Muon (g-2)

P. Ko (KIAS)

What is the (anomalous) magnetic moment ?

– Magnetic Dipole Moment in Classical E&M –

- General Physics : Current Loop $\rightarrow \vec{\mu} = IA\vec{n}$

- Slowly varying EM field couples to

  Total electric charge $Q = \int d^3x\ \rho(\vec{x})$

  Quadratic moment Q_{ij} :

$$Q_{ij} = \int d^3x\ \rho(\vec{x})(x_i x_j - \frac{1}{3}\ \delta_{ij}\vec{x}^2)$$

  Magnetic dipole moment $\vec{\mu}$

- Low Energy Theorem

– Magnetic Moment in QFT –

- Fundamental Properties of a particle :
  - Mass M
  - Spin s : Bosons and Fermions with Diff. Statistics
  - Charge Q : integral multiples of e_p
  - Magnetic Dipole Moment (MDM) μ
- MDM vs. Spin

$$\vec{\mu} = g\frac{e\hbar}{2mc}\ \vec{s}$$

- Pointlike Dirac particle : $g = 2$ (e.g., electron, muon, quark etc.)

cf. $g = 2.79 \ \ (-1.86)$ for proton (neutron)

$\rightarrow$ Indicate p and n are composite particles

- Quantum Corrections modifies g at loop levels.

$$a_\mu \equiv \left(\frac{g-2}{2}\right)_\mu$$

$\rightarrow$ Anomalous Magnetic Dipole Moment

  - Theoretically well understood

    (except for the hadronic contibutions)

  - Measured with high precisions
  - Indirect Probe of New Physics Beyond the SM

– How to measure it ? –

- Relativistic Muon in a Ring with $\vec{B}$ field
- Cyclotron Frequency :

$$\omega_c = \frac{eB}{mc\gamma} = \frac{2\mu_B B}{\hbar\gamma}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}, \qquad \beta = |\vec{\beta}| = |\frac{\vec{v}}{c}|$$

- Spin Precession Frequency (BMT equation) :

  [V. Bargmann, L. Michel and V.L. Telegdi,

  Phys. Rev. Lett. **2**, 435 (1959)]

$$\omega_s = \frac{eB}{mc\gamma}\left[\,1 + \left(\frac{g-2}{2}\right)\,\gamma\,\right]$$

- Additional Precession due to $\vec{E}$ field

$$\Delta\omega_a = \frac{e}{mc}\,\left(\frac{1}{\gamma^2-1} - a\right)\vec{\beta}\times\vec{E}$$

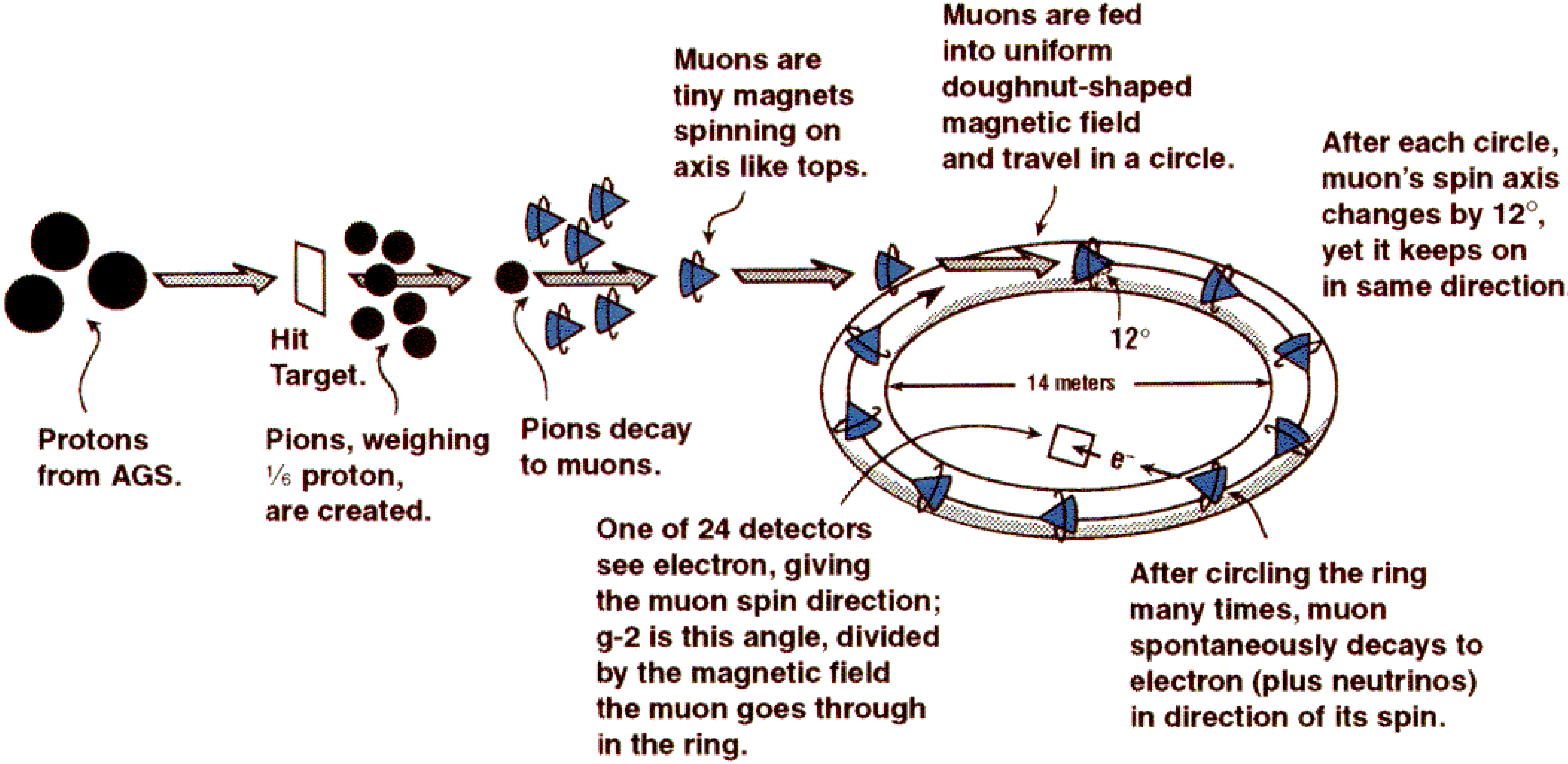
  $\rightarrow$ Can be made zero by choosing $\gamma^2 = 1 + 1/a$

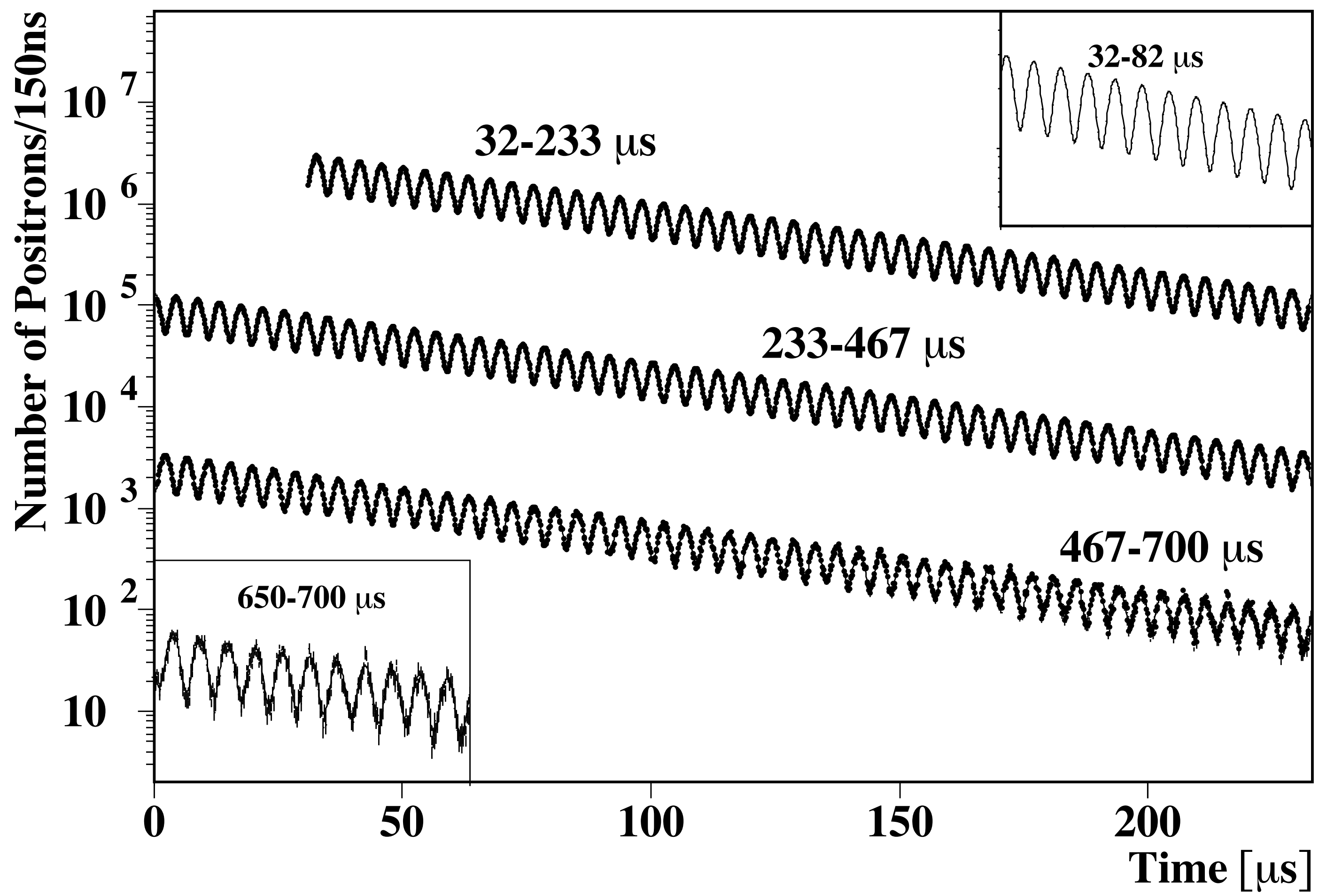
  Magic Muon Momentum

  3.1GeV/c, a.k.a. magic momentum, with $\gamma \approx 29.3$

- Muon decay distribution carries the informations of the muon polarization
  - $(V - A) \times (V - A)$ Nature of Muon Decay
  - The highest energy electron tends to come along the muon polarization

LIFE OF A MUON:
THE g-2 EXPERIMENT




Number of Positrons/150ns
10 7
10 6
10 5
10 4
10 3
10 2
10
0
50
100
150
200
Time [μs]
32-233 μs
233-467 μs
467-700 μs
32-82 μs
650-700 μs

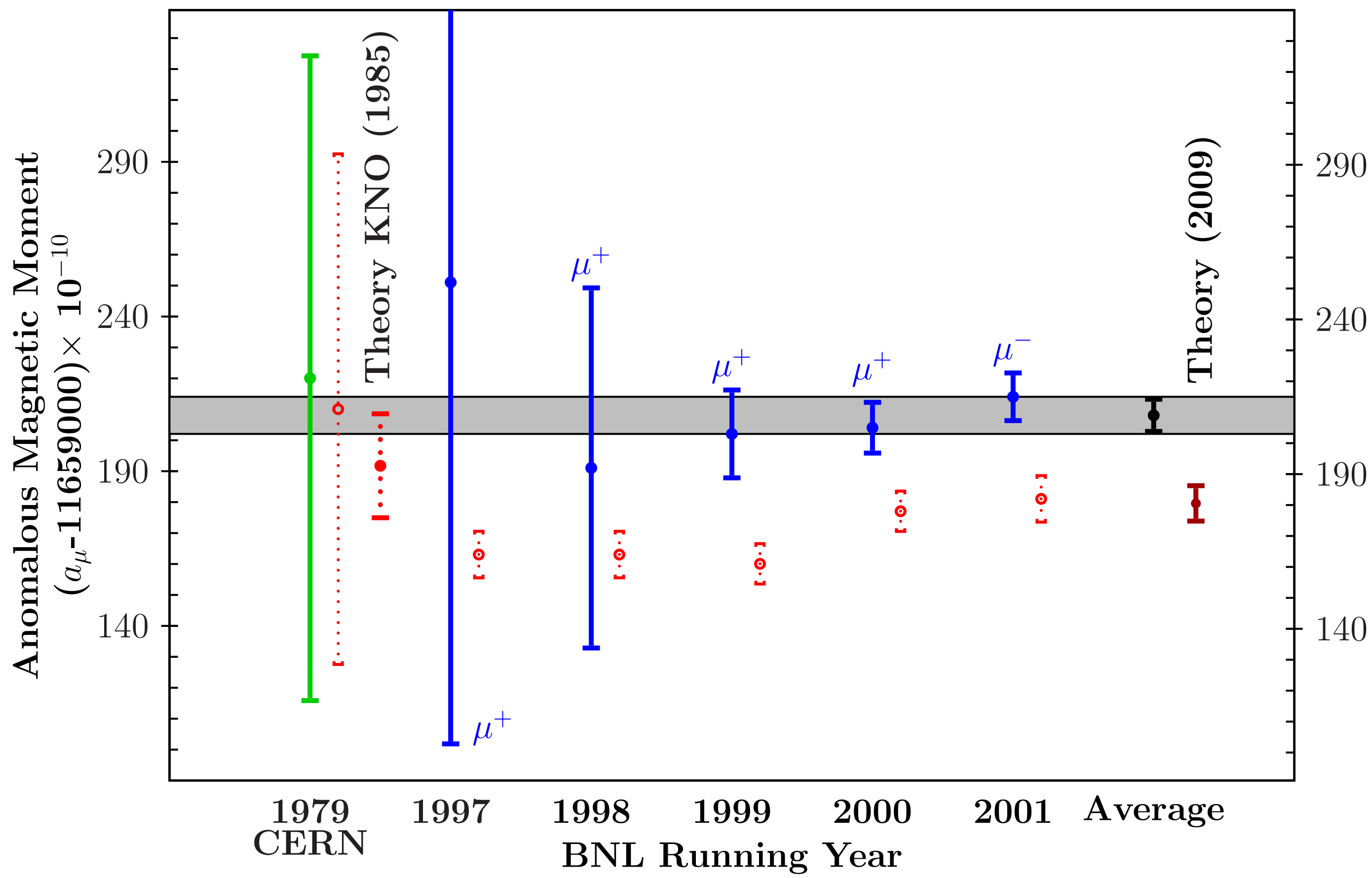


Fig. 7. Results for the individual E821 measurements, together with the new world average and the theoretical prediction. The CERN result is shown together with the theoretical prediction by Kinoshita et al. 1985, at about the time when the E821 project was proposed.The dotted vertical bars indicate the theory values quoted by the experiments.

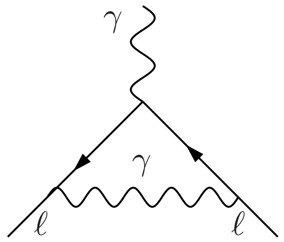


Fig. 8. The universal lowest order QED contribution to a_ℓ.

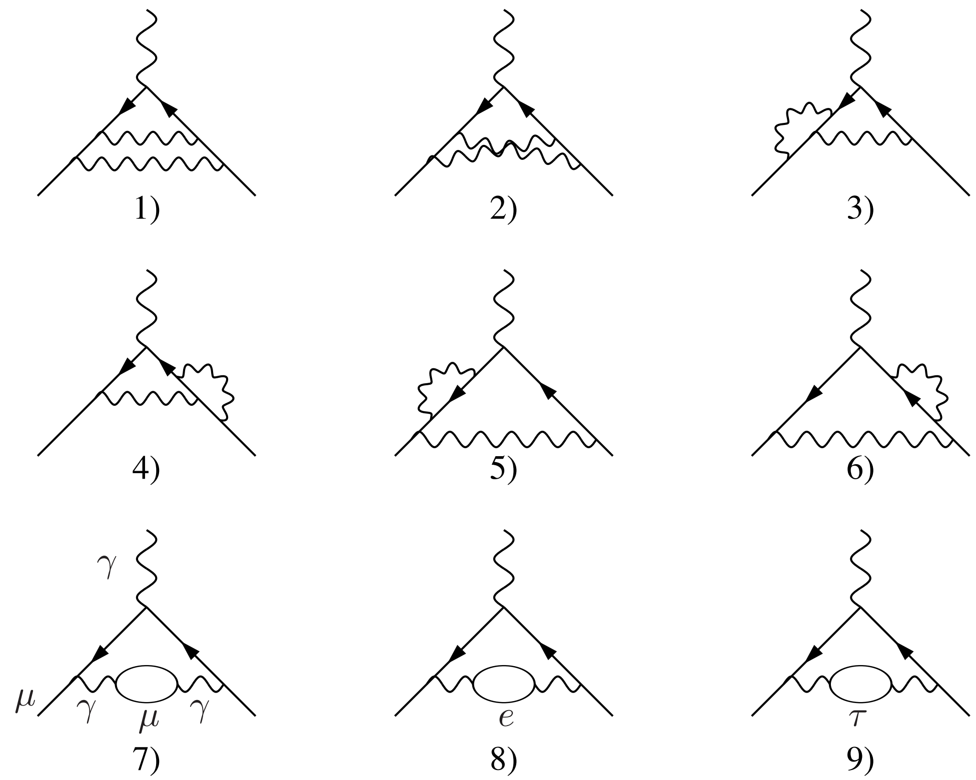


Fig. 9. Diagrams 1-7 represent the universal second order contribution to a_μ, diagram 8 yields the "light", diagram 9 the "heavy" mass dependent corrections.

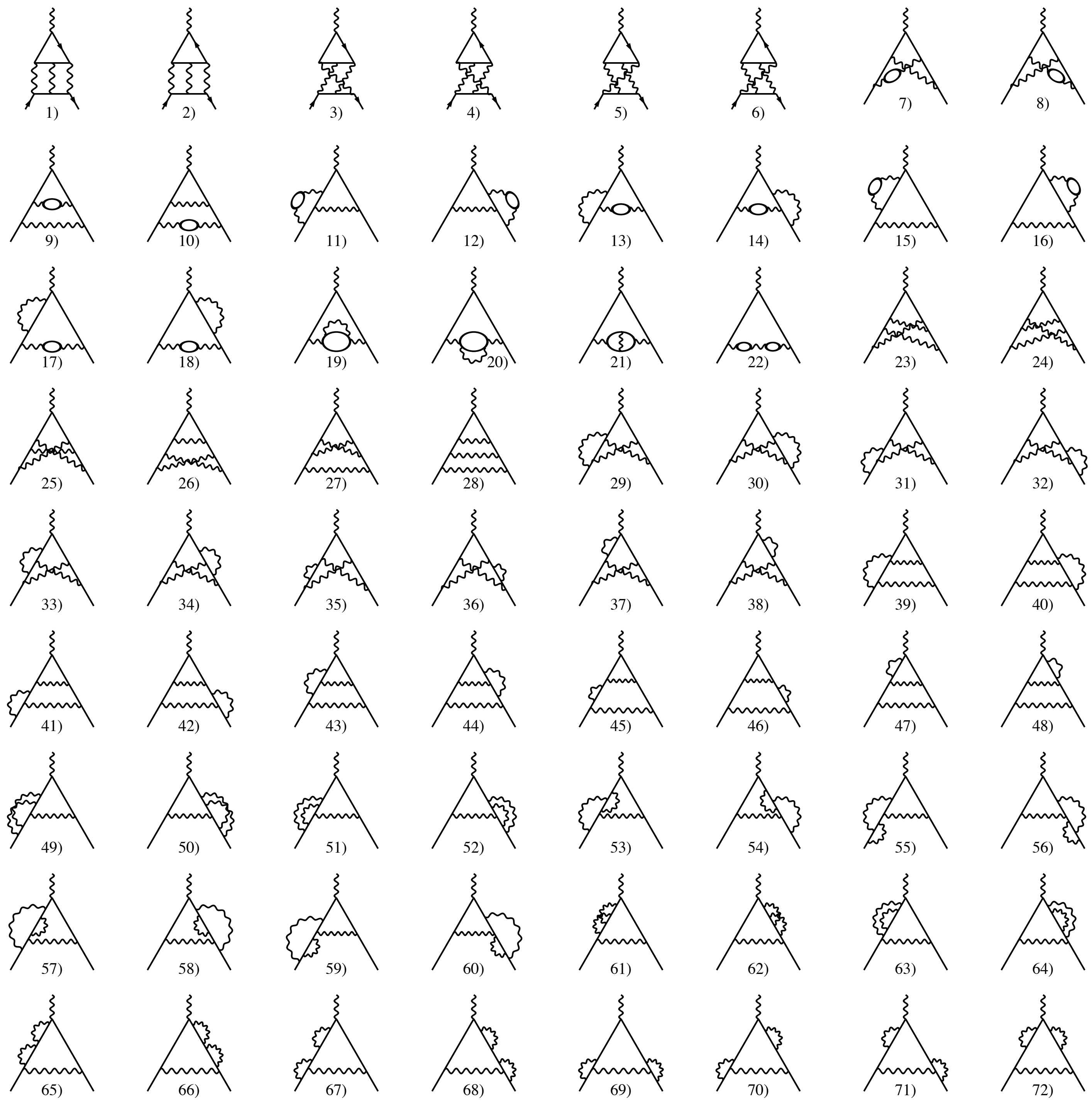


Fig. 10. The universal third order contribution to a_μ. All fermion loops here are muon–loops. Graphs 1) to 6) are the light–by—light scattering diagrams. Graphs 7) to 22) include photon vacuum polarization insertions. All non–universal contributions follow by replacing at least one muon in a closed loop by some other fermion.

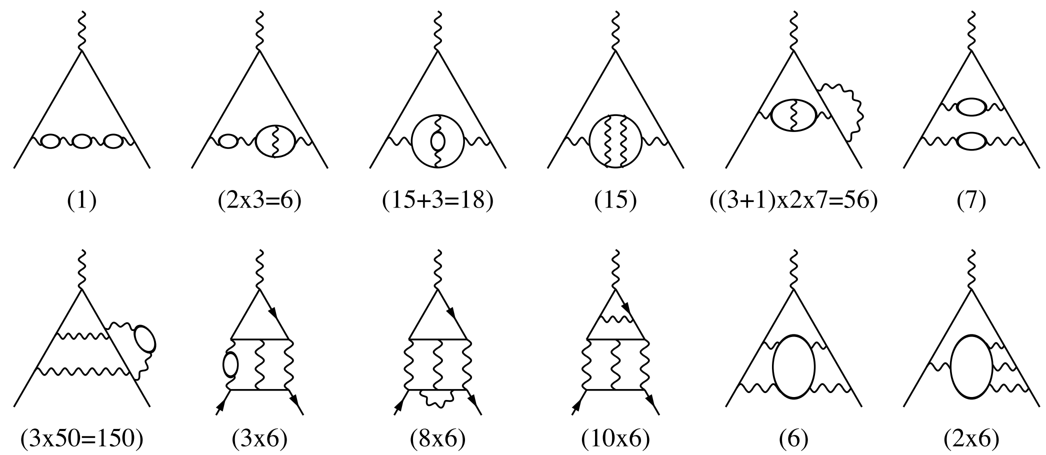


Fig. 11. Some typical eight order contributions to a_ℓ involving lepton loops. In brackets the number of diagrams of a given type if only muon loops are considered. The latter contribute to the universal part.

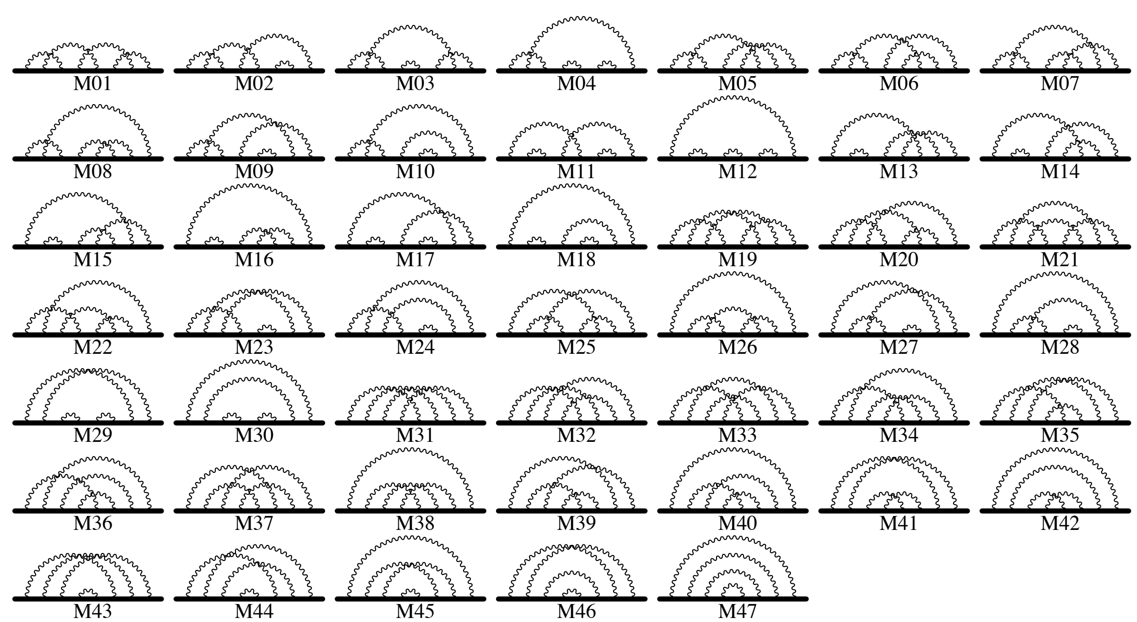


Fig. 12. 4-loop Group V diagrams. 47 self-energy-like diagrams of $M_{01} - M_{47}$ represent 518 vertex diagrams [by inserting the external photon vertex on the virtual muon lines in all possible ways]. Reprinted with permission from [108]. Copyright (2007) by the American Physical Society].

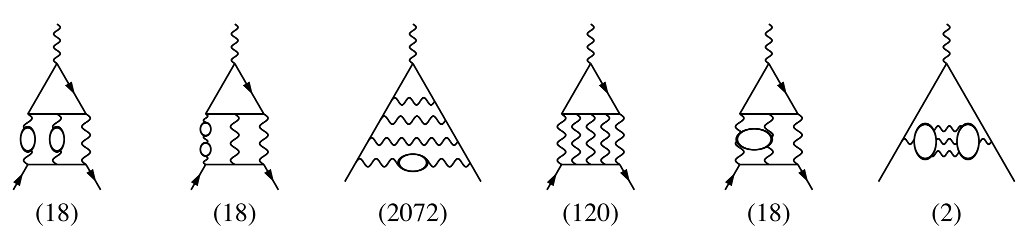


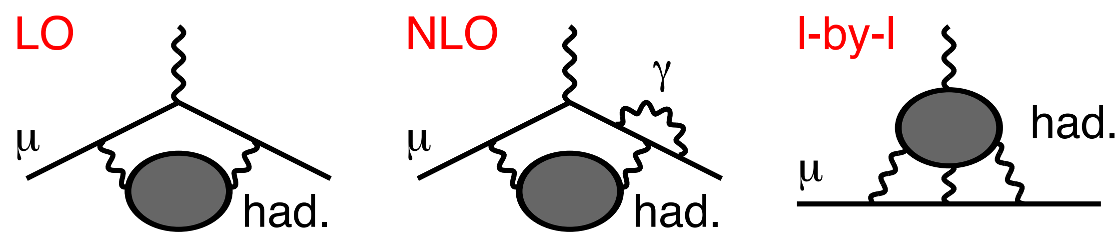
Fig. 13. Typical tenth order contributions to a_ℓ including fermion loops. In brackets the number of diagrams of the given type.

Standard Model Prediction for Muon $g-2$

QED contribution	11 658 471.809 (0.016) $\times 10^{-10}$	Kinoshita & Nio
EW contrib.	15.4 (0.2) $\times 10^{-10}$	Czarnecki et al
Hadronic contrib.		
LO hadronic	**689.4 (4.0)** $\times 10^{-10}$	HLMNT09
NLO hadronic	-9.8 (0.1) $\times 10^{-10}$	HLMNT09
light-by-light	10.5 (2.6) $\times 10^{-10}$	Prades, de Rafael & Vainshtein
Theory TOTAL	**11 659 177.3 (4.8)** $\times 10^{-10}$	
Experiment	**11 659 208.9 (6.3)** $\times 10^{-10}$	world avg
Exp − Theory	**31.6 (7.9)** $\times 10^{-10}$	**4.0 σ discrepancy**

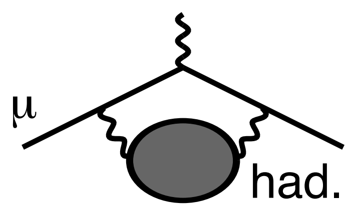
(Numbers taken from HLMNT09 (`arXiv:1001.5401`))

n.b.: hadronic contributions:

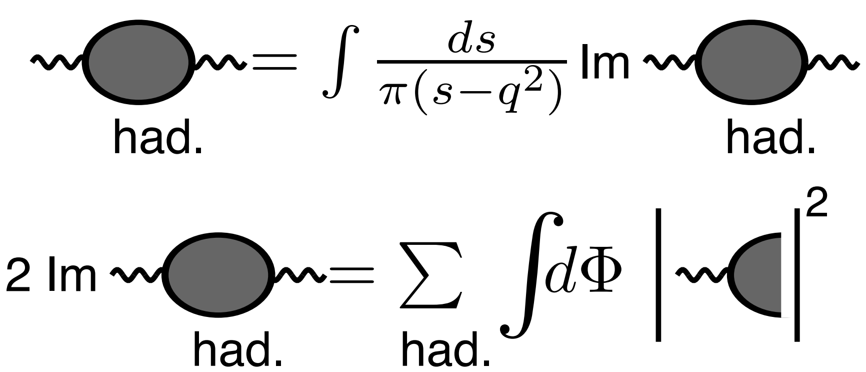


LO Hadronic Contribution

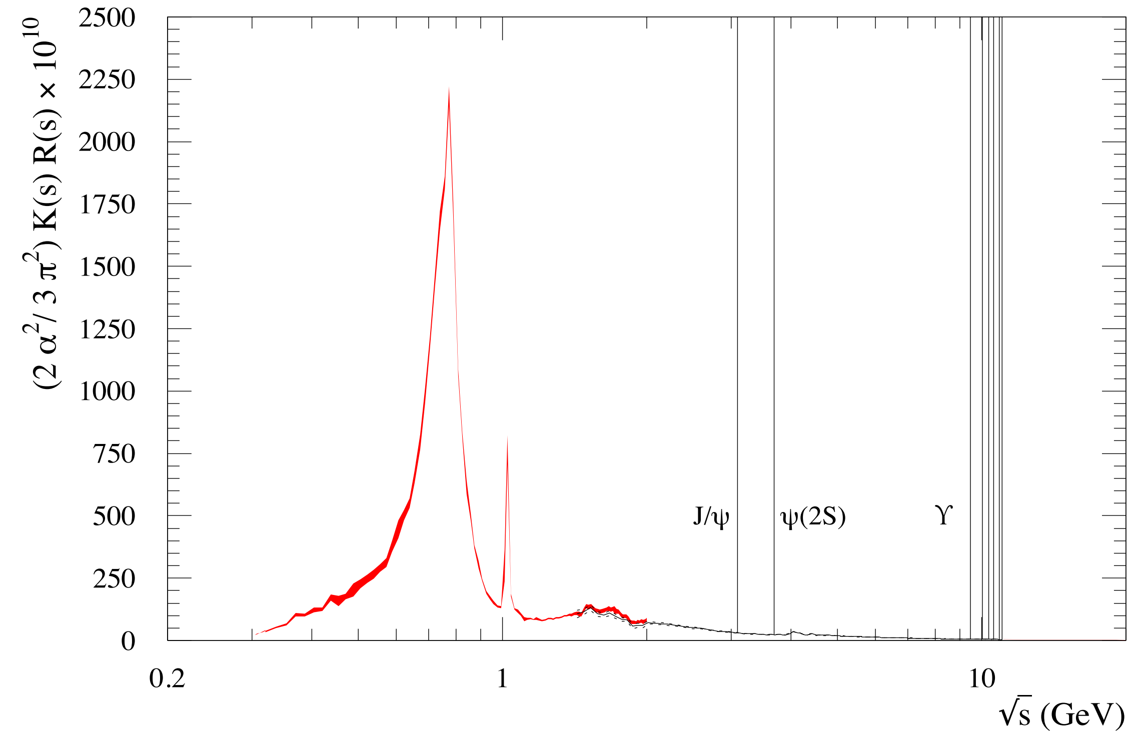
The diagram to be evaluated:



pQCD not useful. Use the dispersion relation and the optical theorem.

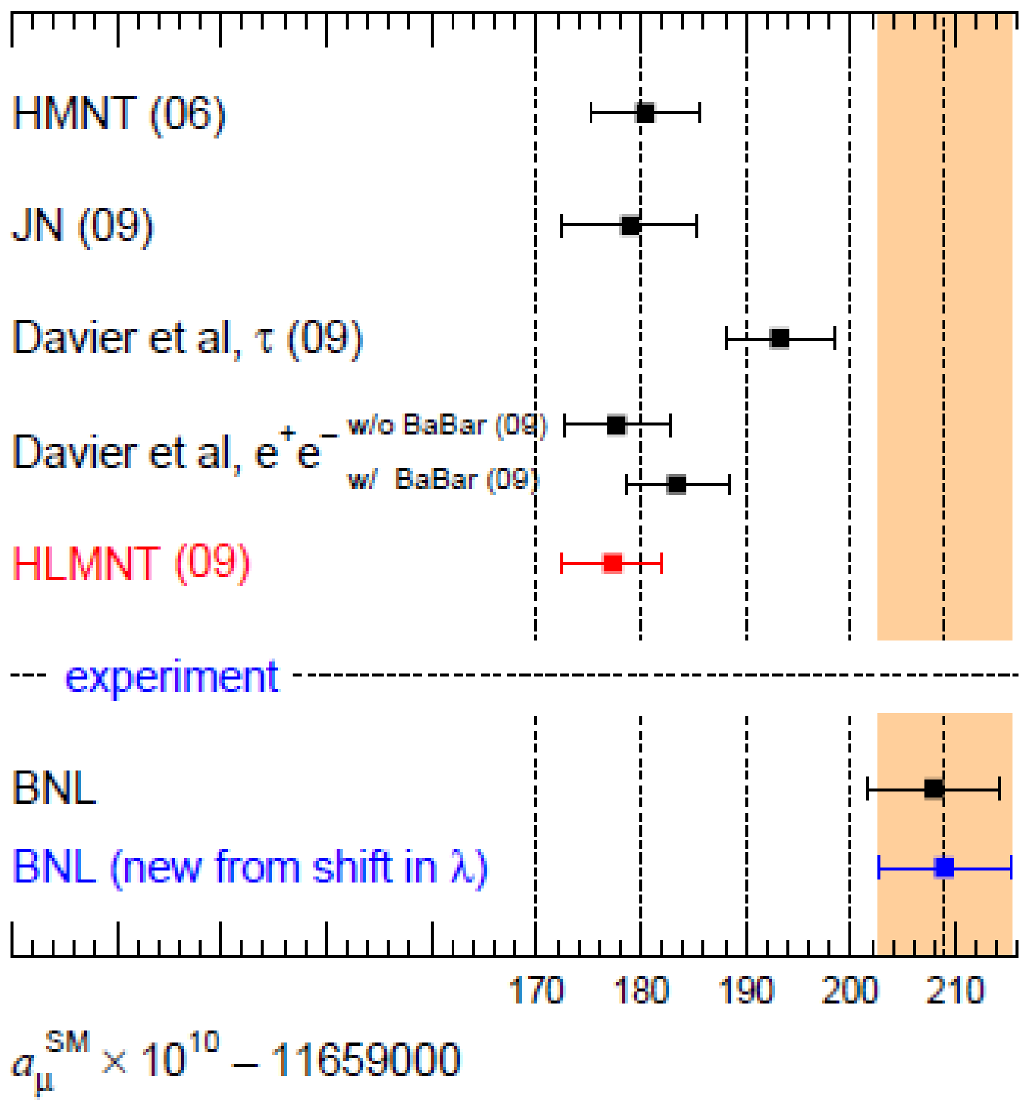


$$\boxed{a_\mu^{\mathrm{had,LO}} = \frac{m_\mu^2}{12\pi^3}\int_{s_{\mathrm{th}}}^{\infty} ds\ \frac{1}{s}\hat{K}(s)\sigma_{\mathrm{had}}(s)}$$



- Weight function $\hat{K}(s)/s = \mathcal{O}(1)/s \Longrightarrow$ Lower energies more important
- We have to rely on exp. data for $\sigma_{\mathrm{had}}(s)$ $\Longrightarrow$ Good data crucial

$a_\mu^{\rm SM}$ compared to BNL world av.



Davier et al.: $1.8/3.9/3.1\,\sigma$

JN 09: $3.2\,\sigma$ $[179.0 \pm 6.5]$

Recent changes

TH: Improved LO hadronic (from e^+e^-):

[New data from CMD-2, SND, KLOE, BaBar, CLEO, BES. Combination of excl. (BaBar RadRet) and incl. data below 2 GeV.]

$(6894 \pm 46) \cdot 10^{-11} \longrightarrow (6894 \pm 40) \cdot 10^{-11}$

TH: Use of recent L-by-L compilation [PdeRV]: $a_\mu^{\rm L-by-L} = (10.5 \pm 2.6) \cdot 10^{-10}$

EXP: Small shift of BNL's value due to CODATA's shift of muon to proton magn. moment ratio:

Was $a_\mu = 116\ 592\ 080(63) \times 10^{-11}$

$\rightarrow a_\mu = 116\ 592\ 089(63) \times 10^{-11}$ $(0.5 ppm)$

▶ With this input HLMNT get:

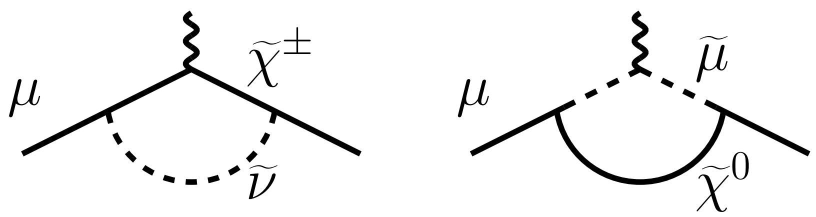
$a_\mu^{\rm EXP} - a_\mu^{\rm TH} = (31.6 \pm 7.9) \cdot 10^{-10}$, $\sim 4.0\sigma$

T.Teubner, talk at Phipsi09, Oct, 2009

SUSY Contributions?

Is the 4.0σ deviation due to SUSY?

Dominant **SUSY contributions**:



which is, very roughly, given by

$$a_\mu^{\mathrm{SUSY}} = (\mathrm{sgn}\ \mu)\frac{\alpha(M_Z)}{8\pi \sin^2\theta_W}\frac{m_\mu^2}{\widetilde{m}^2}\tan\beta,$$

where $\widetilde{m}$ is the SUSY scale.

Numerically,

$$a_\mu^{\mathrm{SUSY}} = (\mathrm{sgn}\mu) \times 13 \times 10^{-10} \times \left(\frac{100\mathrm{GeV}}{\widetilde{m}}\right)^2 \tan\beta$$

In order for this to be $15.8 \le a_\mu^{\mathrm{SUSY}} \times 10^{10} \le 47.4$ (2 σ range),

$$\boxed{\widetilde{m} = \mathbf{170} - \mathbf{640}\ \mathrm{GeV}}$$

for $\tan\beta = 10 - 50$. (**Rough estimates**)

Summary

- muon g-2 : Competition between Th and Exp, one of the best precision test of the SM
- Stringent tests for Lorentz tranformation (time dilation)