QCD Lecture [Day 1]

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3rd France-Asia Particle Physics School (FAPPS11)
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ÉCOLE DE PHYSIQUE
des HOUCHES

Disclaimer

- Related to QCD, there are really rich/many interesting topics.
- I'm just an experimental physicist who has been working at HERA(ep) and LHC (pp) experiments.
  ➔ Apparently, it's not possible to cover whole of QCD aspects, and most probably my selected topics/ways of explanation are biased toward experimental side.

It is like that teacher of driving school is not a F1 driver, nor a hyper-engineer of car.
..but he/she can tell how to drive up to a sufficient level that one can enjoy daily life with car.

Plan for 3 days lectures

- **Day-1: Basis of QCD**
- Day-2: Proton structure @ lepton-hadron collision
- Day-3: Jets @ hadron-hadron collision

QCD knowledge necessary for doing physics at LHC

I will try to tell some QCD topics which I think (based on my experiences) would be good to know to enjoy daily life in particle physics field.

For happy daily life (in particle physics life)

Introduction

- **We are at FAPPS = France-Asia Particle Physics School**
- **You are supposed to hear about QCD**

Particle Physics

Have learned @ lecture by
Prof. V. Ruhlman-Kleider

► Its ultimate goal is (I think):
-- To find out the elementary building blocks of the matter, and
-- To find out the theory that governs the interactions between them.

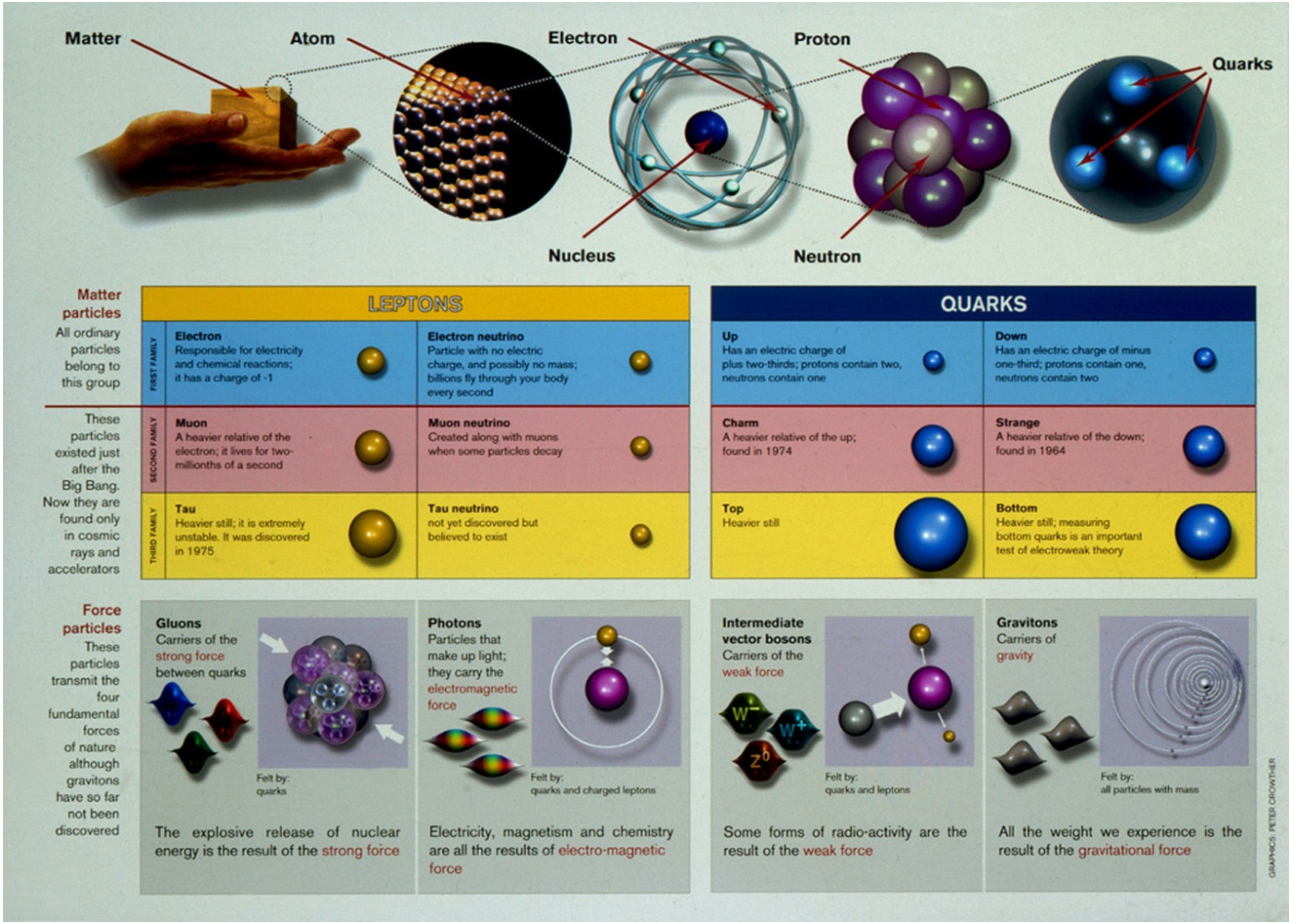
"Imagine that the gods are playing some great game like chess, let's say, and you don't know the rules of the game, but you're allowed to look at the board, at least from time to time, in a little corner, perhaps, and from these observations you try to figure out what the rules of the game are, what the rules of the pieces moving are." (*R.P.Feynman*)

► Out understanding on this deep and fundamental question has been dramatically improved in these ~100 years namely since the discovery of electron

➔ Our current state-of-art is （➔Next page）

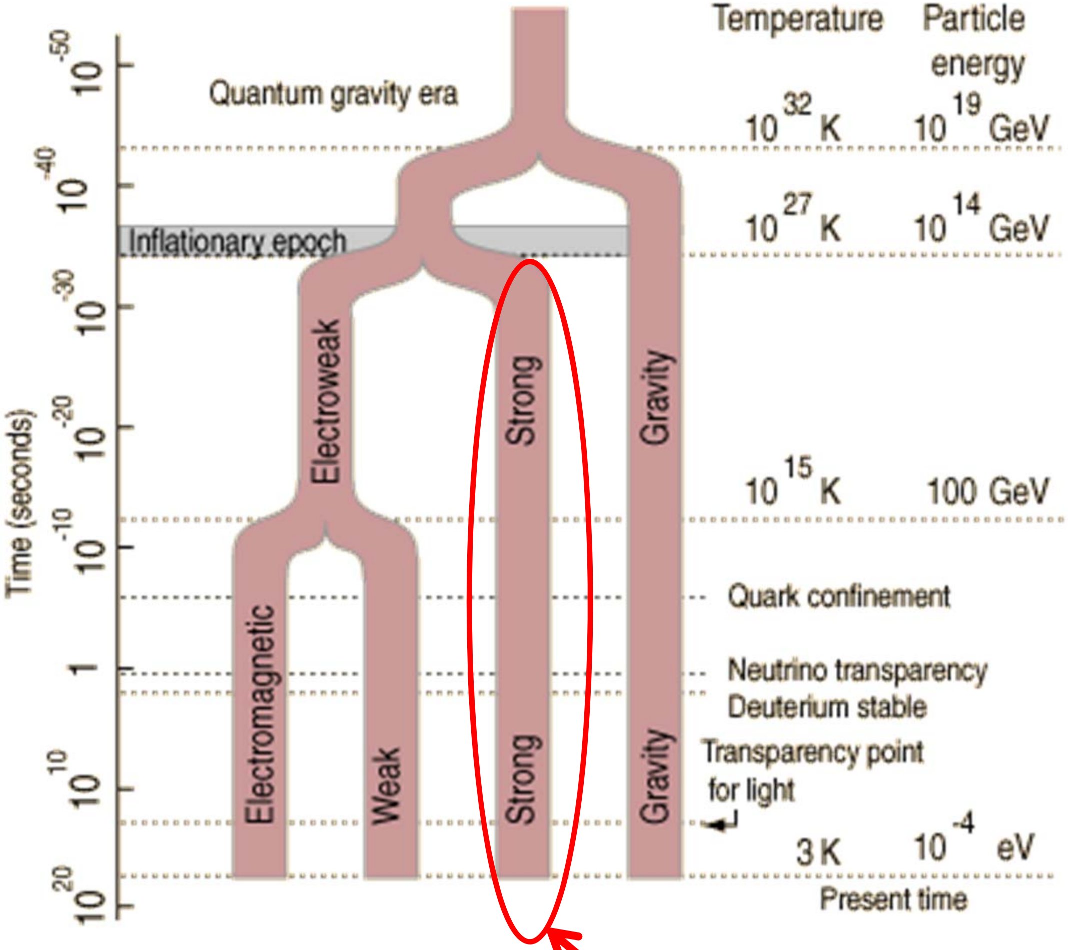
Particles [“Players”]

Have learned @ lecture by
Prof. V. Ruhlman-Kleider



Standard Model [“Rules”]

Have learned @ lecture by Prof. V. Ruhlman-Kleider



► Electro-magnetic, Weak, Strong interactions can be described in a unified frame work of the Gauge Field Theory

► Energy scale: EM and Weak are unified at O(100 GeV)

► The Standard Model (SM) gives excellent description to the current experimental data

Quantum Chromo Dynamics (QCD) which this lecture covers. (about “Strong” force)

Color

- **QCD = Quantum Chromo Dynamics**

What is color ?

Meson spectroscopy

For more,
→ Lectures by
Prof. S. Olsen

► Spin ↑↓ Spin 0 (“Singlet”)

↑↑ Spin 1 (“Triplet”)

$$\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$$

► Isospin (charge for the symmetry under the exchange of u and d quarks)

Meson: Spin-even strong interacting particles
→ Bound state of quark and anti-quark

1st generation	2nd generation	3rd generation	Charge
u (~3 MeV)	c (~1200 MeV)	t (~173000 MeV)	2/3 e
d (~4 MeV)	s (~120 MeV)	b (~4200 MeV)	-1/3 e

Isospin triplet	Spin	
	0	1
$u\bar{d}$	π^+	ρ^+
$u\bar{u} - d\bar{d}$	π^0	ρ^0
$d\bar{u}$	π^-	ρ^-
	~135 MeV	~770 MeV

Isospin singlet	Spin	
	0	1
$u\bar{u} + d\bar{d}$	η^0	ω^0
	~550 MeV	~780 MeV

Baryon spectroscopy

For more,
→ Lectures by
Prof. S. Olsen

► Isospin (charge for the symmetry under the exchange of u and d quarks)

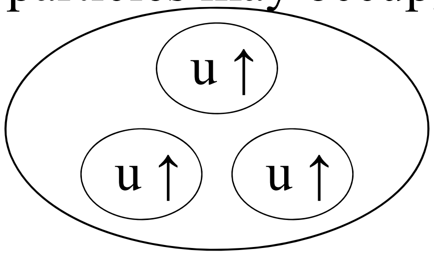
Baryon: Spin-odd strong interacting particles
→ Bound state of 3 quarks

$$\frac{1}{2} \otimes \frac{1}{2} \otimes \frac{1}{2} = \frac{1}{2} \oplus \frac{1}{2} \oplus \frac{3}{2}$$

	Spin 1/2
uud	p
udd	n
	~940 MeV

	Spin 3/2
uuu	Δ^{++}
uud	Δ^{+}
udd	Δ^{0}
ddd	Δ^{-}
	~1230 MeV

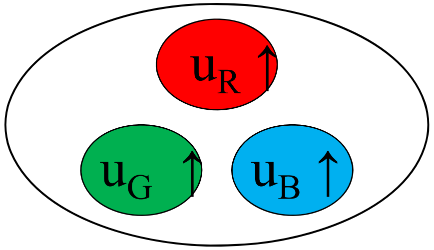
→ Big success in classfying hadrons (mesons,baryons)
→ However, this violates the Pauli exclusion principle: "No two identical spin-1/2 particles may occupy the same quantum state simultaneously".



"Color"

► A new internal degree of freedom, "color"

-- 3 quantum states: **R**, **G**, **B**



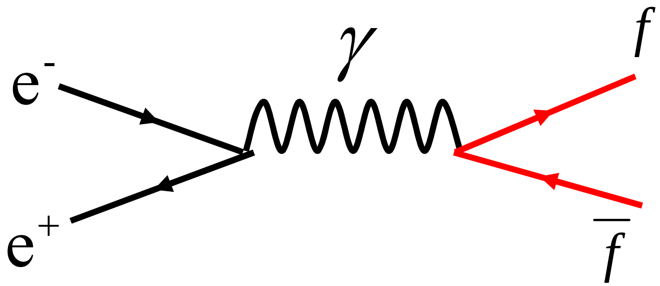
1st generation			2nd generation			3rd generation			Charge
u_R	u_G	u_B	c_R	c_G	c_B	t_R	t_G	t_B	2/3 e
(~3 MeV)			(~1200 MeV)			(~17300 MeV)			
d_R	d_G	d_B	s_R	s_G	s_B	b_R	b_G	b_B	-1/3 e
(~4 MeV)			(~120 MeV)			(~4200 MeV)			

Note: this "color" is nothing to do with the usual color, named just from analogy

Bound state must be "color white"

► Existence of color

-- Let's consider $e^+e^- \rightarrow X$



$$\sigma(Electro-magnetic) \propto Q^2$$

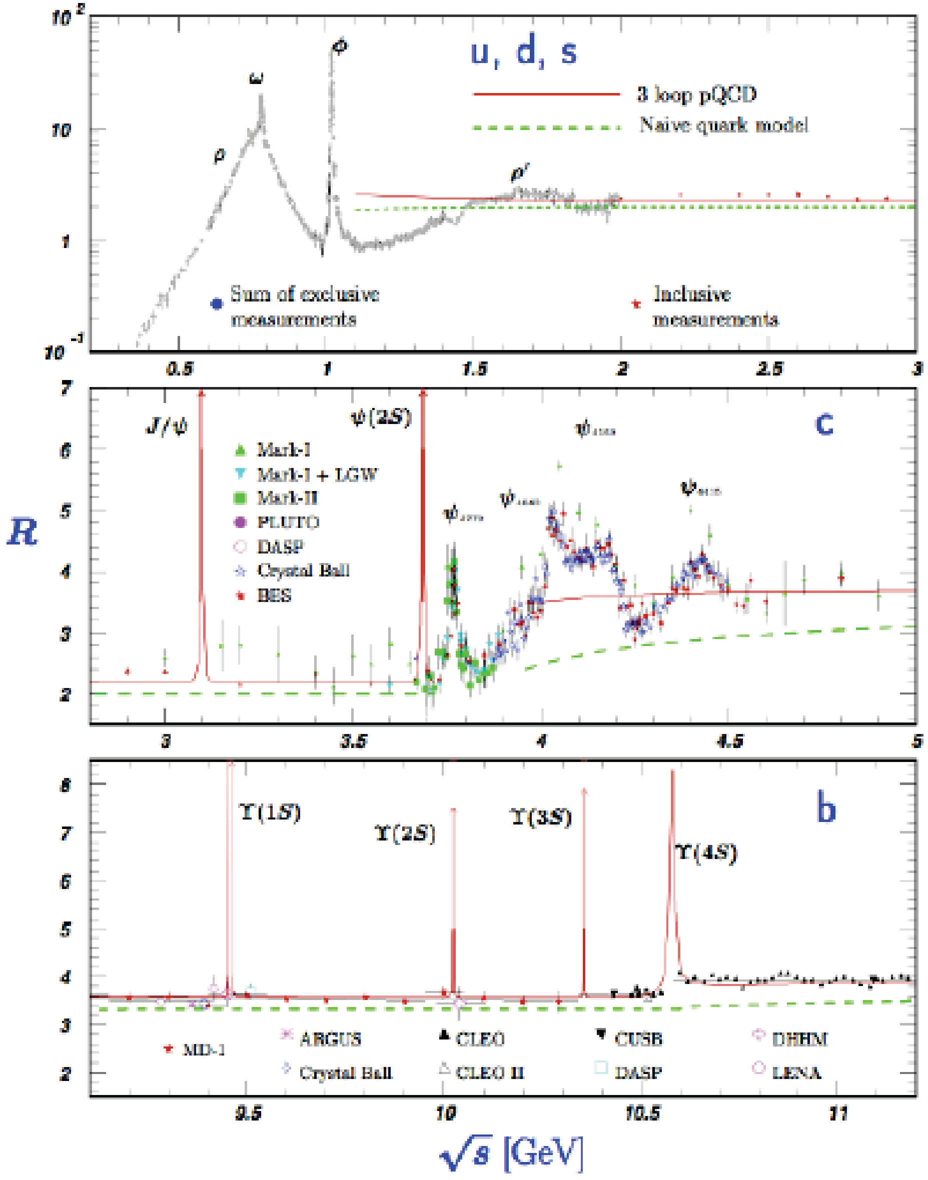
$$R = \frac{\sigma(e^+e^- \rightarrow hadrons)}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = Nc \sum_q Q_q^2$$

Quark charge

Nr of "colors" (if any)

Sum over quarks

"R-ratio"



$$R_{u,d,s} = \left[\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2\right] \times 3$$

$$R_{u,d,s,c} = R_{u,d,s} + \left[\left(\frac{2}{3}\right)^2\right] \times 3$$

$$R_{u,d,s,c,b} = R_{u,d,s,b} + \left[\left(-\frac{1}{3}\right)^2\right] \times 3$$

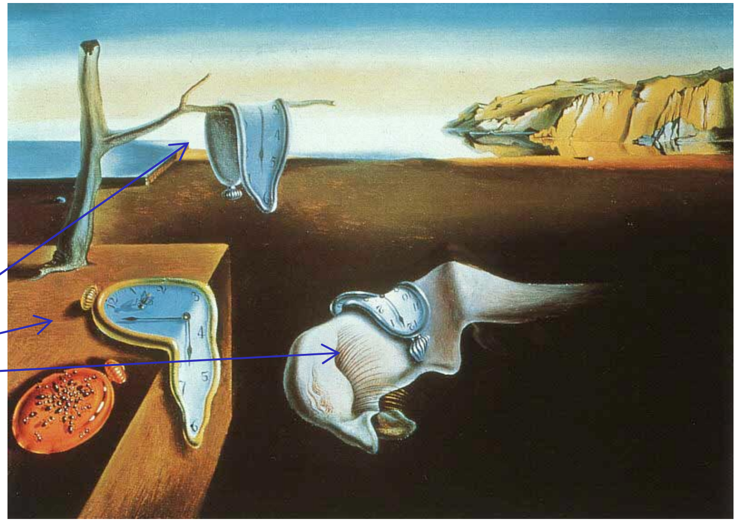
3 colors are there indeed !

QCD

- **Formulate dynamics between colors in the framework of Quantum Field Theory → QCD**

Analogy to QED

Have learned @ lecture by Prof. V. Ruhlman-Kleider



► Guiding principle for all field theories:
Local gauge invariance
- QED: U(1) gauge symmetry

Invariant under phase transformation
(aka "offset") at each of space-time point

$$e(x) \to e^{'}(x) = e^{ie\theta(x)} e(x)$$

Phase difference between space-time points is cancelled by gauge field (massless gauge boson)

$$\frac{\partial}{\partial x^{\mu}} e(x) \to \left[\frac{\partial}{\partial x^{\mu}} + ieA_{\mu}(x) \right] e(x)$$

EM field
Quanta of field ➔ Photon

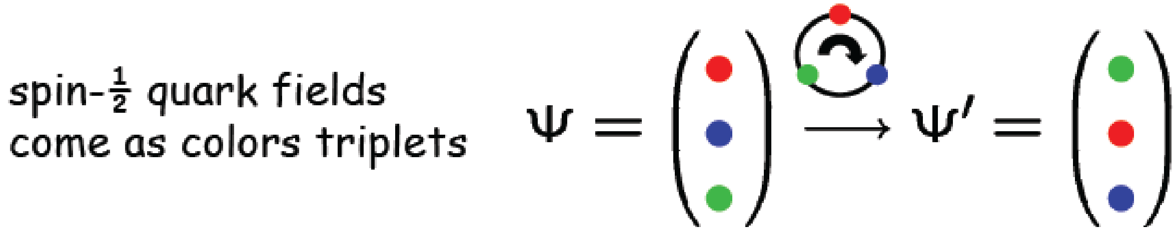
$$A_{\mu}(x) \to A_{\mu}(x) - \frac{\partial}{\partial x^{\mu}} \theta(x)$$

Local gauge invariance ➔
① Vector field (EM field) of massless particle (photon)
② Universal coupling between field and matter

$$\alpha_{QED} \approx \frac{1}{137}$$

QCD

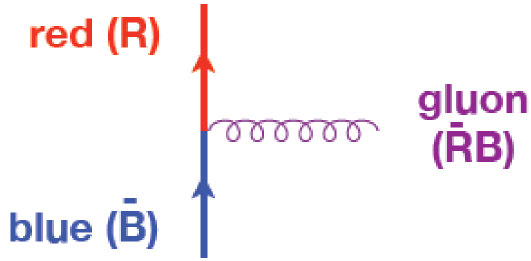
► Invariance under local SU(3) rotation in color space



► Similarly to QED, local SU(3) invariance dictates:

-- Field quanta: 8 massless spin-1 "gluon"s

* Gluons carry one color charge and one anti-color charge



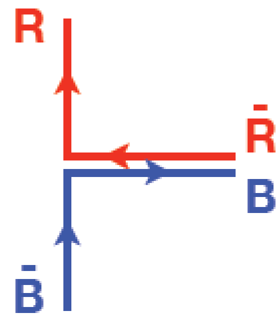
$\bar{R}B$ $\bar{R}G$

$\bar{G}B$ $\bar{G}R$

$\bar{B}R$ $\bar{B}G$

$(\bar{R}R - \bar{B}B)/\sqrt{2}$

$(\bar{R}R + \bar{B}B - 2\,\bar{G}G)/\sqrt{6}$



-- Universal coupling constant: α_s

Color matrices

Gell-Mann matrix

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda^8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

► The color charge of gluon is represented by a "matrix" in color space: $T^A = \frac{1}{2}\lambda^A$

-- T^A with A=1…8

"Generator" of SU(3) group

- Algebra: $[T^A, T^B] \equiv T^A T^B - T^B T^A = i f^{ABC} T^C$

"non-abelian"

-- f^{ABC} contains SU(3) group structure

$$\sum_{C,D} f^{ACD} f^{BCD} = C_A \delta^{AB}$$

$$\sum_A T^A T^A = C_F \cdot 1$$

-- Under normalization of $T_r(T^A T^B) = T_F \delta^{AB}$

Specific group structure (SU(3))

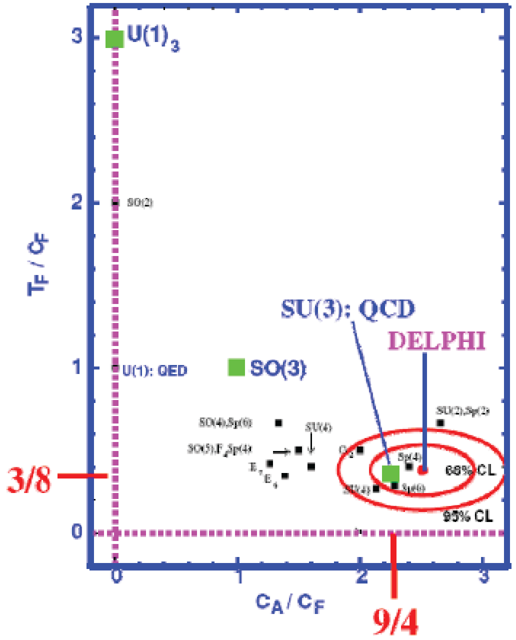
$$C_A = N = 3$$

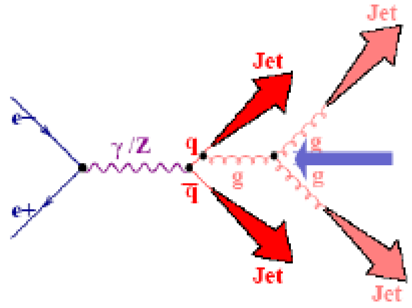
$$C_F = \frac{N^2 - 1}{2N} = \frac{4}{3}$$

$$T_F = \frac{1}{2}$$

Experimental data supporting SU(3)

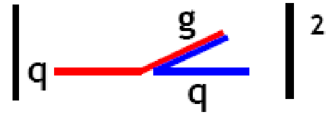
► 4 jets production in e^+e^- collisions



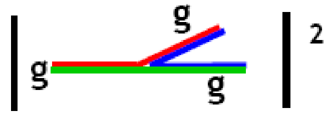


angular correlations between four jets depend on

$$\frac{C_A}{C_F} = \frac{9}{4},\ \frac{T_F}{C_F} = \frac{3}{8}$$



~ $C_F = 4/3$



~ $C_A = 3$

Specific group structure (SU(3))

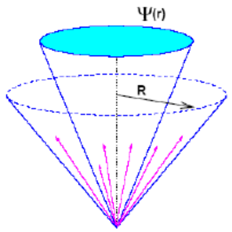
$$C_A = N = 3$$

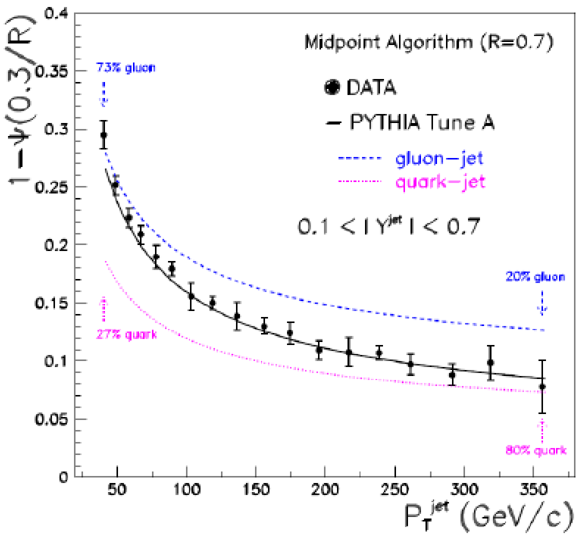
$$C_F = \frac{N^2 - 1}{2N} = \frac{4}{3}$$

$$T_F = \frac{1}{2}$$

► Jet shapes

-- Gluon jet is wider than quark jet



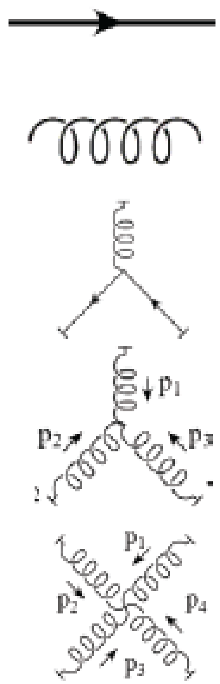


Non-abelian

► Induces self-interaction between gluons

QCD Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\mathrm{QCD}} &= \bar{\Psi}(i\partial_\mu\gamma^\mu - m)\Psi \\
&- (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \\
&- g\bar{\Psi}A^a_\mu T_a \gamma^\mu \Psi \\
&- \frac{1}{2} g(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu) f_{abc} A^{\mu b} A^{\nu c} \\
&- \frac{1}{4} g^2 f_{abc} A^b_\mu A^c_\nu f_{ade} A^{\mu d} A^{\nu e}
\end{aligned}
$$



like in QED

non abelian

(we ignore here complications due to the gauge-fixing term)

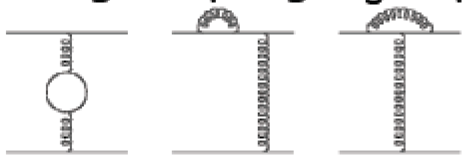
..leads to Asymptotic freedom

rough qualitative picture (for the moment):

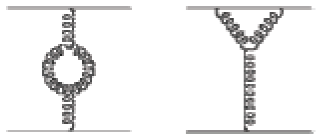
we get back to this when we discuss renormalization!

Gross, Wilczek; Politzer ('73/'74) (Nobel prize 2004)

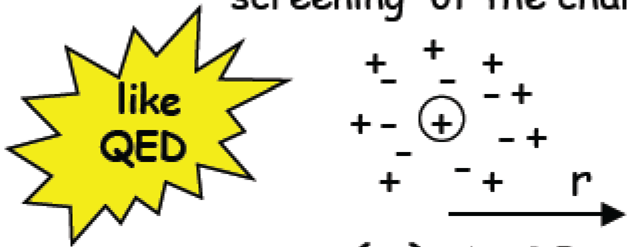
value of strong coupling α_s depends on distance (i.e. energy)

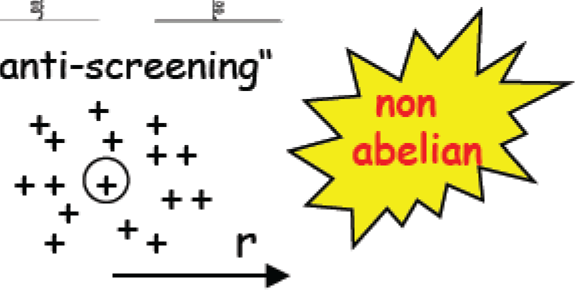


"screening" of the charge



"anti-screening"





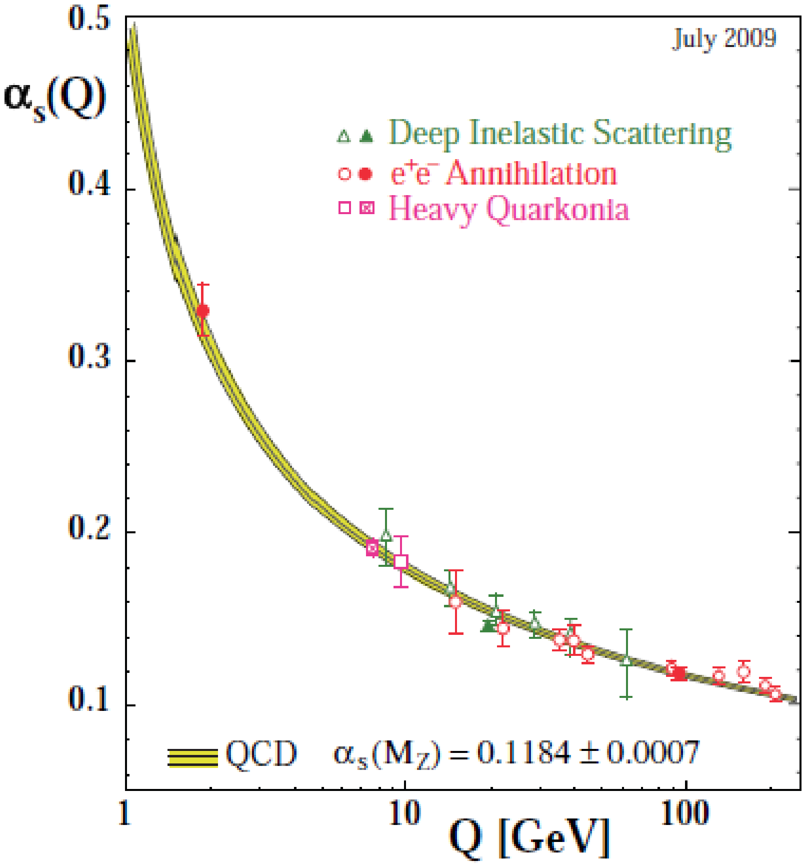
$\alpha_s(r)\uparrow$ if $r\downarrow$ $\qquad$ $\alpha_s(r)\downarrow$ if $r\downarrow$

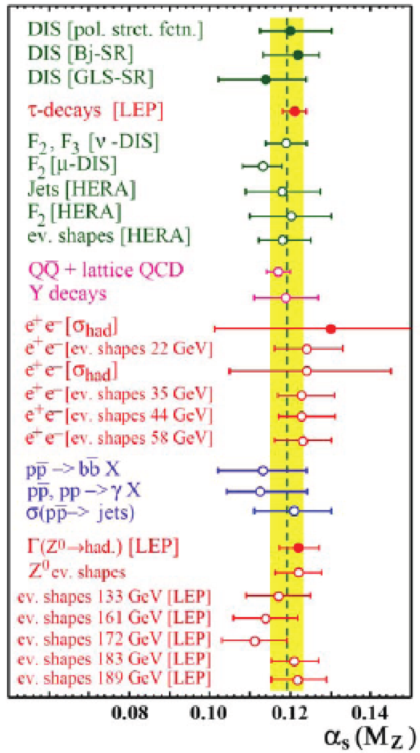
Who wins? $$\alpha_s(Q^2) = \frac{g^2}{4\pi} \approx \frac{4\pi}{(11-\frac{2}{3}N_f)\ln(Q^2/\Lambda^2)} \qquad Q \sim 1/r$$

coupling at some reference scale Q_0

Strong coupling constant: α_s

S. Bethke, arXiv:0908.1135



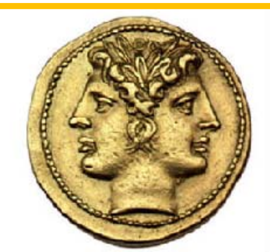


$r \uparrow \;\rightarrow\; \alpha_s \uparrow$

confinement

non-perturbative hadronic structure

e.g. Lattice QCD



asymptotic freedom

hard scattering cross sections

perturbative methods

interplay

$r \downarrow \;\rightarrow\; \alpha_s \downarrow$

M. Stratmann @ Wako Spin Fest, 2005

Confinement

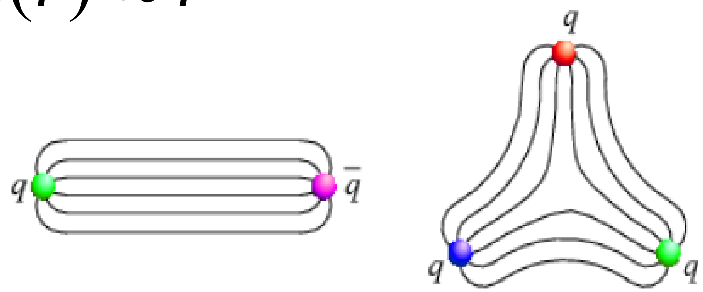
► Coupling (force) is stronger as longer distance. $\phi(r) \propto r$

→ Color is confined as:

-- Gluon field is screened out and confined into hadrons

"Hadrons are to be color-white"

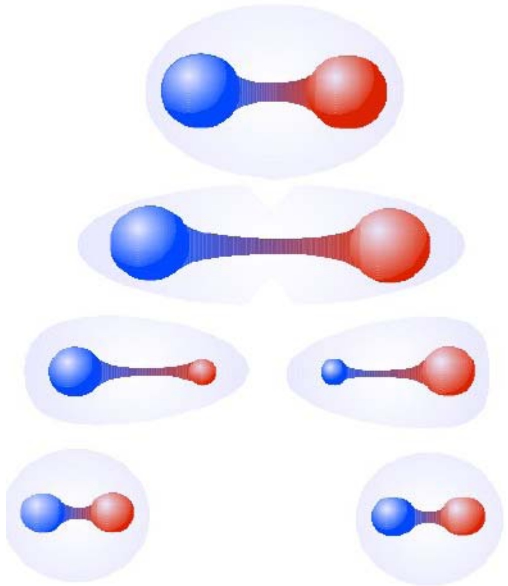
-- Energy injected into a hadron does not separate quarks but goes into creating quark-antiquark pairs, hence hadrons



http://www.scholarpedia.org/

"Jet"

Quarks and gluons kicked by hard scatter form a "spray" of collinear hadrons

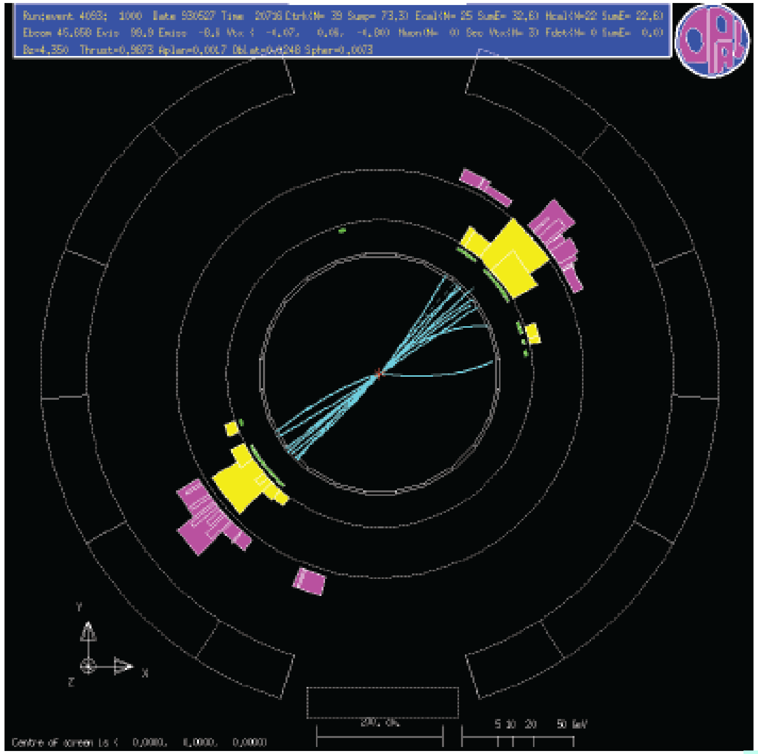


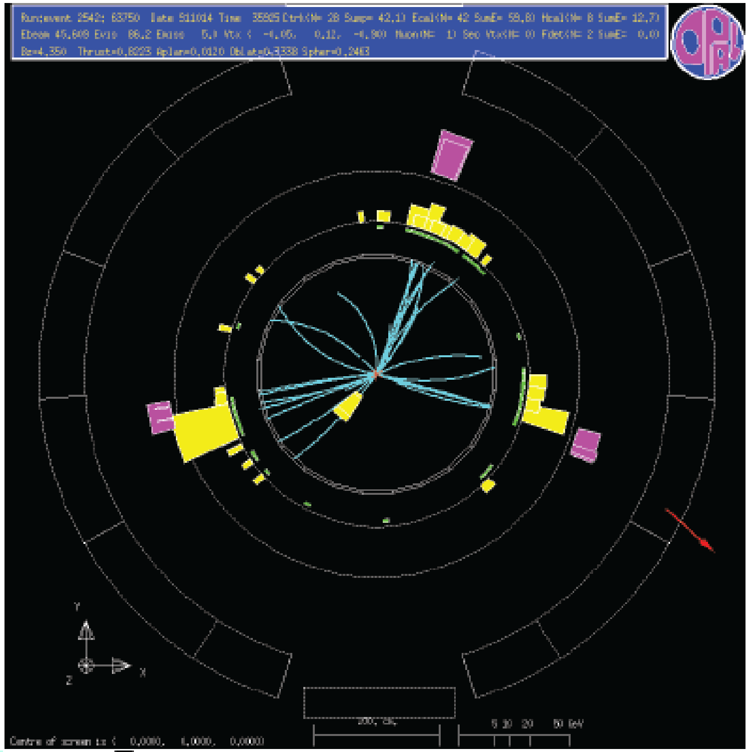
Thomas Lippert, NIC-ZAM, Jülich, for the SESAM Collaboration)

Quarks/gluons cannot be detected as they are; "confined" into hadrons

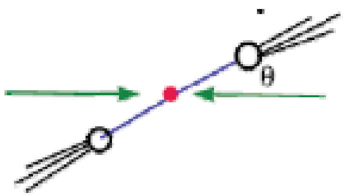
Existence of gluon

► Nevertheless, gluon existence can be “seen” in e.g. ee → X

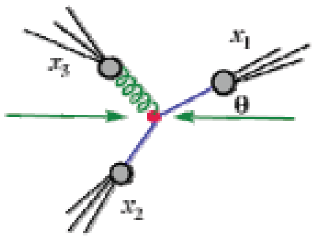




About ~10 % have 3rd jet



Corresponds to a rough measurement on α_s



Perturbative QCD

- **Have looked:**
  -- SU(3) group structure supported in data
  -- Running α_s, giving asymptotic freedom
  -- Confinement qualitatively explained

➔ Possibility to get "precise" prediction e.g for hadron collider processes ?

(Flush overview of Day-2 and Day-3 topics)

Perturbative QCD (pQCD)

► Perturbative expansion $\sigma = \sigma_0 + \alpha\sigma_1 + \alpha^2\sigma_2 \ldots$

-- If α is small, there is a chance that calculation up to finite order shows a good approximation; $\sigma_0 >> \alpha\sigma_1 >> \alpha^2\sigma_2 \ldots$

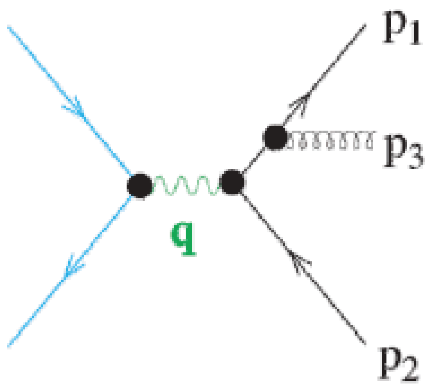
0th order $\sigma^{(0)} = \sigma_0.$

1st order $\sigma^{(1)} = \sigma_0 + \alpha\sigma_1$

2nd order $\sigma^{(2)} = \sigma_0 + \alpha\sigma_1 + \alpha^2\sigma_2$

"k-Factor":
=Ratio between $\sigma^{(n)}$ vs $\sigma^{(n+1)}$

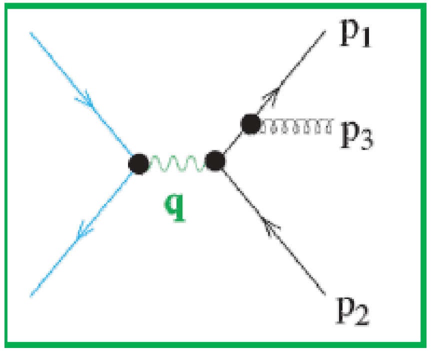
► Asymptotic freedom : α_s getting smaller at shorter distance
➔ Enables us to compute interactions of quarks and gluons at short-distances ?



$$\frac{d\sigma}{dx_1 dx_2} = \sigma_0 \frac{\alpha_S}{2\pi} C_F \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

x_1: fractional momentum of p_1 wrt p_0
x_2: fractional momentum of p_2 wrt p_1

→ Diverges at x_1=1, x_2=1…



Collinear/Soft singularities

$$\frac{d\sigma}{dx_1 dx_2} = \sigma_0 \frac{\alpha_s}{2\pi} C_F \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

$\sigma_0 = \frac{4\pi\alpha_{em}}{s} \sum e_q^2$

2 & 3 collinear

collinear singularities:

$(1-x_1) \rightarrow 0$

(gluon collinear to antiquark)

$(1-x_2) \rightarrow 0$

(gluon collinear to quark)

3
soft

soft singularity:

gluon soft: $x_3 \longrightarrow 0$

⇕

$x_1 \rightarrow 1, \quad x_2 \rightarrow 1$

➔ These singularties arise from interactions at long distance, and called as infrared divergence

*** Appear when 2→3 kinematics reduces to 2→2 kinematics**

➔ Not indicating general failure of pQCD; the observable we try to calculate here is not **infrared-safe**

Factorization

► Firstly, there are meaningful infrared-safe observables which are insensitive to indistinguishable 2→2, 2→3 origin of long-distance interactions

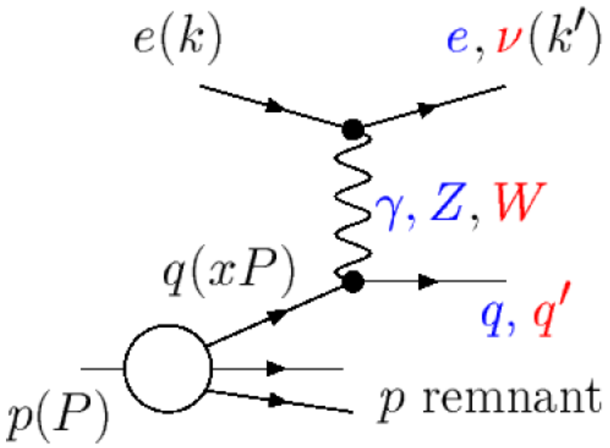
E.g e e → hadrons

$$R = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_C \sum_q Q_q^{\,2}$$

Fully inclusive, infrared-safe by definition.

► Factorization:

-- If we can factorize the physical observable into a calculable infrared safe and a non-calculable but universal piece



$$\sigma(ep) \propto \sum \int_0^1 dx\, d\sigma(eq_i \to eq_i; x)\, f(q_i; x)$$

Can be regarded as a probability to find quark (with x) inside proton
→ "Proton structure"
(Lecture on Day 2)

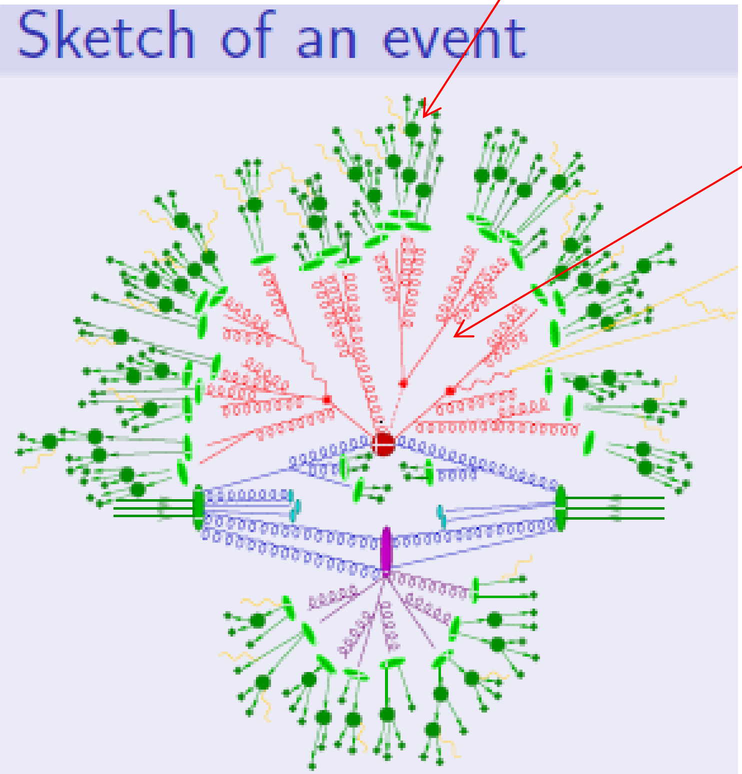
Note: Factorization has to be proven for each process

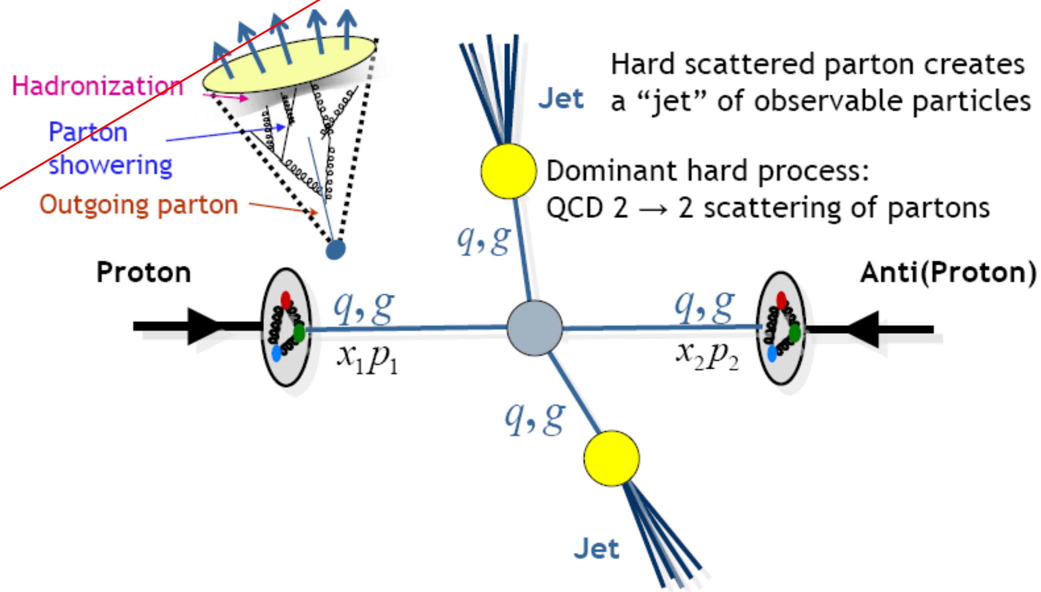
Up to detector level

For more,
→ Lectures by
Prof. Kurihara

► Even if factorization holds, detectors are a long-distance away and experiments only see hadrons (not free quarks/gluons)

→ To establish connection between theory and experiment, Monte Carlo technique is used to simulate "parton shower" and "hadronization"





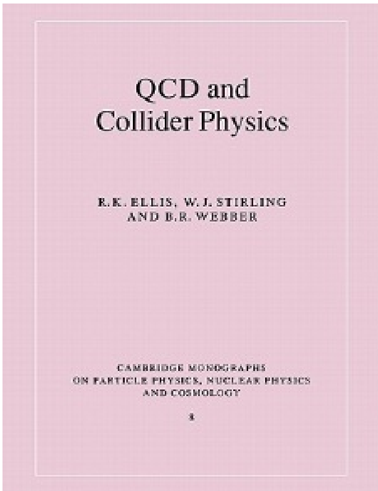
→ Jets at hadron colliders (Lecture on Day-3)

Wrap up

Topics discussed

► Quarks for hadron spectroscopy
► Basics of QCD
    -- SU(3) giving non-abelian self interactions
    -- Asymptotic freedom
    -- Confinement
► Possibility of precise determination power of pQCD but also its difficulty

References



- QCD and Collider Physics (Cambridge press)
  -- R.K.Ellis, W.J.Stirling, B.R.Webber

End of Day-1