L}, \quad C = (-1)^{L} \to 0^{++}, 1^{--}, 2^{++}, \text{etc.}$$

- No place for π (with 0^{-+})
- Observed J^{PC} clearly says that quarks are spin-1/2 fermions, not scalars
- Exotic mesons don't follow the above assigment

Effective Lagrangian Approach

- If new physics scale is high enough, it is legitimate to integrate out the heavy d.o.f.
- The low energy physics can be described in terms of effective lagrangian :

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm ren} + \sum_{d=5}^{\infty} \frac{\mathcal{O}^{(d)}}{\Lambda_d^{d-4}}$$

  where all the operators in $\mathcal{L}_{\rm eff}$ are made of light d.o.f. with their local gauge symmetries
- Effects of the nonrenormalizable operators $\sim (E/\Lambda_d)^{d-4}$ relative to the amplitude from $\mathcal{L}_{\rm ren}$
- EFT is useful, as long as $E \ll \Lambda_d$, since we can keep only a few of the NR operators

SM as an EFT: Below e^+e^- Threshold

- Only relevant quantum dof is photon A_μ
- If E increases, we need to include more and more NR operators
- Eventually, unitarity will be broken $\rightarrow$ We have to include new d.o.f.'s in the EFT, and redefine the EFT with more d.o.f.
- QED at $E \ll 2m_e$: A_μ, local $U(1)$ and P, C

$$\mathcal{L}_{\mathrm{EET}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{e^4}{(4\pi)^2 \Lambda^4} F^4 + \ldots$$

  where $\Lambda \sim m_e$
- This effective lagrangian describes $\gamma\gamma$ scattering, the cross section of which will break unitarity when E reaches $2m_e$

SM as an EFT: Below e^+e^- Threshold

- The cross section grows like $\sim s^3$:

$$\sigma(\gamma\gamma \to \gamma\gamma) \sim \frac{e^8}{\Lambda^8} s^3$$

  and see at which energy scale unitarity is violated
- Unitarity will be restored due to a new process that opens up: $\gamma\gamma \to e^+e^-$
- One has to redefine the effective lagrangian near e^+e^- threshold, by including the electron/positron fields explicitly

Digress on Unitarity

- Unitarity is the most profound thing in QM
- Scattering Operator S is unitary:

$$\langle f|S|i\rangle = S_{fi} = \delta_{fi} + i(2\pi)^4\delta^4(p_i - p_f)T_{fi}$$

- Unitarity: $S^\dagger S = SS^\dagger = 1$

$$T_{fi} - T^*_{fi} = i(2\pi)^2 \sum_n \delta^4(p_f - f_n)T_{fn}T^*_{in}$$

- If interaction is weak, we can ignore the RH $\rightarrow$ T becomes Hermitian $T_{fi} = T^*_{if}$
- Optical theorem for $f = i$:

$$2\mathrm{Im}T_{ii} = (2\pi)^4 \sum_n |T_{in}|^2\delta^4(P_i - P_n)$$

Rayleigh Scattering: Why is Sky Blue ?

- Photon scattering with neutral atom A where

$$E_\gamma \ll \Delta E_{n1} \equiv E_n - E_1$$

$\rightarrow$ Elastic scattering of light on neutral atoms

- Atom is described by nonrelativistic Schrödinger wave function ψ_A with dim 3/2:

$$\mathcal{L} = \psi_A^\dagger \left(i\frac{\partial}{\partial t} - H \right) \psi_A + \frac{e^2}{\Lambda^3} \psi_A^\dagger \psi_A F_{\mu\nu} F^{\mu\nu} + \ldots.$$

- $\Lambda \sim \Delta E_{21}, r_0$??
- Note that photon couples to a neutral atom. How ???

- No coupling of photon to neutral objects only at renormalizable level
- Photon couples to neutral particle at nonrenormalizable level due to quantum fluctuation can involve charged particles in the loop
- Likewise, gluons can couple to photons
- γA scattering cross section :

$$\sigma(\gamma A \to \gamma A) \sim \frac{e^4}{\Lambda^6} E_\gamma^4$$

  for $E_\gamma \ll \Delta E_{2,1}$
- Blue light scatters more than red light $\to$ Sky is blue, and we can enjoy the beautiful sunrise/sunset in red

Van der Waals Force

- Potential between neutral atoms are described by two-photon exchange diagrams from the previous lagrangian $\psi_A^\dagger \psi_A F^2$
- Additional contact interaction has to be considered:

$$\frac{1}{\Lambda^2}\left(\psi_A^\dagger \psi_A\right)^2$$

- Calculate the two contributions and discuss what is the form of the force between two neutral atoms (Van der Waals interaction) ?
- What is a in the exponent in $V(r) \sim r^a$?
- What if we consider the neutral atom relativistically ?

QED as an EFT below $\mu^+\mu^-$ threshold

- QED at $2m_e \leq E \ll 2m_\mu$: $A_m u$, e, $\bar{e}$, local $U(1)$ and P, C

$$
\begin{aligned}
\mathcal{L}_{\text{Eff}} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{e}(iD - m_e)e \\
&+ \frac{e^4}{(4\pi)^2\Lambda_1^4}F^4 + \frac{e}{(4\pi)^2\Lambda_2}\overline{e}\sigma^{\mu\nu}eF_{\mu\nu}
\end{aligned}
$$

  where $\Lambda_1 \sim m_\mu$, and $\Lambda_{2,3} \sim O(1)$ TeV or larger (see later discussions on these points)

- NP scale in each NR operator is independent (different from each other) in general, since the origin can be different
- Scale for F^4 is now $\sim m_\mu$, unlike the previous case

QED as an EFT below $\mu^+\mu^-$ threshold

- Additional $1/(4\pi)^2$ suppression for NR operators generated at one-loop level, compared with NR operators generated at tree level, even if their operator dim's are the same
- If we impose $SU(2)_L \times U(1)_Y$ instead of $U(1)_{\rm em}$, the Λ_2 term should be replaced by

$$\frac{e}{(4\pi)^2\Lambda_2^2}\overline{e_L}\sigma^{\mu\nu}He_RF_{\mu\nu} \to \frac{ev}{\sqrt{2}(4\pi)^2\Lambda_2^2}\overline{e_L}\sigma^{\mu\nu}e_RF_{\mu\nu}$$

and the effect becomes smaller for the same Λ_2, or the bound on Λ_2 becomes stronger

QED as an EFT above $\mu^+\mu^-$ threshold

- QED at $E \ll 2m_\pi$: A_μ, e, $\bar{e}$, μ, $\bar{\mu}$, local $U(1)$ and P, C

$$
\begin{aligned}
\mathcal{L}_{\rm Eff} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{e}(iD - m_e)e + \overline{\mu}(iD - m_\mu)\mu \\
&+ \frac{e^4}{(4\pi)^2\Lambda_1^4}F^4 + \frac{e}{(4\pi)^2\Lambda_2}\overline{e}\sigma^{\mu\nu}eF_{\mu\nu} + \frac{e}{(4\pi)^2\Lambda_3}\overline{\mu}\sigma^{\mu\nu}\mu F_{\mu\nu} \\
&+ \frac{e}{(4\pi)^2\Lambda_4}\overline{e}\sigma^{\mu\nu}\mu F_{\mu\nu} + \frac{e^2}{\Lambda_5^2}(\overline{e}e)(\overline{e}\mu) + H.c.
\end{aligned}
$$

where $\Lambda_1 \sim m_\pi$, $\Lambda_{2,3} \gtrsim XX$ TeV , and $\Lambda_{4,5} \gtrsim$ TeV or larger (see later discussions on these points)