QCD Lecture [Day 2]

Kunihiro Nagano (KEK)

3rd France-Asia Particle Physics School (FAPPS11)
Les Houches, France, 12/Oct/2011

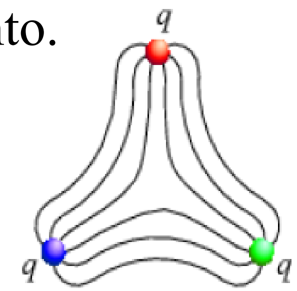
ÉCOLE DE PHYSIQUE
des HOUCHES

Plan for 3 days lectures

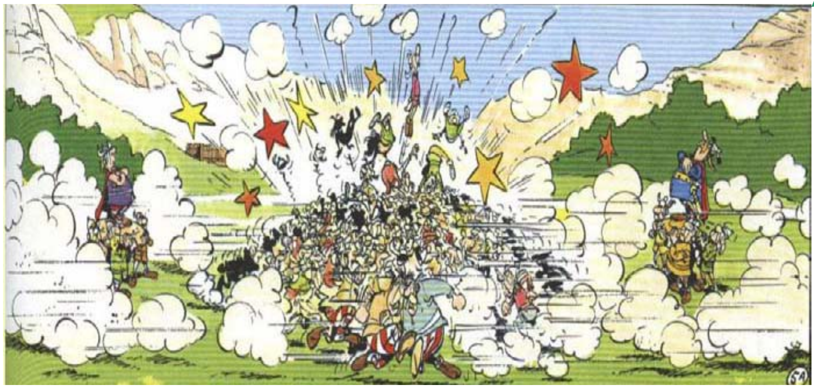
- Day-1: Basis of QCD
- **Day-2: Proton structure @ lepton-hadron collision**
- Day-3: Jets @ hadron-hadron collision

QCD knowledge necessary for doing physics at LHC

► Leant from Day-1: hadrons are composite of quarks and gluons which are "confined" into.



If we do not know about the inside of proton exactly, LHC (proton-proton collisions) will become like:



i.e. don't know what's happening ☺

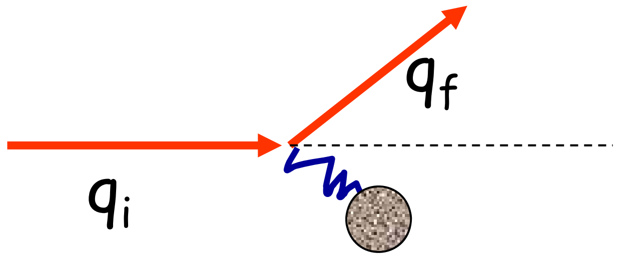
Day-2 is to know about inside proton

Introduction

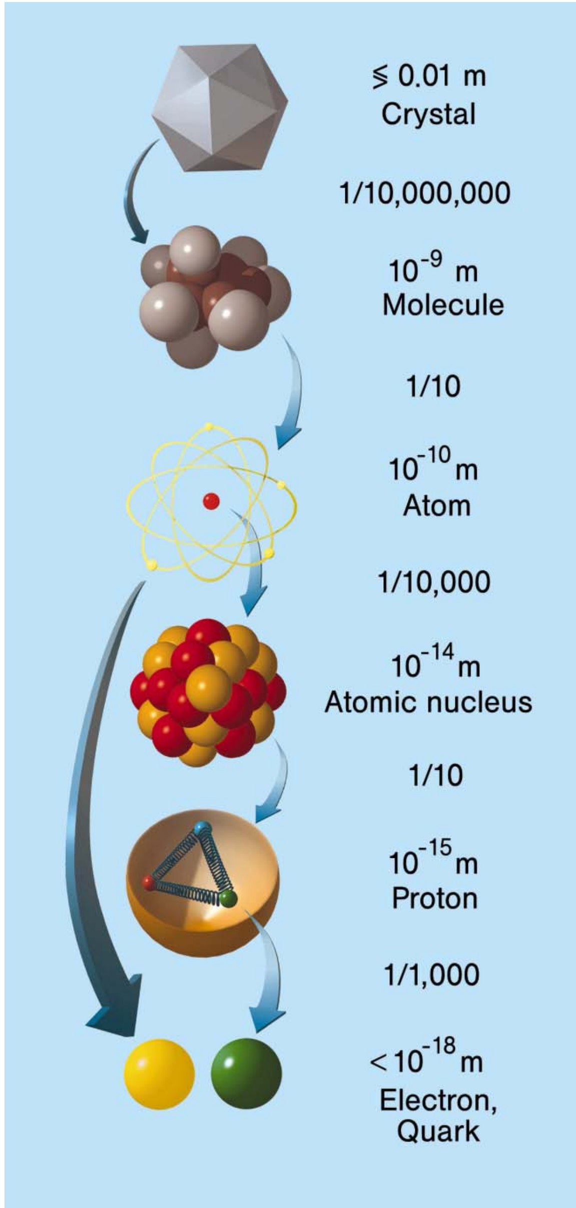
- **How to look inside the matter ?**

How to "look" into the structure of matter

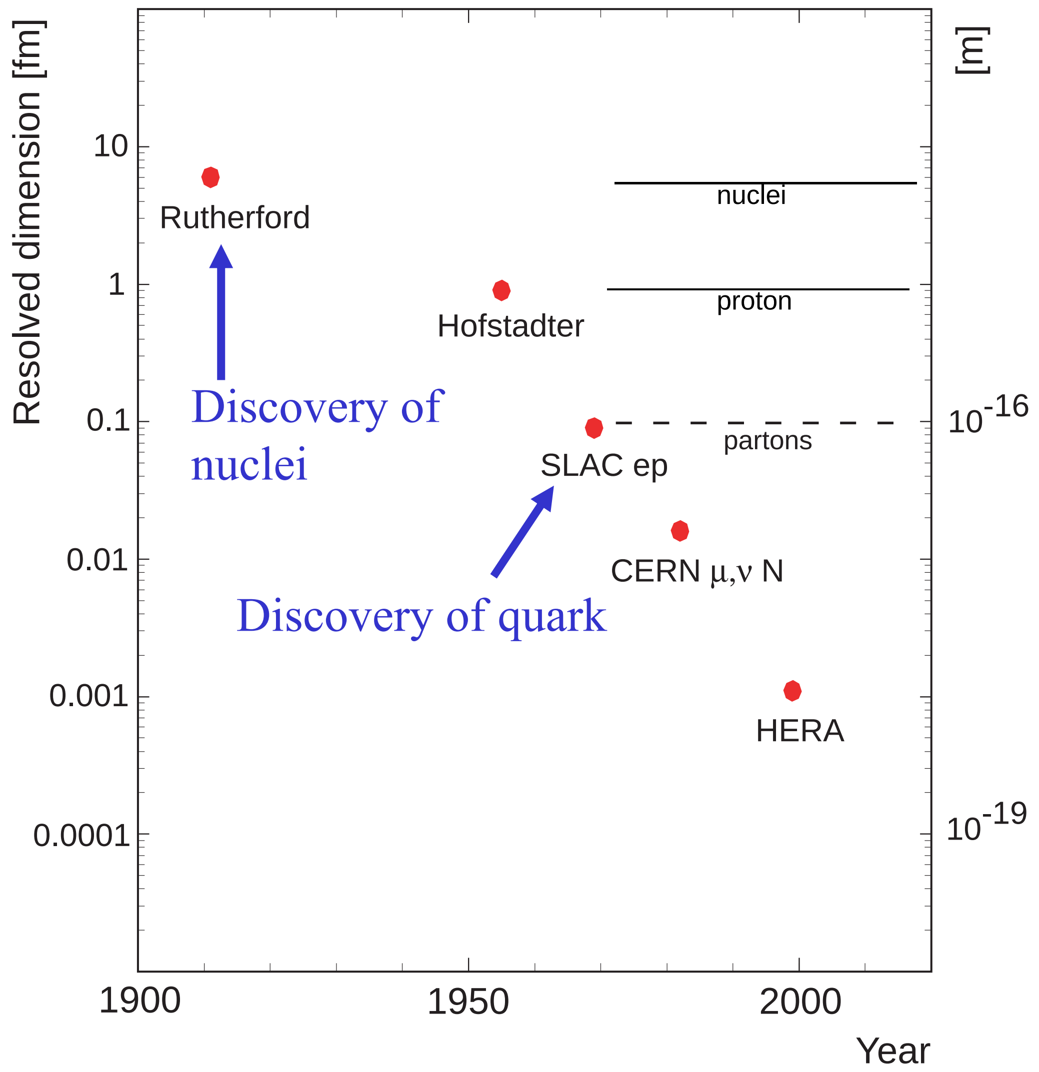
Spatial resolution
~ (Wavelength)$^{-1}$ → $\hbar/q$
⇒ By scattering with high energy particle



Optical microscope
↓
Electron microscope
↓
X-ray sources
↓
α, β-rays from isotopes
Ex. Discovery of nuclei by Rutherford



Spatial resolution ~ (Wave length)$^{-1}$ → $\hbar/q$



Will be introduced later in detail.

Quark-Parton model

- **How to describe "structure" inside proton ?**
  → How proton is composed of quarks/gluons ?

How to “describe” the structure of matter

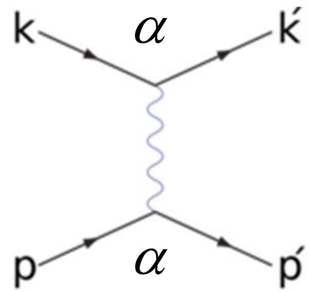
► First, let’s consider two spin-1/2 point-like particles scattering: “no structure”

-- Kinematics: 1 degree-of-freedom (elastic scattering), e.g. scattering angle
→ Better to be Lorentz-invariant →

$$y \equiv \frac{2pq}{(p+q)^2} = 1 - \cos^2(\frac{\theta}{2})$$

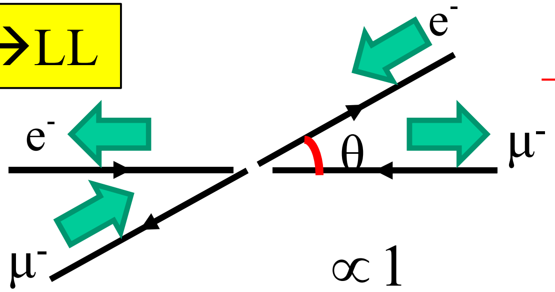
$$q \equiv k' - k$$

$$Q^2 \equiv -q^2$$



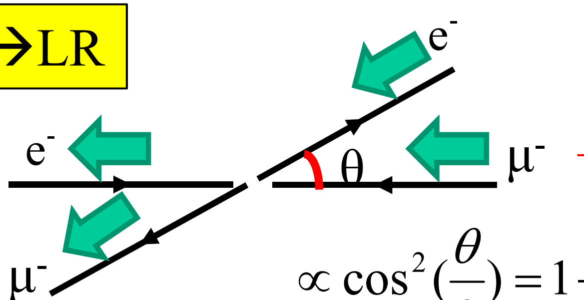
-- EM e-μ scattering : Helicity conservation vs. Angular momentum conservation

LL→LL



$\propto 1$

LR→LR



$\propto \cos^2(\frac{\theta}{2}) = 1 - y$

coupling

$$\frac{d\sigma}{dy} = \frac{2\pi\alpha^2}{Q^4}\left[1 + (1-y)^2\right]s$$

In terms of y (by single quantity)

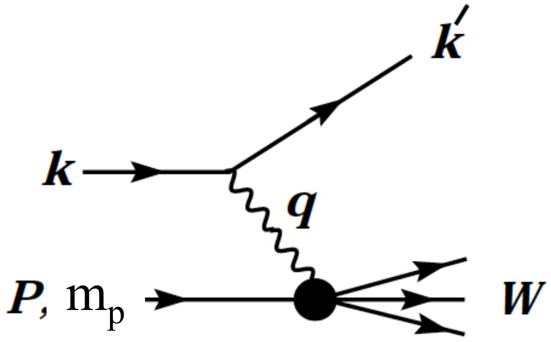
Propagator term

$$s \equiv (p+k)^2$$

Center of mass energy squared

How to "describe" the structure of matter -cont'd-

- ► With large momentum transfer Q^2, proton cannot be stay intact; breaks up into many hadrons: **"Deep Inelastic Scattering (DIS)"**
  -- Kinematics: 2 degree of freedoms, scattering angle and hadronic mass



$$x \equiv \frac{Q^2}{2pq}$$

$$W^2 - m_p{}^2 = (\frac{1}{x} - 1)Q^2$$

- For the case of EM e-p scattering, i.e. trying to look inside proton with EM probe.
  ➔ Intuitively, cross section can be expected to be:

$$\frac{d^2\sigma}{dxdy} = \frac{2\pi\alpha^2}{Q^4}\left[1 + (1-y)^2\right]s \times F_2$$

coupling

In terms of x and y (by two quantities)

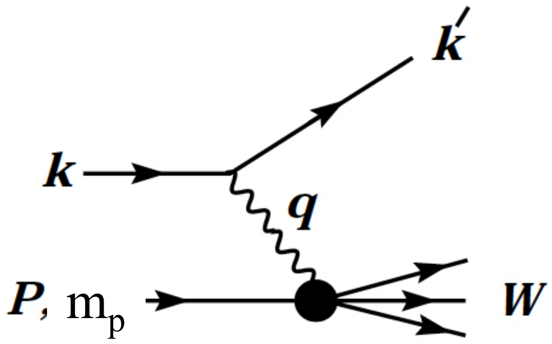
Propagator term

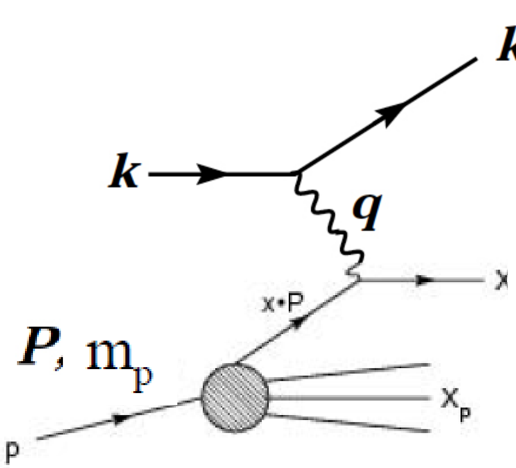
Structure functions (F_2) to parameterize proton structure; how different from point-like case.

Quark-Parton model

- Proton is consisted of "partons" one of which goes into a (hard-)scattering
- The other partons are just "spectators" : similar to the impulse approximation
  - ➔ Linear superposition of (hard-)scattering of each parton

► If parton is massless spin-1/2 particle:





① Massless ➔ x is the **momentum fraction (wrt. proton) of the parton**

"Bjorken x"

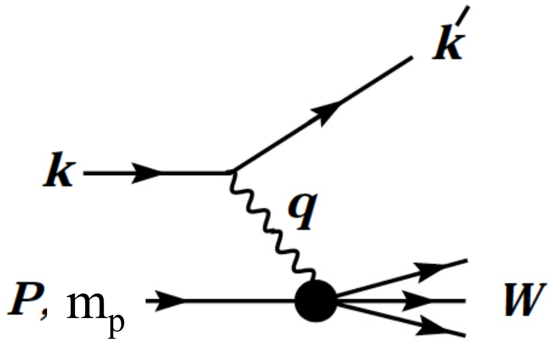
$$(\eta p + q)^2 = 0$$ ← **Massless**

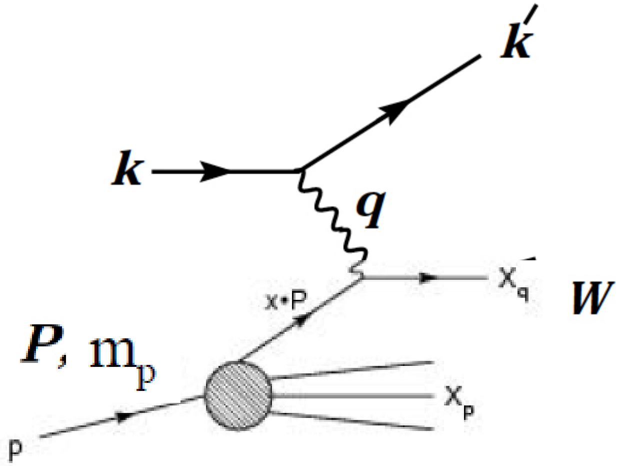
If we call momentum fraction as η

$$\rightarrow \eta = \frac{Q^2}{2pq} = x$$

Quark-Parton model -cont'd-

► If parton (inside spin-1/2 proton) is massless spin-1/2 particle:





② Spin 1/2 ➔ Structure function F_2 is (charge-squared weighted) sum of spin ½ parton's existing probability

$$\sum \left(\frac{2\pi\alpha^2}{Q^4}\left[1+(1-y)^2\right]s \right) e_i^{\,2} q_i \qquad \frac{d^2\sigma}{dxdy} = \frac{2\pi\alpha^2}{Q^4}\left[1+(1-y)^2\right]s \times F_2$$

$$F_2 = \sum_i e_i^{\,2} x q_i(x)$$

→ q(x): (Existing) Probability density function of parton q with momentum fraction x

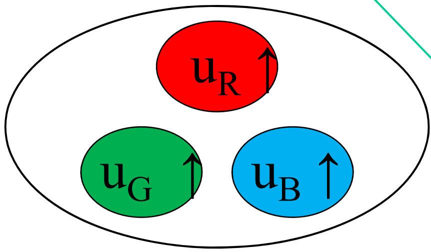
"Parton distribution function (PDF)"

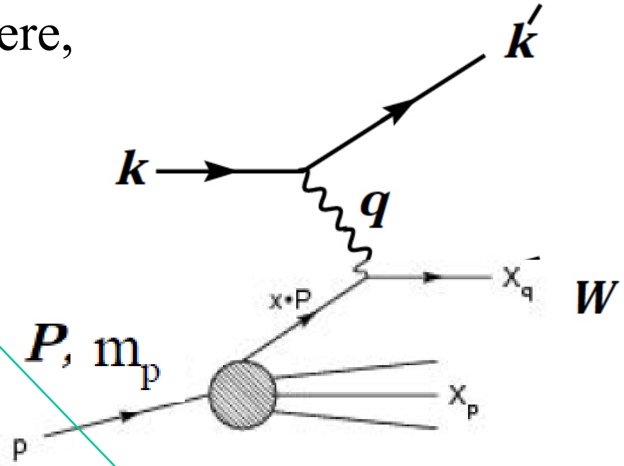
Quark-Parton model -cont'd-

► If proton structure (parton composition) is static:

-- If point-like parton is simply just there,

-- and their existence probability is just a matter





③ Cross section and F_2 will be a function of only x → **"Scaling"**

$$F_2(x) = \sum_i e_i^2 x q_i(x)$$

Here I changed dx → dQ^2 from previous page

$$\frac{d^2\sigma}{dxdQ^2} = \frac{2\pi\alpha^2}{xQ^4}\left[1+(1-y)^2\right]\times F_2(x)$$

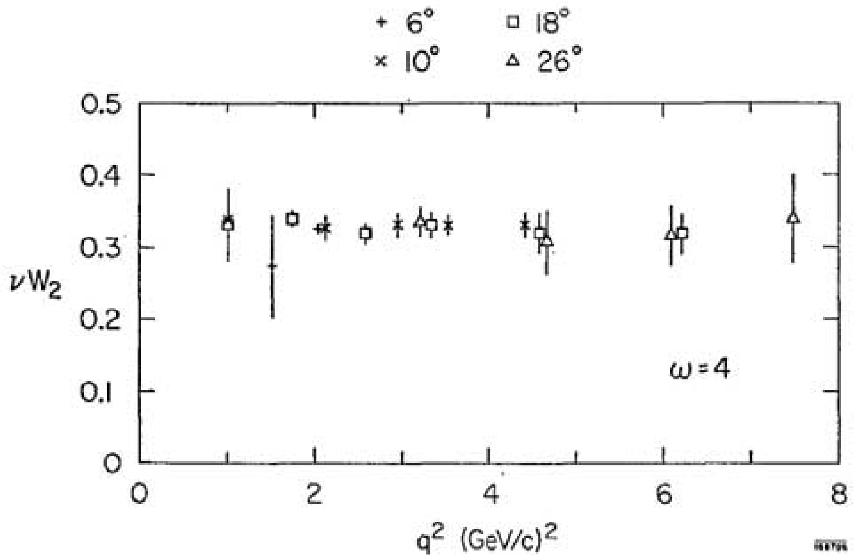
2 degree-of-freedom kinematics ➔ 1 degree-of-freedom

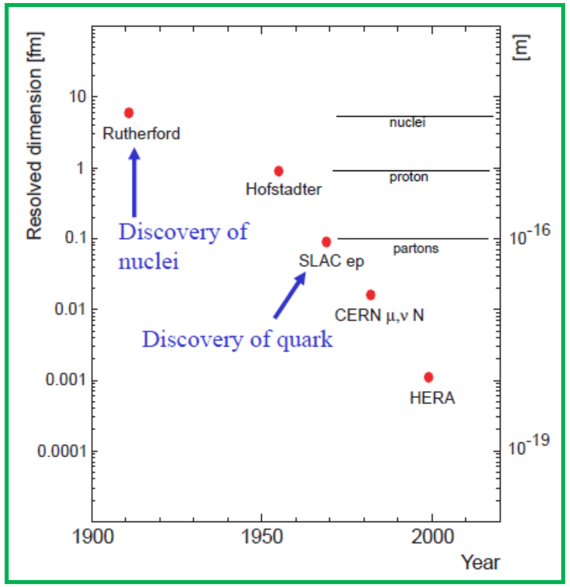
i.e. Not depending on Q^2

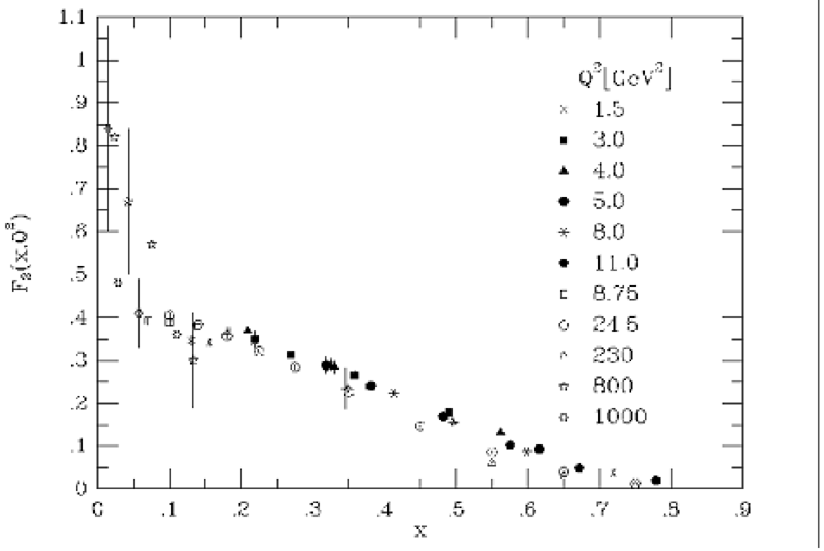
Q^2 is the spatial resolution to "look" structure.
➔ Structure stays same although we increase resolution

Bjorken Scaling

► Structure function F_2 measured at Q^2 range: $1 < Q^2 < 8$ GeV^2







Bjorken scaling shown up to $Q^2 \sim 10$ GeV^2
→ Validity of Quark-Parton model
"Discovery of quarks"

Scaling violation

- **Quark-parton model describes proton structure by means of:**
  -- PDFs; existence probability of each parton
- **Quark-parton model gives a "static" view of proton**
  -- No dependence on spatial resolution Q^2

⇩

Dynamical view of proton

Dynamical picture of QCD

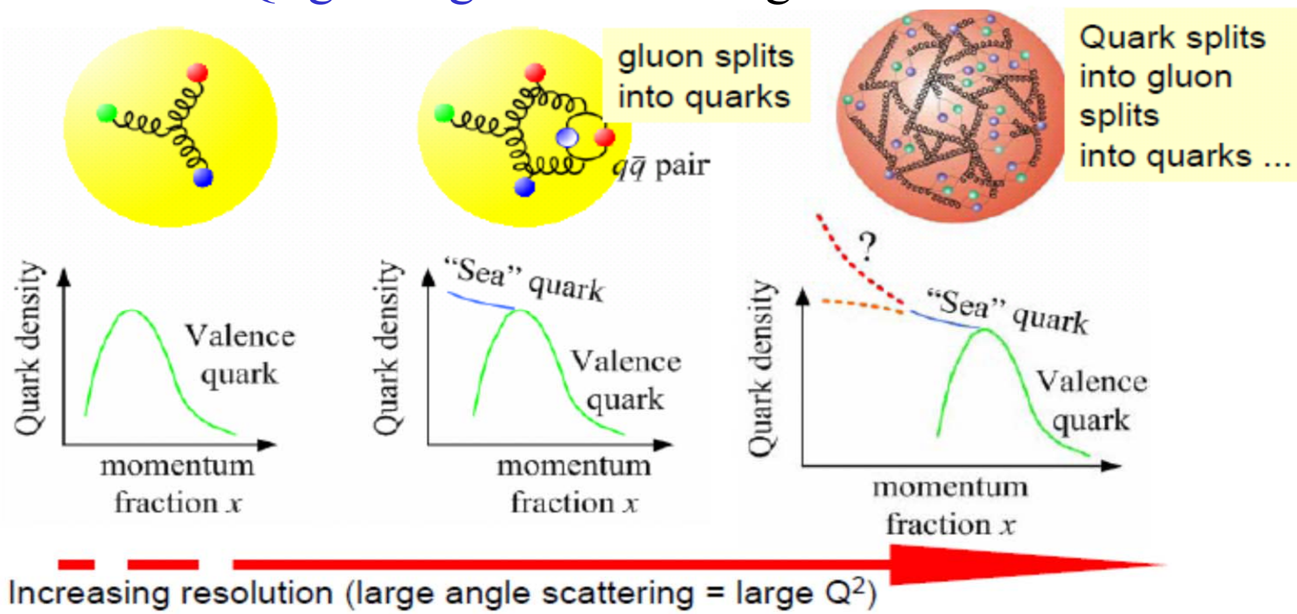
► Increased spatial resolution Q^2 → Shorter interaction time τ_{int} $\quad \boxed{\tau_{int} \approx 1/Q^2}$

-- Gluon splits into a pair of quark and anti-quark, which in turn recombines back to gluon later. Such is repeated every short time scale.

→ With high Q^2, hard scattering can occur with such instantly-lived quark

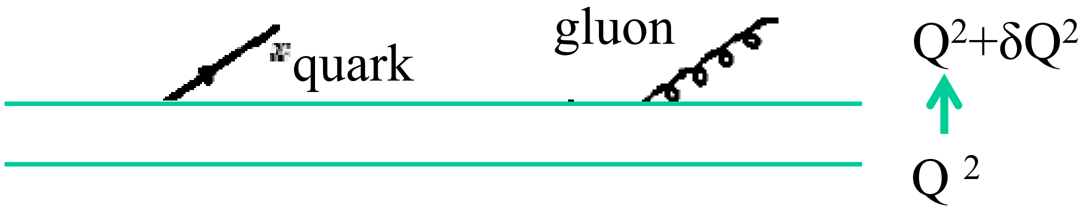
➔ Taking a "snap-shot" of dynamic picture of proton

-- With EM interaction (γ-probe), gluon cannot be seen directly (cannot directly interact with γ), but is indirectly seen as "increase of quarks with smaller x as Q^2 gets higher" : "Scaling violation"



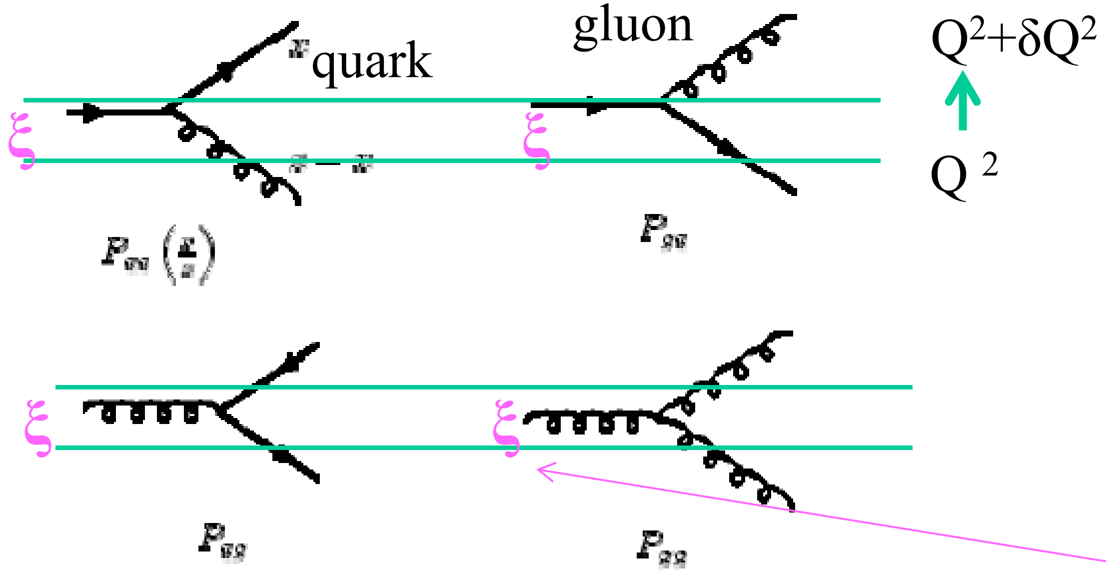
How the looking is changed as the scale goes

► How quark and gluon PDFs evolve as the scale Q^2 goes



How the looking is changed as the scale goes

► How quark and gluon PDFs evolve as the scale Q^2 goes



Called: Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equation
(→ Visit this later again.)

$$\frac{\partial}{\partial \ln Q^2}\begin{pmatrix} q_i(x) \\ g(x) \end{pmatrix} = \frac{\alpha_s}{2\pi}\sum_i \int_x^1 \frac{d\xi}{\xi}\begin{pmatrix} P_{q_i q_j} & P_{q_i g} \\ P_{g q_j} & P_{gg} \end{pmatrix}\begin{pmatrix} q_j(\xi) \\ g(\xi) \end{pmatrix}$$

Slowly (log) changes wrt Q^2
➔ Measurement in wide Q^2 coverage necessary

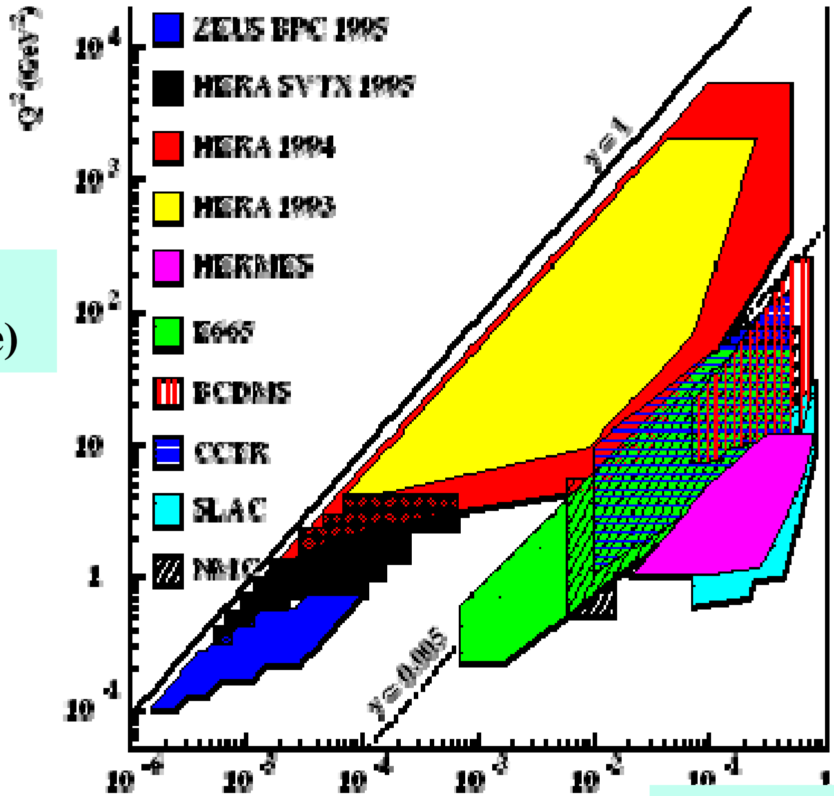
Sum up all quark flavours

Integrate all momentum higher than x

Deep inelastic scattering (DIS) experiments

- $Q^2_{MAX} = s$: center-of-mass energy squared
- Fixed target vs collider kinematics

Q²
(scale)



Bjorken x

- HERA: world's only e-p collider (Ee=27.5 GeV, Ep=920 GeV)

$$\sqrt{s} = 320\,\text{GeV}$$

-- Operated until year 2007

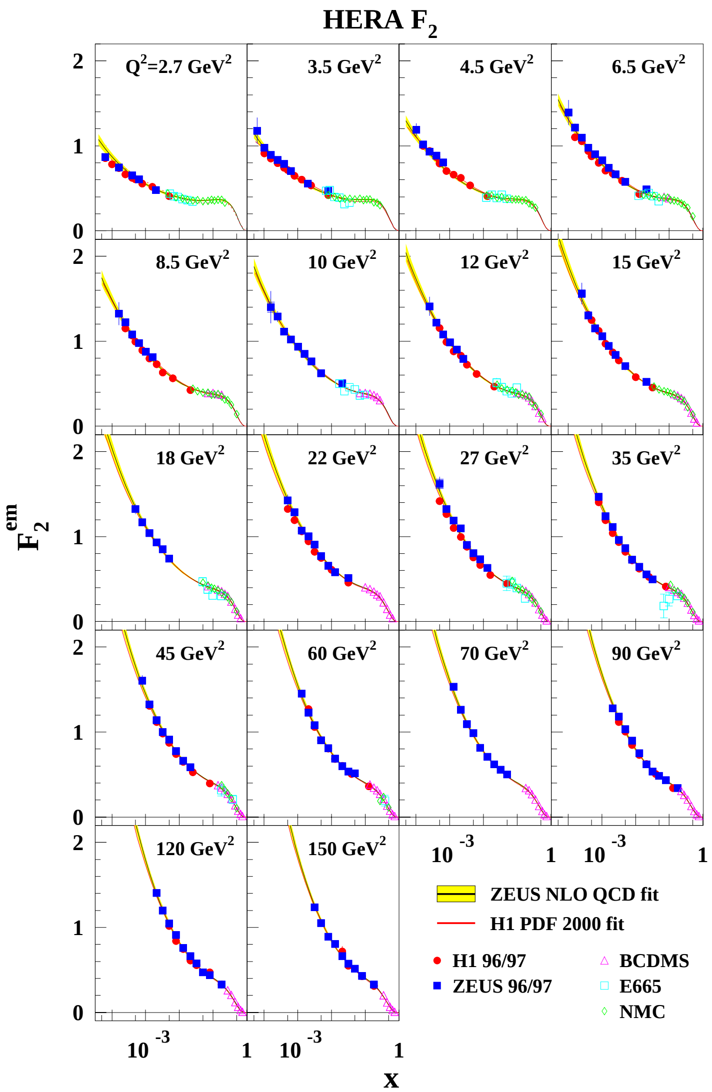
$$Q^2_{MAX} = s \sim 10^5\, GeV^2$$

$$\lambda_{MAX} \sim 1/1000\, r_{proton}$$

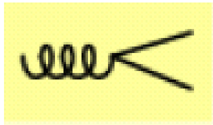
(corresponds to ~50 TeV incident beam on fixed target)

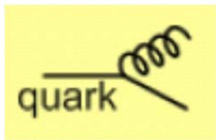
Structure function measurements

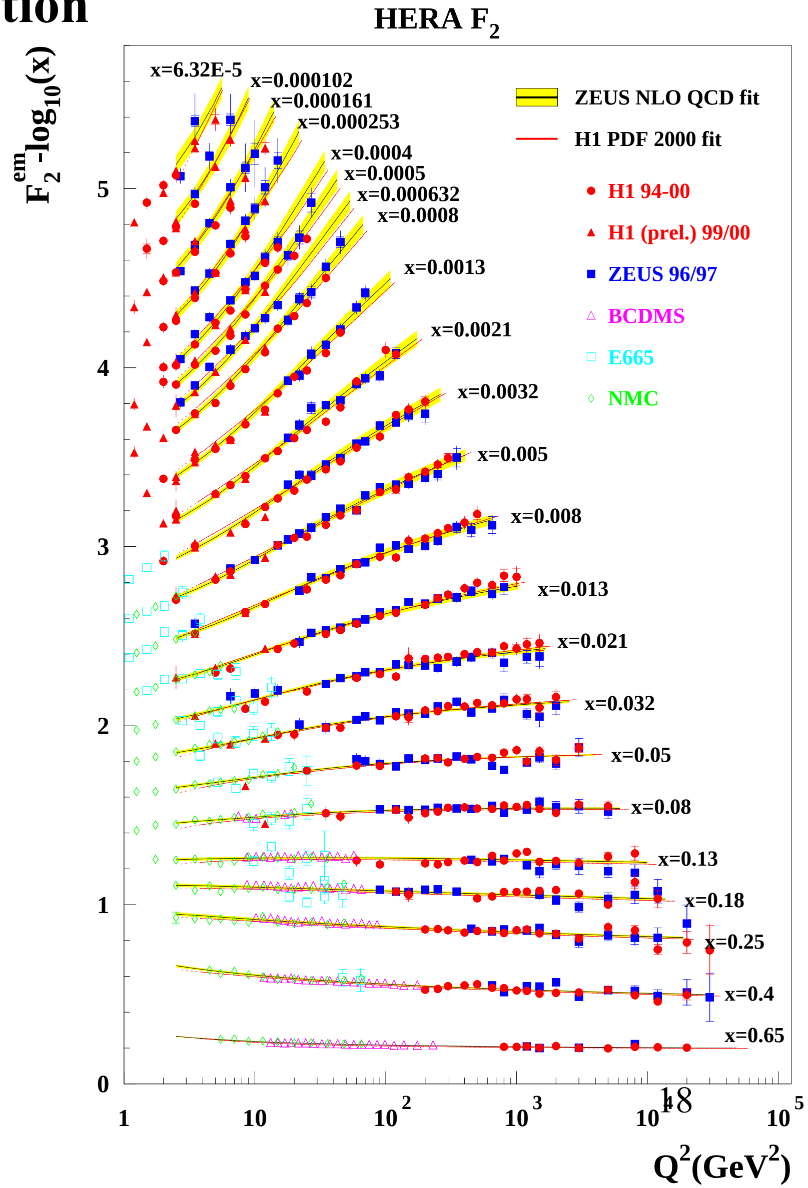
- "Strong Rise" of F_2



- Scaling violation







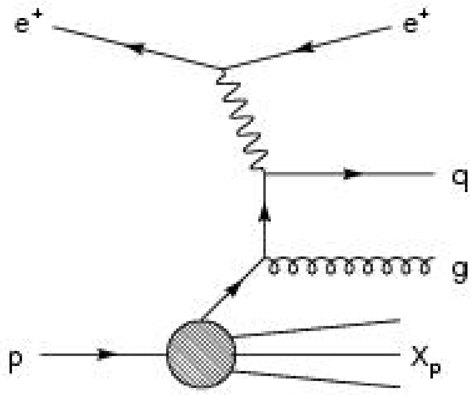
Factorization and revisit of DGLAP evolution

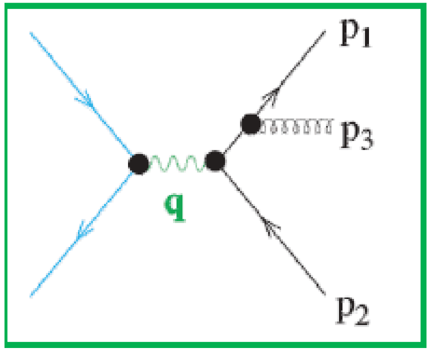
- **QCD predicts a dynamical picture of proton, namely its structure's evolution wrt. log of Q^2 (spatial resolution)**

➔ **Where this "lnQ^2" comes from ?**

DIS at Leading Order QCD

► Let's consider leading order QCD effect to DIS





Slides @ Day-1

Reminder:
Collinear/Soft singularities

$$\frac{d\sigma}{dx_1 dx_2} = \sigma_0 \frac{\alpha_s}{2\pi} C_F \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

$$\sigma_0 = \frac{4\pi\alpha_{em}}{s} \sum e_q^2$$

2 & 3 collinear

collinear singularities:

$(1 - x_1) \to 0$

(gluon collinear to antiquark)

$(1 - x_2) \to 0$

(gluon collinear to quark)

3
soft

soft singularity:

gluon soft: $x_3 \to 0$

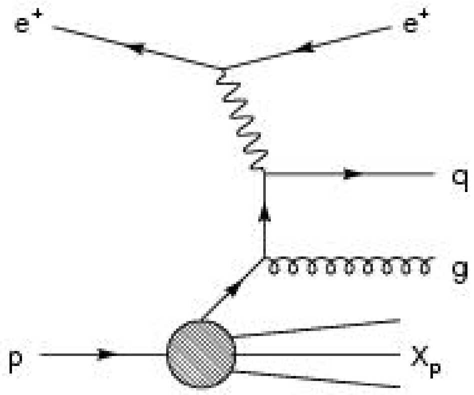
$\Updownarrow$

$x_1 \to 1, \quad x_2 \to 1$

➔ These singularities arise from interactions at long distance, and called as infrared divergence

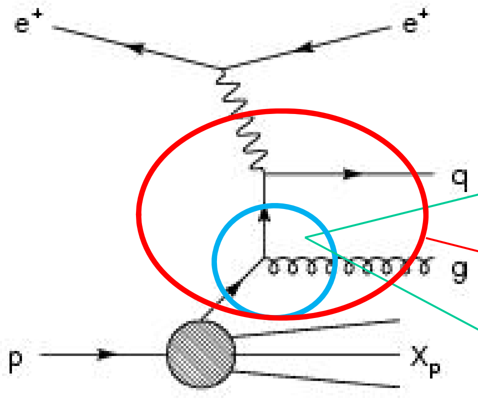
DIS at Leading Order QCD

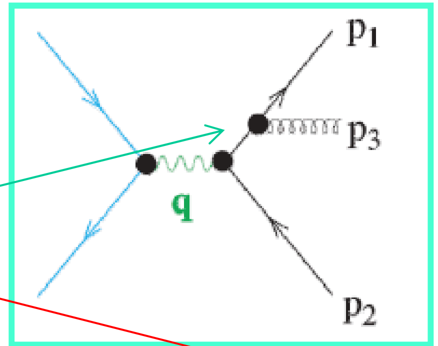
► Let's consider leading order QCD effect to DIS



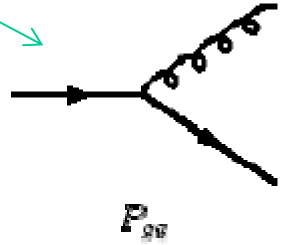
DIS at Leading Order QCD

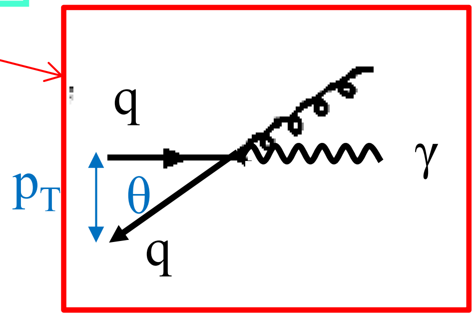
► Let's consider leading order QCD effect to DIS





- What is the origin of collinear divergence





$$\int \frac{d\Omega}{1-\cos\theta}$$

At small angle:

$$p_t^{\,2} = p^2 \sin^2\theta$$

$$\approx p^2\theta^2$$

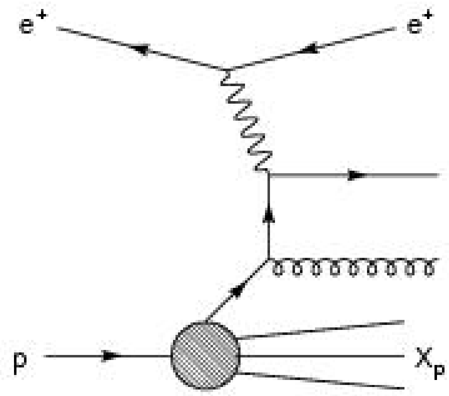
$$p_t^{\,2}(\max) \approx Q^2$$

$$\int_0^{p_t^{\,2}(\max)} \frac{dp_t^{\,2}}{p_t^{\,2}} \longrightarrow \int_{\kappa^2}^{p_t^{\,2}(\max)} \frac{dp_t^{\,2}}{p_t^{\,2}} = \ln\left(\frac{Q^2}{\kappa^2}\right) + C$$

If we introduce k^2 cutoff

$\ln Q^2$ dependence originates from here

Factorization



► Let's consider leading order QCD effect to DIS

$\kappa^2 \to 0$

Splitting function (Probability of q→qg) × **Collinear divergence**

$$F_2(x,Q^2) = x\sum_q e_q^{\,2}\left[q_o^{\,2}(x) + \frac{\alpha_S}{2\pi}\int_x^1 \frac{d\xi}{\xi} q_0(\xi)\{P(\frac{x}{\xi})\ln(\frac{Q^2}{\kappa^2}) + C(\frac{x}{\xi})\}\right]$$

Naïve Quark-Parton Model

Everything else that can be calculable

► Factorize at scale μ_F

$$\ln(\frac{Q^2}{\kappa^2}) = \ln(\frac{Q^2}{\mu_F^{\,2}}) + \ln(\frac{\mu_F^{\,2}}{\kappa^2})$$

$$F_2(x,Q^2) = x\sum_q e_q^{\,2}\int_x^1 \frac{d\xi}{\xi} q(\xi,\mu_F^{\,2}) \times \{\delta(1-\frac{x}{\xi}) + \frac{\alpha_S}{2\pi} P(\frac{x}{\xi})\ln\frac{Q^2}{\mu_F^{\,2}} + C'\}$$

$\kappa^2 \to 0$

$$q(x,\mu_F^{\,2}) = q_0(x) + \frac{\alpha_S}{2\pi}\int_x^1 q_0(\xi)\{P(\frac{x}{\xi})\ln\frac{\mu_F^{\,2}}{\kappa^2} + C''\}$$

Arbitrary choice to split C btw F_2 and PDF

Factorization -cont'd-

► Arbitrary choice on "C" → Factorization scheme

-- $\overline{\text{MS}}$, DIS schemes, etc.

► PDF absorbs collinear divergence

-- Cannot be fully calculated

-- However, its variation with μ_F is given by

DGLAP evolution equation

$$\frac{\partial q_i(x,\mu_F{}^2)}{\partial \ln \mu_F{}^2} = \frac{\alpha_S}{2\pi}\int_x^1 \frac{d\xi}{\xi} q_i(\xi,\mu_F{}^2) P(\frac{x}{\xi})$$

Take derivative with $\ln \mu_F{}^2$

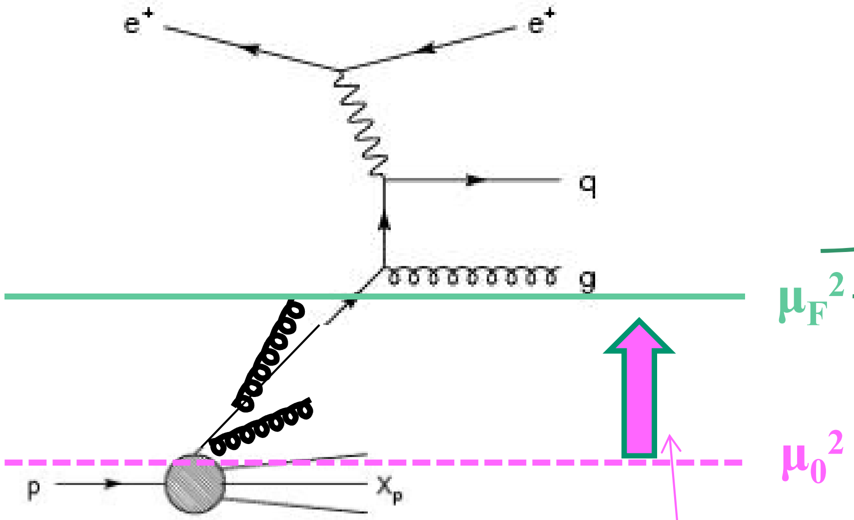
$$F_2(x,Q^2) = x\sum_q e_q{}^2 \int_x^1 \frac{d\xi}{\xi} q(\xi,\mu_F{}^2)\times\{\delta(1-\frac{x}{\xi}) + \frac{\alpha_S}{2\pi}P(\frac{x}{\xi})\ln\frac{Q^2}{\mu_F{}^2} + C'\}$$

$\kappa^2 \to 0$

$$q(x,\mu_F{}^2) = q_0(x) + \frac{\alpha_S}{2\pi}\int_x^1 q_0(\xi)\{P(\frac{x}{\xi})\ln\frac{\mu_F{}^2}{\kappa^2} + C''\}$$

Arbitrary choice to split C btw F_2 and PDF

What's happened by factorization



Hard process: scale > μ_F^2

-- Infrared-safe, perturbative calculation possible

$$\hat{\sigma}(eq_i)$$

$$\sigma_i = \int_0^1 dx \{ \hat{\sigma}(eq_i) \times q_i(x) \}$$

Soft process: scale < μ_F^2

-- Non-perturbative effects absorbed in PDF

$$q_i(x)$$

-- If PDF is given at a certain scale (μ_0^2) PDF at μ_F^2 can be extrapolated by DGLAP evolution equation

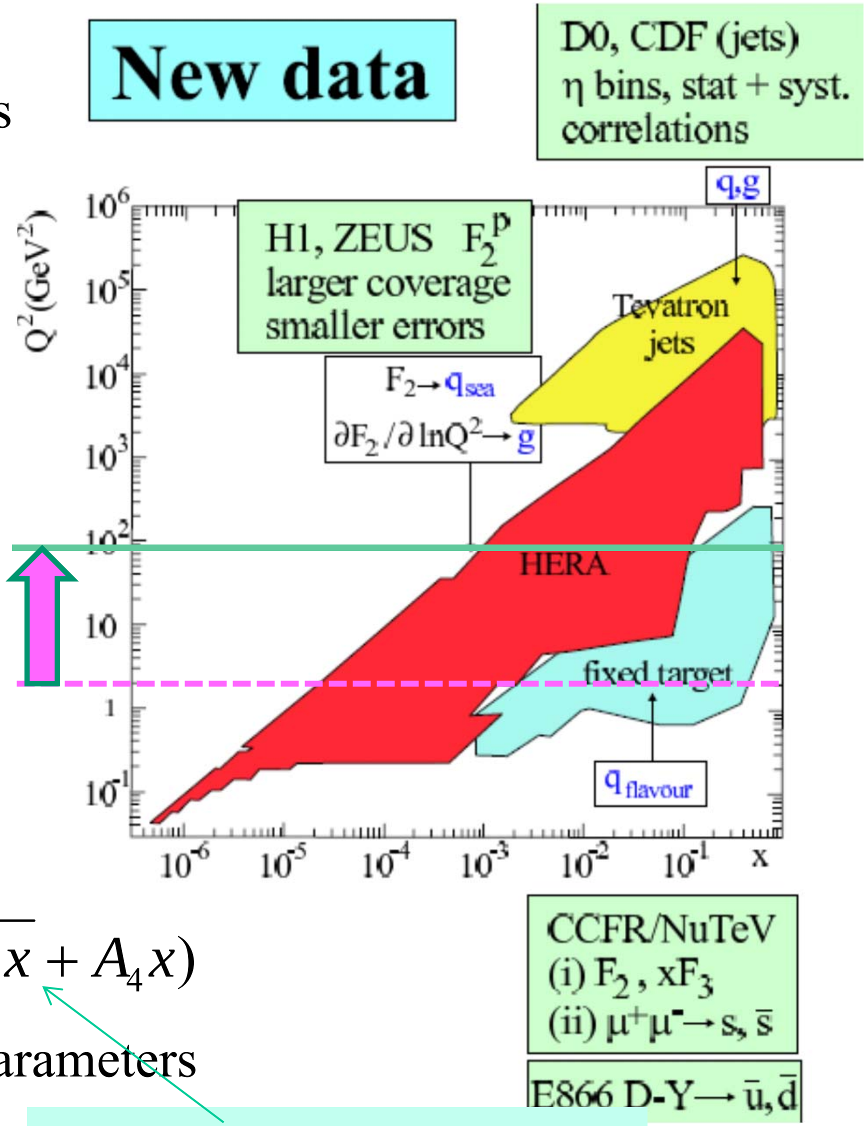
Determination of PDF

- **Factorization technique allows us to split out un-calculable collinear divergences due to long-range.**
  -- PDFs to absorb it.
- **Nevertheless, QCD can predict how PDFs should evolve once they are given at a certain starting scale.**

➔ How to determine such "PDFs at a certain starting scale" ?

Determination of PDF

► Determine PDFs by fitting measurements



$$F_2^{meas.}(x, Q^2)$$

$$F_2^{theory}(x, Q^2; A_i) \leftarrow q_i(\mu_F{}^2; A_i) \quad \mu_F{}^2$$

DGLAP ↑

$$q_i(\mu_0{}^2; A_i) \quad \mu_0{}^2$$

- Parameterize PDFs by using some functional form e.g.

$$q_i(\mu_0{}^2) = A_0 x^{A_1} (1-x)^{A_2} (1 + A_3\sqrt{x} + A_4 x)$$

and assume some initial values for the parameters

Governs high-x behavior

Governs low-x behavior

"Smoothing function" to connect low-high x smoothly

Determination of PDF

► Determine PDFs by fitting measurements

$$F_2^{meas.}(x,Q^2)$$

$$F_2^{theory}(x,Q^2;A_i) \leftarrow q_i(\mu_F^2;A_i)$$

DGLAP

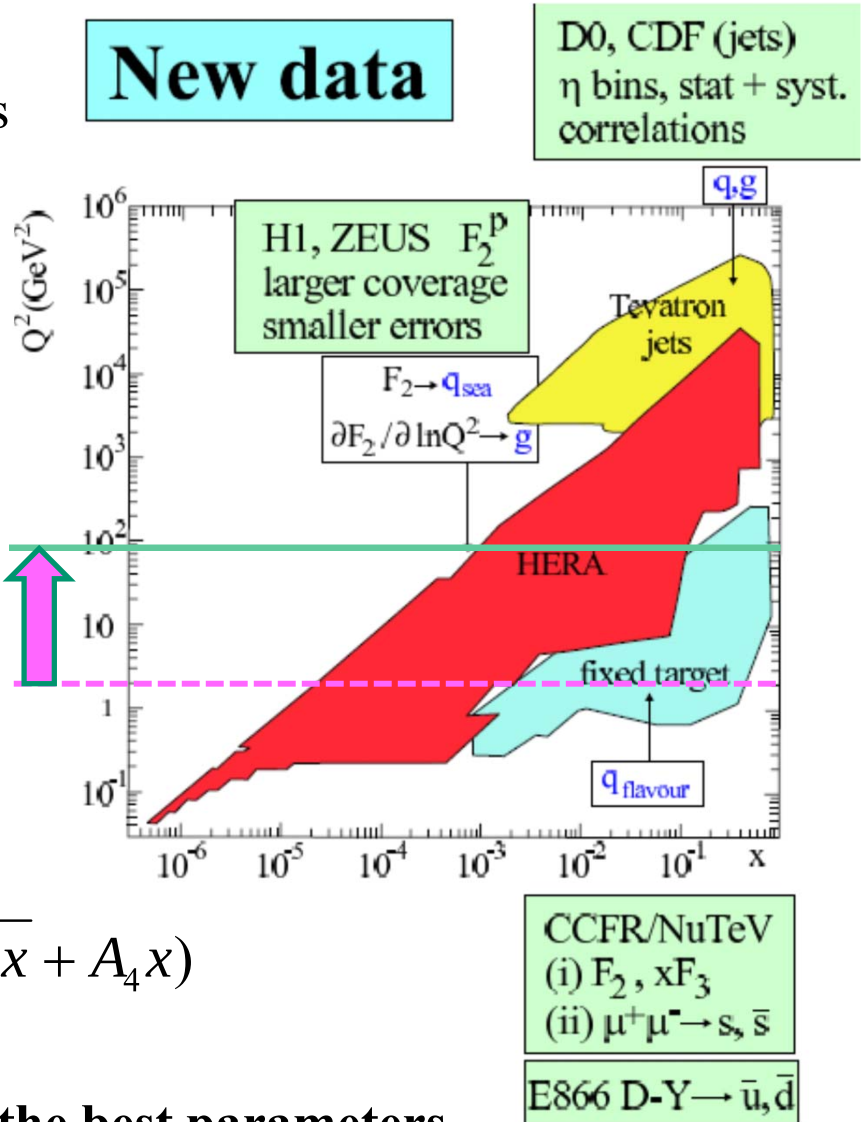
$$q_i(\mu_0^2;A_i)$$

● Parameterize PDFs by using some functional form e.g.

$$q_i(\mu_0^2) = A_0 x^{A_1}(1-x)^{A_2}(1+A_3\sqrt{x}+A_4 x)$$

update parameters, and repeat

→ With these iterations, find out the best parameters which describe the data best



PDF parameterization [An example]

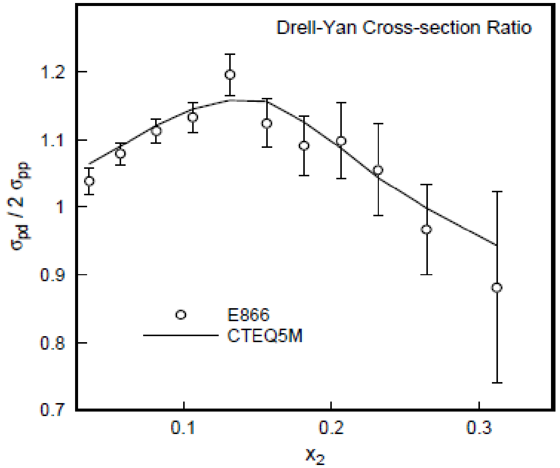
$$S_G = \int_0^1 dx \frac{F_2^p(x) - F_2^n(x)}{x} = \frac{1}{3} - \frac{2}{3}\int_0^1 dx\,[\bar{d}(x) - \bar{u}(x)]$$

- Flavor decomposition with u_V, d_V, g, q_{sea}, $\bar{d} - \bar{u}$

$$u_V \equiv u - \bar{u} \qquad \bar{u} = u_{sea} \qquad \bar{d} = d_{sea} \qquad \bar{s} = s$$

$$d_V \equiv d - \bar{d}$$

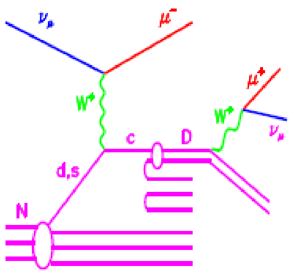
$$q_{sea} = u_{sea} + d_{sea} + s + \bar{u} + \bar{d} + \bar{s}$$



- Constraints
  - -- Number sum rule $\int_0^1 u_V dx = 2 \qquad \int_0^1 d_V dx = 1$
  - -- Momentum sum rule $\int_0^1 (xu_V + xd_V + xg + xq_{sea}) = 1$

- Assumptions
  - -- $s = \frac{1}{4}(\bar{u} + \bar{d})$

-- νN(CCFR etc) di-muon data



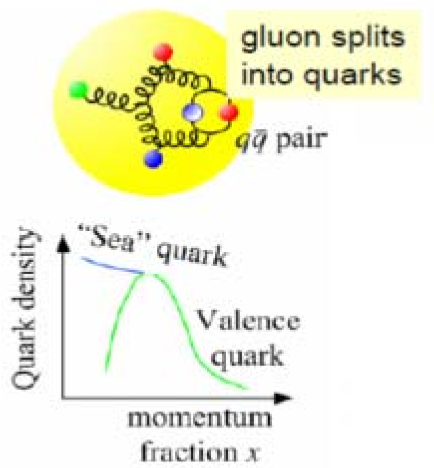
di-muon	NuTeV	CCFR	Combined
Neutrino	5012	5030	10042
Anti-Nu	1458	1060	2518

* High stats & high precision data
* Best constraints on strange quark

➔ **In total, > ~10 parameters left free to be determined by the fit**

Complementarity of data

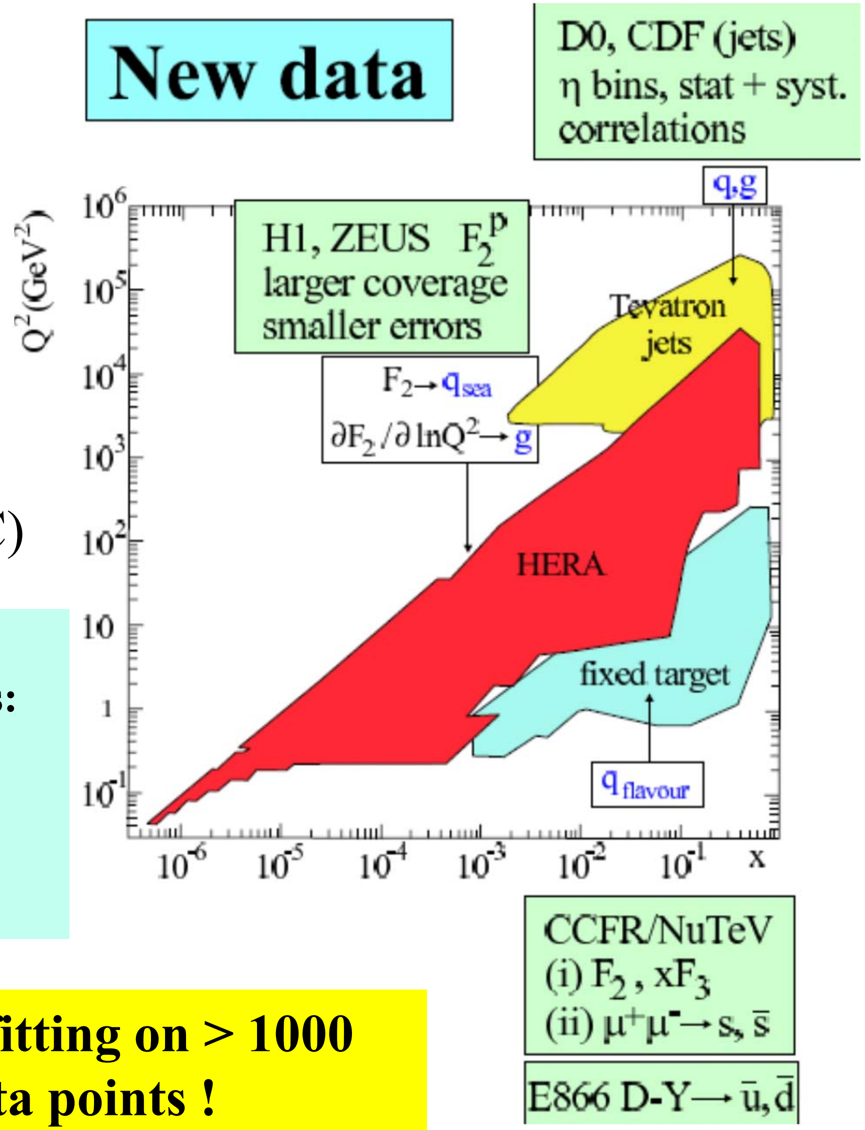
- ► HERA data
  - -- At low x (10^{-4} to 10^{-1})
    - * Sea quarks
    - * Gluon via scaling violation
- ► Fixed targets DIS data
  - -- Valence at high x
- ► Hadron-hadron data (TEVATRON, LHC)
  - -- Gluon at high x



Various analyses by various groups:
-- MRS
-- CTEQ
-- HERA PDF
-- NNPDF …

"Global" fitting on > 1000 precise data points !

New data



Uncertainties of PDFs

- ► Experimental errors
  - -- Statistical uncertainties ("random")
  - -- Systematical uncertainties ("correlated")
    - * Correlation between data points; one systematic source e.g. HERA luminosity should move all HERA data up/down simultaneously

Diagonarized PDF error matrix
➔ LHPDF PDF error sets: 1…20 etc.

You need to run your MC with ~20 times with different "PDF error sets" to evaluate PDF systematic

- ► Theoretical model assumption
  - -- Order (LO, NLO, NNLO….)
  - -- Choice of μ_0^2
  - -- Choice of functional form : CTEQ uses $1 + A_3 x^{A_4}$ etc., NNPDF does not use function
  - -- Treatment of heavy-flavor quarks
    - * variable flavor number scheme, fixed flavor number scheme, etc…
  - -- Cut on data sets (to define pQCD safe region)
    - * $W^2 > 20\ \text{GeV}^2$, $Q^2 > 4\ \text{GeV}^2$

$$q_i(\mu_0^{\,2}) = A_0 x^{A_1}(1-x)^{A_2}(1 + A_3\sqrt{x} + A_4 x)$$

NN ↑ Neural Net

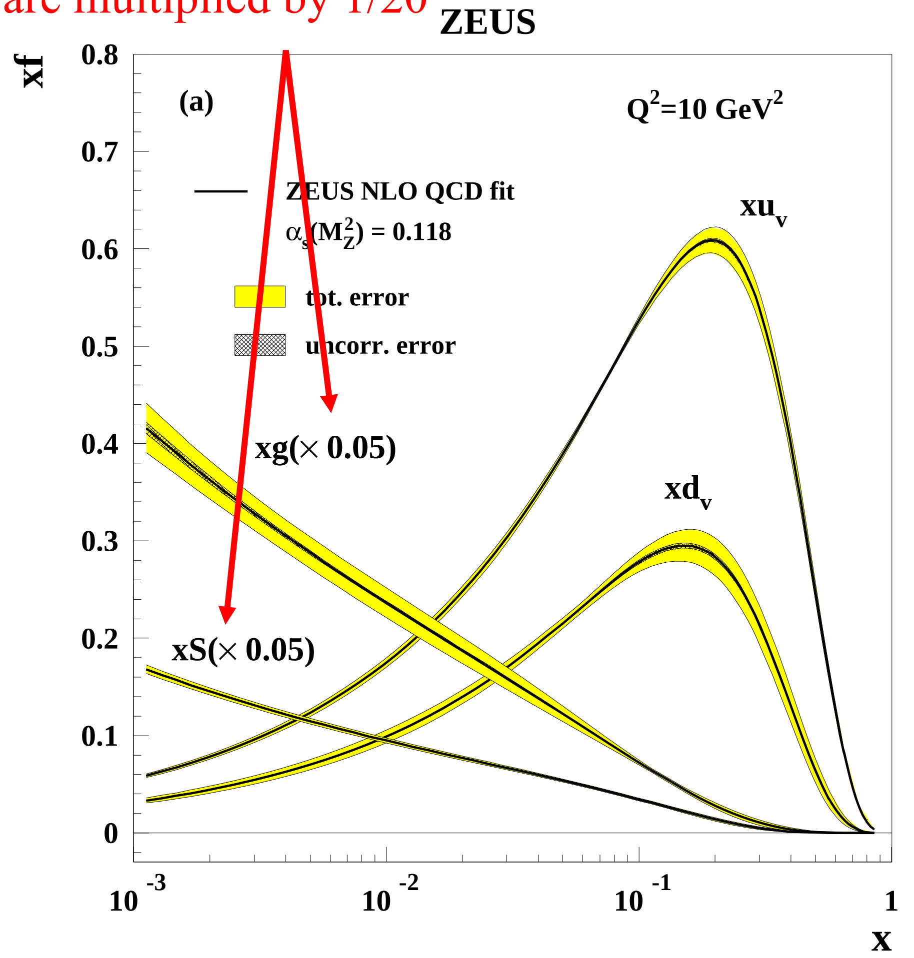
Comparing CTEQ vs MRS is not a "correct" method to evaluate systematic error. (Just to give a "feeling" of it ; better than not to do)

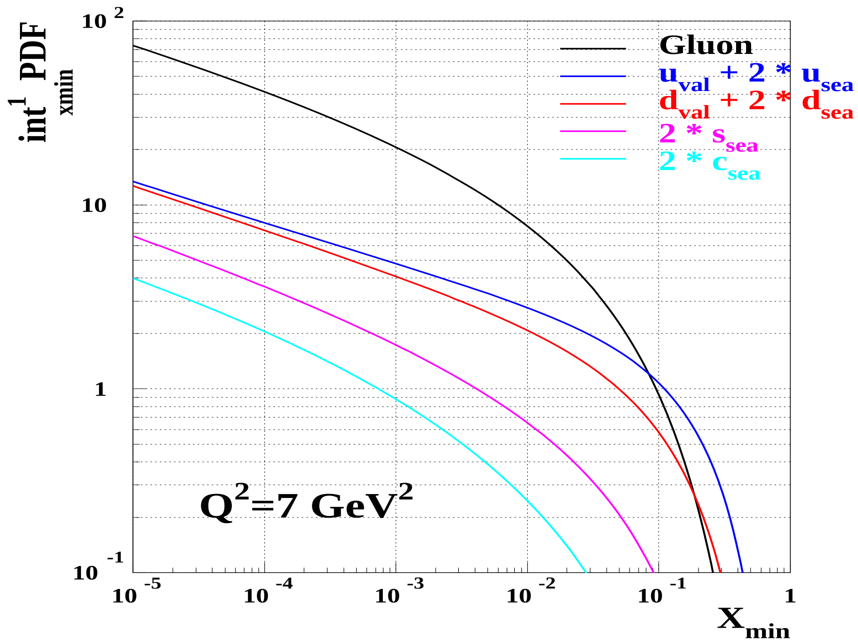
PDF

$$\int_{x_{min}}^{1} f_q(x)dx$$

I.e. how many quarks are there with x>xmin

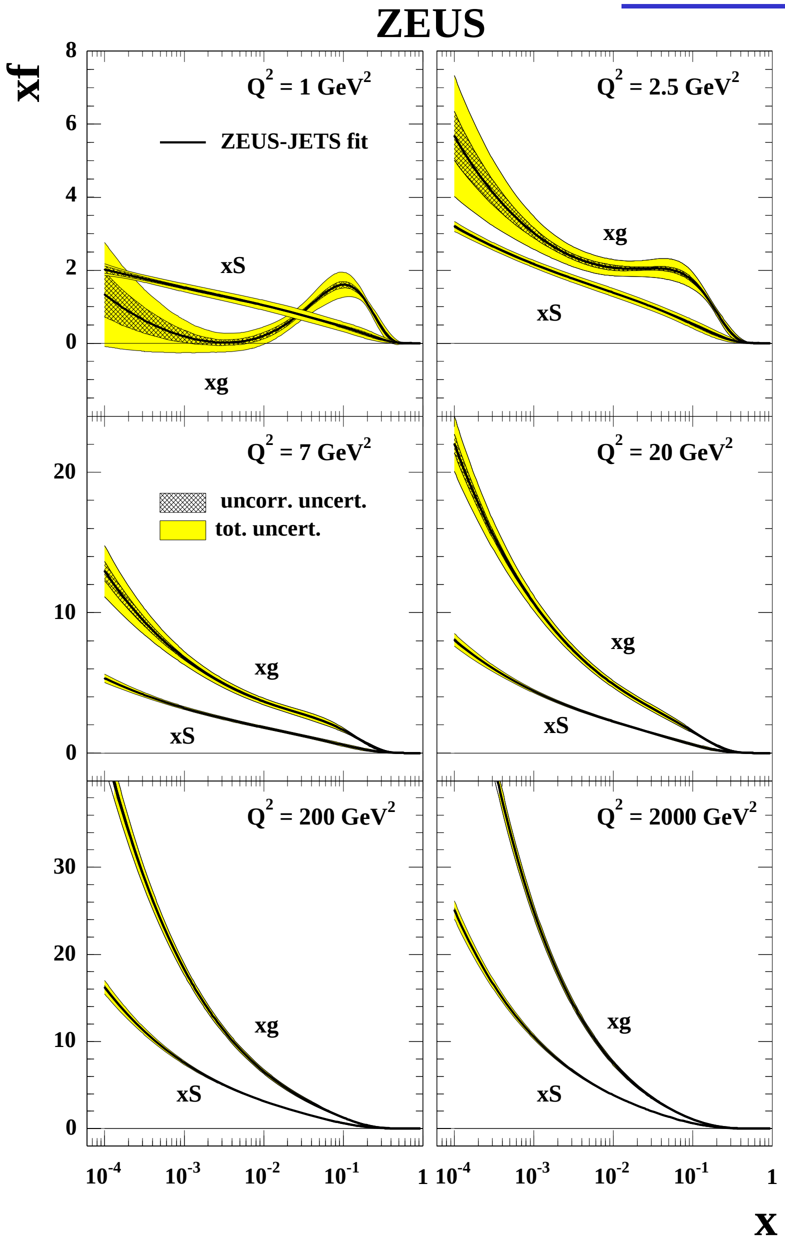
Note that gluon and sea quarks are multiplied by 1/20

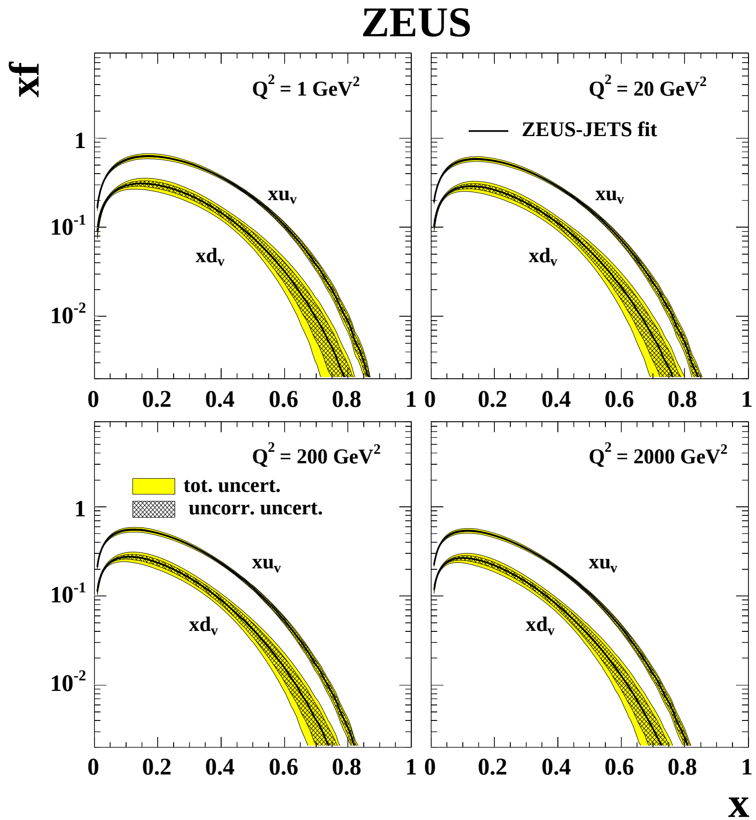




There are many gluons and quarks with small x inside proton.

PDF -cont'd-





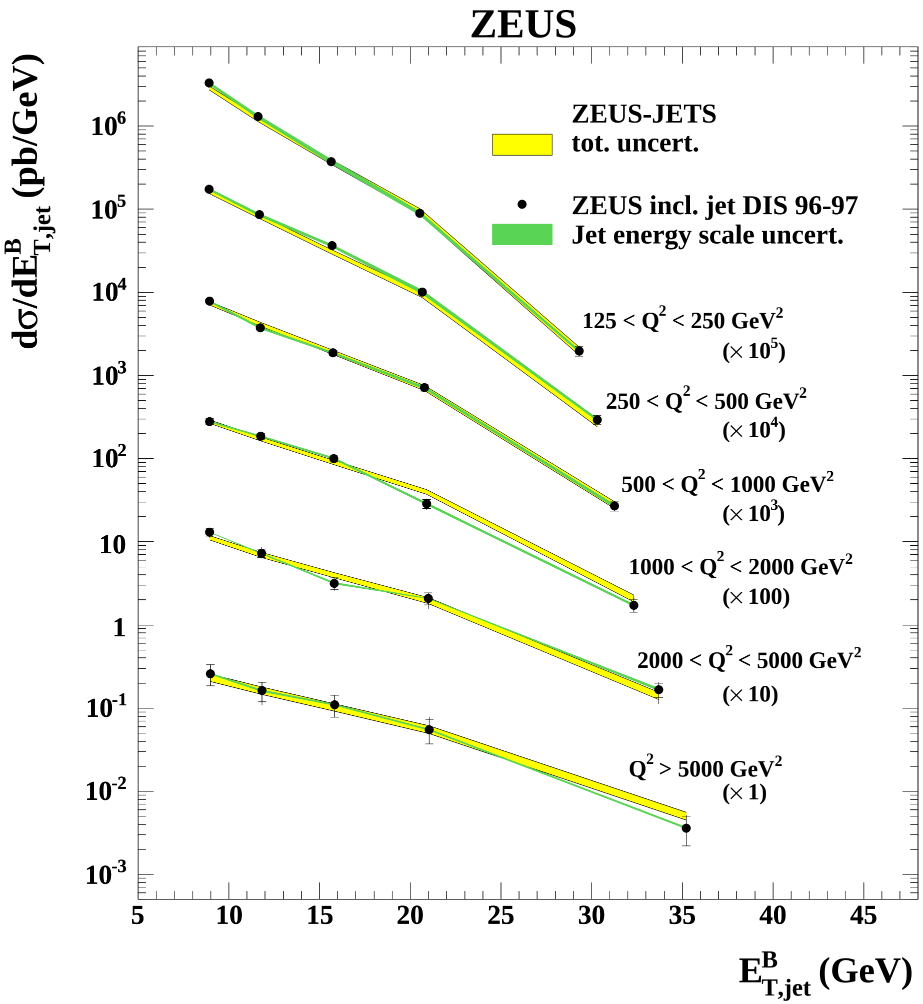
Evolution as Q^2 goes.

→ Sea/gluons grow rapidly.

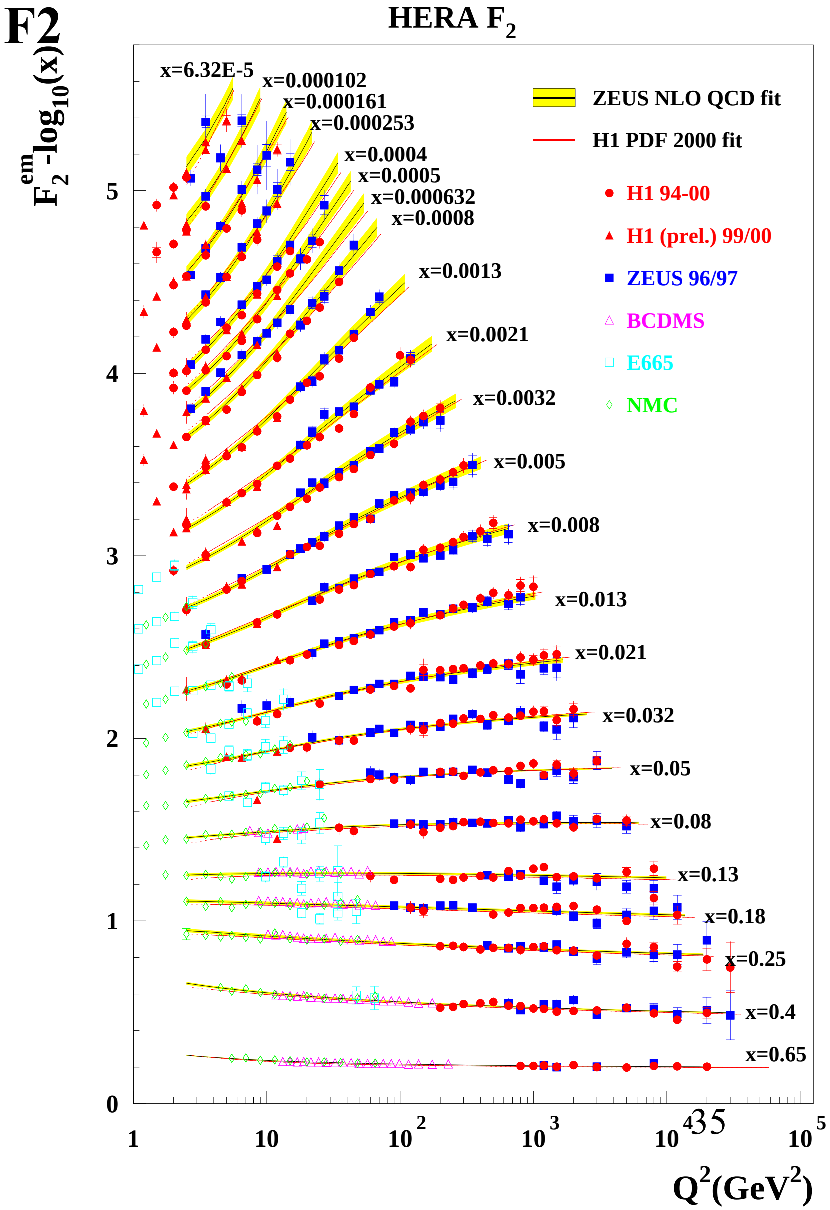
→ Their relative uncertainty gets smaller.

Description of data

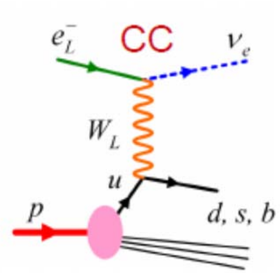
- Jet production cross section



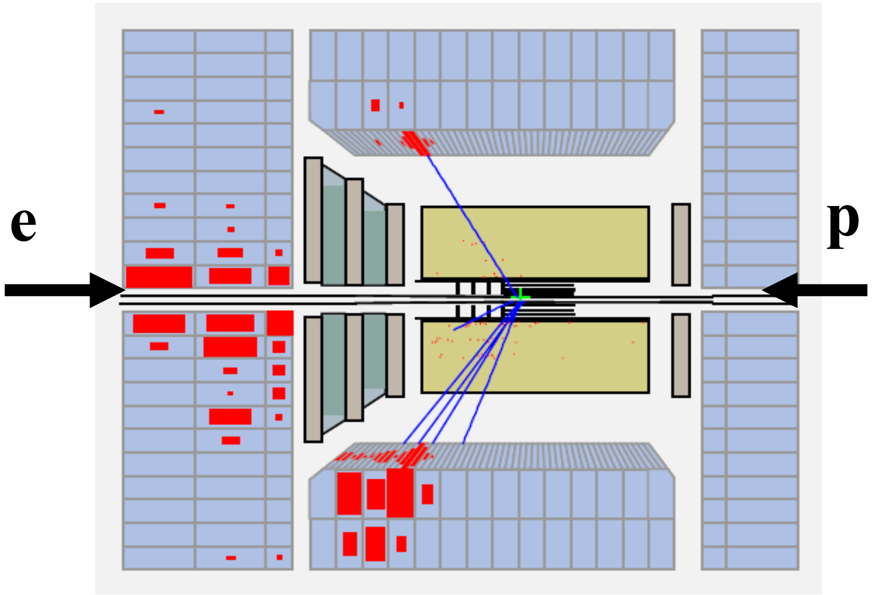
- F2



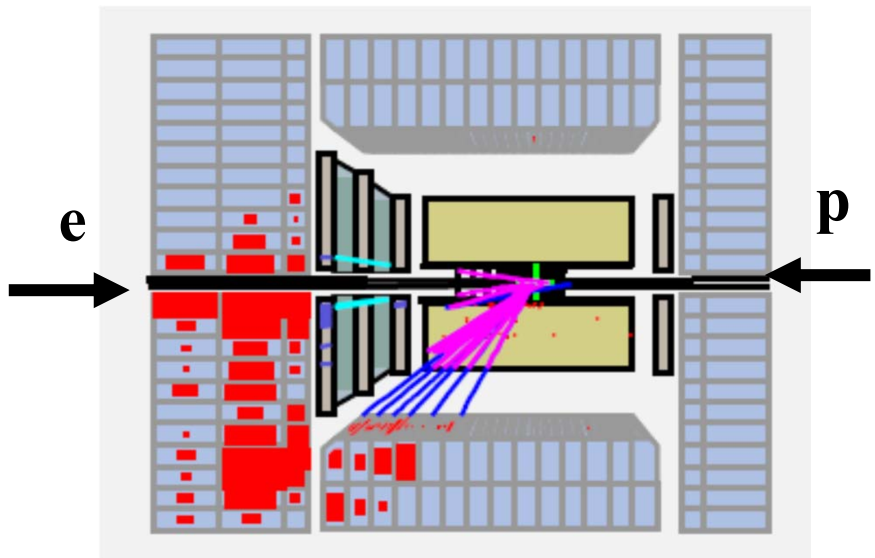
At the HERA ultimate Q^2 region



NC Event



CC Event



- Selection: presence of high energy scattered electron

  $E'_e > 10$ GeV

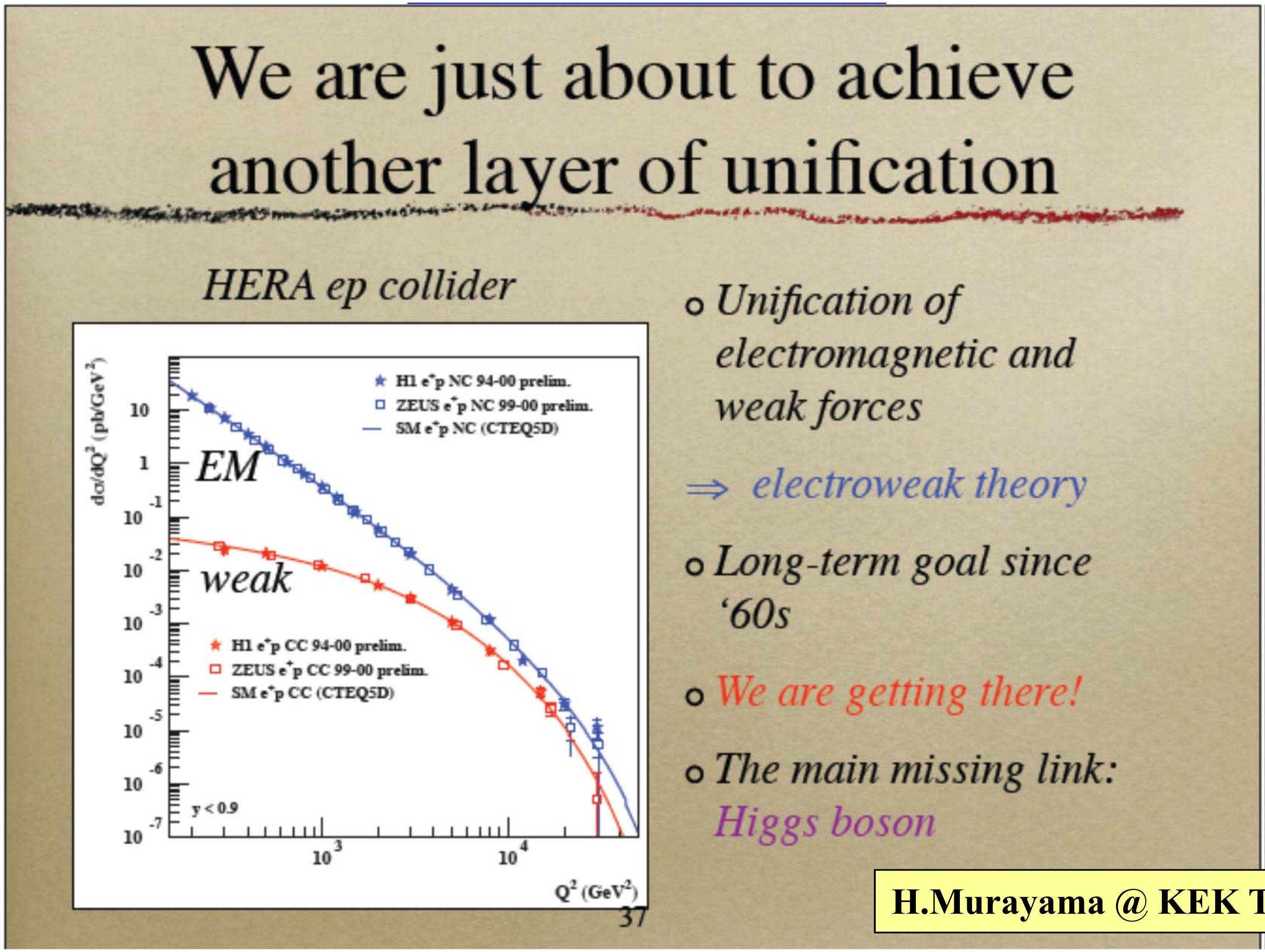
- Kinematics well reconstructed using electrons and/or hadrons

- Selection: presence of large missing transverse momentum: $P_{T,miss}$

  $P_{T,\ miss} > 12$ GeV

- Kinematics reconstructed using hadrons only

EW unification



H.Murayama @ KEK TC 2007

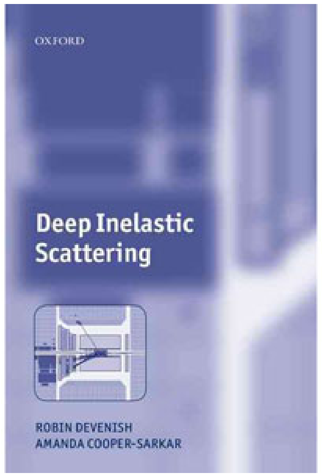
- NC and CC cross sections become similar at EW scale
  ➔ "EW unification" (Differences remained are mainly due to PDFs)

Wrap up

Topics discussed

► Structure function to describe proton structure
► QCD inspired Quark-Parton Model
  -- Scaling violation with DGLAP evolution
  -- Factorization
► How to determine PDFs
  -- Global fitting and its error
➔ How these descriptions reproduce data well

References



● Deep Inelastic Scattering (Oxford press)
  -- R. Devenish, A. Cooper-Sarkar

End of Day-2