

Electroweak interaction

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1. Basics and history
2. The Standard Model
3. Tests of the Standard Model



Lecture 3

Tests of the Standard Model

1. Introduction
2. The electroweak observables, definitions and measurements
3. Standard Model calculations beyond tree-level
4. Tests of the Standard Model

The pre-LEP era

- SM tree-level relations between g , G_F and α lead to: \times

$$M_W = \sqrt{\frac{\pi\alpha}{\sqrt{2}G_F}} \frac{1}{\sin\vartheta_W} \quad M_Z = \sqrt{\frac{\pi\alpha}{\sqrt{2}G_F}} \frac{1}{\sin\vartheta_W \cos\vartheta_W}$$

- Comparison with experimental data, 1/2:

$$G_F = 1.16637(1) \cdot 10^{-5} \text{ GeV}^{-2} \quad \Rightarrow \quad \sqrt{\frac{\pi\alpha}{\sqrt{2}G_F}} = 37.2805 \text{ GeV}$$

$$\alpha^{-1} = 137.035999679(94)$$

- before the W and Z boson discovery:

DIS, 1982 $\sin^2\vartheta_W = 0.229 \pm 0.010$

\Rightarrow predictions:

$$M_W = 77.9 \pm 1.7 \text{ GeV}$$

$$M_Z = 88.7 \pm 1.4 \text{ GeV}$$

- first mass measurements (UA1 & UA2, 1983):

$$M_W = 81 \pm 5 \text{ GeV}$$

$$M_Z = 91.9 \pm 1.9 \text{ GeV}$$

agreement

The pre-LEP era

- o Comparison with experimental data, 2/2:
 - o final mass measurements by the UA experiments (1989/92):

$$\begin{aligned} M_W &= 80.84 \pm 0.22 \pm 0.81 \pm 0.17 \text{ GeV} \\ M_Z &= 91.74 \pm 0.28 \pm 0.92 \pm 0.12 \text{ GeV} \end{aligned} \quad (1\%)$$

⇒ hence the prediction:

$$\sin^2 \vartheta_W = 1 - M_W^2 / M_Z^2 = 0.2234 \pm 0.007 \quad (3\%)$$

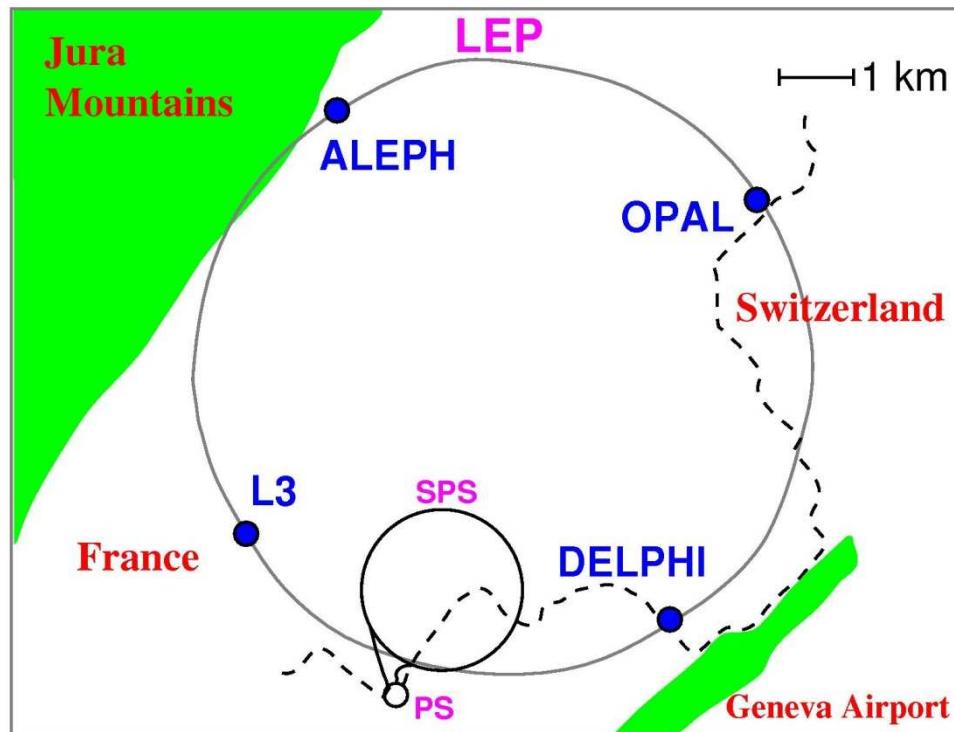
- o DIS, 1990:

$$\sin^2 \vartheta_W = 0.2309 \pm 0.006 \quad (2.5\%)$$

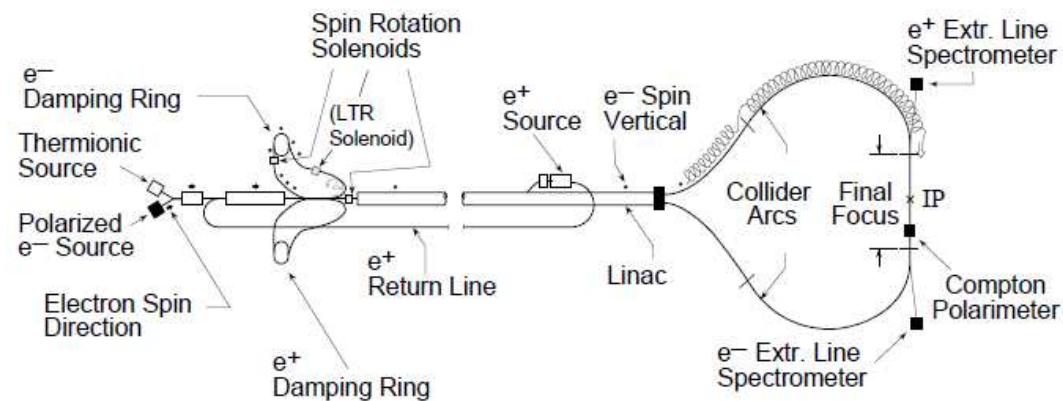
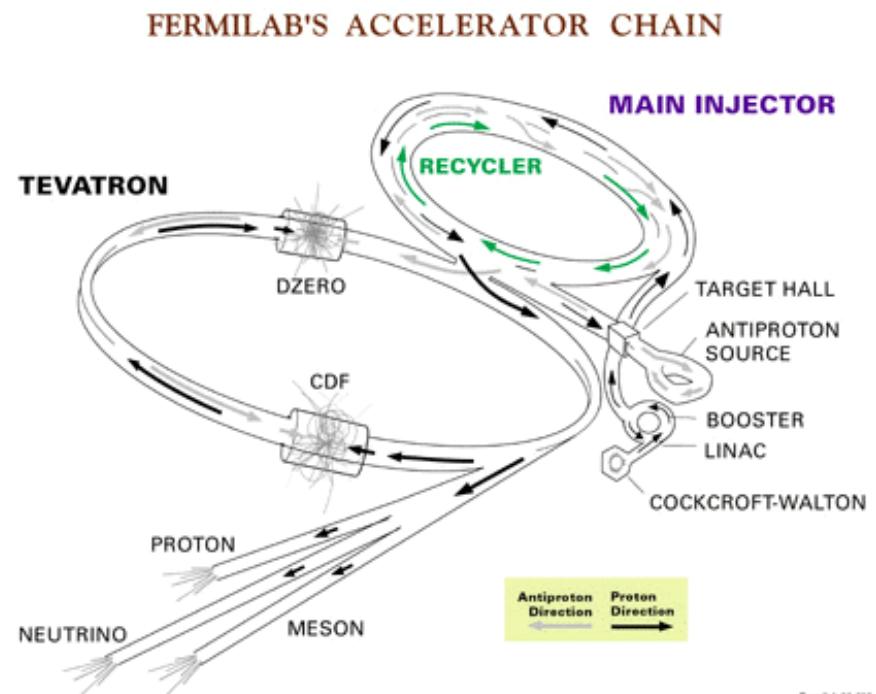
- ⇒ The tree-level predictions of the SM of electroweak interactions are confirmed experimentally, with a precision of a few %
⇒ From 1989 onwards: go to higher precision (experience + theory)

LEP, SLC, Tevatron

LEP 1989-2000 e^+e^- 91Gev, 130-208GeV



Tevatron 1987-2011 $p\bar{p}$ ~2TeV



SLC 1988-1998 e^+e^- 91GeV

Now, EW precision
physics carried over
at LHC

The era of precision tests

- Most precise mass measurements:

$$M_W = 80.399 \pm 0.023 \text{ GeV} \quad (0.03\%) \quad (\text{LEP2, Tevatron})$$

$$M_Z = 91.1875 \pm 0.0021 \text{ GeV} \quad (0.0023\%) \quad (\text{LEP1, final})$$

- SM tree-level predictions: X

$$M_W = \sqrt{\frac{\pi\alpha}{\sqrt{2}G_F}} \frac{1}{\sin\vartheta_W} = \frac{M_Z}{\sqrt{2}} \left[1 + \sqrt{1 - \frac{4\pi\alpha}{\sqrt{2}G_F M_z^2}} \right]^{\frac{1}{2}} = \frac{M_Z}{\sqrt{2}} \left[1 + \sqrt{1 - \frac{4(37.2805)^2}{M_z^2}} \right]^{\frac{1}{2}}$$

⇒ prediction:

$$M_W = 80.939 \pm 0.0026 \text{ GeV}$$

⇒ strong disagreement with data : higher orders to be included !

2. Observables

The W mass

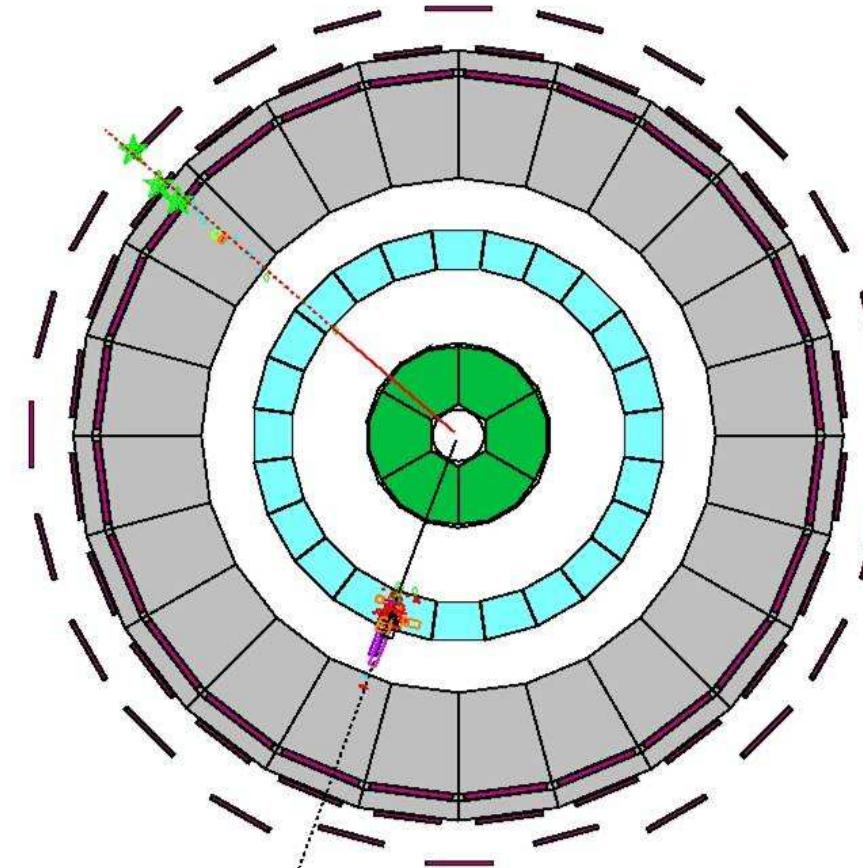
- o SM tree-level prediction:

$$M_W = \sqrt{\frac{\pi\alpha}{\sqrt{2}G_F}} \frac{1}{\sin\vartheta_W}$$

$$= \frac{M_Z}{\sqrt{2}} \left[1 + \sqrt{1 - \frac{4\pi\alpha}{\sqrt{2}G_F M_Z^2}} \right]^{\frac{1}{2}}$$

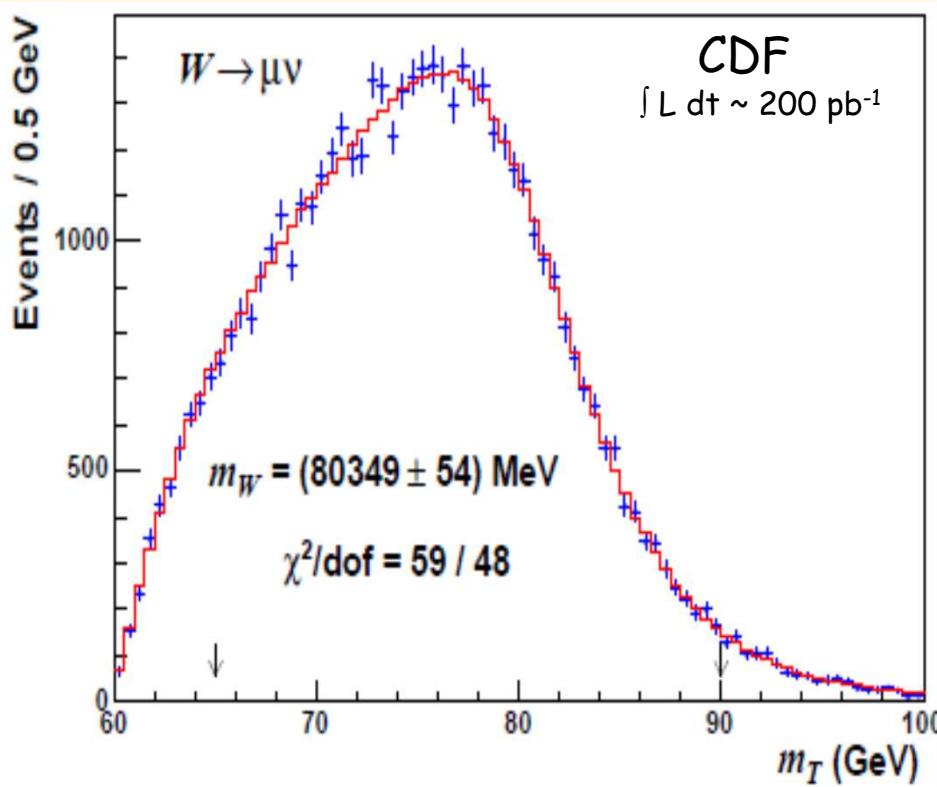
- o W experimental production:
 - Tevatron: W events
 - LEP2: WW pairs
- o W mass measurement:
 - from reconstructed event final-state kinematic properties
 - from $\sigma_{WW}(\sqrt{S})$ at LEP2

LEP: $e^+e^- \rightarrow W^+W^- \rightarrow e^+\nu\mu^-\bar{\nu}$



- W mass measurement: main method is that using distributions of reconstructed W mass estimators

Tevatron: $p\bar{p} \rightarrow W \rightarrow \mu\nu$



W boson mass (GeV)

TEVATRON $M_W = 80.420 \pm 0.031$

LEP2 $M_W = 80.376 \pm 0.033$

Average $M_W = 80.399 \pm 0.023$

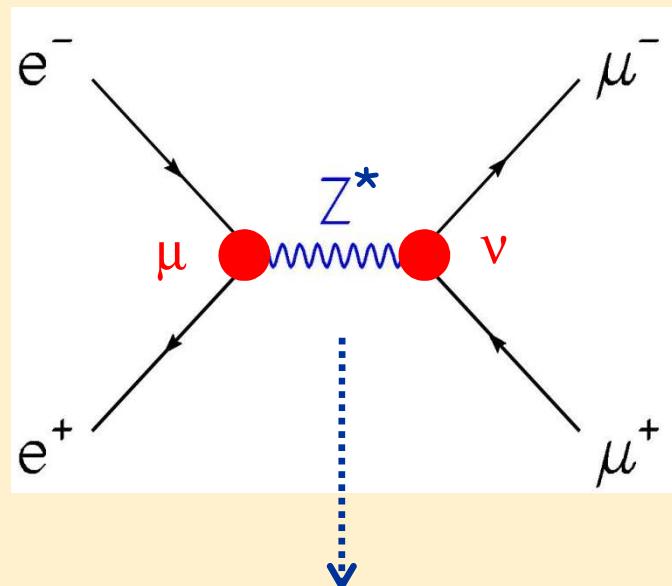
$(\chi^2 / \text{DoF} : 0.9 / 1)$

LEP EWWG-2011

'First run II measurement of the W boson mass',
Phys. Rev. D 77, 112001 (2008).

The Z mass and lineshape

- Most precisely measured in e^+e^- collisions at $\sqrt{S} \sim 91$ GeV
- SM tree-level prediction for $e^+e^- \rightarrow Z^* \rightarrow \text{two fermions}$



$$-i \frac{g_{\mu\nu} - k_\mu k_\nu / M_Z^2}{k^2 - M_Z^2 + i \Gamma_Z M_Z}$$

reference mass total width

$\bullet = -i \frac{g}{\cos \vartheta_W} \gamma^\mu \left(\frac{g_v^f - g_a^f \gamma_5}{2} \right)$

$$g_v^f = T_f^3 - 2Q^f \sin^2 \vartheta_W \Rightarrow g_v^{e,\mu,\tau} \approx -0.04$$

$$g_a^f = T_f^3 \Rightarrow g_a^{e,\mu,\tau} = -1/2$$

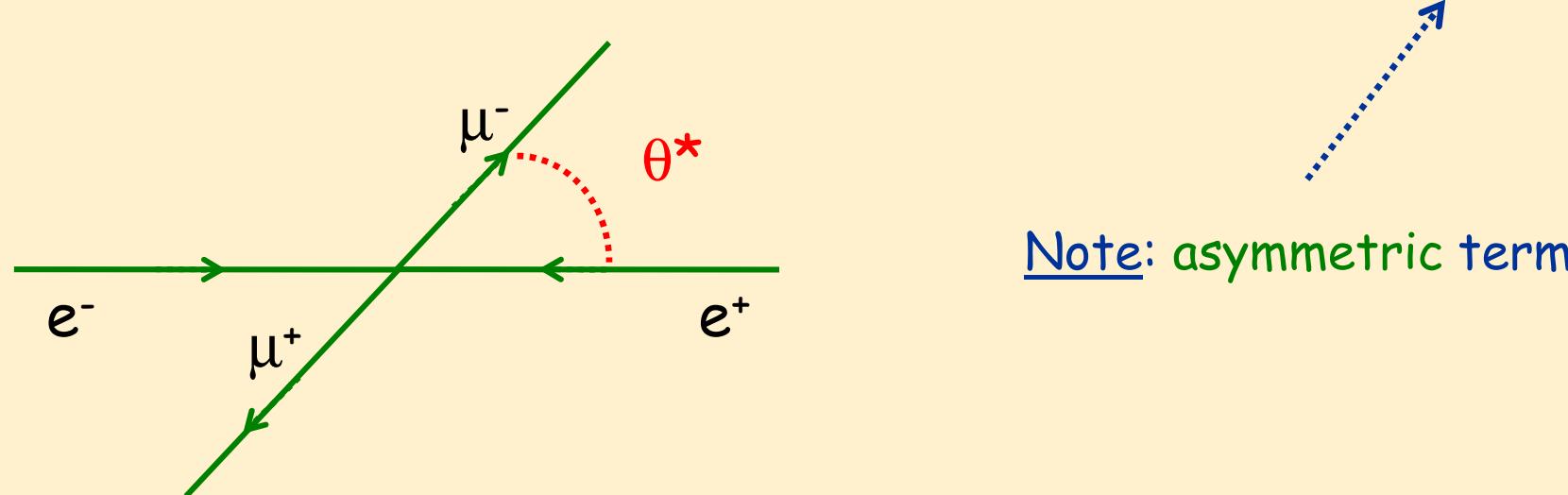
= Propagator in unitary gauge
& Breit-Wigner approximation

Note: $k^2 = (M_{Z^*})^2 = (\sqrt{S})^2$

✓ SM tree-level prediction for $e^+e^- \rightarrow Z^* \rightarrow$ two fermions

- o Feynman diagram computation (COM frame, m_{leptons} neglected)
leading to non polarized differential cross-section:

$$\frac{d\sigma_\mu}{d \cos \theta^*}(\sqrt{s}) = \frac{1}{8\pi} \left(\frac{g^2}{8 \cos^2 \vartheta_W} \right)^2 \frac{s}{(s - M_Z^2)^2 + \Gamma_Z^2 M_Z^2} \\ \left\{ (g_v^{e2} + g_a^{e2})(g_v^{\mu2} + g_a^{\mu2})(1 + \cos^2 \theta^*) + 8 g_v^e g_a^e g_v^\mu g_a^\mu \cos \theta^* \right\}$$



- o Note: for quark final-states: $\times C_q$ (colour factor)

✓ SM tree-level prediction for $e^+e^- \rightarrow Z^* \rightarrow$ two fermions

o Total production cross-section:

$$\sigma_\mu(\sqrt{s}) = \frac{1}{3\pi} \left(\frac{g^2}{8 \cos^2 \vartheta_W} \right)^2 \frac{s(g_v^{e^2} + g_a^{e^2})(g_v^{\mu^2} + g_a^{\mu^2})}{(s - M_Z^2)^2 + \Gamma_z^2 M_z^2}$$

o At the Z pole (i.e. $\sqrt{s} = M_Z$):

$$\sigma_\mu^0 \equiv \sigma_\mu(M_Z) = \frac{1}{3\pi} \left(\frac{g^2}{8 \cos^2 \vartheta_W} \right)^2 \frac{(g_v^{e^2} + g_a^{e^2})(g_v^{\mu^2} + g_a^{\mu^2})}{\Gamma_z^2}$$

hence

$$\sigma_\mu(\sqrt{s}) = \frac{s \Gamma_z^2}{(s - M_Z^2)^2 + \Gamma_z^2 M_z^2}$$

normalization
factor

energy dependence (from the propagator):
Breit-Wigner shape, function of M_Z, Γ_Z

✓ **SM tree-level prediction for $e^+e^- \rightarrow Z^* \rightarrow$ two fermions**

- o Z pole cross-section:

$$\sigma_\mu^0 \equiv \sigma_\mu(M_Z) = \frac{1}{3\pi} \left(\frac{g^2}{8 \cos^2 \vartheta_W} \right)^2 \frac{(g_v^{e^2} + g_a^{e^2})(g_v^{\mu^2} + g_a^{\mu^2})}{\Gamma_z^2}$$

- o Comparison with **partial width** tree-level computations:

$$\Gamma_{f\bar{f}} = \frac{C_f}{48\pi} \left(\frac{g}{\cos \vartheta_W} \right)^2 (g_v^{f^2} + g_a^{f^2}) M_Z \quad C_e = 1, \quad C_q = 3$$

hence

$$\sigma_\mu^0 = \frac{12\pi}{M_Z^2} \frac{\Gamma_{e\bar{e}} \Gamma_{\mu\bar{\mu}}}{\Gamma_z^2}$$

lineshape normalization (from vertices i.e. couplings) is function of M_Z , Γ_Z and Z partial widths

- o Similar expressions for other final-states

SM tree-level prediction for $e^+e^- \rightarrow Z^* \rightarrow \text{two fermions}$

X

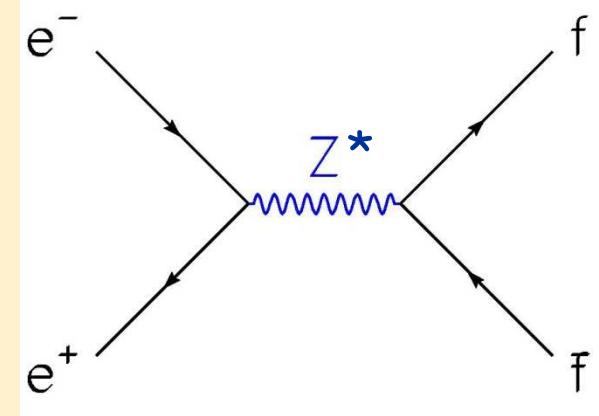
- o Any fermion type:

$$\sigma_f(\sqrt{s}) = \frac{s\Gamma_Z^2}{(s - M_Z^2)^2 + \Gamma_Z^2 M_Z^2} \sigma_f^0 \quad \text{and} \quad \sigma_f^0 = \frac{12\pi}{M_Z^2} \frac{\Gamma_{e\bar{e}}\Gamma_{f\bar{f}}}{\Gamma_Z^2}$$

- o Hadronic final-state (Σ over quarks \neq top) as a reference:

$$\sigma_f(\sqrt{s}) = \frac{s\Gamma_Z^2}{(s - M_Z^2)^2 + \Gamma_Z^2 M_Z^2} \sigma_{had}^0 \times \frac{\Gamma_{f\bar{f}}}{\Gamma_{had}}$$

- o e^+e^- collisions : $\sqrt{s} = 2 E_{\text{beam}} = M_{Z^*}$
 \Rightarrow direct measurement of the Z lineshape, accurate if E_{beam} precisely monitored



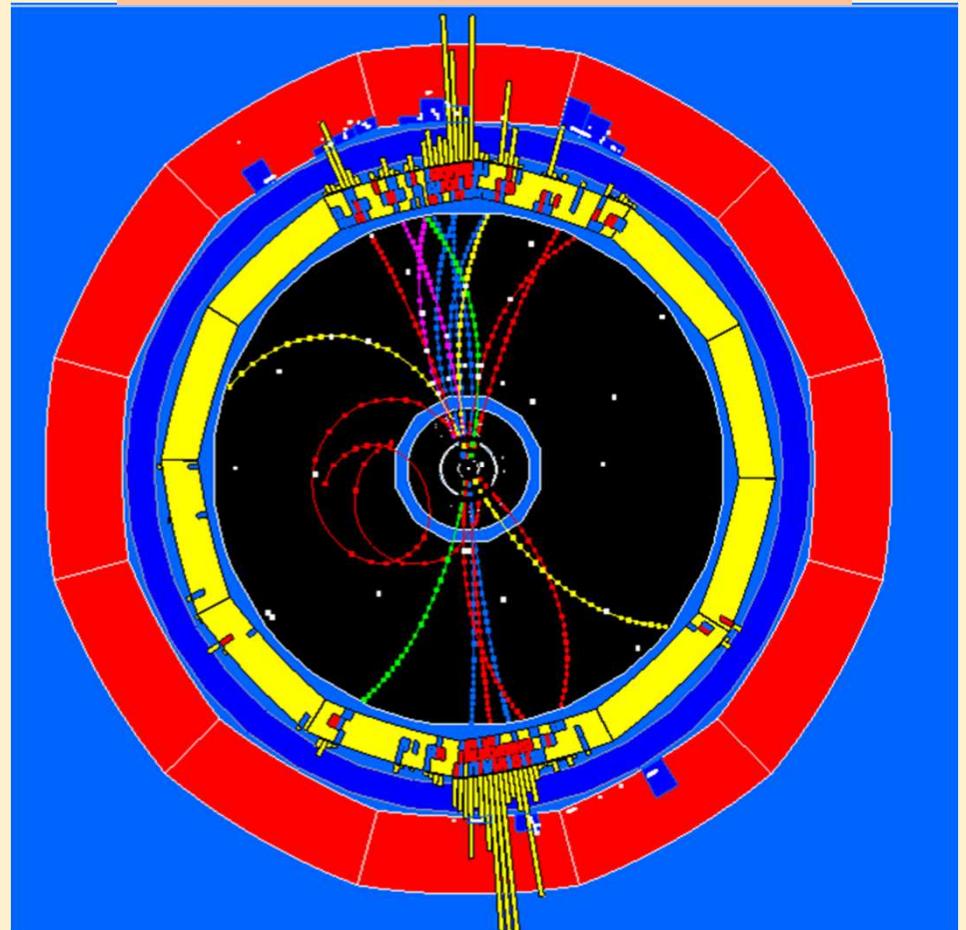
Z lineshape measurement method

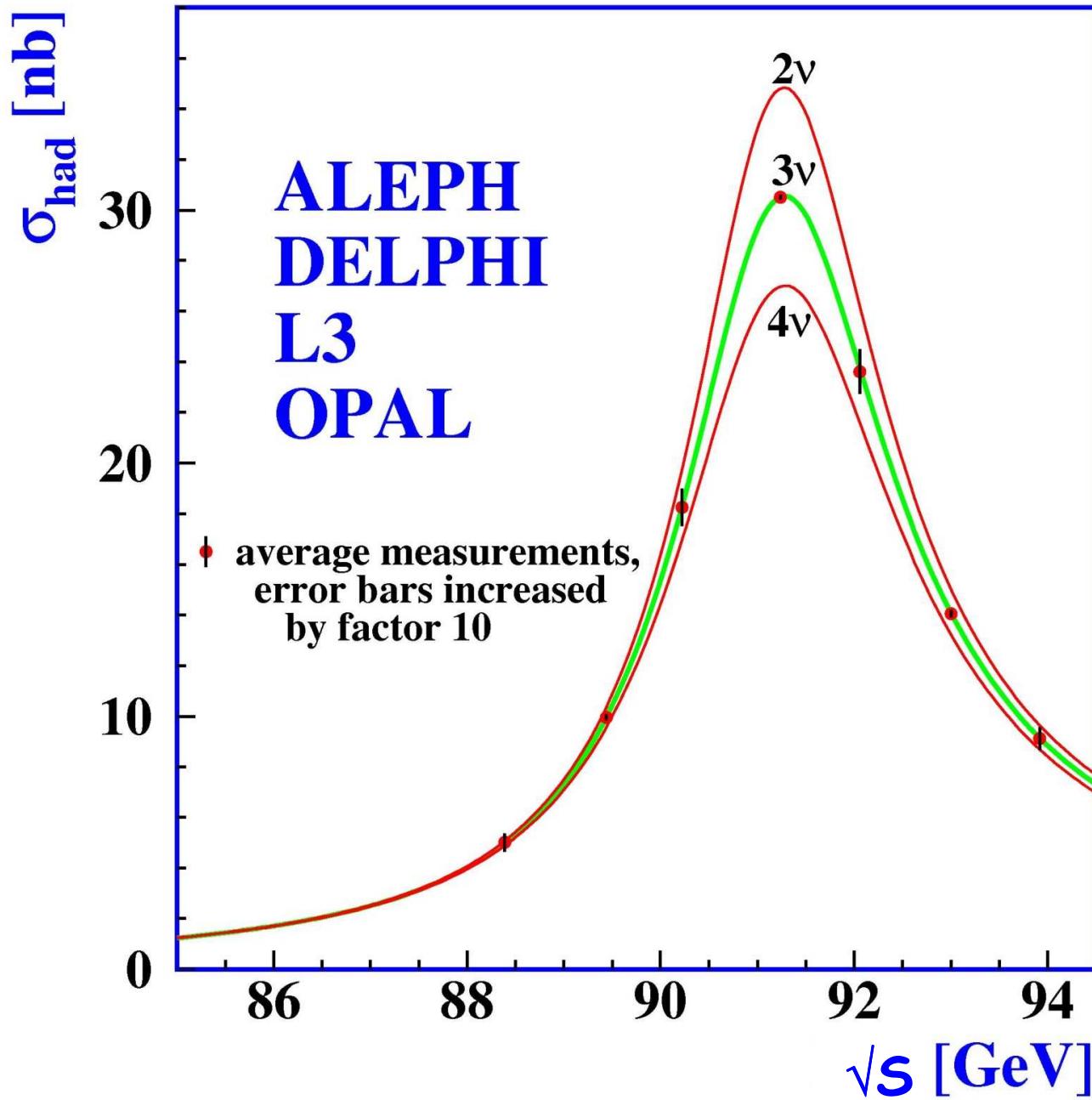
- o Split up two-fermion events according to final-state
- o In each sample, measure event rate as a function of $\sqrt{s}=2E_{\text{beam}}$
- o Fit SM prediction for *Z* lineshape onto data,
i.e. at tree level:

$$\sigma_f(\sqrt{s}) = \frac{s\Gamma_Z^2}{(s - M_Z^2)^2 + \Gamma_Z^2 M_Z^2} \sigma_{\text{had}}^0 \times \frac{\Gamma_{ff}}{\Gamma_{\text{had}}}$$

$\Rightarrow M_Z, \Gamma_Z, \sigma_{\text{had}}^0$ and $\Gamma_{ff}/\Gamma_{\text{had}}, f=e,\mu,\tau,\text{hadrons},b,c$

LEP: $e^+e^- \rightarrow Z \rightarrow q\bar{q} \rightarrow \text{hadrons}$

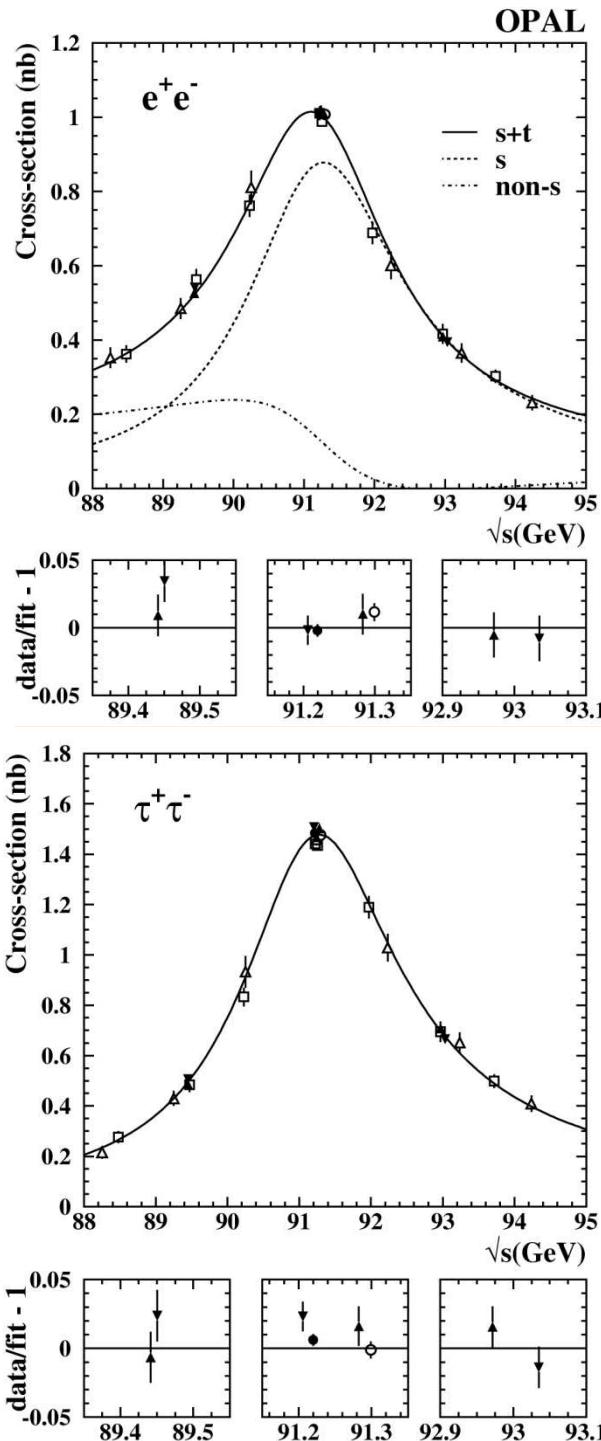
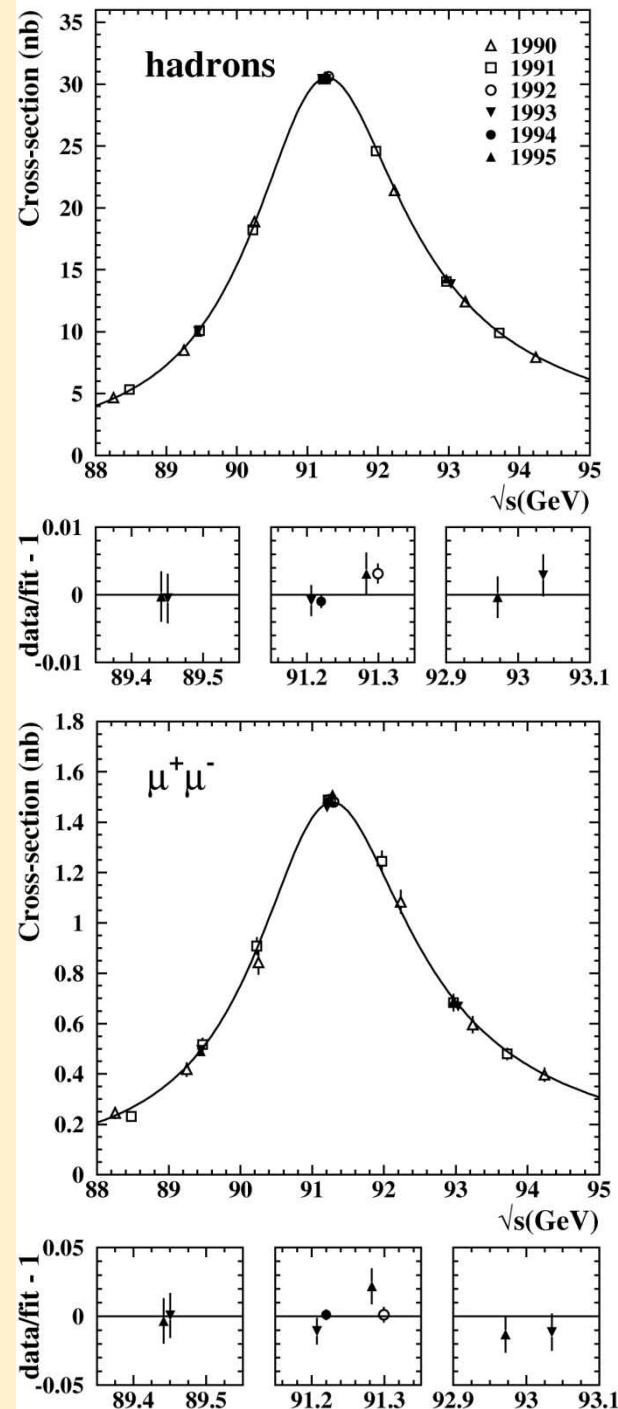




At LEP:
 2 MeV accuracy
 on \sqrt{s}

\downarrow

Z lineshape
 measurement
 = "counting
 experiment"



Other examples of
2 fermion
final-states

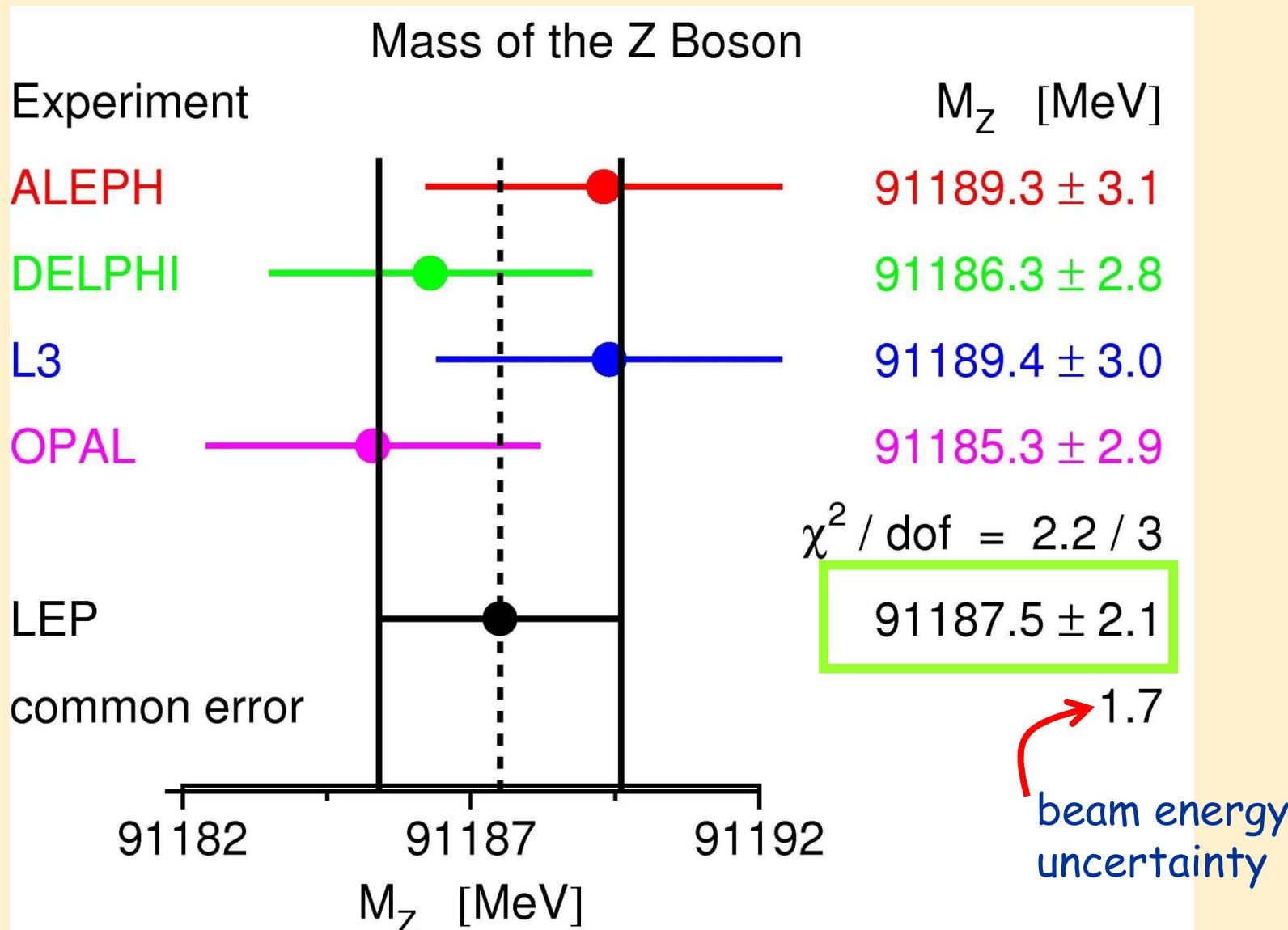
Overall, lineshape
measured for Z into

hadrons,
 $ee, \mu\mu, \tau\tau$,
 bb and cc

final-states

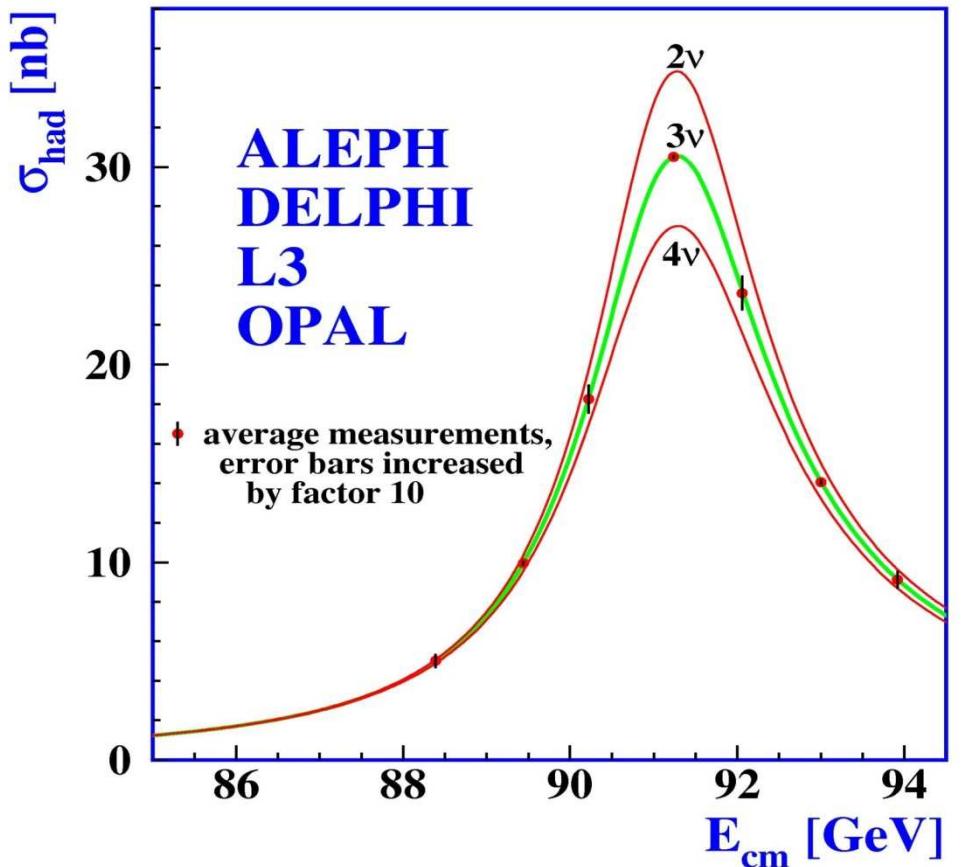
OPAL collaboration,
Eur. Phys. J.
C19 (2001) 587

Final M_Z measurement at LEP



v

Back to the number of light ν 's



- o tree-level SM prediction:

$$\sigma_f(\sqrt{s}) = \frac{s \Gamma_Z^2}{(s - M_Z^2)^2 + \Gamma_Z^2 M_Z^2} \sigma_f^0$$

$$\sigma_f^0 = \frac{12 \pi}{M_Z^2} \frac{\Gamma_{e\bar{e}} \Gamma_{f\bar{f}}}{\Gamma_Z^2}$$

⇒ total width, and hence
normalization, depend on N_ν

- o data best agree with $N_\nu = 3$

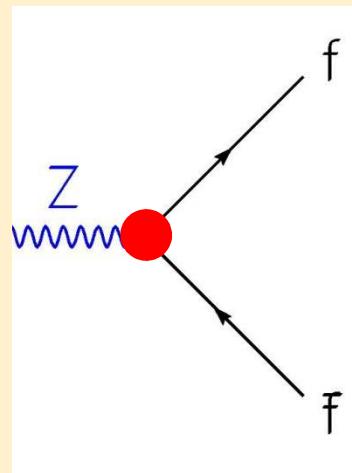
- Precise measurement of N_ν : from the width measurements

$$N_\nu = \left(\frac{\Gamma_{inv}}{\Gamma_{I^+/-}} \right)_{\text{exp}} \times \left(\frac{\Gamma_{I^+/-}}{\Gamma_{\nu\bar{\nu}}} \right)_{SM} = \left(\frac{\Gamma_Z - 3\Gamma_{I^+/-} - \Gamma_{\text{hadrons}}}{\Gamma_{I^+/-}} \right)_{\text{exp}} \times \left(\frac{\Gamma_{I^+/-}}{\Gamma_{\nu\bar{\nu}}} \right)_{SM} = 2.9840 \pm 0.0082$$

$5.943 \pm 0.016 \quad (0.3\%)$ $0.5022 \pm 0.0002 \quad (0.04\%)$

Asymmetries

- o Most precisely measured in e^+e^- collisions at $\sqrt{S} \sim 91$ GeV
- o Origin: parity violation in neutral weak interaction



$$\begin{aligned}
 \bullet &= -i \frac{g}{\cos \vartheta_W} \gamma^\mu \left(\frac{g_V^f - g_A^f \gamma_5}{2} \right) \\
 &= -i \frac{g}{\cos \vartheta_W} \gamma^\mu \left[\left(\frac{g_V^f + g_A^f}{2} \right) \left(\frac{1 - \gamma_5}{2} \right) + \left(\frac{g_V^f - g_A^f}{2} \right) \left(\frac{1 + \gamma_5}{2} \right) \right] \\
 &\quad \text{left-handed fermions} \qquad \qquad \qquad \text{right-handed fermions}
 \end{aligned}$$

\Rightarrow if $g_a^f \neq 0$, Z couplings to right and left fermions are different
 \Rightarrow **asymmetries** in Z couplings to fermions : not as large as for the W but more interesting (depend on $\sin^2 \theta_W$)

v

Asymmetry observables

- Forward-backward asymmetry: asymmetry of the Z decay product angular distribution (\forall fermion)

$$A_{FB}^{0,f} \equiv \frac{n(\theta^* < 90^\circ) - n(\theta^* > 90^\circ)}{n(\theta^* < 90^\circ) + n(\theta^* > 90^\circ)}$$
$$A_{FB}^{0,f} = 3A_e A_f \quad A_f = 2 \frac{g_v^f g_a^f}{(g_v^f)^2 + (g_a^f)^2}$$

"asymmetry parameter"

- Polarisation asymmetry: asymmetry of the rates of the Z decay polarised final-states (can be measured only for τ 's)

$$P_-^\tau \equiv \frac{n(\tau_R) - n(\tau_L)}{n(\tau_R) + n(\tau_L)} = -A_\tau \quad \text{more generally} \quad P_-^\tau(\cos \theta^*)$$

- Left-right asymmetry: asymmetry of the cross-sections with incident polarized electron beams

$$A_{LR} \equiv \frac{\sigma(e_L) - \sigma(e_R)}{\sigma(e_L) + \sigma(e_R)} = A_e \quad \text{more generally} \quad A_{LR}^{FB,f}, f = e, \mu, \tau, b, c$$

Example: A_{FB} asymmetry

angular distribution at pole
(slide 10)

$$\frac{d\sigma_\mu}{d \cos \theta^*} \propto (g_v^{e^2} + g_a^{e^2})(g_v^{\mu^2} + g_a^{\mu^2})(1 + \cos^2 \theta^*) + 8 g_v^e g_a^\mu g_v^\mu g_a^\mu \cos \theta^*$$

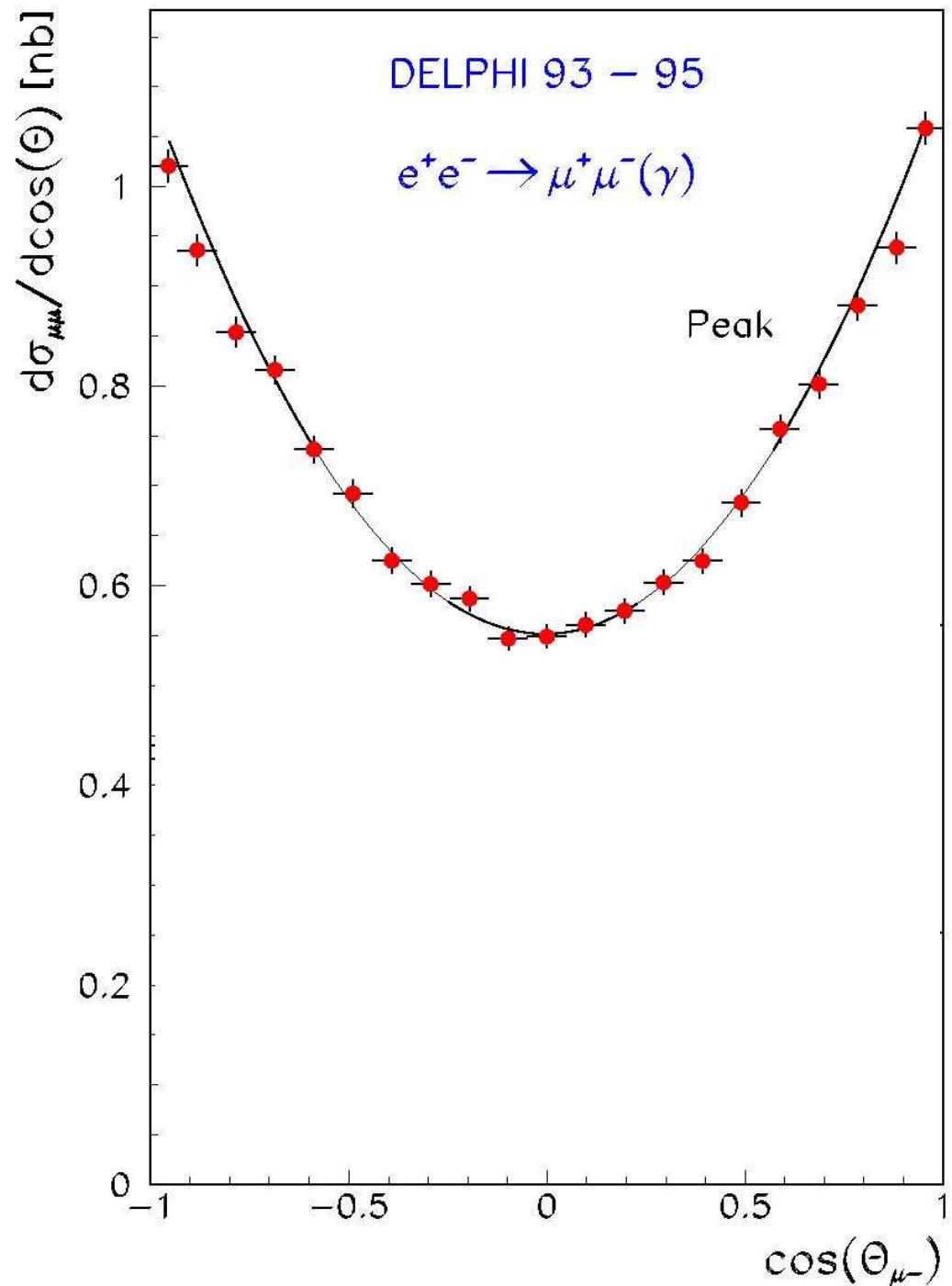
symmetric

asymmetric

as a function of A_{FB} :

$$\frac{d\sigma_\mu}{d \cos \theta^*} \propto (1 + \cos^2 \theta^* + \frac{8}{3} A_{FB}^{0,\mu} \cos \theta^*)$$

tiny effect: $A_{FB}^{0,I} \sim 0.02$



Summary 1: electroweak observables

- o W mass:

$$M_W = \sqrt{\frac{\pi\alpha}{\sqrt{2}G_F}} \frac{1}{\sin\vartheta_W} = \frac{M_Z}{\sqrt{2}} \left[1 + \sqrt{1 - \frac{4\pi\alpha}{\sqrt{2}G_F M_Z^2}} \right]^{\frac{1}{2}} \rightarrow M_W$$

- o Z lineshape:

$$\sigma_f(\sqrt{s}) = \frac{s \Gamma_Z^2}{(s - M_Z^2)^2 + \Gamma_Z^2 M_Z^2} \sigma_f^0 \quad \sigma_f^0 = \frac{12\pi}{M_Z^2} \frac{\Gamma_{e\bar{e}} \Gamma_{f\bar{f}}}{\Gamma_Z^2}$$

$\rightarrow \Gamma_Z, M_Z, \sigma_{\text{had}}^0, R_I = \Gamma^{\text{had}} / \Gamma^I, R_q = \Gamma^q / \Gamma^{\text{had}}$ ($I=e,\mu,\tau, q=b,c$)

- o Asymmetries:

$$A_{FB}^{0,f} = \frac{n(\theta^* < 90^\circ) - n(\theta^* > 90^\circ)}{n(\theta^* < 90^\circ) + n(\theta^* > 90^\circ)} = 3A_e A_f, \quad A_f = 2 \frac{g_V^f g_a^f}{(g_V^f)^2 + (g_a^f)^2} \rightarrow A_{FB}^{0,f} \quad (f=e,\mu,\tau,b,c)$$

$$P_\tau = \frac{n(\tau_R) - n(\tau_L)}{n(\tau_R) + n(\tau_L)} = -A_\tau \quad \rightarrow P_\tau(\cos\theta^*) \rightarrow A_\tau, A_e$$

$$A_{LR} = \frac{\sigma(e_L) - \sigma(e_R)}{\sigma(e_L) + \sigma(e_R)} = A_e \quad \rightarrow A_{LR}^{FB,f} \rightarrow A_I, A_b, A_c$$

Summary 2: electroweak measurements

Z boson

These results
are all final

Physics Report Volume 427
Nos. 5-6 (May 2006) 257.

measurement	experiments	relative accuracy
$M_Z = 91.1875 \pm 0.0021 \text{ GeV}$	LEP	$2.3 \cdot 10^{-5}$
$\Gamma_Z = 2.4952 \pm 0.0023 \text{ GeV}$	LEP	$9.2 \cdot 10^{-4}$
$\sigma_h^0 = 41.540 \pm 0.037 \text{ nb}$	LEP	$8.9 \cdot 10^{-4}$
$R_l = 20.767 \pm 0.025$	LEP	10^{-3}
$A_{FB}^{0,l} = 0.01714 \pm 0.00095$	LEP	5.5 %
$R_b = 0.21629 \pm 0.00066$	LEP+SLD	0.3 %
$R_c = 0.1721 \pm 0.0030$	LEP+SLD	1.7 %
$A_{FB}^{0,b} = 0.0992 \pm 0.0016$	LEP+SLD	1.6 %
$A_{FB}^{0,c} = 0.0707 \pm 0.0035$	LEP+SLD	5.0 %
$\mathcal{A}_b = 0.923 \pm 0.020$	SLD	2.2 %
$\mathcal{A}_c = 0.670 \pm 0.027$	SLD	3.9 %
$\mathcal{A}_\tau = 0.1439 \pm 0.0043$	LEP	3.0 %
$\mathcal{A}_e = 0.1498 \pm 0.0049$	LEP	3.3 %
$\mathcal{A}_l = 0.1513 \pm 0.0021$	SLD	1.4 %

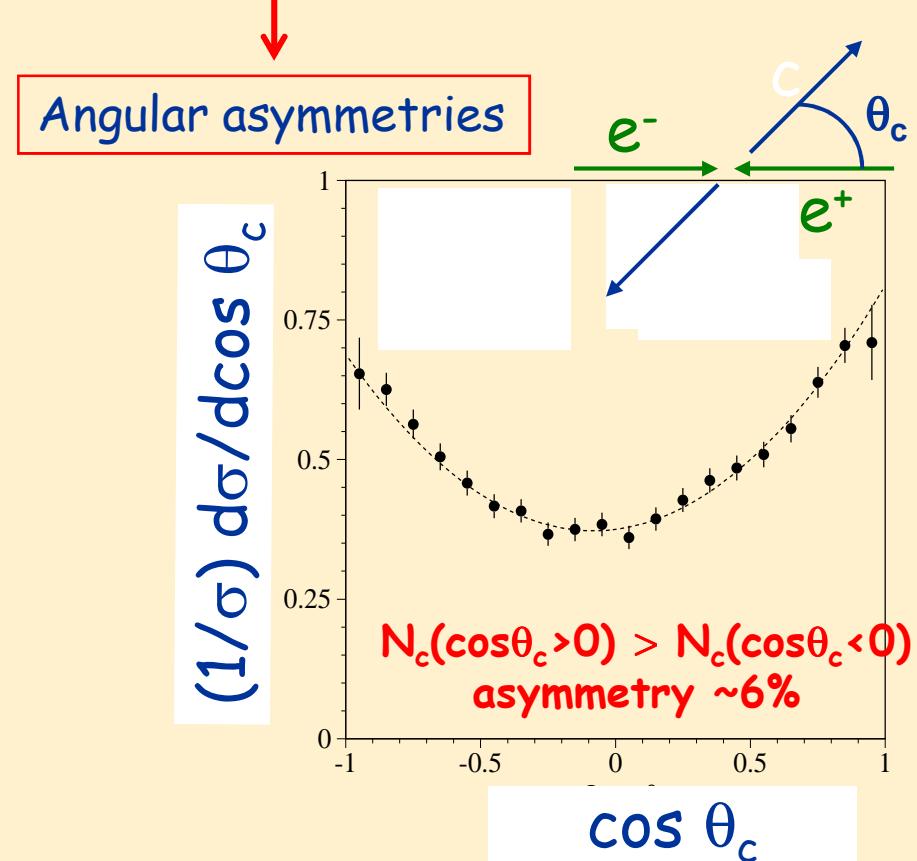
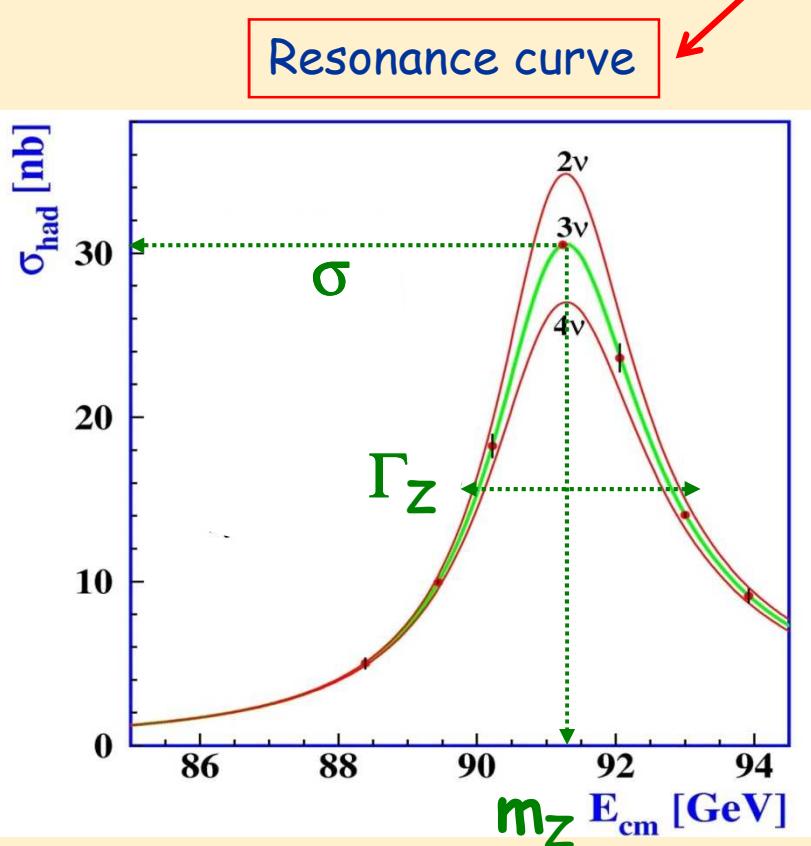
W boson

measurement	experiments	relative accuracy
$M_W = 80.376 \pm 0.033 \text{ GeV}$	LEP 2 (prel.)	$4.1 \cdot 10^{-4}$
$M_W = 80.420 \pm 0.031 \text{ GeV}$	Tevatron runs 1 and 2 (prel.)	$3.9 \cdot 10^{-4}$
$M_{\text{top}} = 173.2 \pm 0.9 \text{ GeV}$	Tevatron runs 1 and 2 (prel.)	$5.2 \cdot 10^{-3}$

top

Summary 3: precision tests of the Standard Model

- Precise measurements of :
 - the **W** and the **top** quark masses
 - the **Z** properties (mass, width, asymmetries)



to be compared with precise predictions of the Standard Model

3. Beyond tree-level

"Radiative corrections"

v

QED corrections

e.g. first order, $e^+e^- \rightarrow \mu^+\mu^-$ process

Weak corrections

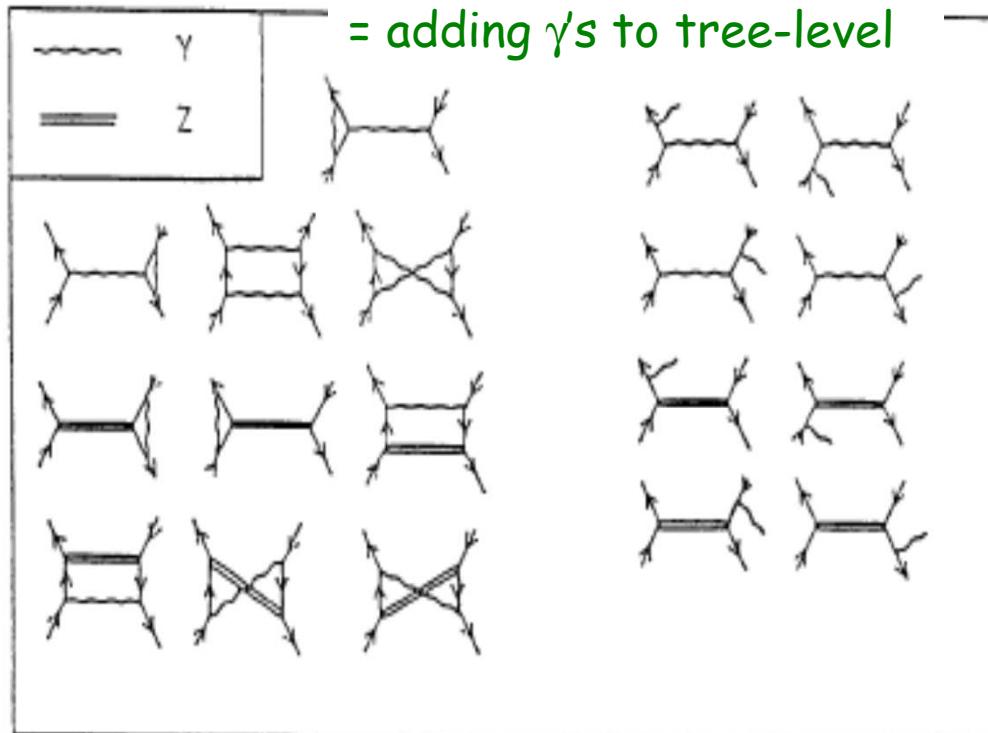


Figure 1: QED corrections to $e^+e^- \rightarrow \mu^+\mu^-$

gauge invariant, finite and large corrections
independent of the non-em part of the theory
depend on experimental details (γ below
experimental sensitivity)

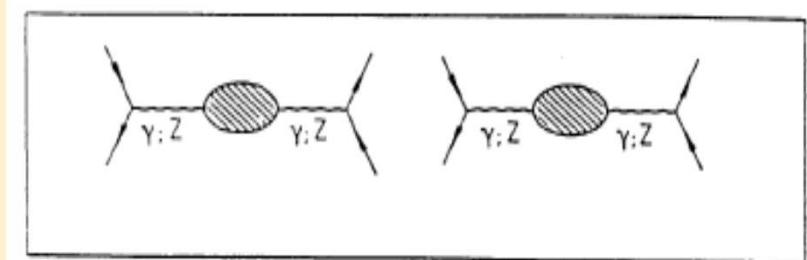


Figure 2: Propagator corrections to $e^+e^- \rightarrow \mu^+\mu^-$
↙ main corrections

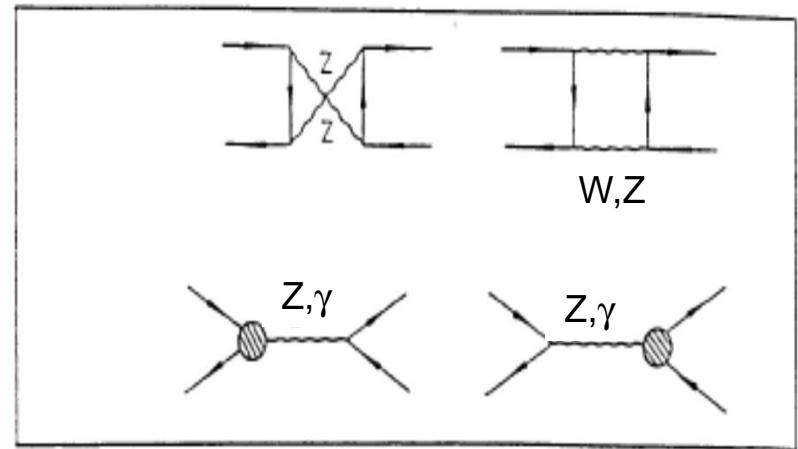


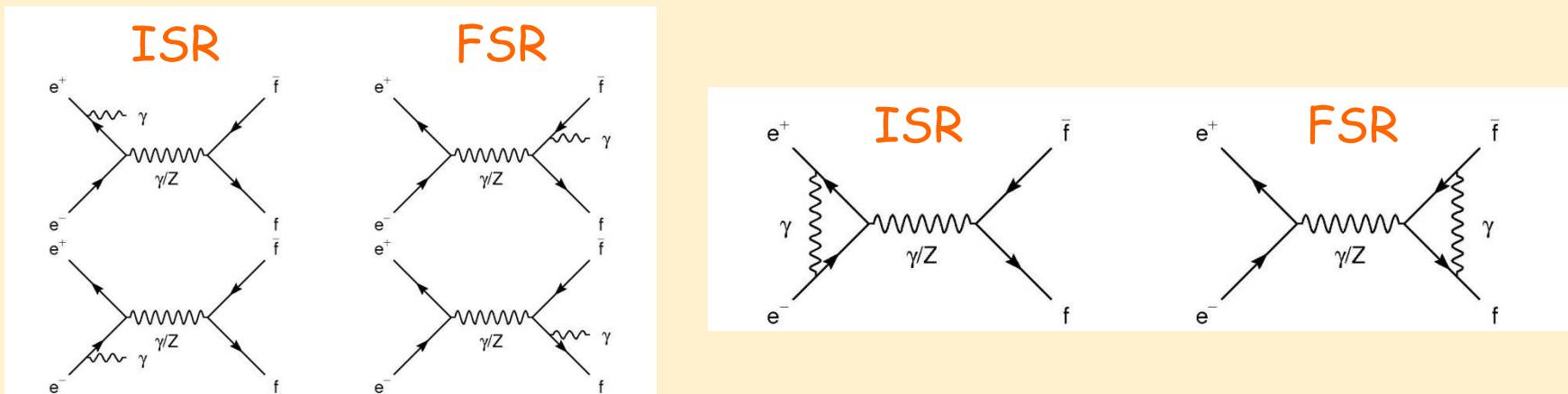
Figure 3: Vertex corrections and box contributions to $e^+e^- \rightarrow \mu^+\mu^-$

independent of experimental details
depend on the electroweak theory content

⇒ TESTS OF THE SM

QED corrections

- Initial-state radiation, final-state radiation & their interference

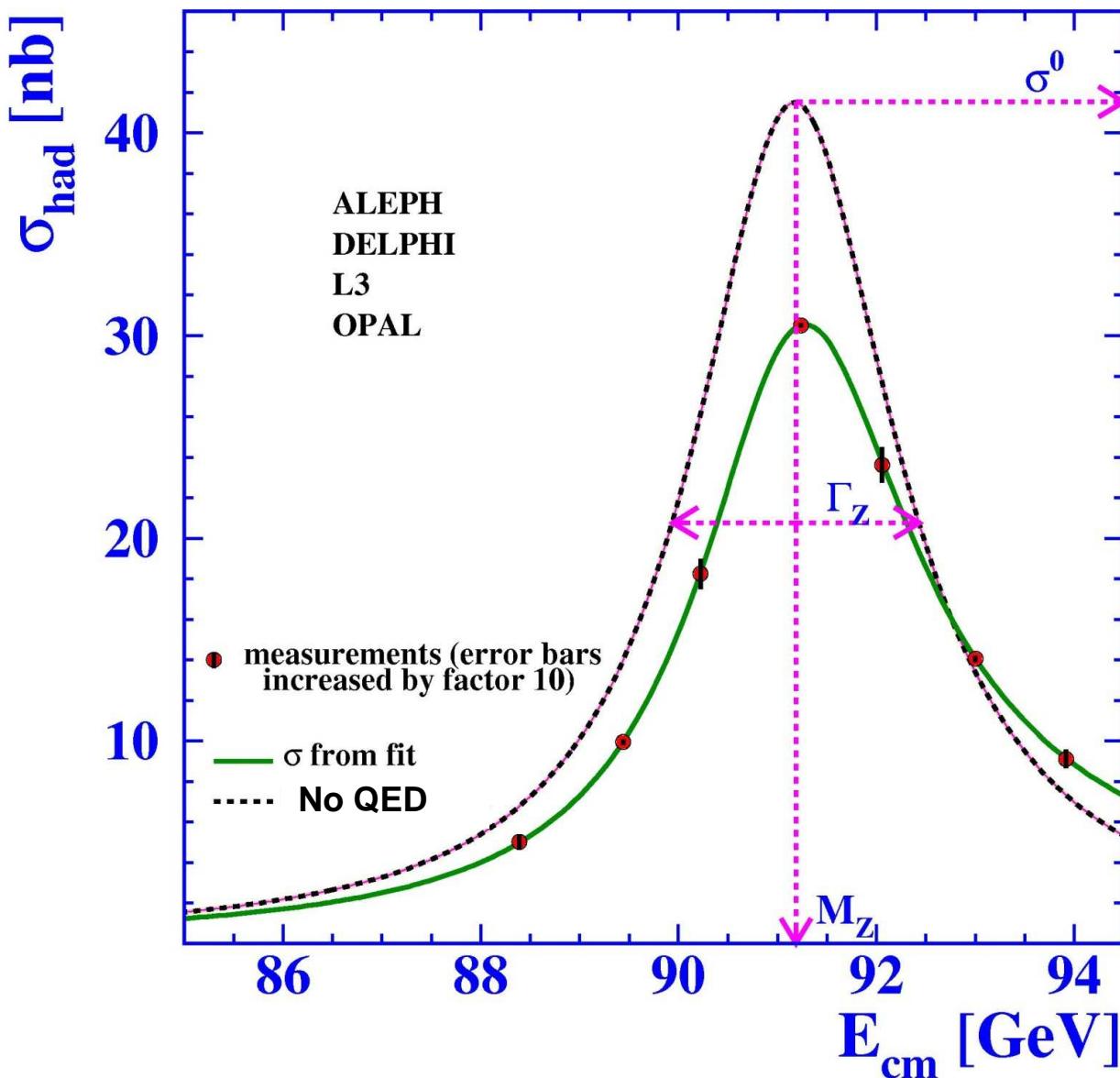


- Main effect on Z lineshape observables: ISR

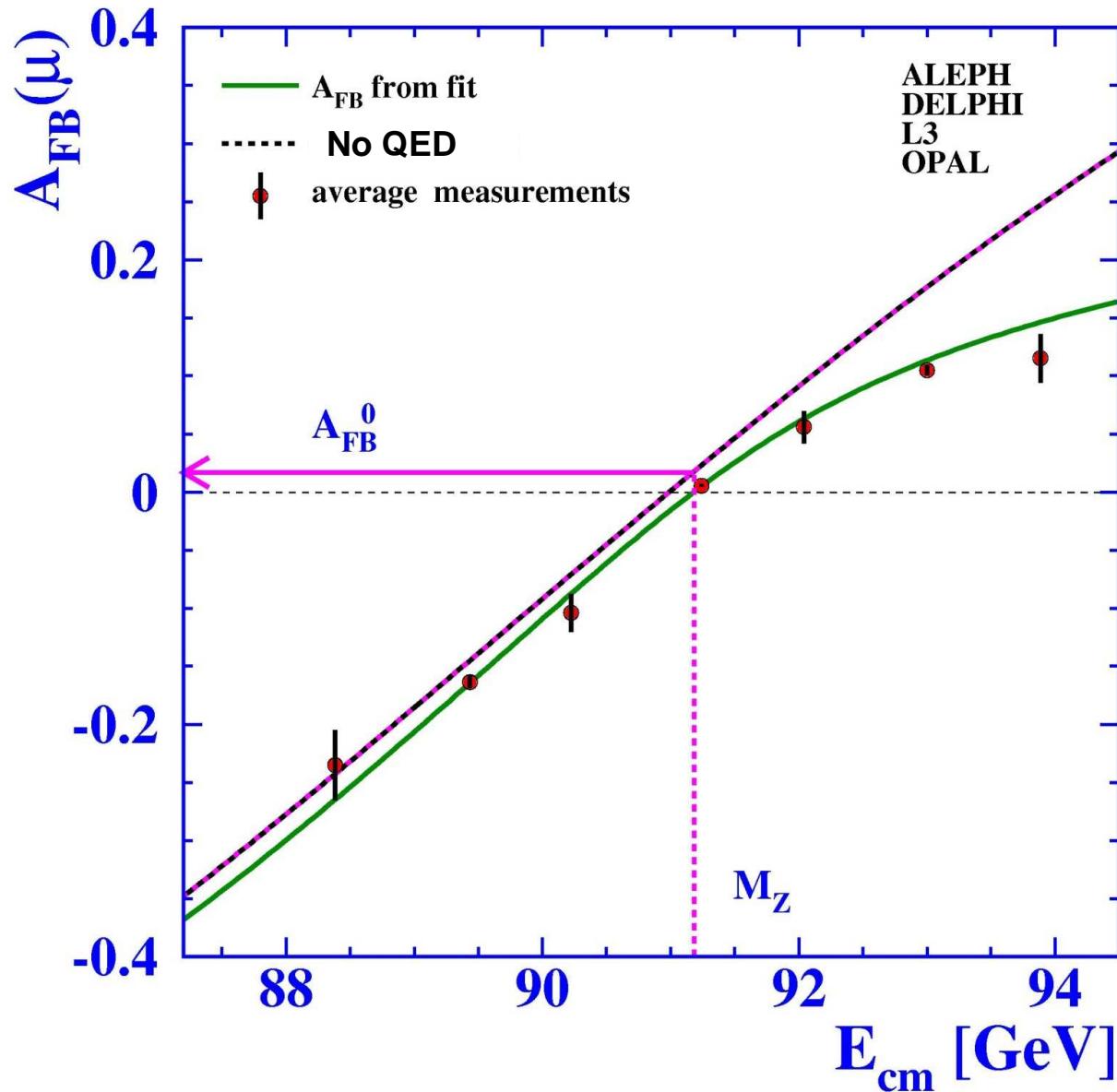
$$\sigma_f^{QEDconvoluted}(\sqrt{s}) = \int_{z_{min}}^1 dz G(z) \sigma_f(\sqrt{sz}) \quad z_{min} = \frac{4m_f^2}{s}$$

cross-section after radiation of a γ with $E_\gamma = \sqrt{s}/2(1-z)$ - see slide 13

- $G(z)$: QED radiator, computed to $O(\alpha^3)$



- o Reduction of the peak cross-section by 36%
- o Upward shift of the cross-section peak by +100 MeV



- o Downward shift of the forward-backward asymmetries,
e.g. at the Z pole, $A_{FB}^{0,\parallel}$ reduced by \sim its tree-level value

Weak corrections

- Example 1 : γ propagator corrections

full
propagator

$$\begin{aligned} \text{full propagator} \rightarrow & \quad \gamma \cdot \gamma = \gamma \\ & + \text{---} \circlearrowleft \text{---} \quad \gamma + \text{---} \circlearrowright \text{---} \quad \gamma + \dots \\ & + \text{---} \circlearrowleft \text{---} \quad \text{---} \circlearrowleft \text{---} \quad \gamma + \dots \end{aligned}$$

→ tree-level

→ irreducible
diagrams

→ reducible
diagrams

hence

$$\begin{aligned} \gamma \cdot \gamma &= \gamma \\ &+ \text{---} \circlearrowleft \text{---} \quad \gamma + \text{---} \circlearrowleft \text{---} \quad \text{---} \circlearrowleft \text{---} \quad \gamma + \dots \end{aligned}$$

Taylor expansion,
can be resummed
to all orders

sum of all irreducible diagrams, computed at a given order

✓

γ propagator corrections

- In the Feynman gauge, after renormalization (in the on-shell scheme):

- tree-level: $D_{\mu\nu}^0(k^2) = -i \frac{g_{\mu\nu}}{k^2}$

- γ self-energy operator ("vacuum polarization tensor"):



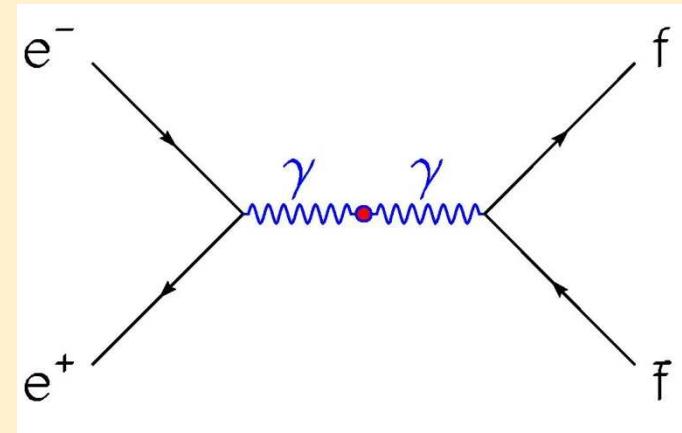
$$\rightarrow \bar{\Pi}^{\mu\nu}(k^2) = -ig^{\mu\nu}\bar{\Sigma}(k^2) = -ig^{\mu\nu}k^2\bar{\Pi}(k^2)$$

\bar{} bar = renormalized
\bar{\Pi} γ self-energy function

- Full, renorm. propagator: $\bar{D}_{\mu\nu}(k^2) = -i \frac{g_{\mu\nu}}{k^2[1 + \bar{\Pi}(k^2)]} = \frac{-ig_{\mu\nu}}{k^2 + \bar{\Sigma}(k^2)}$
 - $\bar{\Pi}(k^2)$: sum of the irreducible diagrams. Known to $O(\alpha^3)$.
 - $\frac{1}{[1 + \bar{\Pi}(k^2)]}$ comes from the resummation of $\bar{\Pi}(k^2)$ to all orders.

γ propagator corrections

- Consequence: "running of α "



renormalized electron charge

$$\overline{e}^2 J_{em,e}^\mu \bar{D}_{\mu\nu}(k^2) J_{em,f}^\nu$$

$$= \frac{\overline{e}^2}{1 + \bar{\Pi}(k^2)} J_{em,e}^\mu D_{\mu\nu}^0(k^2) J_{em,f}^\nu$$

tree-level propagator

- Only change w.r.t. tree-level QED amplitudes: change $\alpha = \overline{e}^2 / 4\pi$

into

$$\alpha(k^2) = \frac{\alpha}{1 + \bar{\Pi}(k^2)}$$

running coupling constant

- $k^2 \sim M_Z^2$: $\alpha(M_Z^2) = \frac{\alpha}{1 - \Delta\alpha}$ Numerically: $\Delta\alpha = 0.05901 \pm 0.00035$
(or $\alpha = 1/137 \rightarrow \alpha(M_Z^2) \sim 1/129$)

Weak corrections

- Example 2: W propagator corrections (Feynman gauge)

- tree level: $D_{\mu\nu}^0(k^2) = -i \frac{g_{\mu\nu}}{k^2 - M_{W,0}^2 + i\epsilon}$
- full, renormalized propagator: bare mass, no physical meaning

$$\bar{D}_{\mu\nu}(k^2) = -i \frac{g_{\mu\nu}}{k^2 - M_W^2 + \bar{\Sigma}_W(k^2)} = \frac{-ig_{\mu\nu}}{\left[1 + \bar{\Pi}_W(k^2)\right] \left(k^2 - M_W^2 + i \frac{\text{Im} \bar{\Sigma}_W(k^2)}{1 + \bar{\Pi}_W(k^2)}\right)}$$

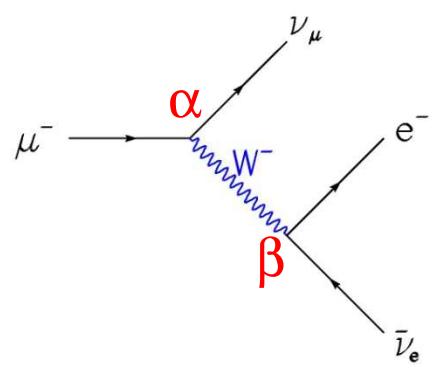
renormalized
(i.e. physical) W mass W self-energy function W width: given by imaginary part of self-energy function, thus tree-level prediction for Γ_W is modified by radiative corrections

$$\text{Re } \bar{\Sigma}_W(k^2) \equiv (k^2 - M_W^2) \bar{\Pi}_W(k^2)$$

(on-shell renormalization scheme)

W propagator corrections

- Consequence: modification of the relation between g and G_F X



$$\begin{aligned}
 \rightarrow -\frac{\bar{g}^2}{8} J_\mu^\alpha \bar{D}_{\alpha\beta}(k^2) J_e^\beta &= -i \frac{\bar{g}^2}{8M_W^2 [1 + \bar{\Pi}_W(0)]} J_\mu^\alpha J_{e,\alpha} \\
 &= -i \frac{G_F}{\sqrt{2}} J_\mu^\alpha J_{e,\alpha} \quad \leftarrow \text{Fermi model}
 \end{aligned}$$

new wrt tree-level

- w.r.t. tree-level:

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} \rightarrow \frac{G_F}{\sqrt{2}} = \frac{\bar{g}^2}{8M_W^2(1 - \Delta r)} \quad \Delta r \equiv -\bar{\Pi}_W(0) = \frac{\text{Re } \bar{\Sigma}_W(0)}{M_W^2}$$

- One-loop result:

$$\Delta r = \Delta \alpha - \frac{\cos^2 \vartheta_W}{\sin^2 \vartheta_W} \Delta \rho + \Delta r_{rem} \quad \text{with} \quad \Delta \rho \propto m_{top}^2$$

and $\Delta r_{rem} = f(\log \frac{M_H}{M_W}, \log \frac{m_{top}}{M_W})$

Back to (simple) test of the W mass prediction

- o Most precise W mass measurement:

$$M_W = 80.399 \pm 0.023 \text{ GeV} \quad (0.03\%) \quad (\text{LEP2, Tevatron})$$

- o SM higher order prediction:

$$M_W = \frac{M_Z}{\sqrt{2}} \left[1 + \sqrt{1 - \frac{4 (37.2805)^2}{M_Z^2 (1 - \Delta r)}} \right]^{\frac{1}{2}}$$

Freitas et al. (2000): $\Delta r \sim 0.037$ at (partial) two-loop order ($M_H = 115 \text{ GeV}$)

\Rightarrow prediction: $M_W = 80.35 \pm 0.04 \text{ GeV}$

\Rightarrow once higher orders are included, SM prediction agrees with data !

✓

Beyond tree-level prediction for M_W

$\Delta r(M_H)$, M_W & m_{top} fixed

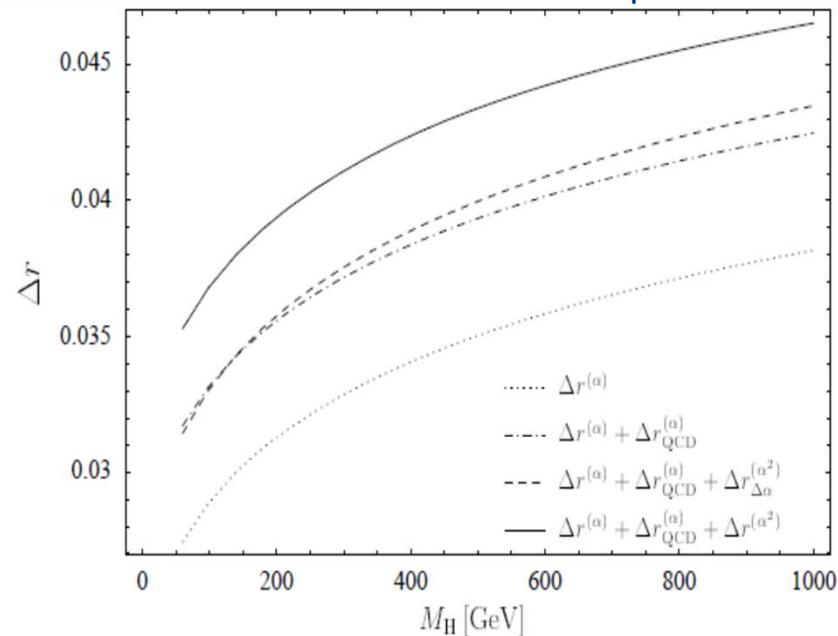
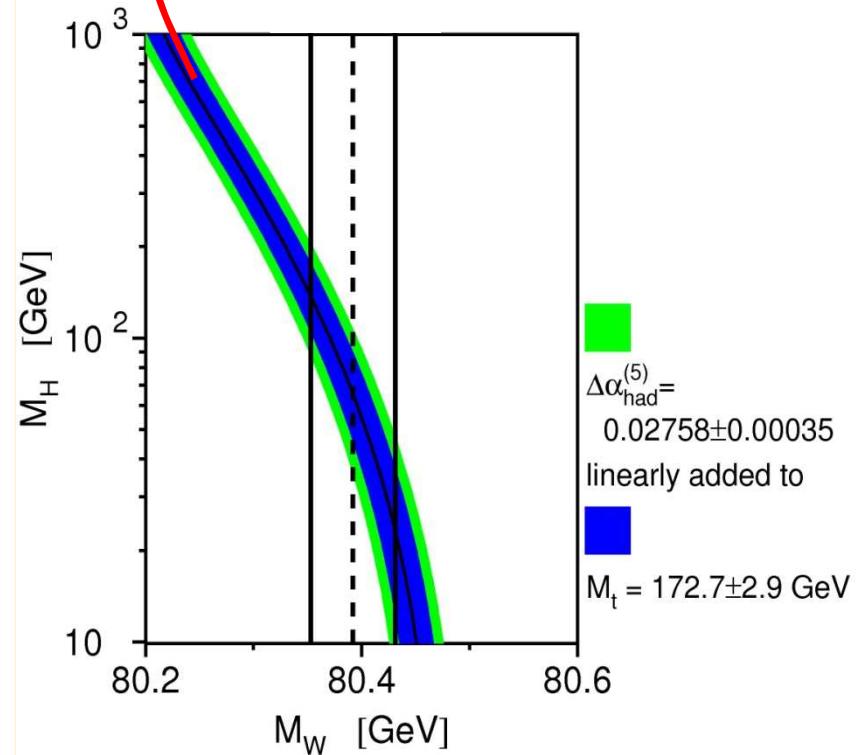


Figure 3: Different contributions to Δr as a function of M_H . The one-loop contribution, $\Delta r^{(\alpha)}$, is supplemented by the two-loop and three-loop QCD corrections, $\Delta r_{QCD}^{(\alpha)} \equiv \Delta r^{(\alpha\alpha_s)} + \Delta r^{(\alpha\alpha_s^2)}$, and the fermionic electroweak two-loop contributions, $\Delta r^{(\alpha^2)} \equiv \Delta r^{(N_f\alpha^2)} + \Delta r^{(N_f^2\alpha^2)}$. For comparison, the effect of the two-loop corrections induced by a resummation of $\Delta\alpha$, $\Delta r_{\Delta\alpha}^{(\alpha^2)}$, is shown separately.

$$M_W = 80.419 \text{ GeV}, m_{top} = 174.3 \text{ GeV}$$

$M_W(M_H)$, m_{top} fixed



LEP EWWG-2005

A.Freitas, W.Hollik, W.Walter, G.Weiglein,
hep-ph/0007091 & Phys.Lett. B495 (2000) 338

Summary 3: weak corrections

- Weak corrections are small (a few %) but depend on crucial SM parameters (m_{top} , m_H), thus allowing precise tests of the SM
- In the on-shell renormalization scheme, tree-level expressions are (moderately) changed as follows:

- W mass:

$$M_W = \frac{M_Z}{\sqrt{2}} \left[1 + \sqrt{1 - \frac{4\pi\alpha}{\sqrt{2}G_F M_Z^2 (1 - \Delta r)}} \right]^{\frac{1}{2}}$$

with:

$$\Delta r = \Delta\alpha - \frac{\cos^2\vartheta_W}{\sin^2\vartheta_W} \Delta\rho + \Delta r_{rem} + \dots \quad \text{and} \quad \Delta r = f(m_{top}^2, \log \frac{M_H}{M_W}, \log \frac{m_{top}}{M_W})$$

Δr known to complete two-loop order and partial three-loop order

✓

Summary 3: weak corrections

- Similarly, close to the Z pole ($k^2 \sim M_Z^2$), tree-level expressions are (moderately) changed as follows :

- Z lineshape:

$$\sigma_f(\sqrt{s}) = \frac{s^2 \Gamma_Z^2}{(s - M_Z^2)^2 + \frac{s^2}{M_Z^4} \Gamma_Z^2 M_Z^2} \sigma_f^0 \quad \sigma_f^0 = \frac{12\pi}{M_Z^2} \frac{\Gamma_{e\bar{e}} \Gamma_{f\bar{f}}}{\Gamma_Z^2}$$

$$\Gamma_{f\bar{f}} = C_f \frac{G_F}{6\pi\sqrt{2}} \left(\frac{1 - \Delta r_{rem} + ..}{1 - \Delta \rho + ..} \right) (g_{v,f}^{eff})^2 + (g_{a,f}^{eff})^2 M_Z^3$$

- Asymmetries: $A_f = 2 \frac{g_{v,f}^{eff} g_{a,f}^{eff}}{(g_{v,f}^{eff})^2 + (g_{a,f}^{eff})^2}$

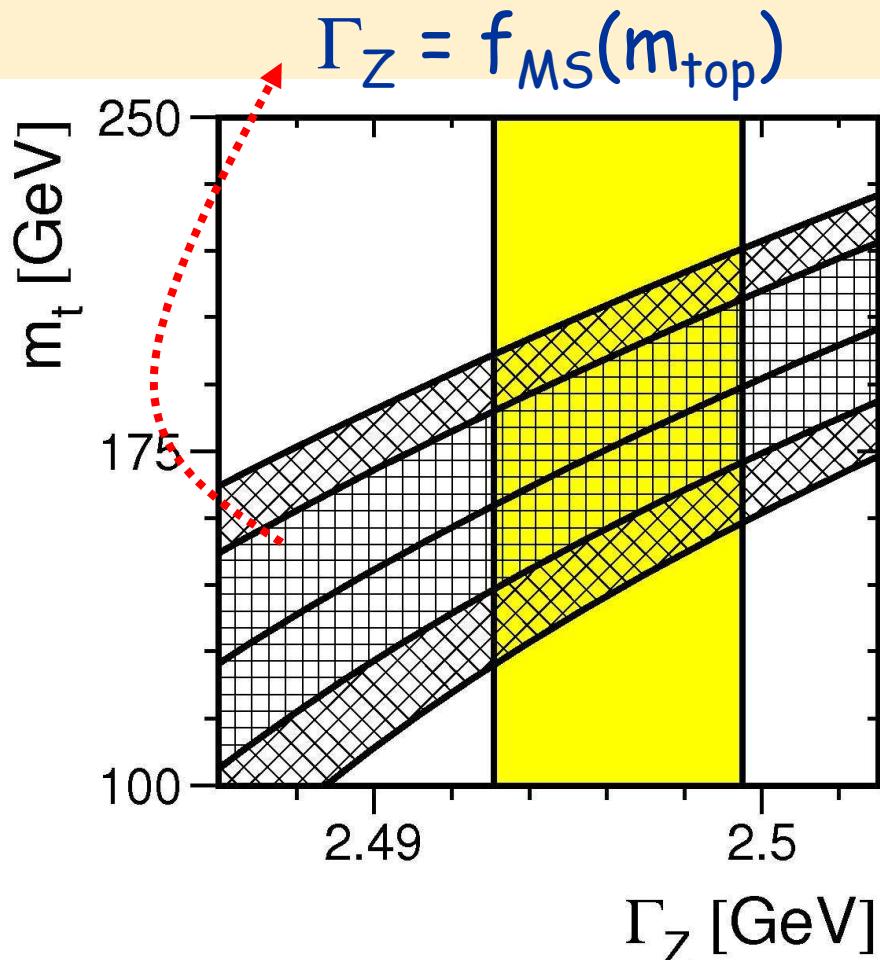
with: $g_{v,f}^{eff} = T_f^3 - 2Q^f \sin^2 \vartheta_{eff} + F_{v,f}^{vertex}$

$$g_{a,f}^{eff} = T_f^3 + F_{a,f}^{vertex}$$

$$\sin^2 \vartheta_{eff} = \sin^2 \vartheta_W + \cos^2 \vartheta_W \Delta \rho + ...$$

These corrections are known to complete one-loop order and partial two- and three-loop orders.

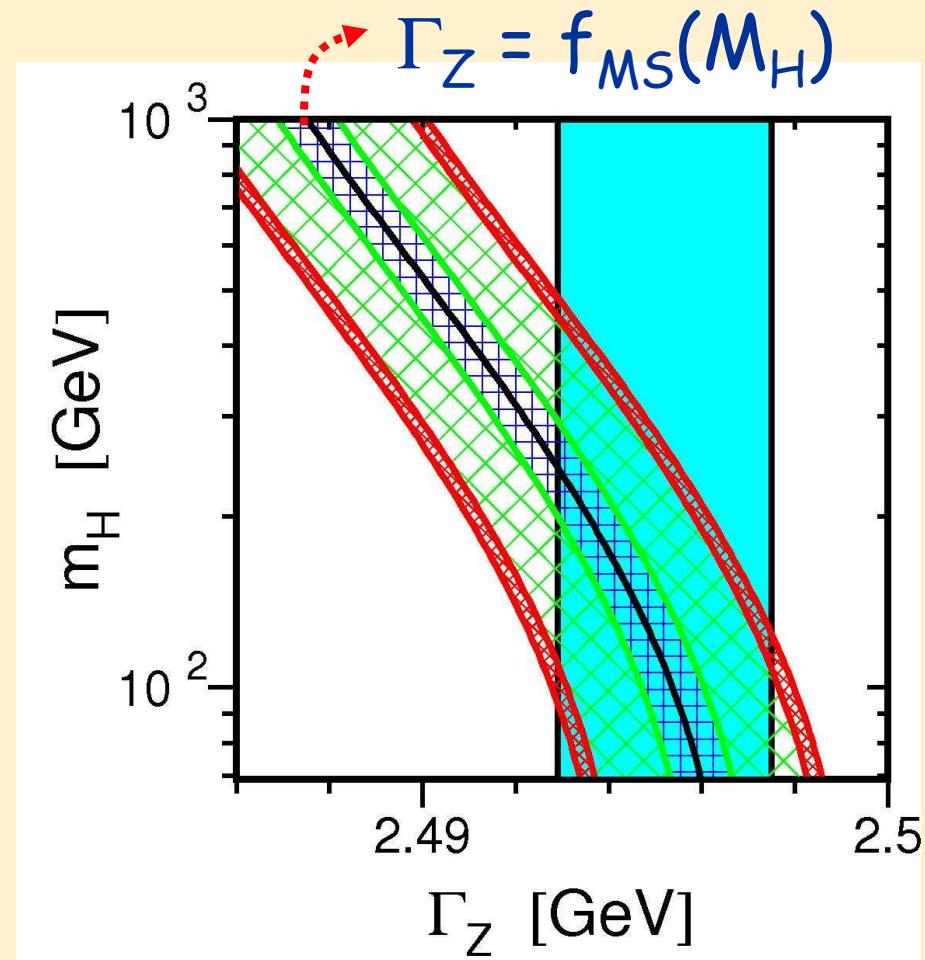
Beyond tree-level prediction for Γ_Z



$60\text{GeV} < M_H < 1000\text{ GeV}$

$\alpha_s = 0.123 \pm 0.006$

LEP EWWG-1995



- $\Delta\alpha_{had}^{(5)} = 0.02758 \pm 0.00035$
- $\alpha_s = 0.118 \pm 0.003$
- $m_t = 172.7 \pm 2.9\text{ GeV}$

LEP EWWG-2005

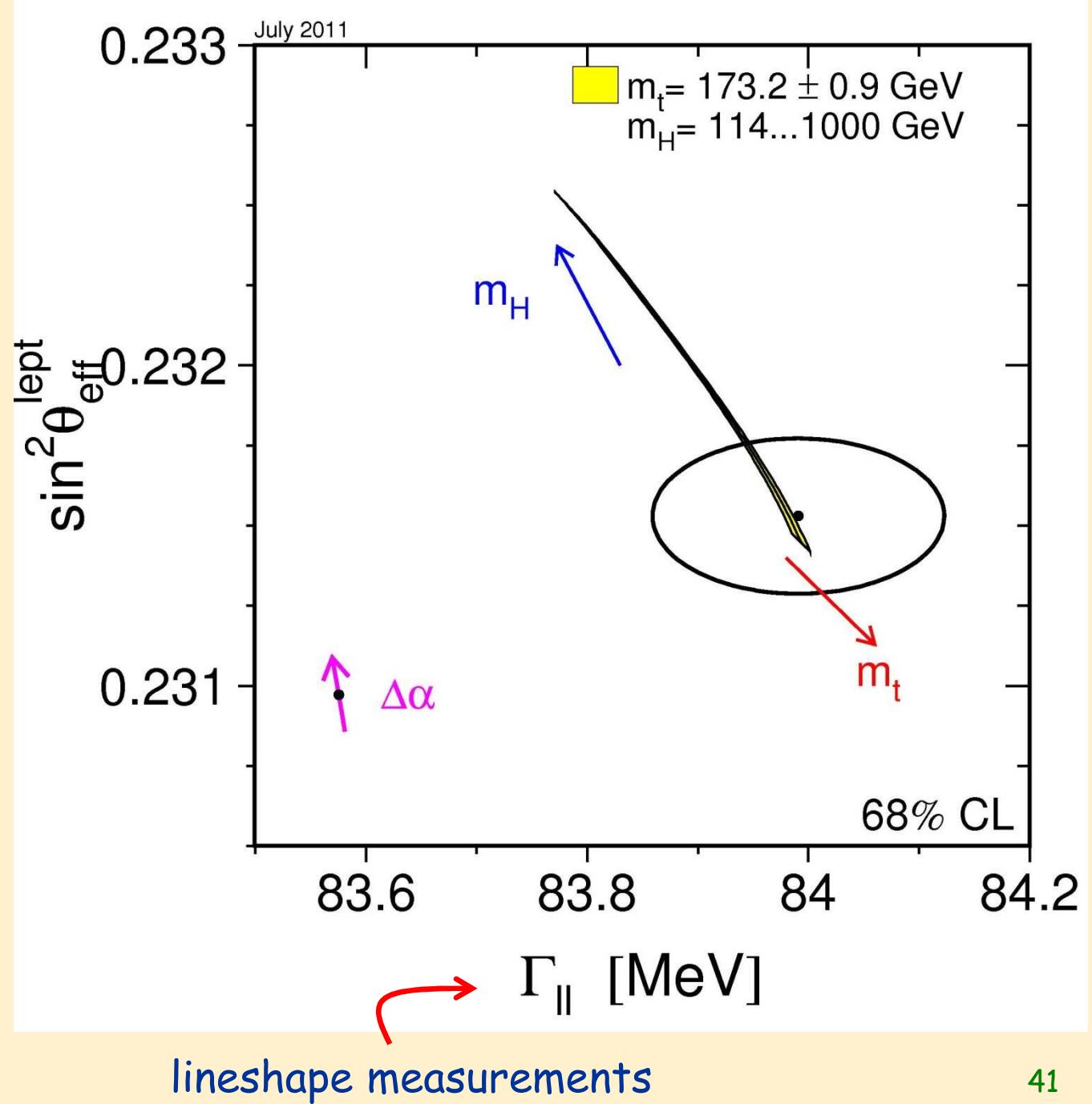
Tests of the SM

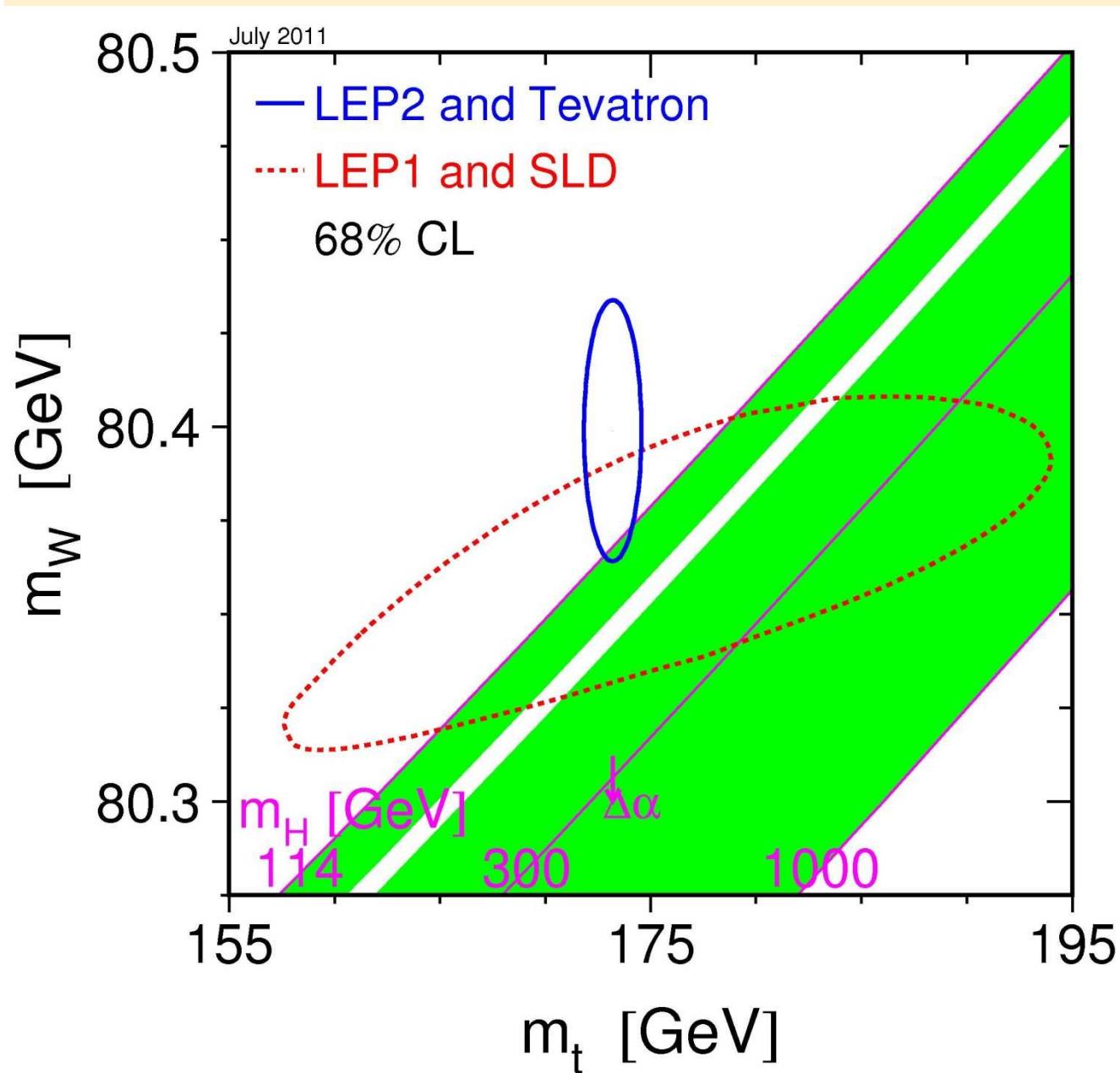
- o For a set of ~20 observables:
 - SM theory → predictions beyond tree-level, $f(m_{top}, M_H)$
 - data → precise (~1% or << 1%) measurements
- o Step 1: test SM internal consistency with subsets of observables
- o Step 2: if SM-data consistency, constrain values of unknown SM parameters, e.g. M_H

Step 1

Asymmetry
measurements,
translated into

$$\sin^2 \vartheta_{\text{eff}}^{\text{lept}} \equiv \frac{1}{4} \left(1 - \frac{g_{v,l}^{\text{eff}}}{g_{a,l}^{\text{eff}}} \right)$$





Step 1

LEP2/Tevatron:
direct mass
measurements

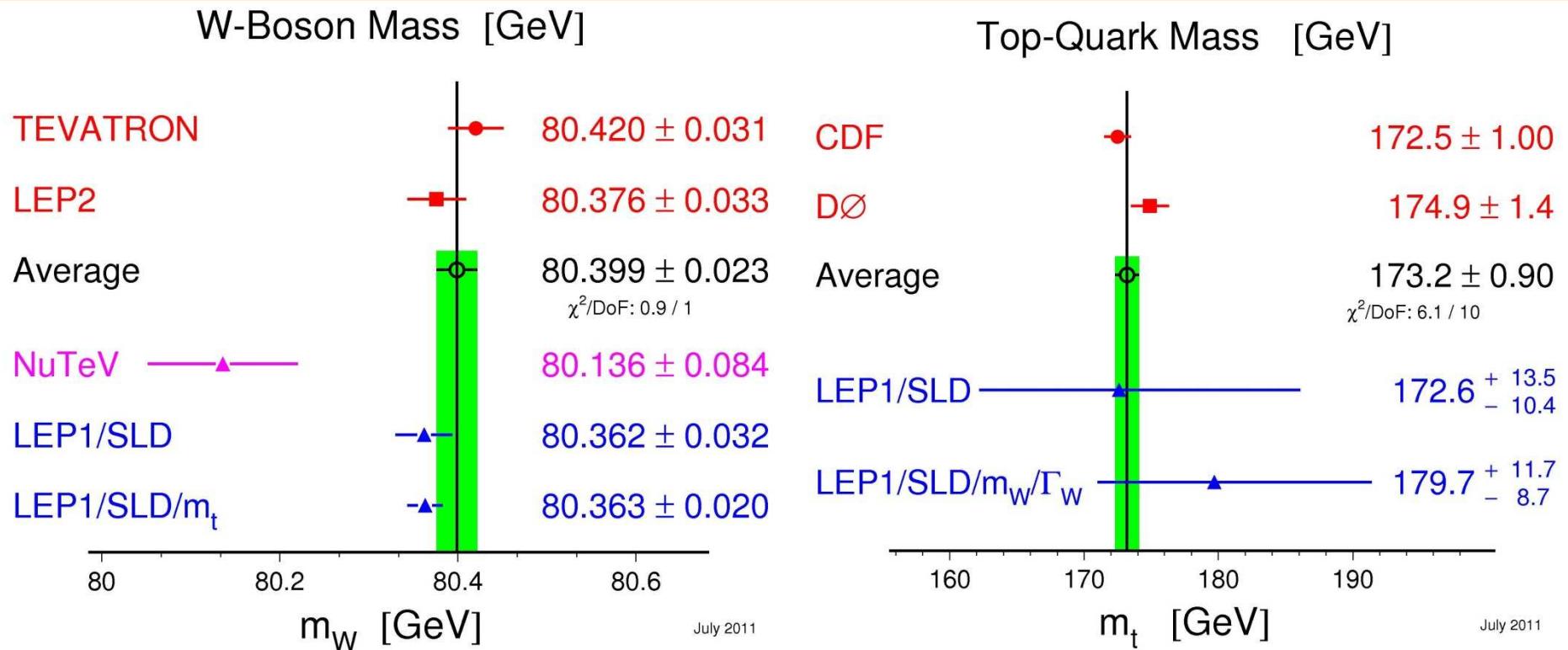
LEP1/SLD:
all lineshape and
asymmetry
measurements,
translated into
indirect
constraints on M_W
and m_{top}

The SM agrees
very well with
precision
EW data

v

Step 1: indirect constraints on M_W and m_{top}

(from various SM fits)



Indirect constraints on M_W and m_{top} from precision data agree with direct measurements

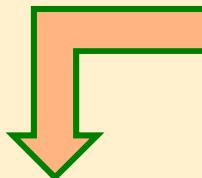
Step 2

SM fit to all
precision data to
constrain M_H value

$(\chi^2/\text{DoF}=17.5/13 \quad 18\%)$



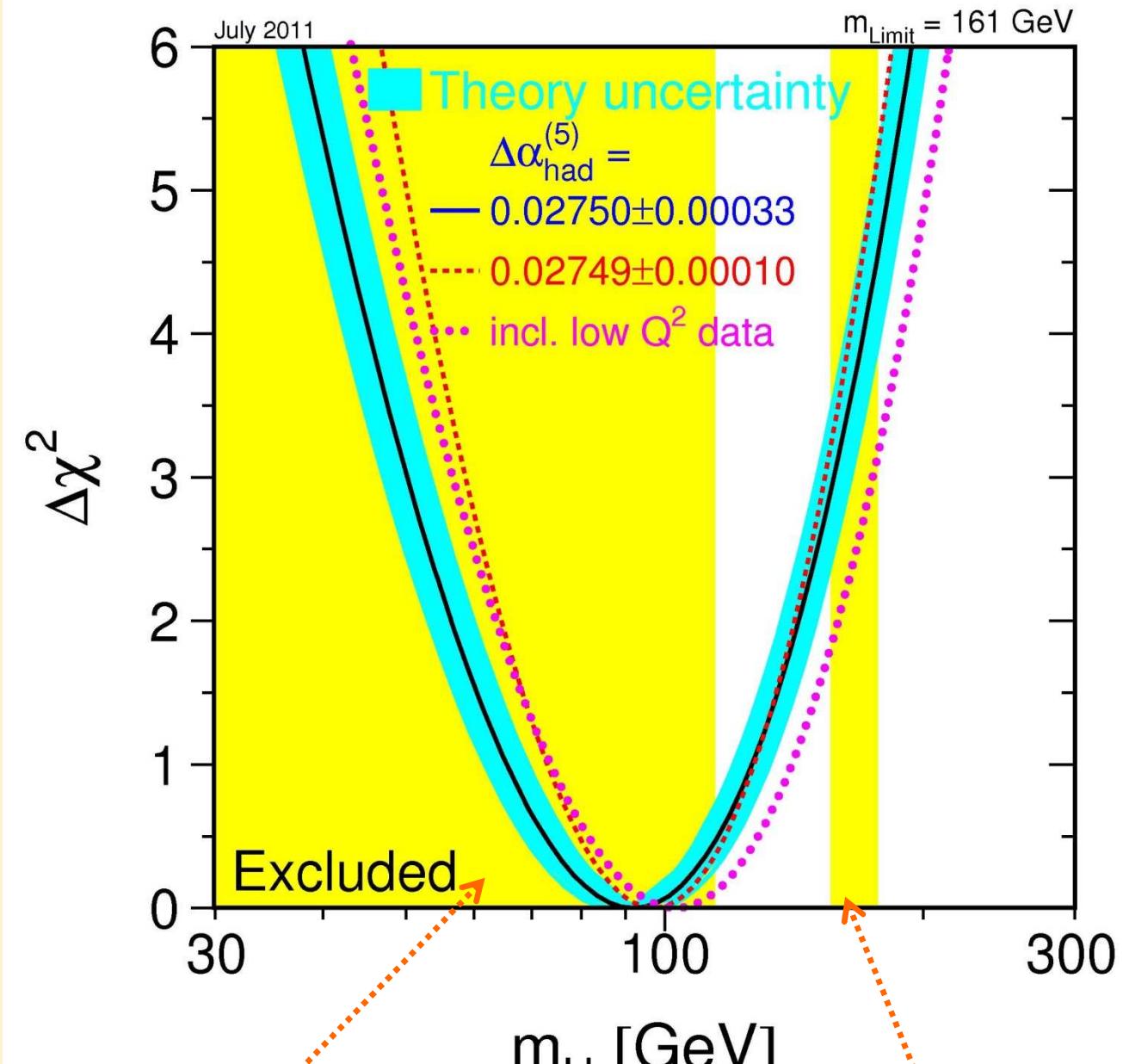
Step 2



Result from
the SM fit to all
precision data :

$m_H < 161 \text{ GeV}$
(95% CL)

95% CL limits from
direct searches:



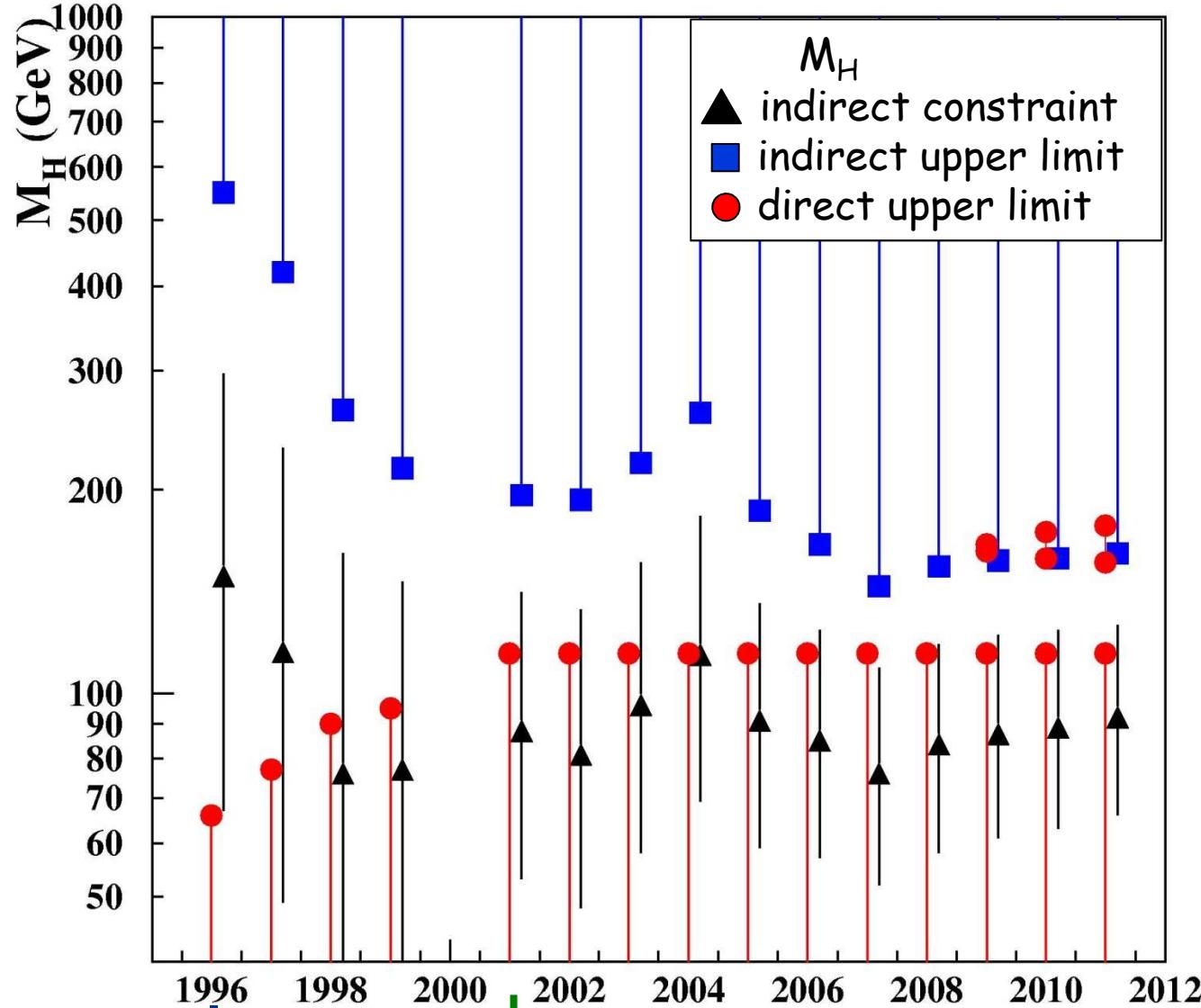
LEP : $m_H > 114.4 \text{ GeV}$

Tevatron: $m_H \notin [156, 177] \text{ GeV}$

✓

Fit of Standard Model parameters

	Z pole data	Z pole data + m_{top}	Z pole data + M_W, Γ_W	Z pole data + $m_{\text{top}}, M_W, \Gamma_W$
$m_{\text{top}}(\text{GeV})$	173 +13 -10	173.2 ± 0.9	179.7 +11.7 -8.7	173.2 ± 0.9
$m_H(\text{GeV})$	118 +203 -64	122 +59 -41	158 +260 -88	92 +34 -26
$\alpha_s(M_Z^2)$	0.1190 ± 0.0027	0.1191 ± 0.0027	0.1190 ± 0.0028	0.1185 ± 0.0026
$\chi^2 / \text{dof (P)}$	16.0/10 (9.9%)	16.0/11 (14%)	17.0/12 (15%)	17.5/13 (18%)
$\sin^2 \theta_{\text{eff}}^{\text{lept}}$	0.23149 ± 0.00016	0.23149 ± 0.00016	0.23142 ± 0.00014	0.23138 ± 0.00013
$\sin^2 \theta_W$	0.22334 ± 0.00062	0.22332 ± 0.00039	0.22288 ± 0.00036	0.22303 ± 0.00028
$M_W(\text{GeV})$	80.362 ± 0.032	80.363 ± 0.020	80.387 ± 0.018	80.378 ± 0.014



m_{top} measurement precise enough to constrain M_H

1st M_W measurement at LEP

2nd order EW corrections

new value of $\Delta\alpha^{(5)}_{had}$

final LEP limit on M_H
new M_W and new m_{top}

2-loop EW corrections

Year

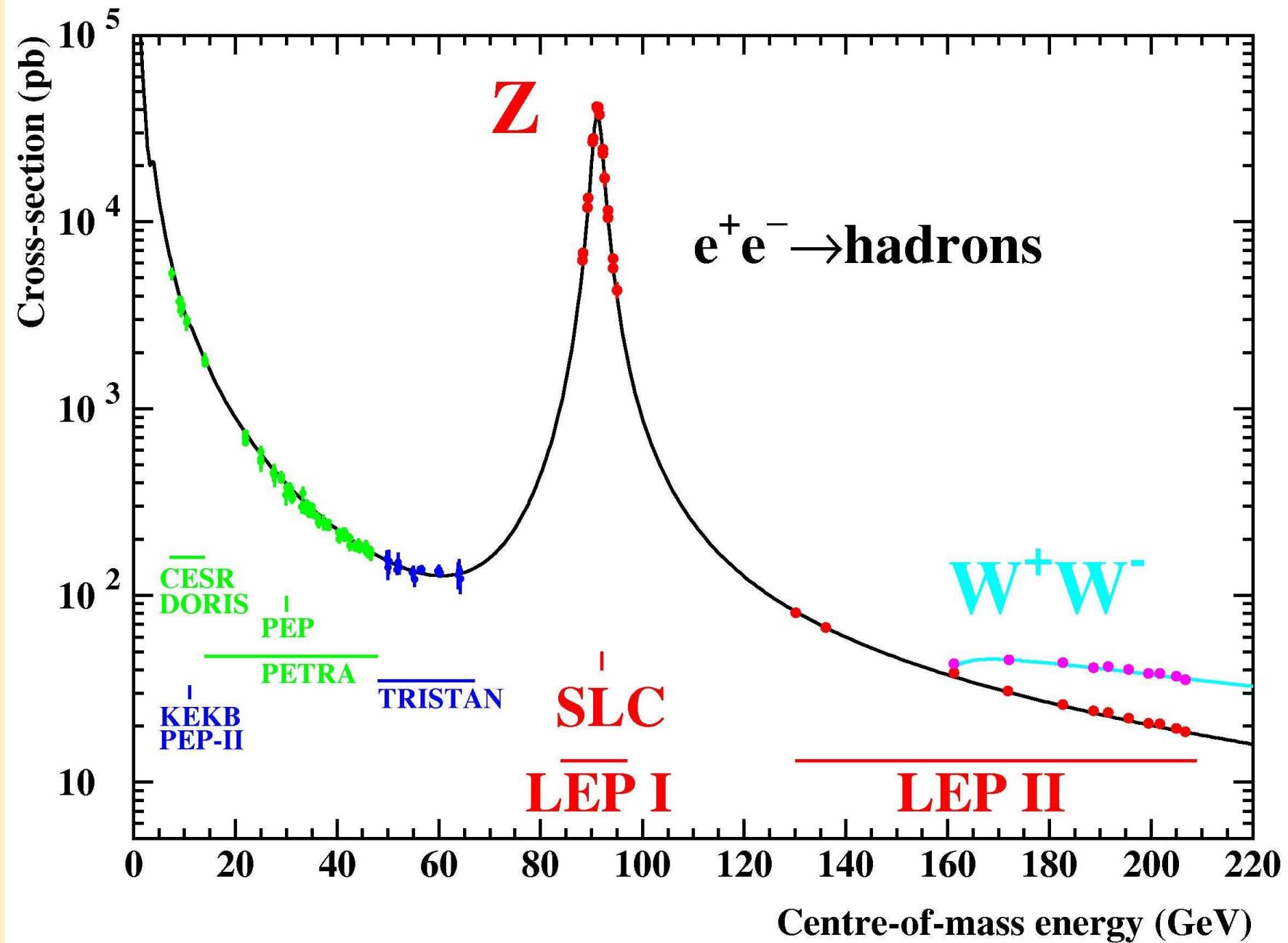
new measurements of m_{top} and/or M_W

Conclusions

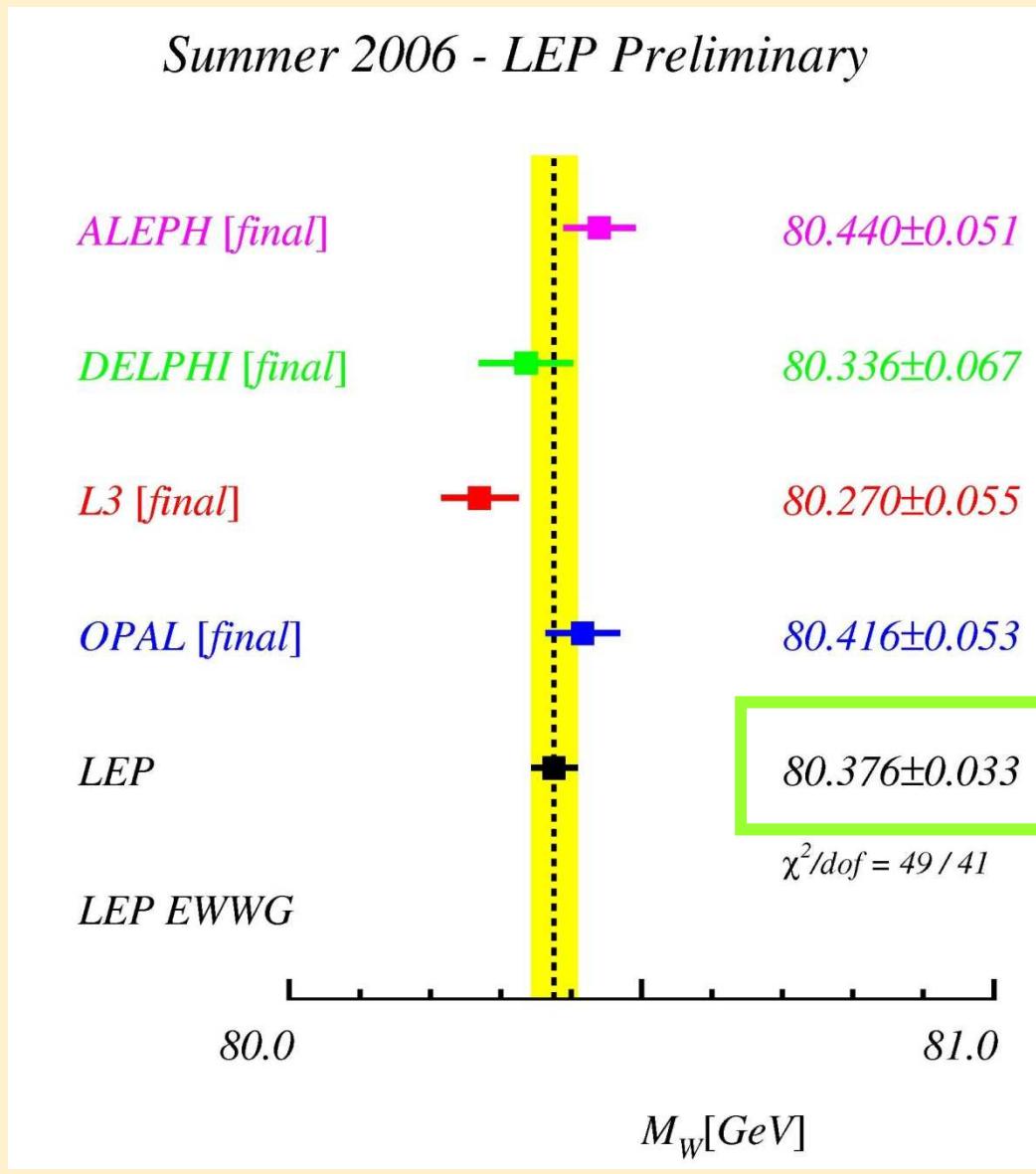
- o The gauge sector of the Electroweak Standard Model has been tested at quantum loop level and found in excellent agreement with data (for a low-mass Higgs boson).
- o Next step: identify the mechanism of spontaneous breaking of the electroweak symmetry (either Higgs mechanism or other).

We expect the SSB sector of the theory to behave like
a low-mass Higgs boson to comply with EW precision data.

BACK-UP SLIDES



M_W at LEP



Mass of the Top Quark

