

# Electroweak interaction

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1. Basics and history
2. The Standard Model
3. Tests of the Standard Model



## Lecture 2

### *The $SU(2) \times U(1)$ Standard Model*

1. Elementary constituents and the SM gauge group
2. The electroweak interaction and the Higgs mechanism
3. Couplings in the electroweak SM
4. Fermion masses
5. First experimental facts in favour of the electroweak SM

## 1. Constituants

# Gauge symmetry group, $G$

- IVB theory with (V-A) charged current:

$$L_{IVB}^{\text{int}}(x) = g(J^\mu W_\mu + h.c.) \quad \text{with e.g. } J_{\text{lept}}^\mu = \bar{\nu}_e \gamma^\mu (1 - \gamma_5) e$$

- QED:

$$L_{QED}^{\text{int}} = q J_{em}^\mu A_\mu \quad \text{with} \quad J_{em}^\mu = -\bar{e} \gamma^\mu e$$

- 3 currents ( $J_{\text{lept}}$ ,  $J_{\text{lept}}^\dagger$ ,  $J_{em}$ )  $\Rightarrow$  3 charges, generators of  $G$ :

$$T_+(t) \equiv \frac{1}{2} \int dx^3 J_{\text{lept}}^0(x) = \frac{1}{2} \int dx^3 \bar{\nu}_e^+ (1 - \gamma_5) e = T_-(t)$$

$$Q(t) = \int dx^3 J_{em}^0(x) = - \int dx^3 e^+ e$$

which verify:

$$[T_+(t), T_-(t)] = \frac{1}{2} \int dx^3 [\bar{\nu}_e^+ (1 - \gamma_5) \nu_e - e^+ (1 - \gamma_5) e] \equiv 2 T_3(t)$$

with  $T_3 \neq Q \Rightarrow T_\pm, Q$  do not form a closed algebra,  $G \neq SU(2)_3$

## Gauge symmetry group, $G$

- o First choice: add another neutral gauge boson coupled to  $T_3$

$\Rightarrow G = SU(2) \times U(1)$  : weak neutral currents predicted

- o Second choice: add a new fermion to modify currents and close the algebra

$\Rightarrow G = SU(2)$  : no weak neutral current

model of Georgi and Glashow (1972). Ruled out by the discovery of weak neutral-currents (1973).

- o Note: in both cases, unitarity is preserved (new diagrams).

# The 12 elementary constituents

Matter → Atom → Electron → Nucleus → Proton → Neutron → Quarks

**6 leptons** → LEPTONS

**6 quarks** → QUARKS

| Matter particles   |               | LEPTONS  |          |   |            | QUARKS   |          |  |   |
|--|---------------|--|----------|---|------------|--|----------|--|---|
| All ordinary particles belong to this group  | FIRST FAMILY  | Electron<br>Responsible for electricity and chemistry because it has a charge                | $e^-$    | Electron neutrino<br>Particle with very little mass; billions fly through our body every second | $\nu_e$    | Up<br>Has an electric charge of plus two-thirds; protons contain two, neutrons contain one | u        | Down<br>Has an electric charge of minus one-third; protons contain two, neutrons contain one | d |
| These particles existed just after the Big Bang. Now they are found only in cosmic rays and accelerators | SECOND FAMILY | Muon<br>A heavier relative of the electron; it is millions of times heavier than an electron | $\mu^-$  | Muon neutrino<br>Created along with muons when some nuclei decay                                | $\nu_\mu$  | Charm<br>A heavier relative of the up quark; found in 1974                                 | c        | Strange<br>A heavier relative of the down quark; found in 1964                               | s |
|  | THIRD FAMILY  | Tau<br>Heavier still; unstable. It was discovered in 1975                                    | $\tau^-$ | Tau neutrino<br>not yet discovered; believed to exist   | $\nu_\tau$ | Top Heavy<br>t or top  | t or top | Bottom<br>Heavier still; the heaviest quark; important test of electroweak theory            | b |

All constituents observed experimentally: from  $e^-$  (1897) to top quark (1995) and  $\nu_\tau$  (2000). So far, no internal structure detected.

# Fermions

- o Fermion "left-handed" and "right-handed" components:

$$\psi_L = \frac{1}{2}(1 - \gamma_5)\psi$$

$$\psi_R = \frac{1}{2}(1 + \gamma_5)\psi$$

chirality operator:  $\gamma_5$       chirality projector:  $\frac{1}{2}(1 \pm \gamma_5)$

$\psi_L$  : negative chirality or left-handed component

$\psi_R$  : positive chirality or right-handed component

ultra-relativistic limit ( $E \gg m$ ): chirality  $\sim$  helicity

- o  $SU(2)$  quantum number assignment (one lepton family):

$$T_+ = \frac{1}{2} \int dx^3 \nu_e^+ (1 - \gamma_5) e = \int dx^3 \nu_{eL}^+ e_L \quad \text{SU(2)<sub>L</sub> doublet} \quad I_L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \quad (T=\frac{1}{2})$$

$$T_- = \int dx^3 e_L^+ \nu_{eL}$$

$$T_3 = \frac{1}{2} \int dx^3 (\nu_{eL}^+ \nu_{eL} - e_L^+ e_L) \quad \text{SU(2)<sub>L</sub> singlet} \quad e_R \quad (T=0)$$

weak  
isospin  
 $\downarrow$

Note: no right component for a massless neutrino

## Fermions

- U(1) quantum number assignment (one-lepton family):

$$Q = -\int dx^3 e^+ e = -\int dx^3 (e_L^+ e_L + e_R^+ e_R)$$

$$Q - T_3 = \int dx^3 [-\frac{1}{2}(\nu_{eL}^+ \nu_{eL} + e_L^+ e_L) - e_R^+ e_R]$$

$$[Q - T_3, T_i] = 0, \forall i = 1, 2, 3$$

⇒ U(1) generator:  $Y \equiv 2(Q - T_3)$  weak hypercharge

- Leptons:

$$T_3(\nu_{eL}) = \frac{1}{2} \quad Q(\nu_e) = 0$$

$$T_3(e_L) = -\frac{1}{2} \quad T_3(e_R) = 0 \quad Q(e) = -1$$

$$Y(l_L) = -1 \quad Y(e_R) = -2$$

- Quarks:

$$T_3(u_L) = \frac{1}{2} \quad T_3(u_R) = 0 \quad Q(u) = \frac{2}{3}$$

$$T_3(d_L) = -\frac{1}{2} \quad T_3(d_R) = 0 \quad Q(d) = -\frac{1}{3}$$

$$Y(q_L) = \frac{1}{3} \quad Y(u_R) = \frac{4}{3} \quad Y(d_R) = -\frac{2}{3}$$

## Summary 1: fermions

- Fermions are described as  $SU(2)_L$  doublets and  $U(1)_Y$  singlets, with the following quantum numbers:

$$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix} \begin{pmatrix} u_L \\ d_L \end{pmatrix} \begin{pmatrix} c_L \\ s_L \end{pmatrix} \begin{pmatrix} t_L \\ b_L \end{pmatrix} \quad T = \frac{1}{2} \quad T_3 = \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix} \quad Y_I = -1 \quad Y_q = \frac{1}{3}$$

$$e_R, \mu_R, \tau_R \quad T=0 \quad T_3=0 \quad Y=-2$$

$$u_R, c_R, t_R \quad T=0 \quad T_3=0 \quad Y=\frac{4}{3}$$

$$d_R, s_R, b_R \quad T=0 \quad T_3=0 \quad Y=-\frac{2}{3}$$

- $T, T_3$ : weak isospin       $Y$ : weak hypercharge

Note: in these lectures, massless neutrinos (no impact on measurements described hereafter) : hence, no singlet neutrinos

## The electroweak interaction

- SU(2)<sub>L</sub> × U(1)<sub>Y</sub> gauge invariant Lagrangian:

$$\mathcal{L} = \bar{\psi} i \gamma^\mu D_\mu \psi - \frac{1}{4} F_{\mu\nu}^i F_i^{\mu\nu} - \frac{1}{4} G_{\mu\nu} G^{\mu\nu}$$

$$D_\mu = \partial_\mu - ig T_i W_\mu^i - ig' \frac{y}{2} B_\mu$$

covariant derivative  
g, g': SU(2)<sub>L</sub> & U(1)<sub>Y</sub> gauge couplings

$$F_{\mu\nu}^i = \partial_\mu W_\nu^i - \partial_\nu W_\mu^i + g \epsilon^{ijk} W_\mu^j W_\nu^k \quad i = 1, 2, 3$$

gauge-field tensors

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

(  $[T_i, T_j] = i \epsilon_{ij}^k T_k$  )

- Intermediate vector bosons: quanta of the SU(2)<sub>L</sub> gauge fields:  $W_\mu^i$ ,  $i=1,2,3$  and U(1)<sub>Y</sub> gauge field:  $B_\mu$
- Note: no mass term (fermions and bosons) in order to preserve gauge invariance

# Higgs mechanism

- o  $SU(2)_L$  complex doublet of scalar fields

$$\Phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} = \begin{pmatrix} \frac{\varphi_1 + i\varphi_2}{\sqrt{2}} \\ \frac{\varphi_3 + i\varphi_4}{\sqrt{2}} \end{pmatrix} \quad Y(\Phi) = 1$$

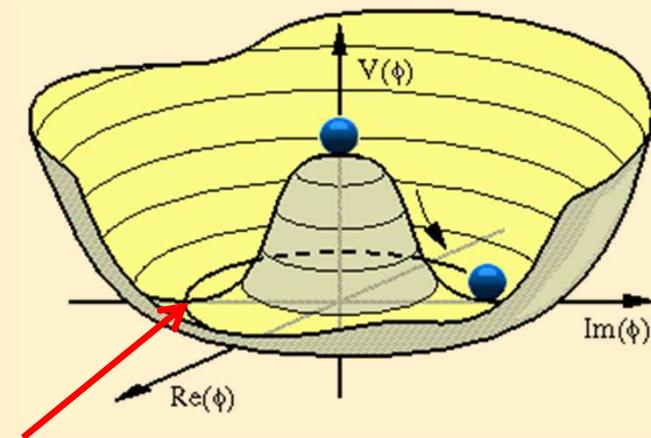
- o Lagrangian additional term:

$$\mathcal{L}_{Higgs} = (D^\mu \Phi)^+ (D_\mu \Phi) - V(\Phi^+ \Phi)$$

$$V(\Phi^+ \Phi) = -\mu^2 \Phi^+ \Phi + \lambda (\Phi^+ \Phi)^2$$

- o  $\mu^2, \lambda > 0$ : infinite number of degenerate vacuum states

X



$$\frac{\partial V}{\partial \Phi^+ \Phi} = 0 \Rightarrow \Phi^+ \Phi|_{vacuum} = \frac{\mu^2}{2\lambda} \quad v^2 \equiv \frac{\mu^2}{\lambda} \quad \text{"Higgs field vacuum expectation value"}$$

Choose one vacuum state  $\Leftrightarrow$  spontaneous symmetry breaking

o Selected vacuum:  $\Phi_0 = \begin{pmatrix} 0 \\ \frac{\nu}{\sqrt{2}} \end{pmatrix}$  X

- Not invariant under  $SU(2)_L \times U(1)_Y$  transformations
- Invariant under  $U(1)_Q = U(1)_{em}$  transformations

$$T_i \Phi_0 \neq 0 \quad Y \Phi_0 \neq 0 \quad Q \Phi_0 = 0$$

thus

$$SU(2)_L \times U(1)_Y \xrightarrow{\Phi_0} U(1)_{em}$$

o Particles: oscillations around selected vacuum

$$\Phi(x) = \Phi_0 + \frac{\eta(x)}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{\nu + \eta(x)}{\sqrt{2}} \end{pmatrix}$$

Note: this is equivalent to choosing the unitarity gauge

$$\Phi'(x) = \Phi_0 + \frac{1}{\sqrt{2}} \begin{pmatrix} \chi_1 + i\chi_2 \\ \chi_3 + i\chi_4 \end{pmatrix} = e^{-\frac{i}{\nu} \zeta^j(x) T_j} \begin{pmatrix} 0 \\ \frac{\nu + \eta(x)}{\sqrt{2}} \end{pmatrix}$$

- o Mass spectrum: arises from  $L_{Higgs} = (D^\mu \Phi)^+ (D_\mu \Phi) - V(\Phi^+ \Phi)$
- o boson masses:

$$D_\mu \Phi = \left( \partial_\mu - ig T_i W_\mu^i - ig' \frac{y}{2} B_\mu \right) \begin{pmatrix} 0 \\ \frac{\nu + \eta(x)}{\sqrt{2}} \end{pmatrix}$$

X  $(D^\mu \Phi)^+ (D_\mu \Phi)$  contains two mass terms :

$$\frac{g^2 \nu^2}{4} \left( \frac{W^{1,\mu} + i W^{2,\mu}}{\sqrt{2}} \right) \left( \frac{W_\mu^1 - i W_\mu^2}{\sqrt{2}} \right) = M_W^2 W^{-,\mu} W_\mu^+$$

$$\Rightarrow M_W = \frac{g \nu}{2}$$

$$\frac{(g^2 + g'^2) \nu^2}{8} \left( \frac{g W^{3,\mu} - g' B^\mu}{\sqrt{g^2 + g'^2}} \right) \left( \frac{g W_\mu^3 - g' B_\mu}{\sqrt{g^2 + g'^2}} \right) = \frac{1}{2} M_Z^2 Z^\mu Z_\mu$$

$$\Rightarrow M_Z = \frac{\sqrt{g^2 + g'^2}}{2} \nu = \frac{M_W}{\cos \vartheta_W}$$

where

$$\sin \vartheta_W \equiv \frac{g'}{\sqrt{g^2 + g'^2}}$$

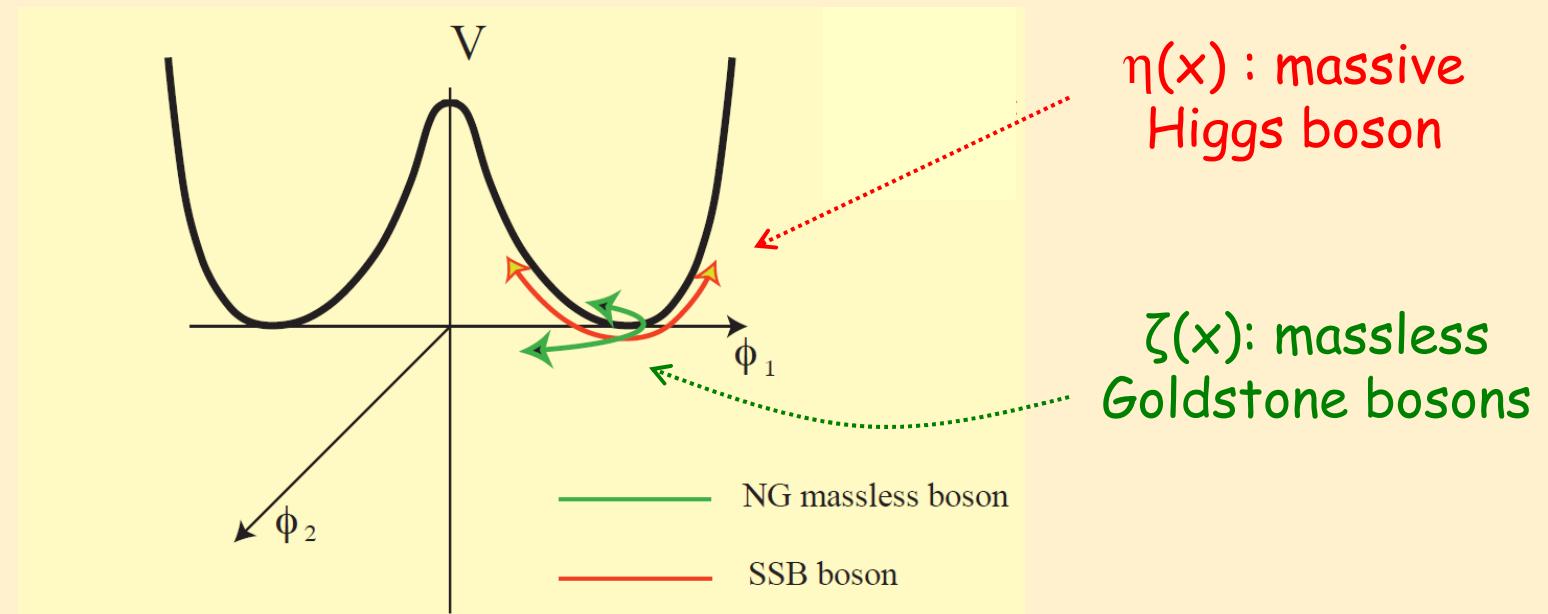
- o Mass spectrum: arises from  $\mathcal{L}_{Higgs} = (D^\mu \Phi)^+ (D_\mu \Phi) - V(\Phi^+ \Phi)$
- o scalar mass:

$$V(\Phi^+ \Phi) = -\mu^2 \Phi^+ \Phi + \lambda (\Phi^+ \Phi)^2$$

$$\Phi(x) = \begin{pmatrix} 0 \\ \frac{\nu + \eta(x)}{\sqrt{2}} \end{pmatrix}$$

X  $V(\Phi^+ \Phi)$  contains one mass term for the  $\eta(x)$  field :

$$\frac{1}{2} (-\mu^2 + \frac{\lambda}{2} 6\nu^2) \eta(x)^2 \equiv \frac{1}{2} M_H^2 \eta(x)^2 \Rightarrow M_H = \sqrt{2}\mu$$



## Summary 2: the electroweak interaction

- o The electroweak interaction is described by an  $SU(2)_L \times U(1)_Y$  gauge theory which is spontaneously broken into  $U(1)_{em}$

- o After SSB:

- o 3 massive gauge bosons

$$W_\mu^\pm = \begin{pmatrix} W_\mu^1 \mp i W_\mu^2 \\ \sqrt{2} \end{pmatrix} \rightarrow W^\pm: M_W = \frac{gv}{2}$$

$$Z_\mu = \begin{pmatrix} g W_\mu^3 - g' B_\mu \\ \sqrt{g^2 + g'^2} \end{pmatrix} \rightarrow Z: M_Z = \frac{\sqrt{g^2 + g'^2}}{2} v = \frac{M_W}{\cos \vartheta_W} \quad \sin \vartheta_W \equiv \frac{g'}{\sqrt{g^2 + g'^2}}$$

- o 1 massless gauge boson

$$A_\mu = \begin{pmatrix} g' W_\mu^3 + g B_\mu \\ \sqrt{g^2 + g'^2} \end{pmatrix} \rightarrow \gamma: M_\gamma = 0$$

- o 1 massive Higgs boson

$$\eta(x) \rightarrow H: M_H = \sqrt{2}\mu = \sqrt{2\lambda}v$$

### 3. Couplings

## Charged current

- Fermion-gauge boson interactions: from  $i\bar{\psi}_L \gamma^\mu D_\mu \psi_L + i\bar{\psi}_R \gamma^\mu D_\mu \psi_R$   
e.g. for one lepton doublet:

$$i\bar{l}_L \gamma^\mu (\partial_\mu - ig T_i W_\mu^i - ig' \frac{y}{2} B_\mu) l_L + i\bar{e}_R \gamma^\mu (\partial_\mu - ig' \frac{y}{2} B_\mu) e_R$$

W-leptons

leads to:

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} (J_\mu^+ W^{+, \mu} + J_\mu^- W^{-, \mu})$$

$$W_\mu^\pm = \left( \frac{W_\mu^1 \mp i W_\mu^2}{\sqrt{2}} \right) \quad J_\mu^+ = \frac{1}{2} \bar{v}_e \gamma_\mu (1 - \gamma_5) e \quad \rightarrow (V-A) \text{ current}$$

- At low-energy : X

$$\mathcal{L}_{cc} \approx \frac{-g^2}{2M_W^2} J_\mu^+ J^{-, \mu} \quad \rightarrow (V-A) \text{ theory with}$$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

- Consequence:

$$\nu = 1/\sqrt{\sqrt{2} G_F} \quad G_F = 1.16637(1) \cdot 10^{-5} \text{ GeV}^{-2} \Rightarrow \nu = 246 \text{ GeV}$$

from  $\mu$  lifetime measurements

## Electromagnetic current

- o Fermion-gauge boson interactions: from  $i\bar{\psi}_L \gamma^\mu D_\mu \psi_L + i\bar{\psi}_R \gamma^\mu D_\mu \psi_R$   
e.g. for one lepton doublet:

$$i\overline{l}_L \gamma^\mu (\partial_\mu - ig T_i W_\mu^i - ig' \frac{y}{2} B_\mu) l_L + i\overline{e}_R \gamma^\mu (\partial_\mu - ig' \frac{y}{2} B_\mu) e_R$$

leads to: doublet singlet

$$\mathcal{L}_{em} = g \sin \vartheta_W J_\mu^{em} A^\mu$$
$$A^\mu = \sin \vartheta_W W_\mu^3 + \cos \vartheta_W B_\mu \quad J_\mu^{em} = -\overline{e} \gamma_\mu e = Q(e) \overline{e} \gamma_\mu e$$

electron field

QED recovered if  $g \sin \vartheta_W = e$   $e$ , electron charge magnitude

- o Note:  $e = \sqrt{4\pi\alpha}$   $\alpha^{-1} = 137.035999679(94)$   
from low-energy measurements

## Weak neutral current

- Fermion-gauge boson interactions: from  $i\bar{\psi}_L \gamma^\mu D_\mu \psi_L + i\bar{\psi}_R \gamma^\mu D_\mu \psi_R$   
e.g. for one lepton doublet:

$$i\overline{l}_L \gamma^\mu (\partial_\mu - ig T_i W_\mu^i - ig' \frac{y}{2} B_\mu) l_L + i\overline{e}_R \gamma^\mu (\partial_\mu - ig' \frac{y}{2} B_\mu) e_R$$

leads to: doublet singlet

$$\mathcal{L}_{NC} = \frac{g}{\cos \vartheta_W} J_\mu^0 Z^\mu \quad Z^\mu = \cos \vartheta_W W_\mu^3 - \sin \vartheta_W B_\mu$$

$$J_\mu^0 = \frac{1}{2} \overline{e} \gamma_\mu (g_v^e - g_a^e \gamma^5) e + \frac{1}{2} \overline{\nu}_e \gamma_\mu (g_v^{\nu} - g_a^{\nu} \gamma^5) \nu_e = J_\mu^3 - \sin^2 \vartheta_W J_\mu^{em}$$

↑ ↑

different couplings for different particles

- Vector and axial couplings of the Z boson:

$$g_v^f = T^3(f) - 2Q(f) \sin^2 \vartheta_W \quad g_a^f = T^3(f)$$

## Summary 3: couplings

- o  $SU(2)_L \times U(1)_Y$  gauge couplings are related to precisely measured constants. At tree level:

- o  $g, M_W \leftrightarrow$  Fermi constant:  $\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$

- o  $g, \sin\theta_W$  (or  $g'$ )  $\leftrightarrow$  fine structure constant:  $g \sin\vartheta_W = e = \sqrt{4\pi\alpha}$

- o Z-fermion vector and axial couplings depend on the fermion type:

$$g_v^f = T^3(f) - 2Q(f) \sin^2 \vartheta_W \quad g_a^f = T^3(f)$$

$T^3(f)$ : quantum number corresponding to weak isospin third component  
for left-handed fermion component

# *Z vector and axial couplings to fermions*

(in the Standard Model at tree-level)

| fermion           | $t_3$ | Q    | $g_v$                                   | $g_a$ | $g_v/g_a$                           |
|-------------------|-------|------|---|-------|-------------------------------------|
| $\nu$             | 1/2   | 0    | 1/2                                     | 1/2   | 1                                   |
| e, $\mu$ , $\tau$ | -1/2  | -1   | $-1/2 + 2 \sin^2 \theta_W \sim -0.04$   | -1/2  | $1 - 4 \sin^2 \theta_W \sim 0.08$   |
| u, c, t           | 1/2   | 2/3  | $1/2 - 4/3 \sin^2 \theta_W \sim 0.19$   | 1/2   | $1 - 8/3 \sin^2 \theta_W \sim 0.38$ |
| d, s, b           | -1/2  | -1/3 | $-1/2 + 2/3 \sin^2 \theta_W \sim -0.35$ | -1/2  | $1 - 4/3 \sin^2 \theta_W \sim 0.70$ |

( $\sin\theta_W \sim 0.23$ )

## Gauge vs mass eigenstates

- Experimental fact: strangeness-conserving and -changing hadronic weak decays have different strengths

- led to Cabibbo theory (1963):

$$e \rightarrow \nu_e : g \quad d \rightarrow u : g \cos \theta_c \quad s \rightarrow u : g \sin \theta_c \quad \sin \theta_c \approx 0.225$$

- in the SM with two families:

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \rightarrow \begin{pmatrix} u_L \\ \cos \theta_c d_L + \sin \theta_c s \end{pmatrix} \quad \begin{pmatrix} c_L \\ s_L \end{pmatrix} \rightarrow \begin{pmatrix} c_L \\ \cos \theta_c s_L - \sin \theta_c d \end{pmatrix}$$

$\Rightarrow$  in the quark sector: SU(2) gauge eigenstates are different from mass eigenstates

(or: weak and strong interactions have different eigenstates)

## Gauge vs mass eigenstates

- In the SM with **three** families:

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \begin{pmatrix} c_L \\ s_L \end{pmatrix} \begin{pmatrix} t_L \\ b_L \end{pmatrix} \rightarrow \begin{pmatrix} u_L \\ d'_L \end{pmatrix} \begin{pmatrix} c_L \\ s'_L \end{pmatrix} \begin{pmatrix} t_L \\ b'_L \end{pmatrix} = \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

weak eigenstates      mass eigenstates  
CKM mixing matrix

- quark mixing can be described as acting **only** on  $d,s,b$  (leads to identical weak currents)
- With **three** families, the mixing matrix may contain **one physical phase** which would provide explanation for the **observed CP violation** in the quark sector

# Quark mixing and currents

- Quark-gauge boson interactions, e.g for one doublet:

$$(\bar{u}_L \quad \bar{d}'_L) i\gamma^\mu (\partial_\mu - ig T_i W_\mu^i - ig' \frac{Y}{2} B_\mu) \begin{pmatrix} u_L \\ d'_L \end{pmatrix} + i \sum_{u,d} \bar{q}_R \gamma^\mu (\partial_\mu - ig' \frac{Y}{2} B_\mu) q_R$$

→ Charged weak current:

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} (J_\mu^+ W^{+, \mu} + h.c.) \quad J_\mu^+ = \frac{1}{2} \bar{u} \gamma_\mu (1 - \gamma_5) d' = \frac{1}{2} \sum_{q=d,s,b} \bar{u} \gamma_\mu (1 - \gamma_5) V_{uq} q$$

⇒ charged currents are flavour non-diagonal

→ Neutral weak current:

$$\mathcal{L}_{NC} = \frac{g}{\cos \vartheta_W} J_\mu^0 Z^\mu \quad J_\mu^0 = \frac{1}{2} \bar{u} \gamma_\mu (g_v^u - g_a^u \gamma^5) u + \frac{1}{2} \bar{d} \gamma_\mu (g_v^d - g_a^d \gamma^5) d$$

with  $g_v^f = T^3(f) - 2Q(f) \sin^2 \vartheta_W$        $g_a^f = T^3(f)$

⇒ neutral currents are flavour conserving

→ Electromagnetic current:

$$\mathcal{L}_{em} = g \sin \vartheta_W J_\mu^{em} A^\mu \quad J_\mu^{em} = Q(u) \bar{u} \gamma_\mu u + Q(d) \bar{d} \gamma_\mu d$$

# Fermion masses

- o Fermion masses: general  $SU(2) \times U(1)$  scalar-fermion Yukawa interactions added to Lagrangian, e.g. for first family:

$$L_{\text{Yukawa}} = f^{(e)} \bar{e}_L \Phi e_R + f^{(u)} \bar{q}_L (i\tau_2 \Phi^*) u_R + f^{(d'd)} \bar{q}_L \Phi d_R + \dots + h.c.$$

$q_L = \begin{pmatrix} u_L \\ d'_L \end{pmatrix}$

Red arrows point to:  
 $\bar{e}_L e_R$  term       $\bar{u}_L u_R$  term       $\bar{d}'_L d_R, \bar{d}'_L s_R, \bar{d}'_L b_R$   
 terms

- o Particles: oscillations around selected vacuum

$$\Phi(x) = \Phi_0 + \frac{\eta(x)}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{\nu + \eta(x)}{\sqrt{2}} \end{pmatrix}$$

- o Masses:

- o leptons and up-quarks, e.g. e or u:  $m_e = \frac{\nu}{\sqrt{2}} f^{(e)}$        $m_u = \frac{\nu}{\sqrt{2}} f^{(u)}$

- o down-quarks, e.g. d:  $m_d = \frac{\nu}{\sqrt{2}} 2 \Re e (f^{(d'd)} V_{ud}^* + f^{(s'd)} V_{cd}^* + f^{(b'd)} V_{td}^*) \equiv \frac{\nu}{\sqrt{2}} f^{(d')}$

## Summary 4: fermion masses

- Quarks: weak interaction eigenstates and mass eigenstates are different

$$\begin{pmatrix} u_L \\ d'_L \end{pmatrix} \begin{pmatrix} c_L \\ s'_L \end{pmatrix} \begin{pmatrix} t_L \\ b'_L \end{pmatrix} \quad \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

- Consequence: charged weak currents are flavour non-diagonal

$$L_{cc} = \frac{g}{\sqrt{2}} (J_\mu^+ W^{+, \mu} + h.c.) \quad J_\mu^+ = \frac{1}{2} \bar{u} \gamma_\mu (1 - \gamma_5) d' = \frac{1}{2} \sum_{q=d,s,b} \bar{u} \gamma_\mu (1 - \gamma_5) V_{uq} q$$

- Fermion masses: described by gauge-invariant Yukawa scalar-fermion interactions:

$$m_f \propto \frac{\nu}{\sqrt{2}} \quad \Rightarrow \text{Higgs-} \bar{f} f \text{ couplings} \propto m_f$$

Note: in these lectures, massless neutrinos. Neutrino masses would induce mixing in the lepton sector → See lectures of Pr. F. Sukeane

# The complete SM Lagrangian

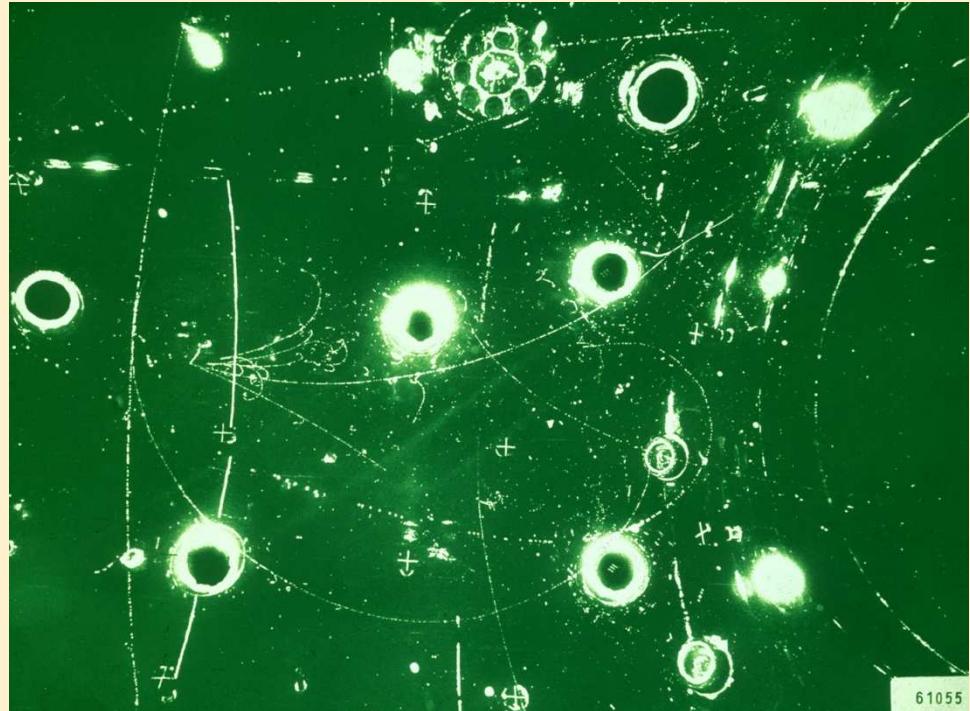
$$\begin{aligned}
L = & -\frac{1}{4}F_{\mu\nu}^i F_i^{\mu\nu} - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} && \text{gauge boson kinetics} \\
& + i \sum_{fermions} \bar{\psi}\gamma^\mu \partial_\mu \psi && \text{and interactions between gauge bosons} \\
& + \frac{g}{\sqrt{2}} \sum_{leptons} \bar{\psi}_\nu \gamma^\mu \frac{(1-\gamma^5)}{2} \psi_l W_\mu^+ + \text{h.c.} && \text{fermion kinetics} \\
& + \frac{g}{\sqrt{2}} \sum_{quarks} \bar{\psi}_{q_u} \gamma^\mu \frac{(1-\gamma^5)}{2} V_{q_u q_d} \psi_{q_d} W_\mu^+ + \text{h.c.} && \text{lepton-W interactions} \\
& + g \sin \theta_W \sum_{fermions} Q_f \bar{\psi} \gamma^\mu \psi A_\mu && \text{quark-W interactions} \\
& + \frac{g}{\cos \theta_W} \sum_{fermions} \bar{\psi} \gamma^\mu \frac{(g_v - g_a \gamma^5)}{2} \psi Z_\mu && \text{fermion-}\gamma\text{ interactions} \\
& + \frac{1}{2} \partial_\mu \eta \partial^\mu \eta + \mu^2 \eta^2 - \frac{\mu^2 v^2}{4} && \text{fermion-Z interactions} \\
& + \frac{g^2 v^2}{4} W^{+\mu} W_\mu^- + \frac{(g^2 + g'^2)v^2}{8} Z^\mu Z_\mu && \text{Higgs boson kinetics,} \\
& + \lambda v \eta^3 + \frac{\lambda}{4} \eta^4 && \text{mass and potential} \\
& + \frac{g^2}{4} (2v\eta + \eta^2) W^{+\mu} W_\mu^- && \text{W and Z boson masses} \\
& + \frac{g^2}{8 \cos^2 \theta_W} (2v\eta + \eta^2) Z^\mu Z_\mu && \text{Higgs boson self-interaction} \\
& + \sum_{fermions} m_f \bar{\psi} \psi && \text{Higgs boson - W interaction} \\
& + \sum_{fermions} \frac{m_f}{v} \eta \bar{\psi} \psi && \text{Higgs boson - Z interaction} \\
& && \text{fermion masses} \\
& && \text{Higgs boson - fermion interactions}
\end{aligned}$$

e → g sin θ<sub>W</sub> → ½ M<sup>2</sup><sub>H</sub> → M<sup>2</sup><sub>W</sub> → ½ M<sup>2</sup><sub>Z</sub>

## First experimental confirmations

- o 1973: neutral current discovery (Gargamelle experiment, CERN)

evidence for neutral current events  $\nu + N \rightarrow \nu + X$  in  $\nu$ -nucleon deep inelastic scattering



- o 1973-1982:  $\sin^2\theta_W$  measurements in deep inelastic neutrino scattering experiments (NC vs CC rates of  $\nu N$  events)

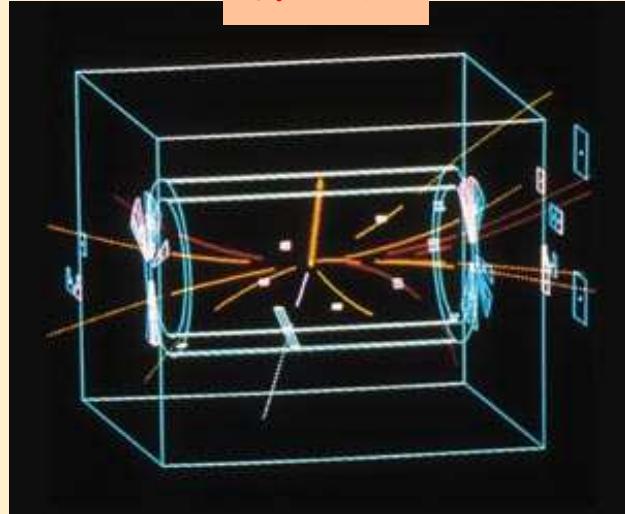
$$\text{e.g. } \frac{\sigma^{NC}(\nu N) - \sigma^{NC}(\bar{\nu} N)}{\sigma^{CC}(\nu N) - \sigma^{CC}(\bar{\nu} N)} = \frac{1}{2}(1 - 2\sin^2\vartheta_W)$$

world average value (PDG, 1982):  $\sin^2\vartheta_W = 0.229 \pm 0.010$  (4%)

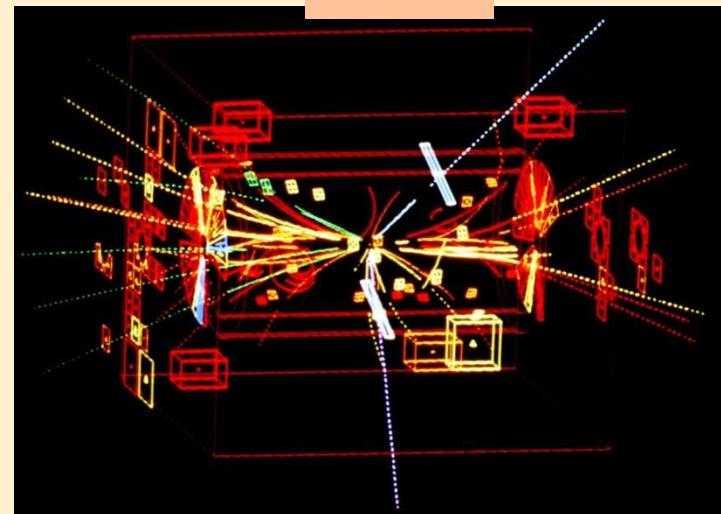
## First experimental confirmations

- o 1983: evidence for massive  $W^\pm$  and  $Z$  bosons (UA1 & UA2 experiments at CERN)

$W \rightarrow e\nu$



$Z \rightarrow ee$



first mass measurements:

$$M_W = 81 \pm 5 \text{ GeV} \quad (\text{UA 1, 6 } W \rightarrow e\nu \text{ events})$$

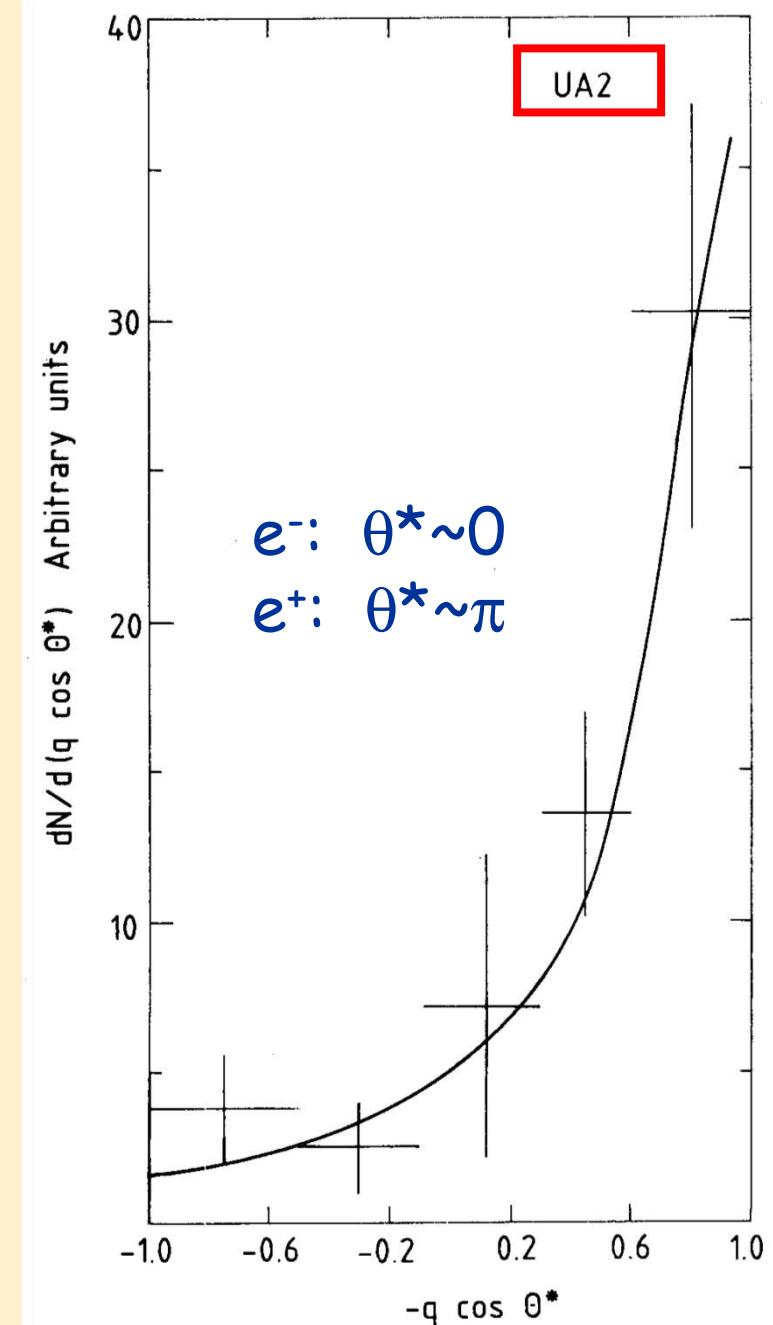
$$M_Z = 91.9 \pm 1.9 \text{ GeV} \quad (\text{UA 2, 4 } Z \rightarrow e^+e^- \text{ events})$$

- o 1987: with more data:  
first measurement of the angular distribution of W decay electrons

in agreement with the V-A structure  
of the charged currents:  $\times$

$$J_\mu^+(We\nu_e) = \frac{1}{2} \bar{\nu}_e \gamma_\mu (1 - \gamma_5) e$$

$$J_\mu^+(Wud) = \frac{1}{2} \bar{u} \gamma_\mu (1 - \gamma_5) V_{ud} d$$



## **BACK-UP SLIDES**

## Gauge transformations in $SU(2)_L \times U(1)_Y$

$$SU(2)_L : \psi_L(x) \longrightarrow \psi_L'(x) = e^{-i\alpha_k(x)T^k} \psi_L(x)$$

$$\psi_R(x) \longrightarrow \psi_R'(x) = \psi_R(x)$$

$$W_\mu^k(x) \longrightarrow W_\mu'^k(x) = W_\mu^k(x) - \frac{1}{g} \partial_\mu \alpha^k(x) + \epsilon^{klm} \alpha^l(x) W_\mu^m$$

$$U(1)_Y : \psi_L(x) \longrightarrow \psi_L'(x) = e^{-i\beta(x)\frac{Y}{2}} \psi_L(x)$$

$$\psi_R(x) \longrightarrow \psi_R'(x) = e^{-i\beta(x)\frac{Y}{2}} \psi_R(x)$$

$$B_\mu(x) \longrightarrow B_\mu'(x) = B_\mu(x) - \frac{1}{g} \partial_\mu \beta(x)$$

with  $[T_i, T_j] = i\epsilon_{ij}^k T_k$