

MC Generation (I) Lecture

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Outline

Today's Lecture

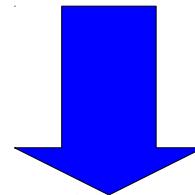
- Introduction
 - ▶ Why we need event generators.
- Ingredients of EG and its structure
 - perturbative part:
matrix element, parton shower, parton distribution function (PDF), matching, ...
 - non-perturb. part:
hadronization, proton remnant, ...

Outline

Tomorrow's exercise

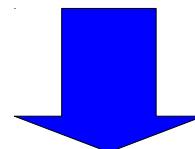
- Topics/Exercises

- Generation of random numbers w/
arbitrary distribution



Event Generation

- Solve an integral equation by the MC method



Parton Shower

Announcement & HomeWork

For 'MC Generation' exercise

Please install "MAXIMA" (& 'Gnuplot' if necessary) into your PC.

- *Maxima* is a Computer Algebra System
- Platform free
 - Windows, MacOSX, Linux,...
- free to use
- If you have "Mathematica" or any other utility (C, FORTRAN, root...), it is OK!

Related URL's

- Official web page:

<http://maxima.sourceforge.net/>

- Download page

<http://maxima.sourceforge.net/download.html>

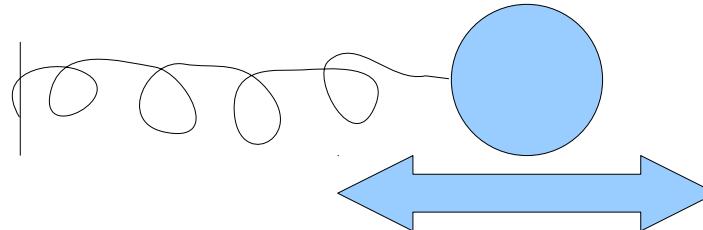
- Manual page:

<http://maxima.sourceforge.net/documentation.html>

Example

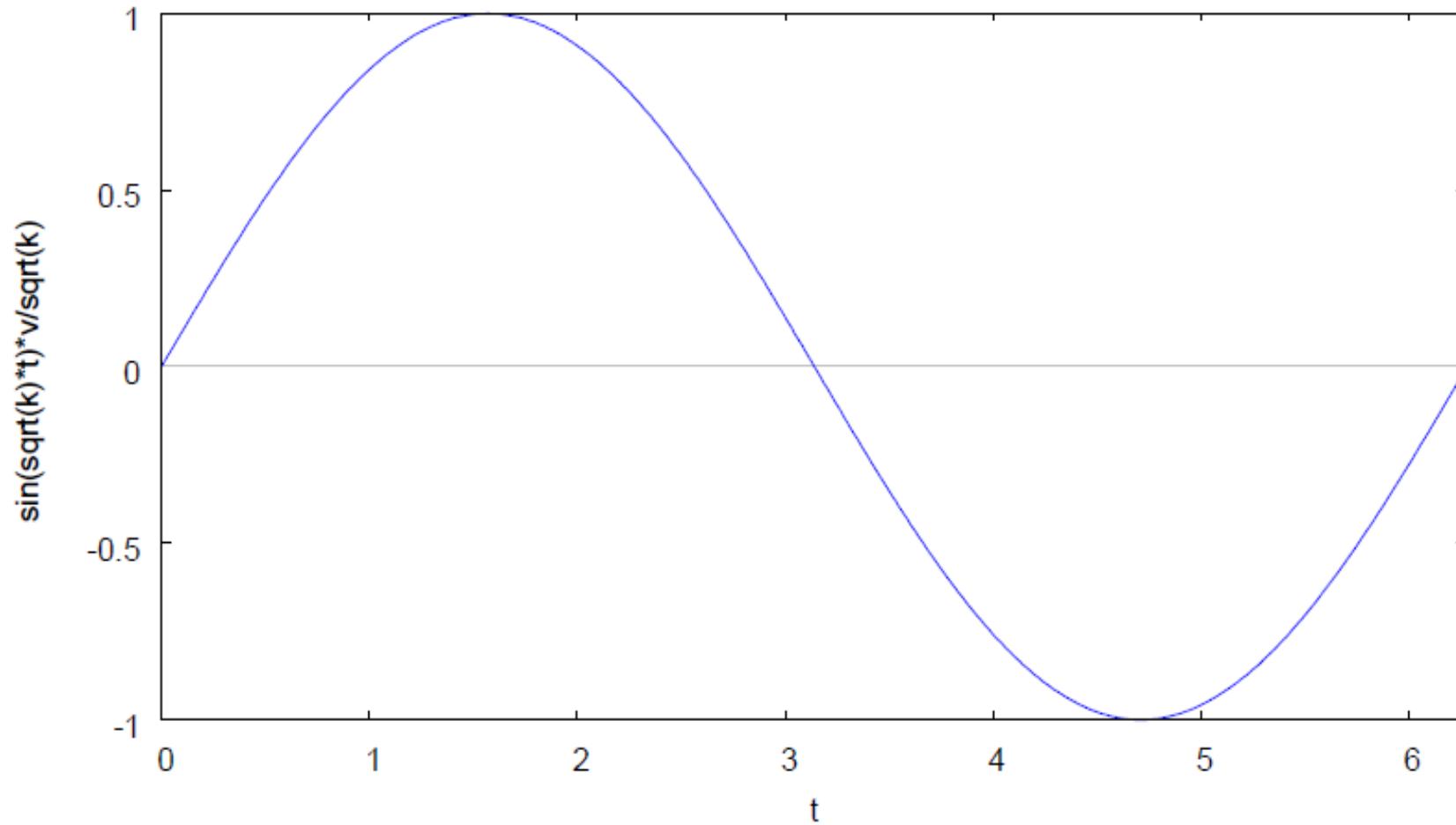
$$F = m \frac{d^2 x}{dt^2}$$

$$F = k x$$



```
(%i1) atvalue(diff(f(t),t),t=0,v);
      atvalue(f(t),t=0,0);
      assume(k>0);
      ho:desolve([diff(f(t),t,2)+k*f(t)=0],[f(t)]);
      k:1;
      v:1;
      plot2d(rhs(ho),[t,0,2*pi]);
(%o1) v
(%o2) 0
(%o3) [k>0]
(%o4) f(t)= $\frac{\sin(\sqrt{k} t)}{\sqrt{k}} v$ 
(%o5) 1
(%o6) 1
(%o7)
```

Example

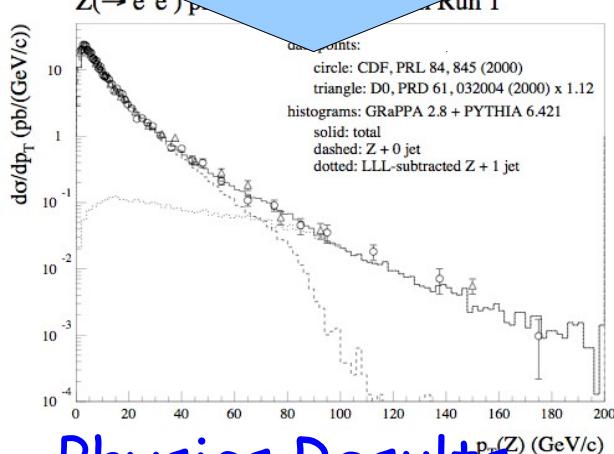


Introduction

Why we need Event Generator?

Theory/Model

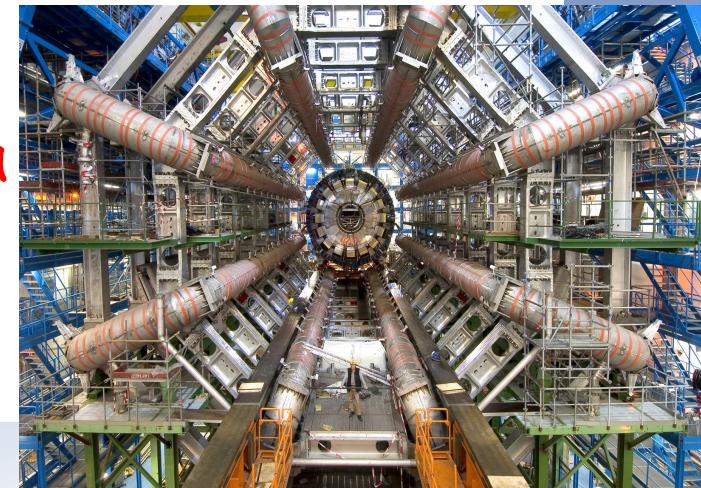
$$\begin{aligned}\tilde{\mathcal{L}} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ & + i\bar{\Psi}\not{D}\Psi + h.c. \\ & + \Psi_i Y_{ij} \Psi_j \Phi + h.c. \\ & + |\not{D}_\mu \Phi|^2 - V(\Phi)\end{aligned}$$



Physics Results



Real data



Detector

Why we need Event Generator?

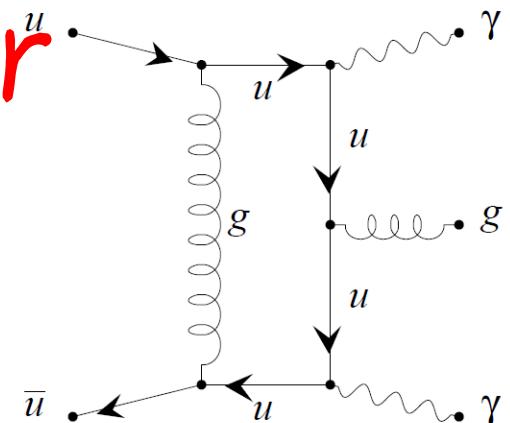
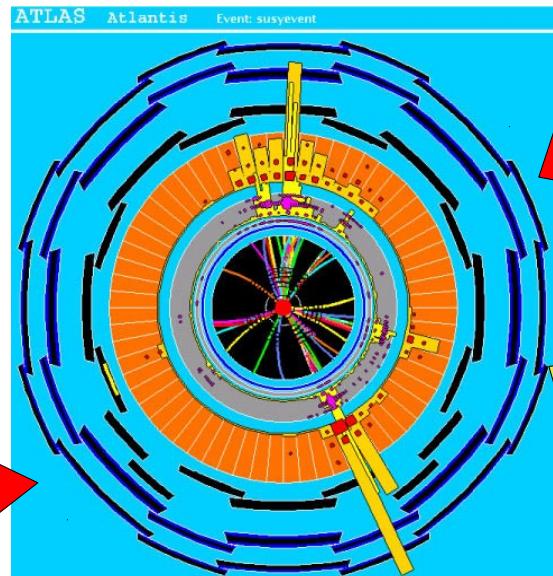
Theory/Model

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ & + i \bar{\psi} \not{D} \psi + h.c. \\ & + \bar{\psi}_i Y_{ij} \psi_j \phi + h.c. \\ & + |\not{D}_\mu \phi|^2 - V(\phi) \end{aligned}$$

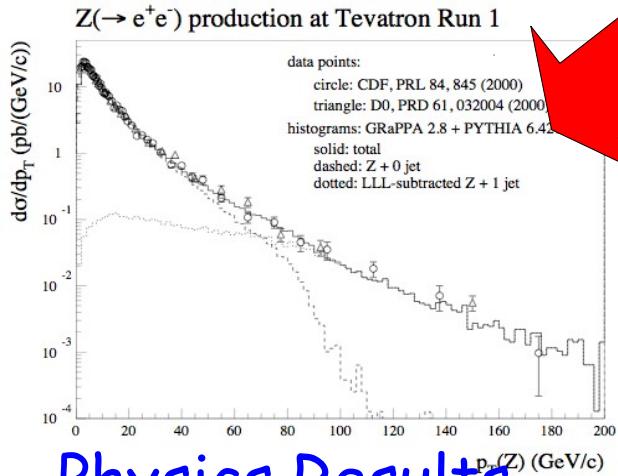
Cross section
calculation



Event Generator

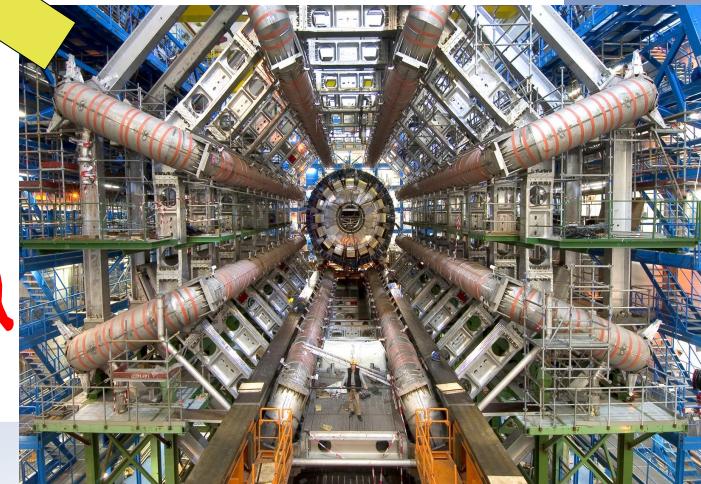


Detector



Physics Results

Real data

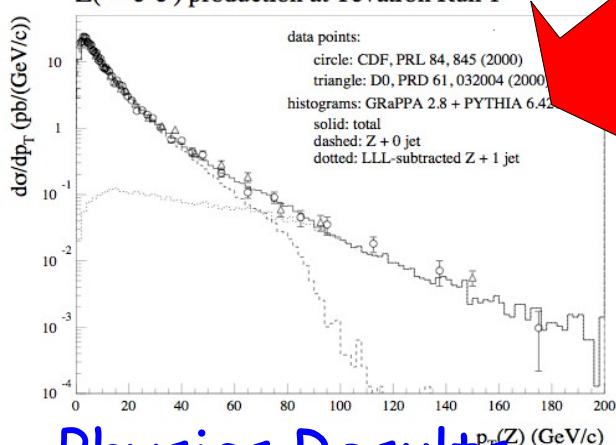


Why we need Event Generator?

Theory/Model

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ & + i \bar{\psi} \not{D} \psi + h.c. \\ & + \bar{\psi}_i \gamma_\mu \psi_i \phi + h.c. \end{aligned}$$

EG data is as important
as real data!

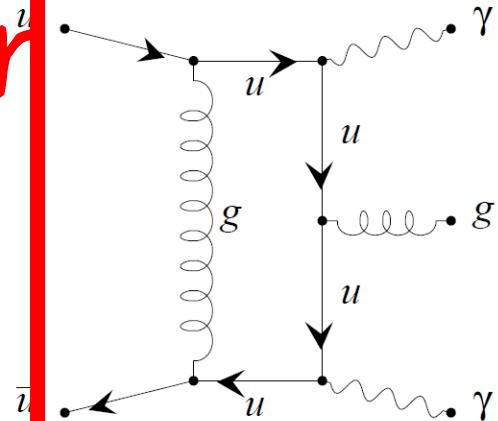
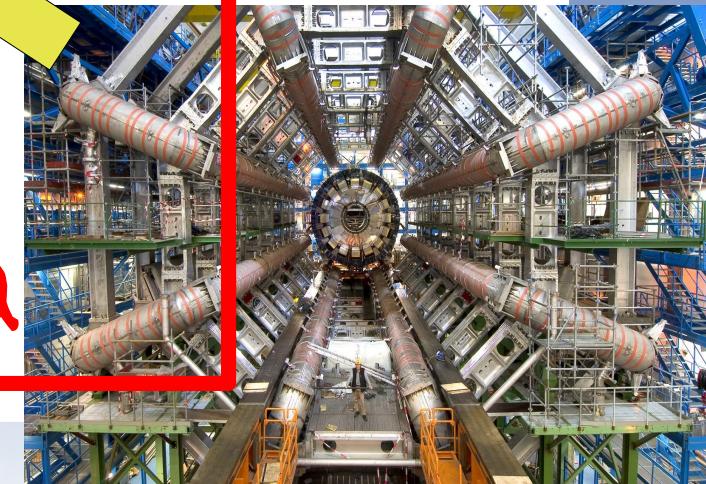


Cross section
calculation

Event Generator



Real data



Detector

Why we need Event Generator?

Theory/Model

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ & + i \bar{\psi} \not{D} \psi + h.c. \\ & + \bar{\psi}_i \gamma_\mu \psi_i \phi + h.c. \end{aligned}$$

$$\delta_{\text{the}} \leq \delta_{\text{exp}} / 10$$

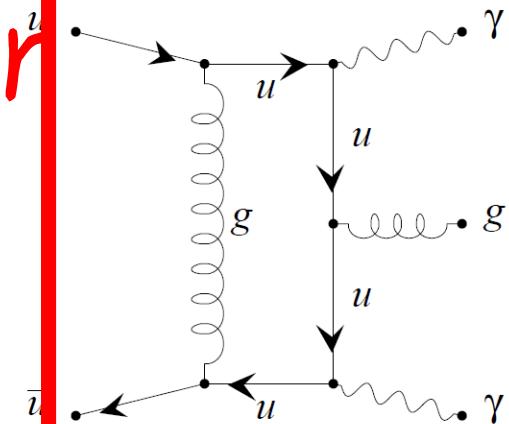
Cross section
calculation

EG data is as important
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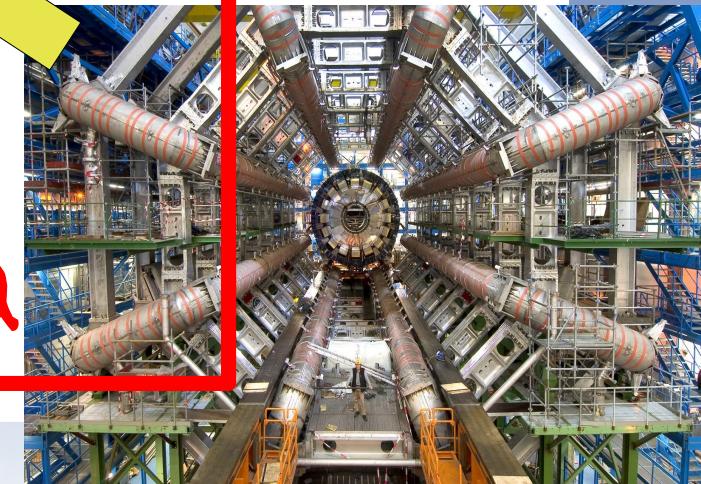


Physics Results

Real data

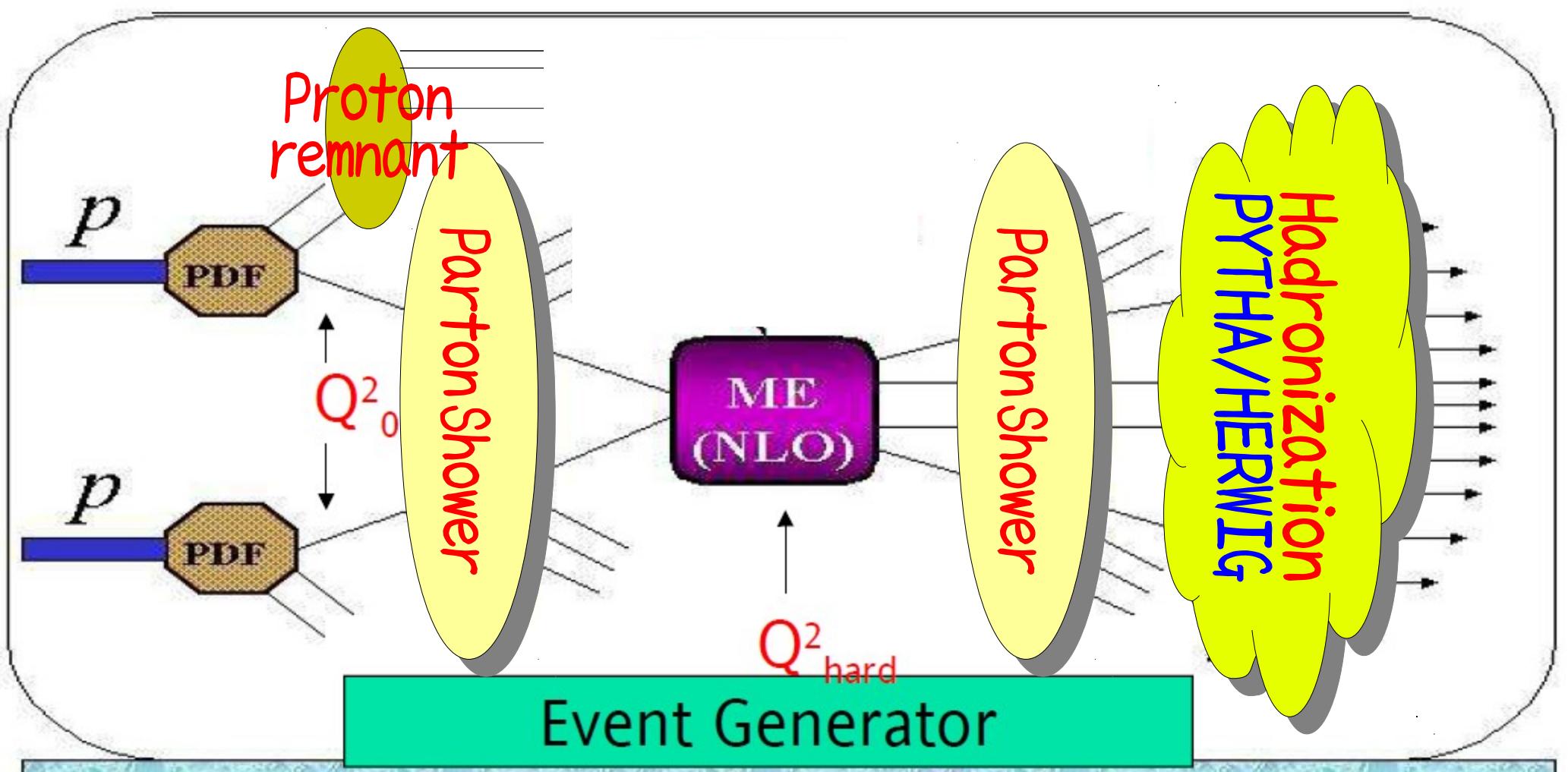


Detector

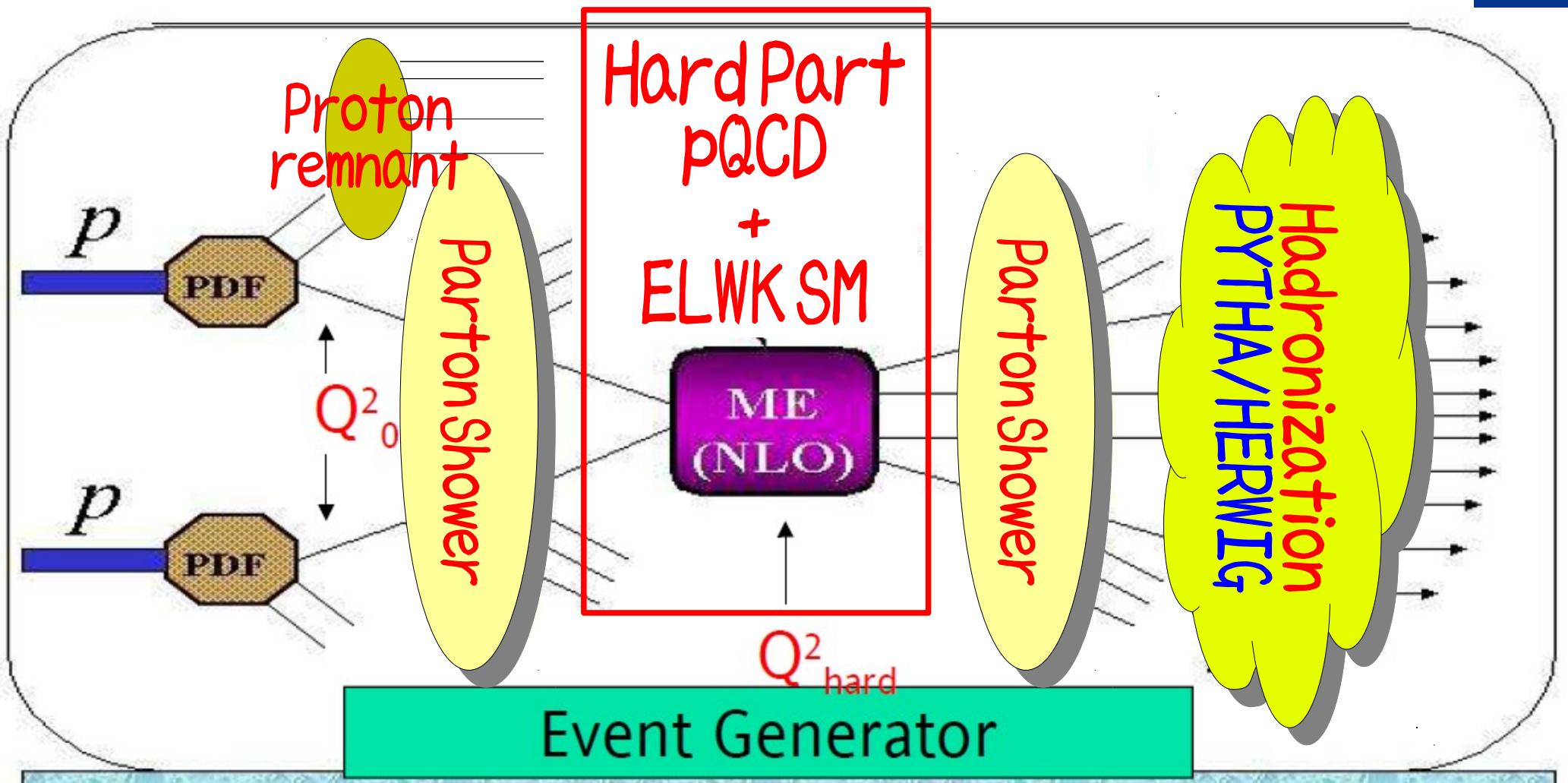


EG Structure

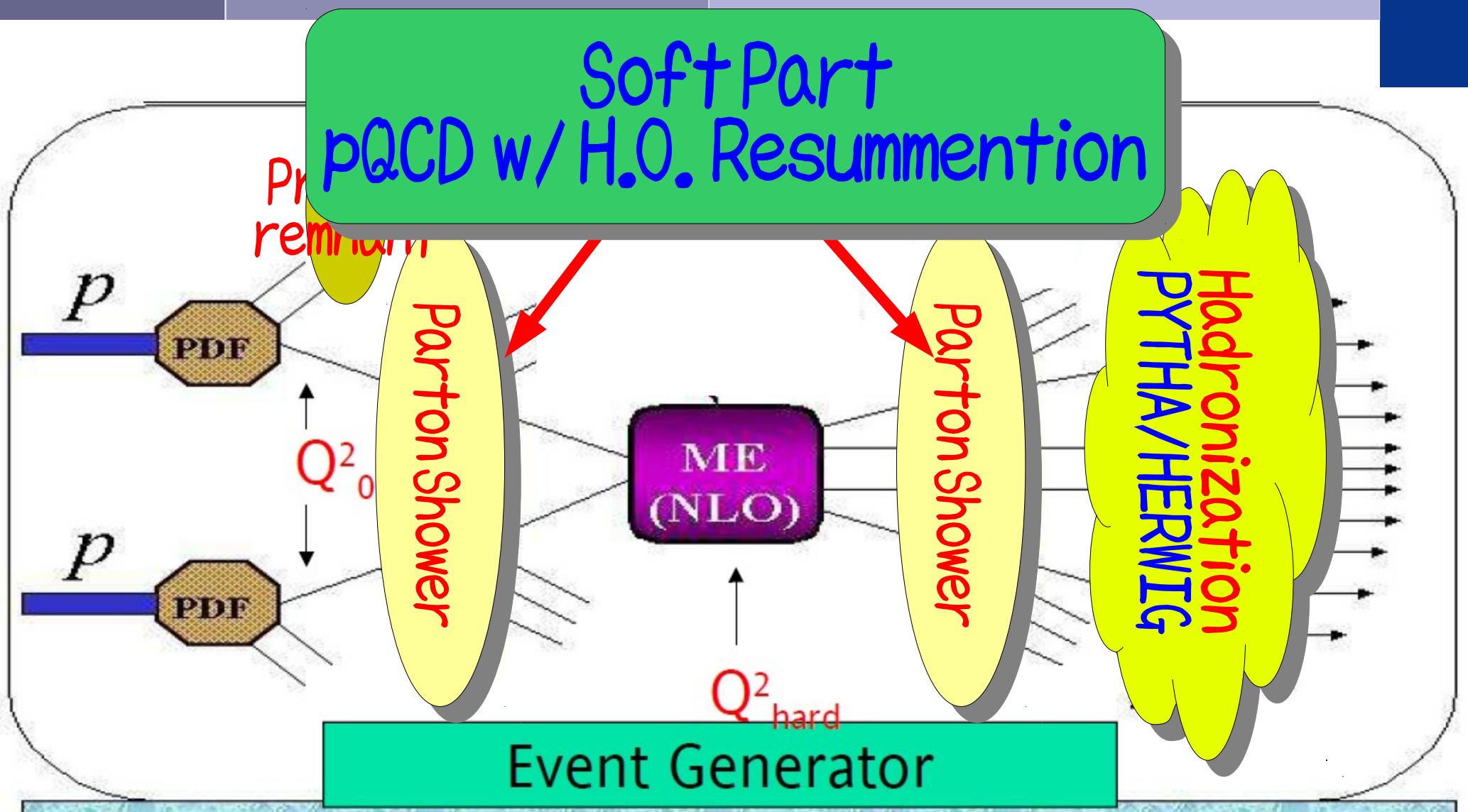
Overview of Event Generator



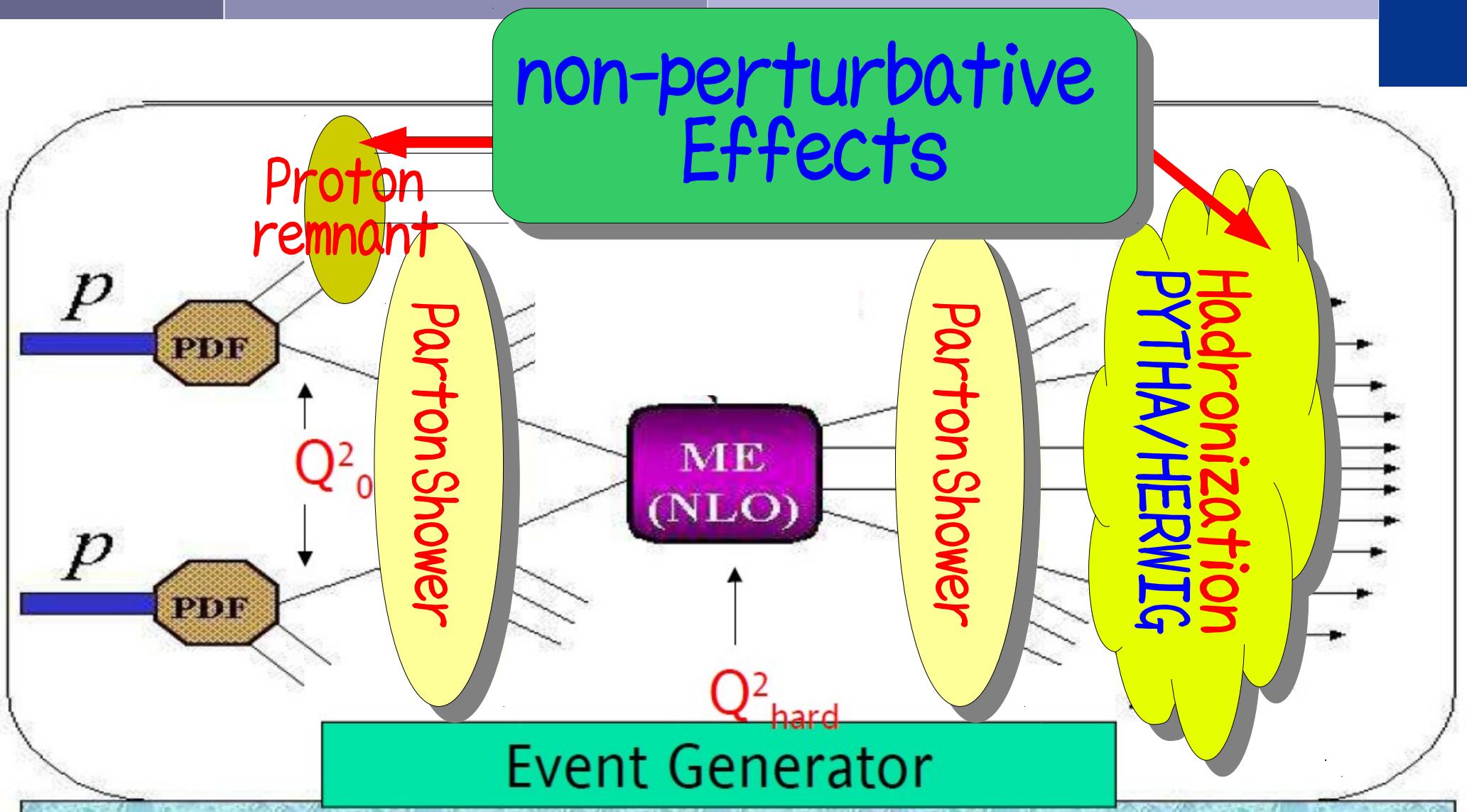
Overview of Event Generator



Overview of Event Generator

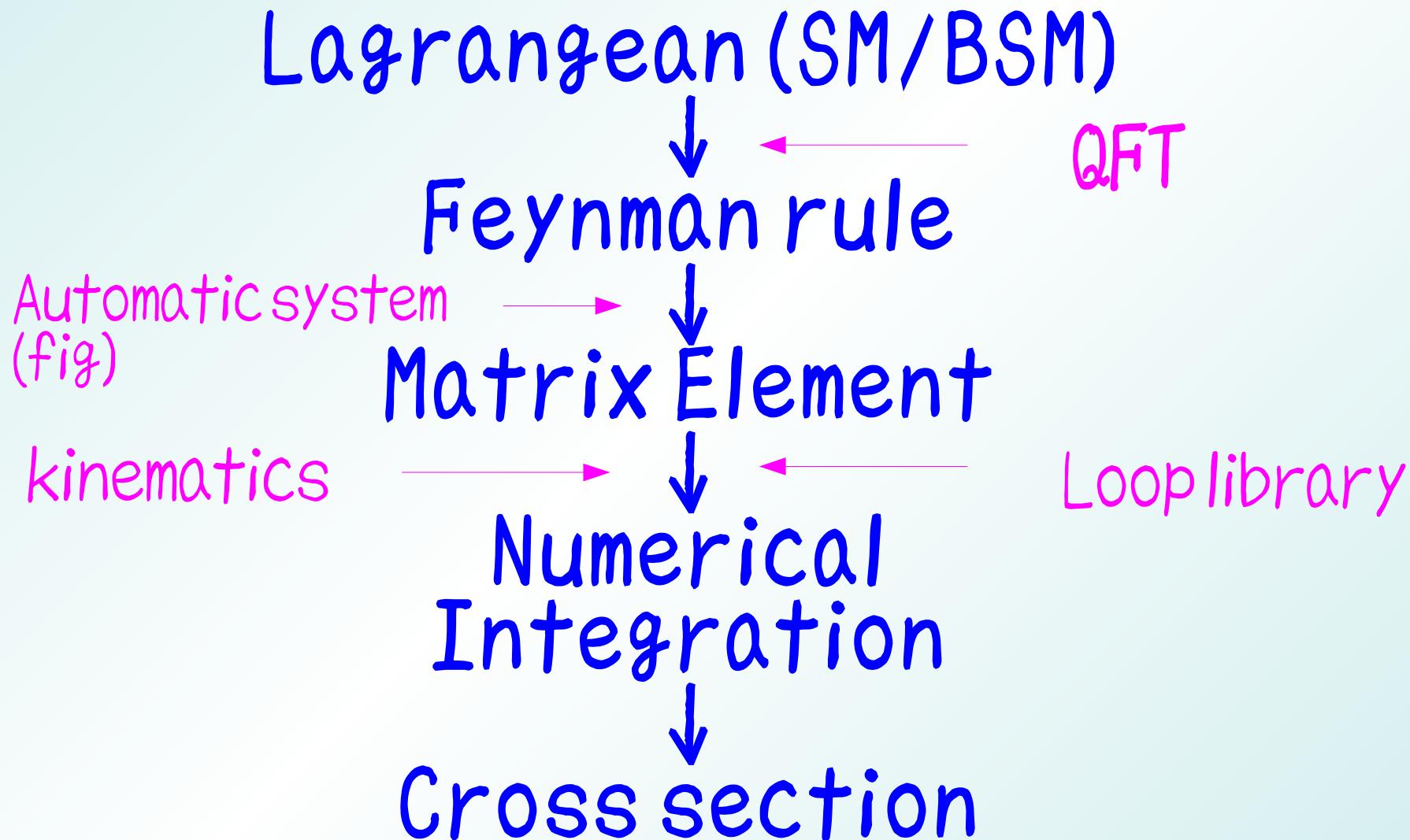


Overview of Event Generator

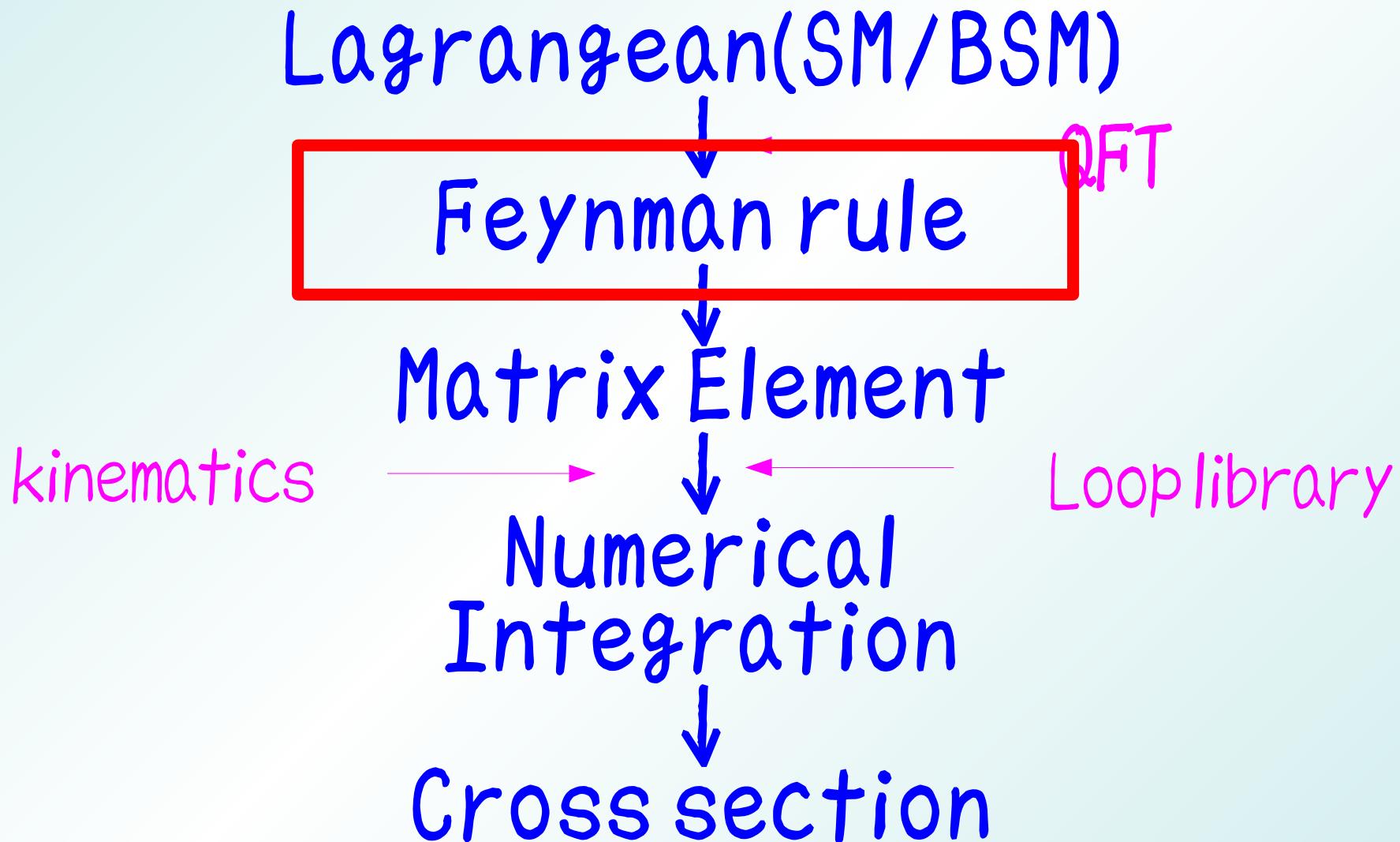


EG Structure: Hard Scattering

ME/Cross section calculations



ME/Cross section calculations



Feynman Rule (QCD)

$$\begin{array}{c} A, \alpha \quad p \quad B, \beta \\ \text{---} \quad \text{---} \quad \text{---} \\ A \quad \quad p \quad \quad B \end{array} \quad \delta^{AB} \left[-g^{\alpha\beta} + (1 - \lambda) \frac{p^\alpha p^\beta}{p^2 + i\varepsilon} \right] \frac{i}{p^2 + i\varepsilon}$$

$$A \quad \quad p \quad \quad B$$

$$\delta^{AB} \frac{i}{p^2 + i\varepsilon}$$

$$a, i \quad p \quad b, j$$

$$\delta^{ab} \frac{i}{(\not{p} - m + i\varepsilon)_{ji}}$$

$$\begin{array}{ccc} & B, \beta & \\ q & \diagdown \quad \diagup & \\ A, \alpha & \quad & C, \gamma \\ \diagup \quad \diagdown & & \\ A, \alpha & & B, \beta \\ \diagdown \quad \diagup & & \\ C, \gamma & & D, \delta \end{array}$$

$$-gf^{ABC} \left[g^{\alpha\beta} (p - q)^\gamma + g^{\beta\gamma} (q - r)^\alpha + g^{\gamma\alpha} (r - p)^\beta \right]$$

(all momenta incoming)

$$\begin{aligned} & -ig^2 f^{XAC} f^{XBD} (g_{\alpha\beta} g_{\gamma\delta} - g_{\alpha\delta} g_{\beta\gamma}) \\ & -ig^2 f^{XAD} f^{XBC} (g_{\alpha\beta} g_{\gamma\delta} - g_{\alpha\gamma} g_{\beta\delta}) \\ & -ig^2 f^{XAB} f^{XCD} (g_{\alpha\gamma} g_{\beta\delta} - g_{\alpha\delta} g_{\beta\gamma}) \end{aligned}$$

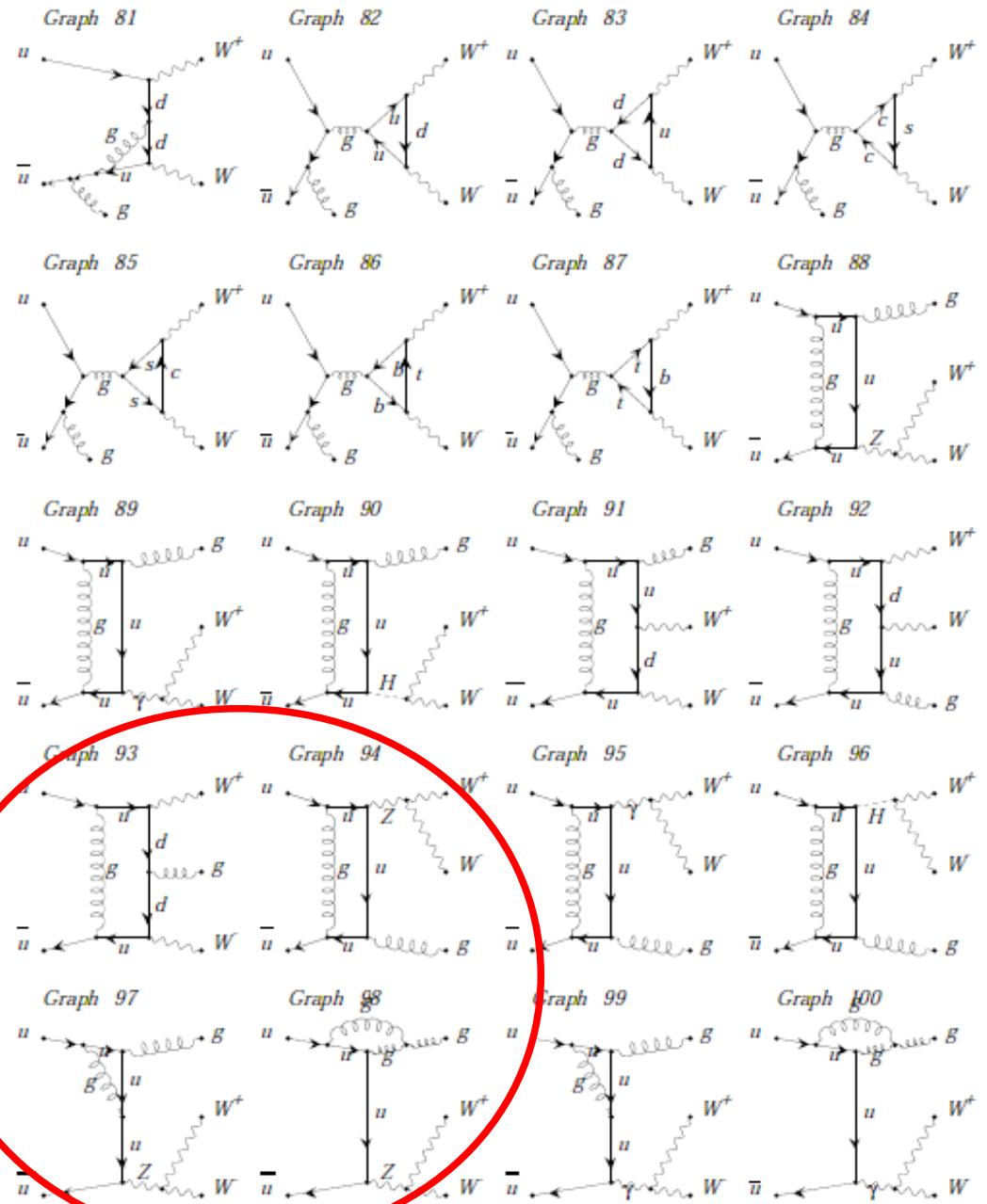
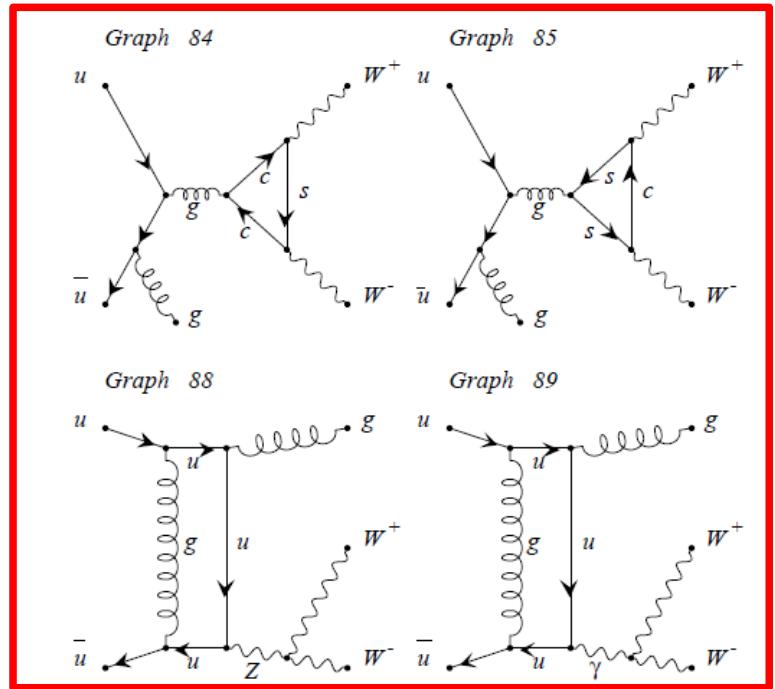
$$\begin{array}{ccc} & A, \alpha & \\ & \diagdown \quad \diagup & \\ B & \quad & C \\ \diagup \quad \diagdown & & \\ & q & \end{array}$$

$$gf^{ABC} q^\alpha$$

$$\begin{array}{ccc} & A, \alpha & \\ & \diagdown \quad \diagup & \\ b, i & \quad & c, j \end{array}$$

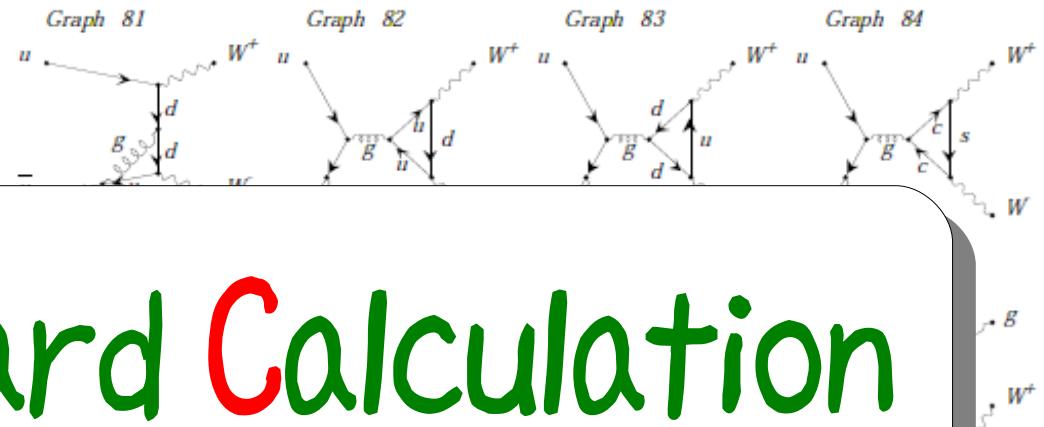
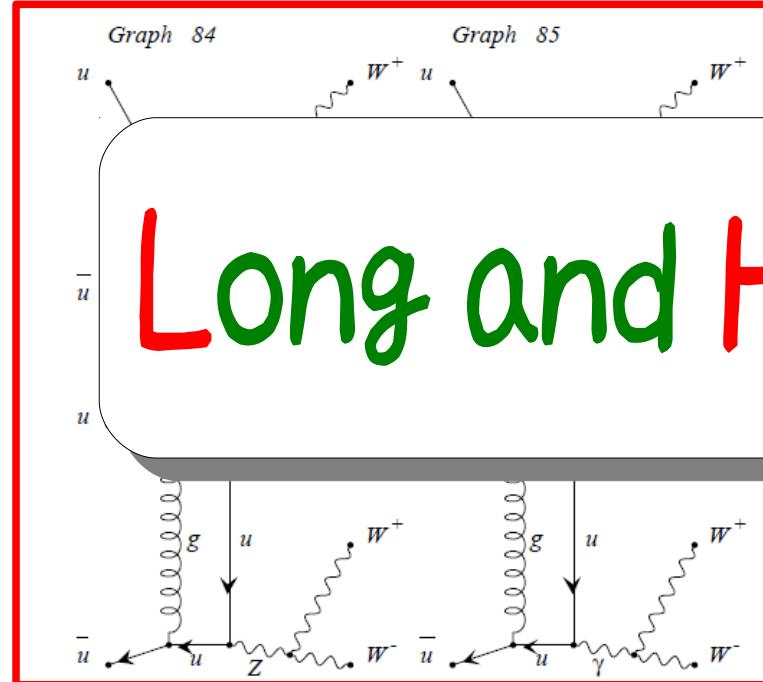
$$-ig (t^A)_{cb} (\gamma^\alpha)_{ji}$$

Feynman diagram: $q\bar{q} \rightarrow WWg$



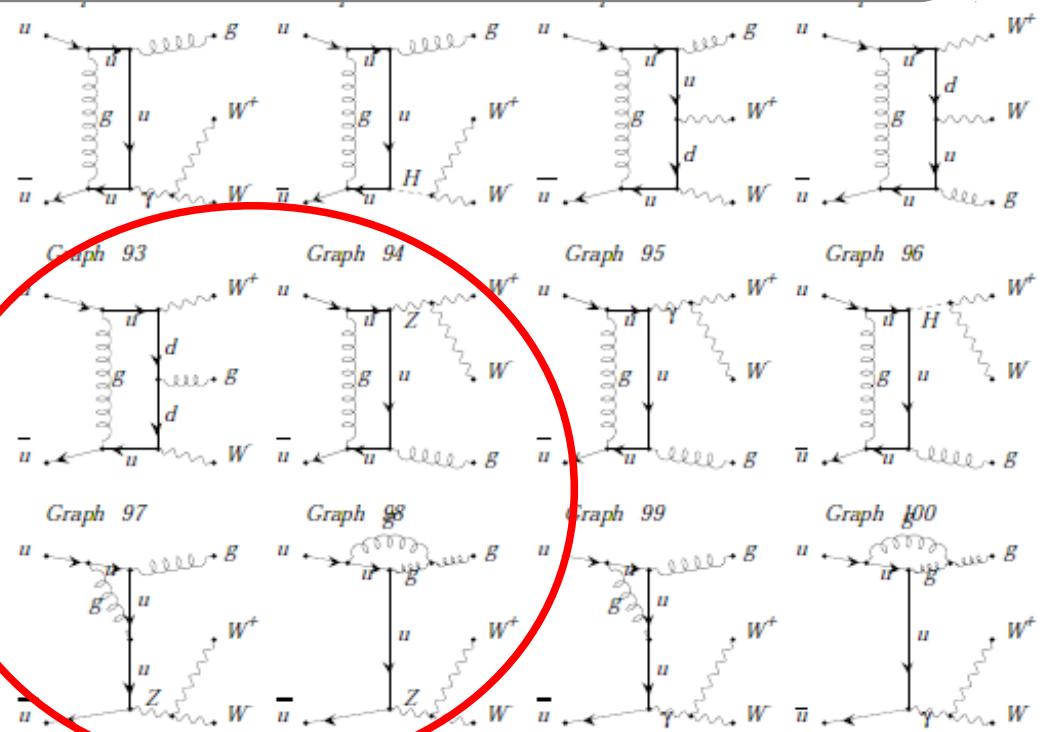
153 diagrams

Feynman diagram: $q\bar{q} \rightarrow WWg$

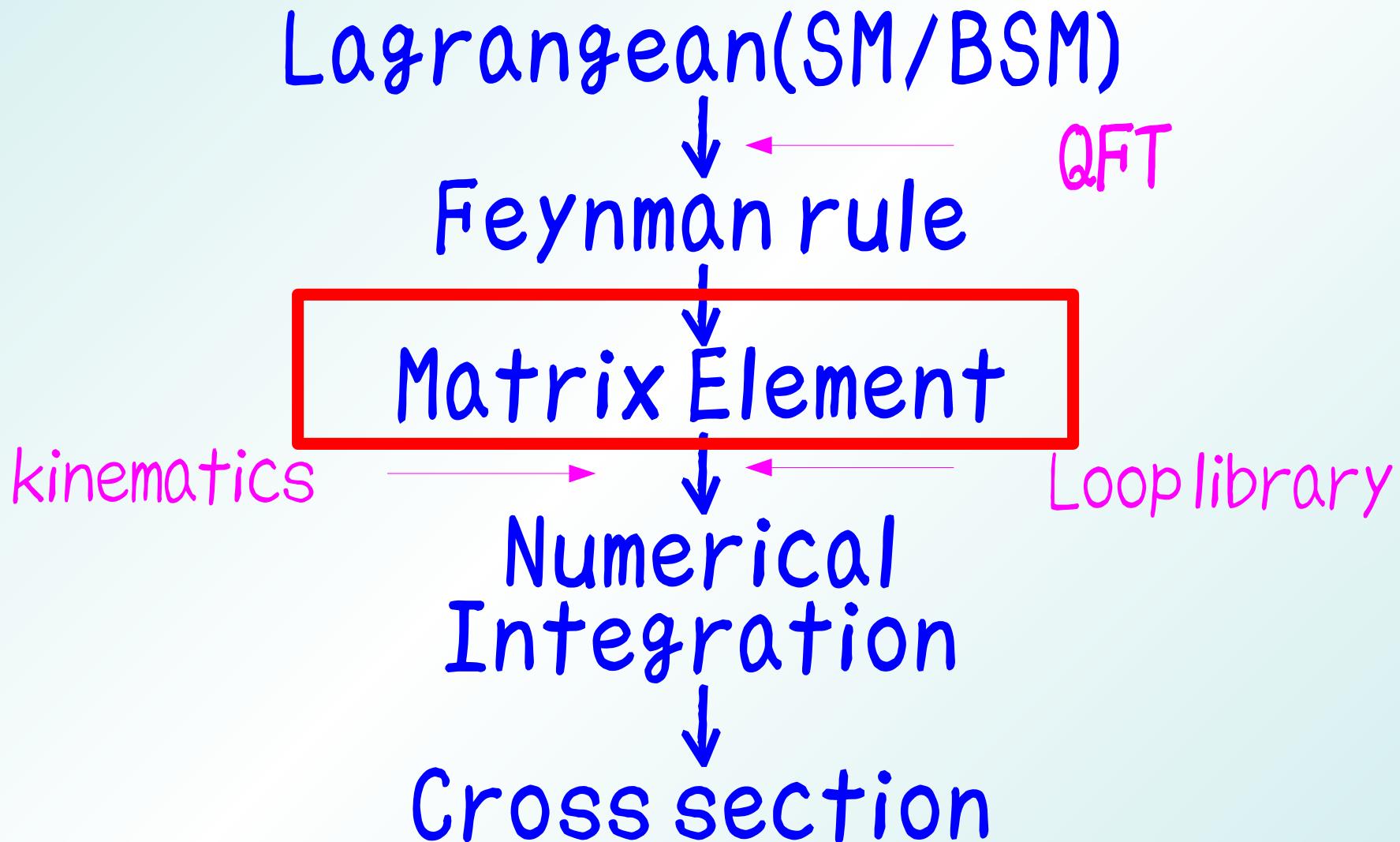


Long and Hard Calculation

153 diagrams



ME/Cross section calculations



Matrix Elements (General Structure)

$$\sigma_{\text{NLO}} = \sigma_{\text{tree}} + \sigma_{\text{loop}} + \sigma_R$$

The diagram shows three Feynman diagrams representing different contributions to the NLO cross section. The first is a tree-level diagram with two incoming red lines and two outgoing red lines. The second is a loop diagram with a central circular vertex connected to four red lines. The third is a more complex diagram with multiple internal lines and vertices, also represented by red lines. Arrows point from each diagram to its corresponding term in the equation below.

$$= \sigma_{\text{tree}} (1 + \delta_V + \delta_{s/c}) + \sigma_{\text{vis}}$$

δ_V : Virtual(loop) correction

$\delta_{s/c}$: Soft/Collinear correction

σ_{vis} : Visible jet cross section

Matrix Elements (IR Structure)

No colored parton in final state at tree level.

after UV-div. renormalized

No IR-divergence

$$\sigma_{\text{NLO}} = [\sigma_{\text{tree}}(1 + \delta_v + \delta_{s/c}) + \sigma_{\text{vis}}] \otimes \text{PDF/PS}$$

Matrix Elements (IR Structure)

No colored parton in final state at tree level.

after UV-div. renormalized

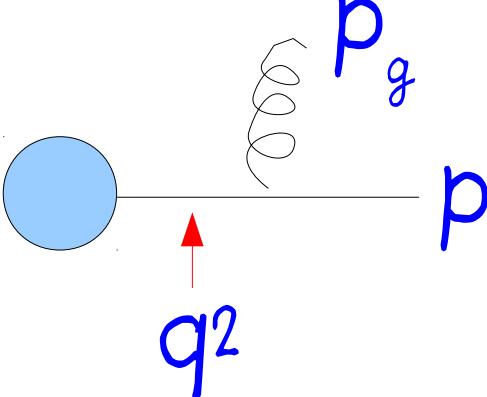
No IR-divergence

$$\sigma_{\text{NLO}} = [\sigma_{\text{tree}}(1 + \delta_v + \delta_{s/c}) + \sigma_{\text{vis}}] \otimes \text{PDF/PS}$$

$1/\epsilon_{\text{IR}}^2, 1/\epsilon_{\text{IR}}$
cancellation

Space/time dimension: $d=4+2\epsilon_{\text{IR}}$

Matrix Elements (IR Structure)


$$p_q \propto \int_0^{q_{max}^2} dq^2 \int_{-1}^1 d\cos\theta \frac{1}{q^2}$$

$$\begin{aligned} q^2 &= (p_q + p_g)^2 \\ &= 2p_q p_g \\ &= E_q E_g (1 - \cos\theta) \end{aligned}$$

0: Colinear divergence

0: Soft divergence

Matrix Elements (IR Structure)

Dimensional Regularization

$$\int_0^{x_1} dx \frac{f(x)}{x} \Rightarrow \int_0^{x_1} dx \frac{f(x)}{x^{1-\epsilon}}$$

Divergent at $x=0$

IR safe if $\epsilon > 0$

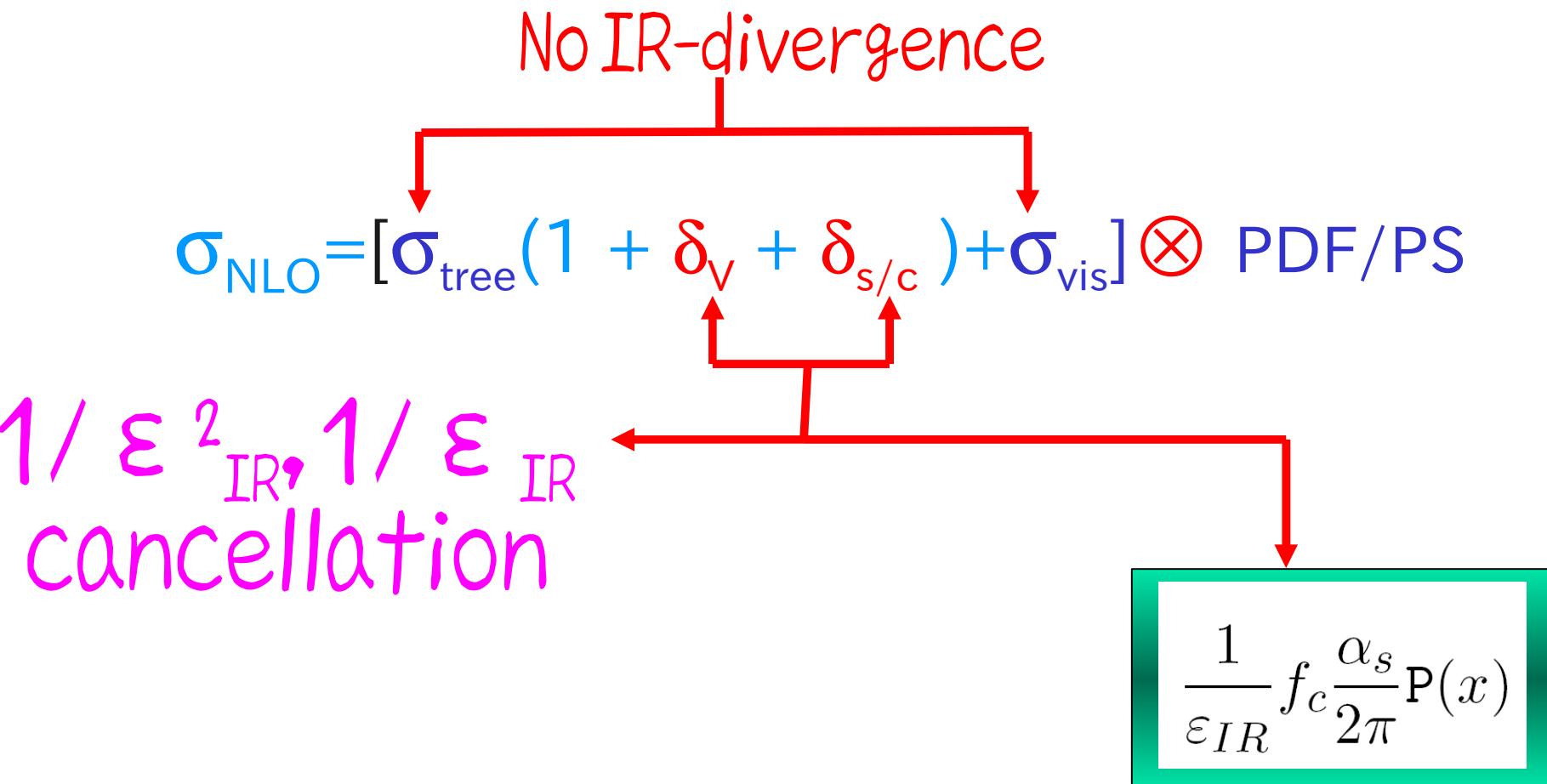
$$\int_0^{x_1} dx \frac{f(x)}{x^{1-\epsilon}} = \int_0^{x_1} \frac{f(x) - f(0)}{x^{1-\epsilon}} + \int_0^{x_1} \frac{f(0)}{x^{1-\epsilon}}$$

IR safe

1/ ϵ pole $\frac{f(0)}{\epsilon}$

Matrix Elements (IR Structure)

No colored parton in final state at tree level.

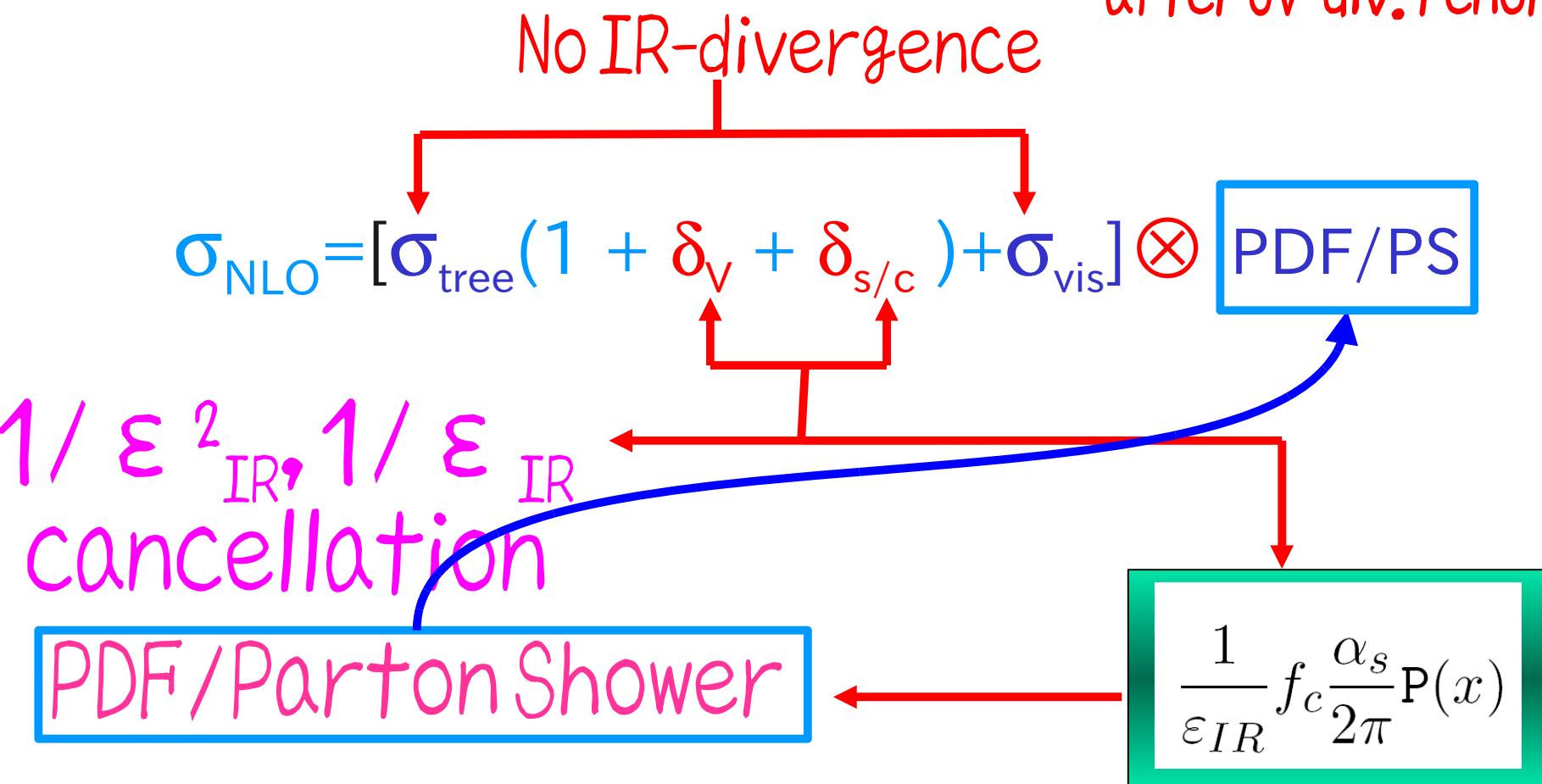


P(x): Splitting function Space/time dimension: d=4+2 ε_{IR}

Matrix Elements (IR Structure)

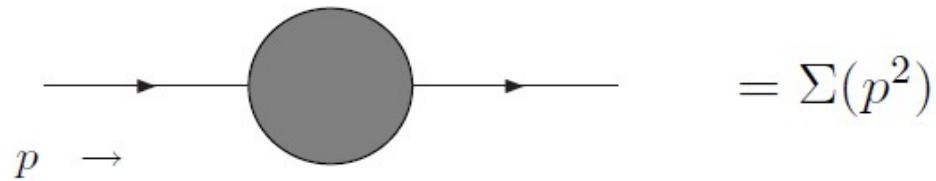
No colored parton in final state at tree level.

after UV-div. renormalized



P(x): Splitting function Space/time dimension: d=4+2 ε_{IR}

Matrix Elements(Loop w/ MS)



on-/off-shell	self-energy
$p^2 \neq 0$	$\Sigma(p^2) = C_F \frac{\alpha_s}{4\pi} \left(1 - \ln \frac{-p^2}{\mu^2}\right)$
$p^2 = 0$	$\Sigma(0) = C_F \frac{\alpha_s}{4\pi} \frac{1}{\varepsilon_{IR}}$

A Feynman diagram showing a loop with two external lines labeled p_1 and p_2 entering from the left and right respectively. A vertical gluon line labeled a enters the loop from the top. The loop is represented by a grey circle.

$$= \Lambda_\mu^a(p_1, p_2, k)$$

$$\begin{aligned} \Lambda_\mu(p_1, p_2, k) &= g \mathbf{T}^a \left(C_F - \frac{1}{2} C_G \right) \frac{\alpha_s}{4\pi} (\mathcal{F}_1^I \gamma^\mu + \mathcal{F}_2^I \frac{p_j^\mu k}{-p_j^2}) \\ &+ g \mathbf{T}^a \frac{1}{2} C_G \frac{\alpha_s}{4\pi} (\mathcal{F}_1^I \gamma^\mu + \mathcal{F}_2^I \frac{p_j^\mu k}{-p_j^2}) \end{aligned}$$

A.5.1 case I

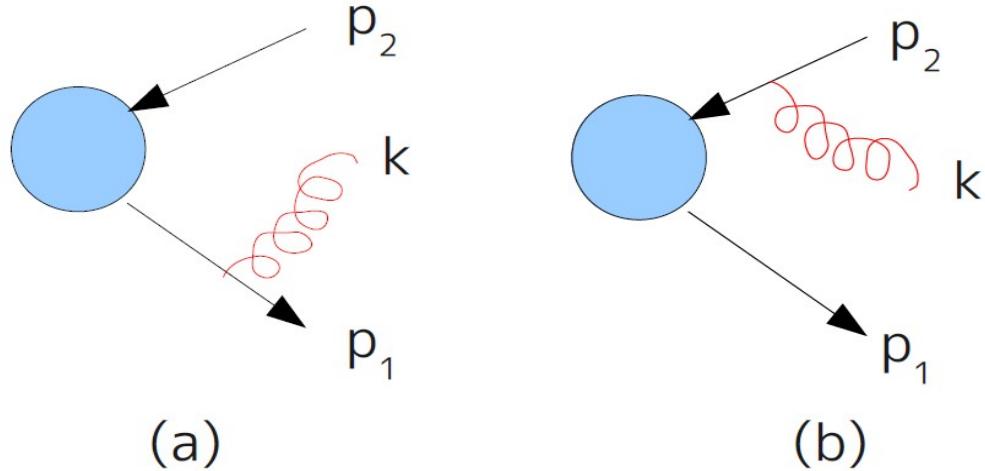
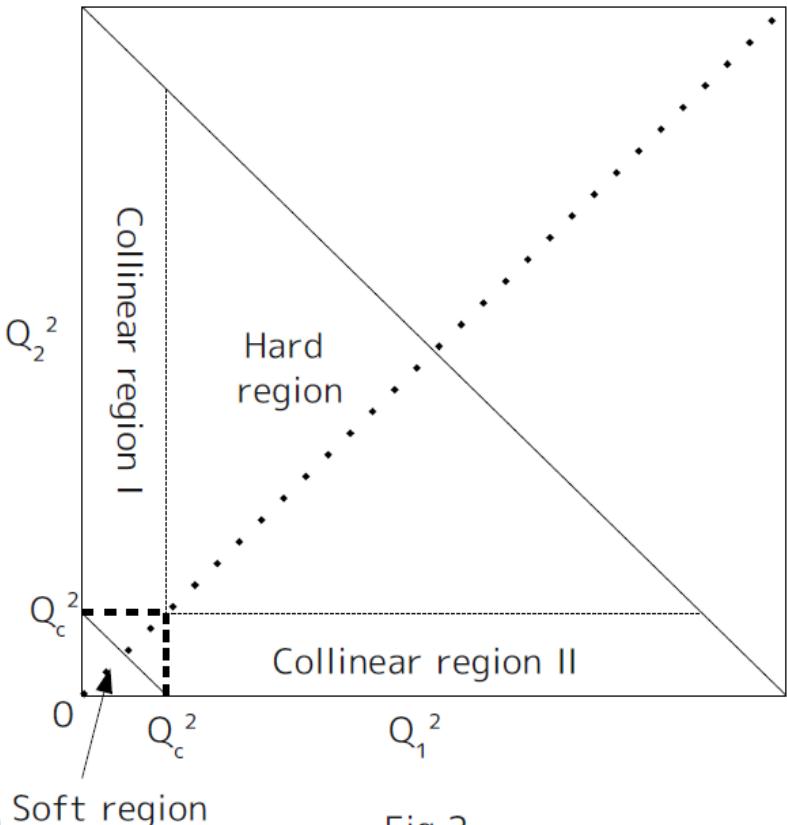
$k^2 = 0$, $p_i^2 = 0$, $p_j^2 = q^2 \neq 0$, and $L = \ln \frac{-q^2}{\mu^2}$, where $(i, j) = (1, 2)$ or $(i, j) = (2, 1)$.

\mathcal{F}_1^I	$\frac{2}{\varepsilon_{IR}} + L - 4$
\mathcal{F}_2^I	$\frac{4}{\varepsilon_{IR}} + 4L - 10$
\mathcal{F}_1^{II}	$-\frac{2}{\varepsilon_{IR}^2} + \frac{3-2L}{\varepsilon_{IR}} + \frac{\pi^2}{6} - L$
\mathcal{F}_2^{II}	$-\frac{2}{\varepsilon_{IR}^2} - \frac{2L}{\varepsilon_{IR}} + \frac{12+\pi^2-6L^2}{6}$

Soft/Collinear correction (final color)

Two types of div.

- Collinear div.
- Soft div.



Lorentz inv./Process indep.

- Phase space slicing
- Subtraction
- Slicing/LLL-Subtraction hybrid

Soft/Collinear correction (final color)

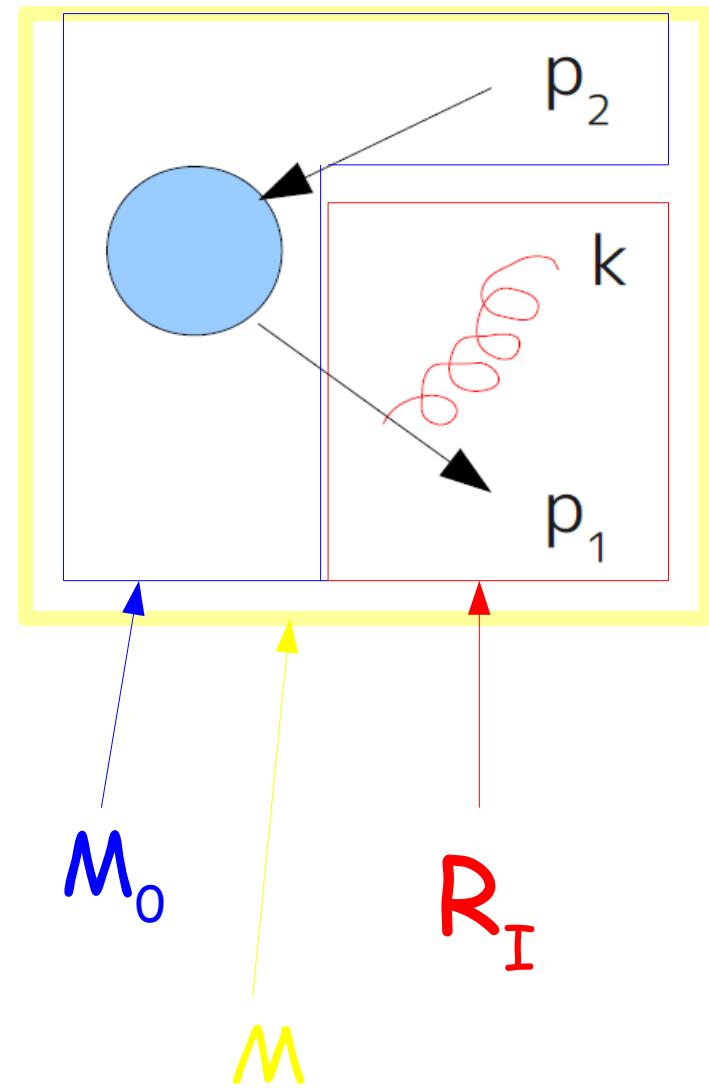
Collinear Part

$$\begin{aligned}\sigma_{n+1} &= \frac{1}{(\text{flux})} \int d\Phi_{n+1} |\mathcal{M}|^2 \\ &\approx \frac{1}{(\text{flux})} \int d\Phi_{n+1} |\mathcal{M}_0|^2 \frac{R_I}{(q_1^2)^2} \\ &= \frac{1}{(\text{flux})} \int d\Phi_n |\mathcal{M}_0|^2 \int \frac{dq_1^2}{2\pi(q_1^2)^2} \int d\Phi_2 R_I \\ &= \frac{1}{(\text{flux})} \int d\Phi_n |\mathcal{M}_0|^2 \otimes \delta_{col},\end{aligned}$$

$$\delta_{col} \equiv \int \frac{dq_1^2}{2\pi(q_1^2)^2} \int d\Phi_2 R_I$$

$$R_I = 2g_s^2 \frac{k_T}{x(1-x)} (P(x) + (1-x)\epsilon_{IR})$$

$$P(x) = \frac{1+x^2}{1-x}$$



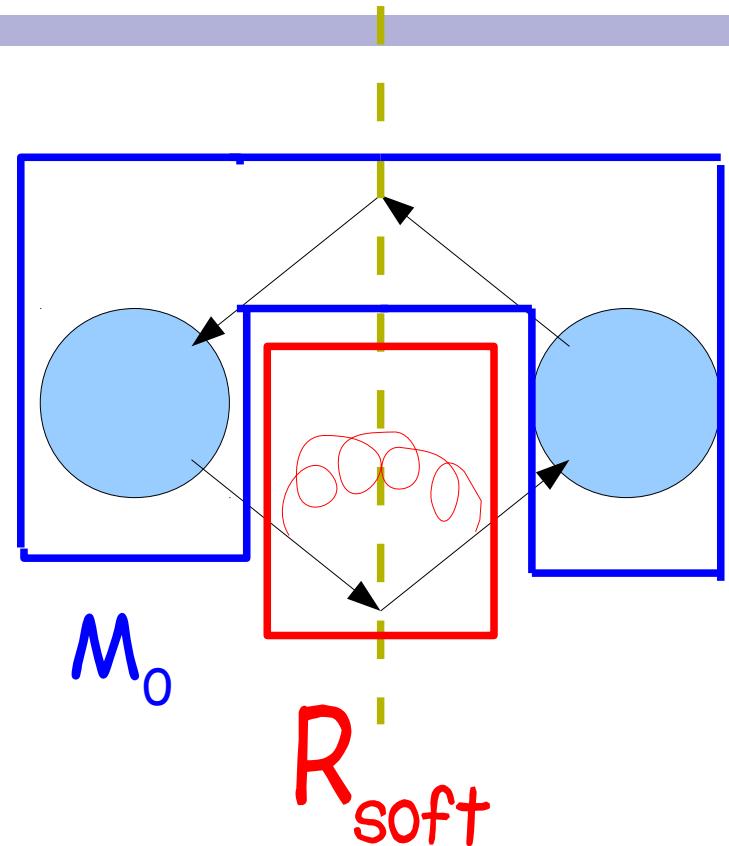
Soft/Collinear correction (final color)

Soft Part

$$\sigma_{n+1} = \frac{1}{(\text{flux})} \int d\Phi_{n+1} |\mathcal{M}|^2$$
$$\approx \frac{1}{(\text{flux})} \int d\Phi_n |\mathcal{M}_0|^2 \otimes \delta_{\text{soft}},$$

$$\delta_{\text{soft}} \equiv \int \frac{dk_0}{\pi} \int d\Phi_2 \frac{s R_{\text{soft}}}{q_1^2 q_2^2}$$

$$R_{\text{soft}} = -4 p_1 \cdot p_2 (1 + \epsilon_{IR})$$



Soft/Collinear correction (final color)

Collinear Part:

$$\delta_{col} = \frac{\alpha_s}{\pi} \frac{1}{\epsilon_{IR}^2} + \frac{\alpha_s}{4\pi} \left(-3 + 4 \log(Q_c^2/\mu_F^2) \right) \frac{1}{\epsilon_{IR}} \\ - \frac{\alpha_s}{4\pi} \left(-7 + \pi^2 + 3 \log(Q_c^2/\mu_F^2) - \log^2(Q_c^2/\mu_F^2) \right)$$

Soft Part:

$$\delta_{soft} = -\frac{\alpha_s}{\pi} \frac{1}{\epsilon_{IR}^2} - \frac{\alpha_s}{\pi} \left(\log(Q_c^2/s) + \log(Q_c^2/\mu_F^2) \right) \frac{1}{\epsilon_{IR}} \\ + \frac{\alpha_s}{12\pi} \left(\pi^2 - 6 \left(\log(Q_c^2/s) + \log(Q_c^2/\mu_F^2) \right)^2 \right)$$

Soft/Collinear correction (final color)

Examples $Z \rightarrow d\bar{d}$

$$\begin{aligned}\Gamma_{Z \rightarrow d\bar{d}} &= \Gamma_0(1 + \delta_{\alpha_s}), \\ \delta_{\alpha_s} &= \delta_v + 2\delta_{col} + \delta_{soft}\end{aligned}$$

Virtual
correction:

$$\begin{aligned}\delta_v &= -\frac{\alpha_s}{\pi} \frac{1}{\epsilon_{IR}^2} + \frac{\alpha_s}{2\pi} \left(3 - 2 \log(s/\mu_F^2) \right) \frac{1}{\epsilon_{IR}} \\ &\quad + \frac{\alpha_s}{2\pi} \left(-8 + \frac{7}{6} \pi^2 + 3 \log(s/\mu_F^2) - \log^2(s/\mu_F^2) \right)\end{aligned}$$

Result: $\delta_{\alpha_s} = \frac{\alpha_s}{2\pi} \left(-1 + \pi^2/3 - 3 \log(Q_c^2/s) - 2 \log^2(Q_c^2/s) \right)$

Soft/Collinear correction (final color)

Examples $Z \rightarrow d\bar{d}$

$$\begin{aligned}\Gamma_{Z \rightarrow d\bar{d}} &= \Gamma_0(1 + \delta_{\alpha_s}), \\ \delta_{\alpha_s} &= \delta_v + 2\delta_{col} + \delta_{soft}\end{aligned}$$

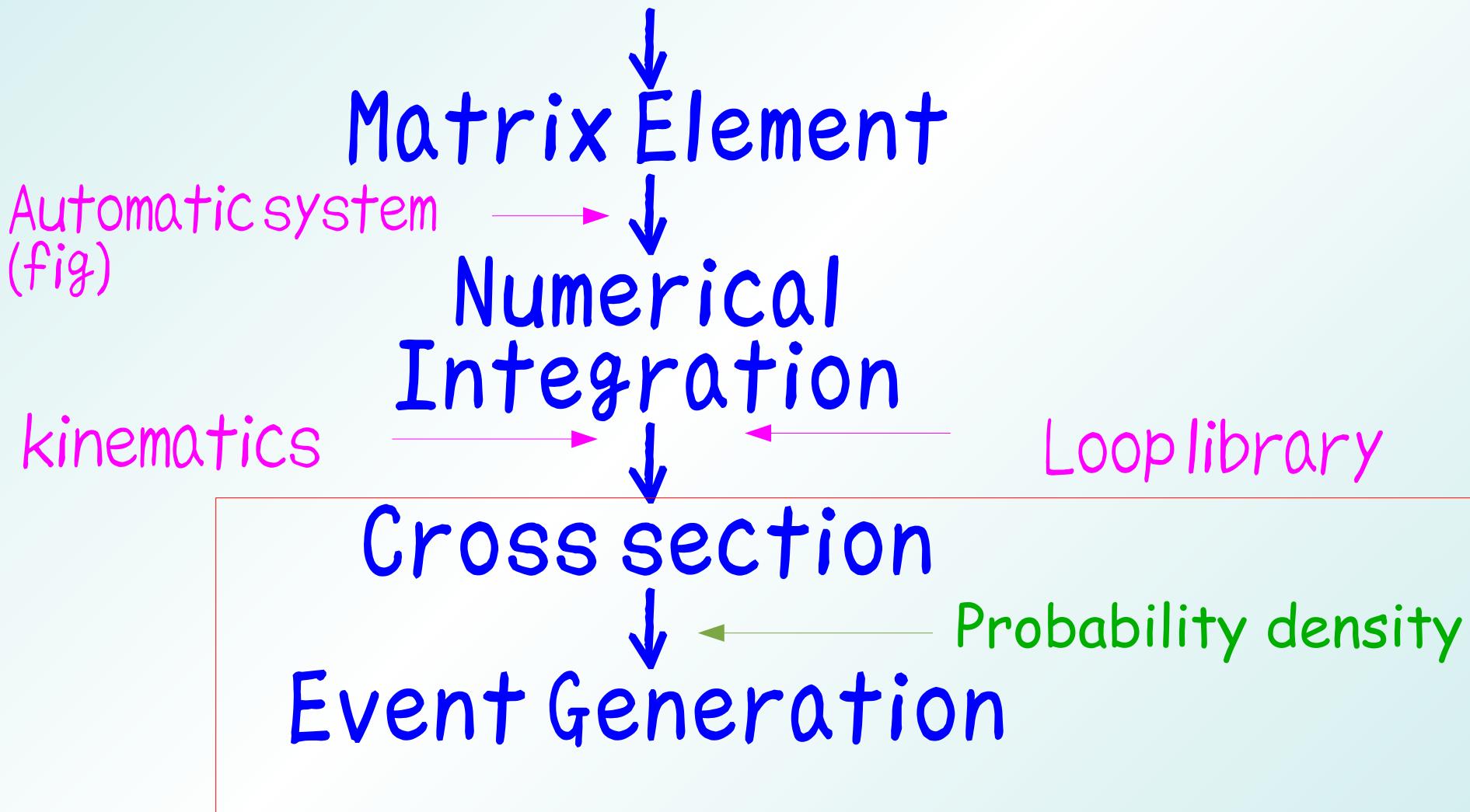
Virtual
correction:

$$\begin{aligned}\delta_v &= -\frac{\alpha_s}{\pi} \frac{1}{\epsilon_{IR}^2} + \frac{\alpha_s}{2\pi} \left(3 - 2 \log(s/\mu_F^2) \right) \frac{1}{\epsilon_{IR}} \\ &\quad + \frac{\alpha_s}{2\pi} \left(-8 + \frac{7}{6}\pi^2 + 3 \log(s/\mu_F^2) - \log^2(s/\mu_F^2) \right)\end{aligned}$$

Result: $\delta_{\alpha_s} = \frac{\alpha_s}{2\pi} \left(-1 + \pi^2/3 - 3 \log(Q_c^2/s) - 2 \log^2(Q_c^2/s) \right)$

+ Hard Corr. \longrightarrow $\Gamma_{\alpha_s} = \Gamma_0 \frac{\alpha_s}{\pi} = 0.01445 \text{ GeV}$

ME/Cross section calculations



What is event generation?

Generation of many sets of values of variables x_1, x_2, \dots, x_n

which reproduce the distribution of function $f(x_1, x_2, \dots, x_n)$.

$$f(x_1, x_2, \dots, x_n) = \frac{d^n \sigma(x_1, x_2, \dots, x_n)}{dx_1 dx_2 \dots dx_n}$$

Event generation = Generation of many sets of Random numbers

with a given specific distribution

set of random # : $\{x_1, x_2, \dots, x_n\} \rightarrow \{\xi_1, \xi_2, \dots, \xi_n\}$

Flat distribution → according to $f(x)$

Random # generation w/ arbitrary dist.

■ Direct Method

A function $f(x)$ defined in $[a, b]$ gives

the probability density $p(x)$:

$$p(x) = \frac{1}{\int_a^b f(y)dy} f(x), \quad \int_a^b p(x)dx = 1$$

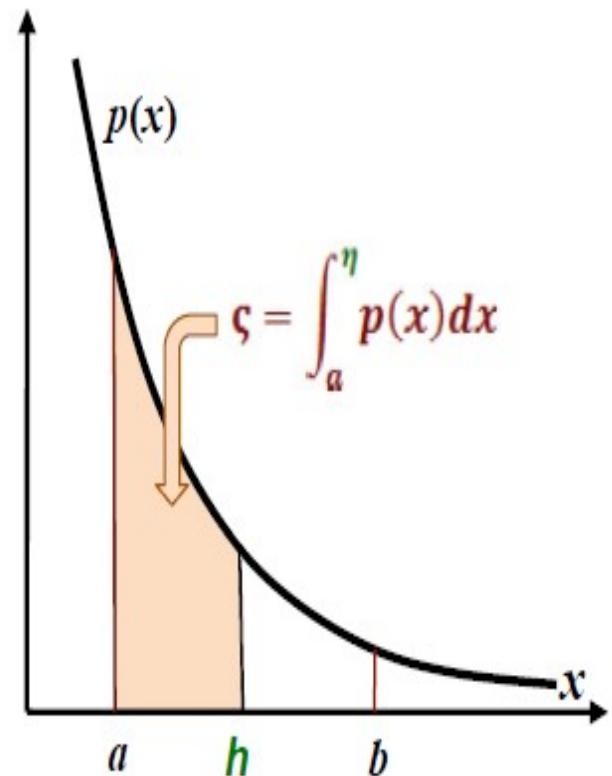
If the variable z , defined by

$$\varsigma = \int_a^y p(x)dx \quad \text{or} \quad \varsigma = \frac{\int_a^y f(y)dy}{\int_a^b f(y)dy},$$

has a uniform distribution in the region $[0,1]$,

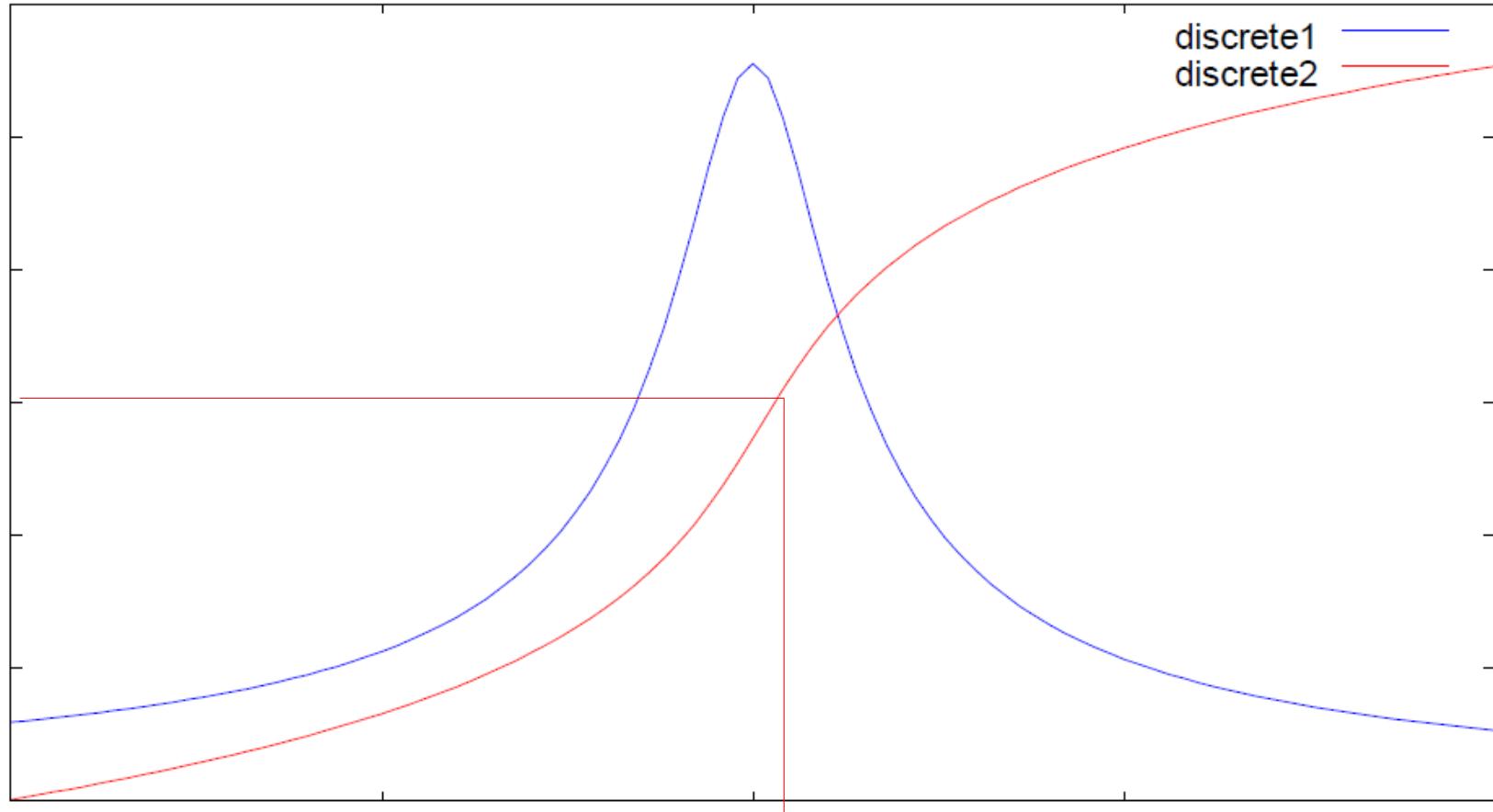
the variable h has a distribution with the probability density $p(x)$.

- [NB] ■ The function $f(x)$ must have the generating function $F(x) = \int f(x)dx$
■ The generating function $y = F(x)$ must have the inverse $x = F^{-1}(y)$.



Random # generation w/ arbitrary dist.

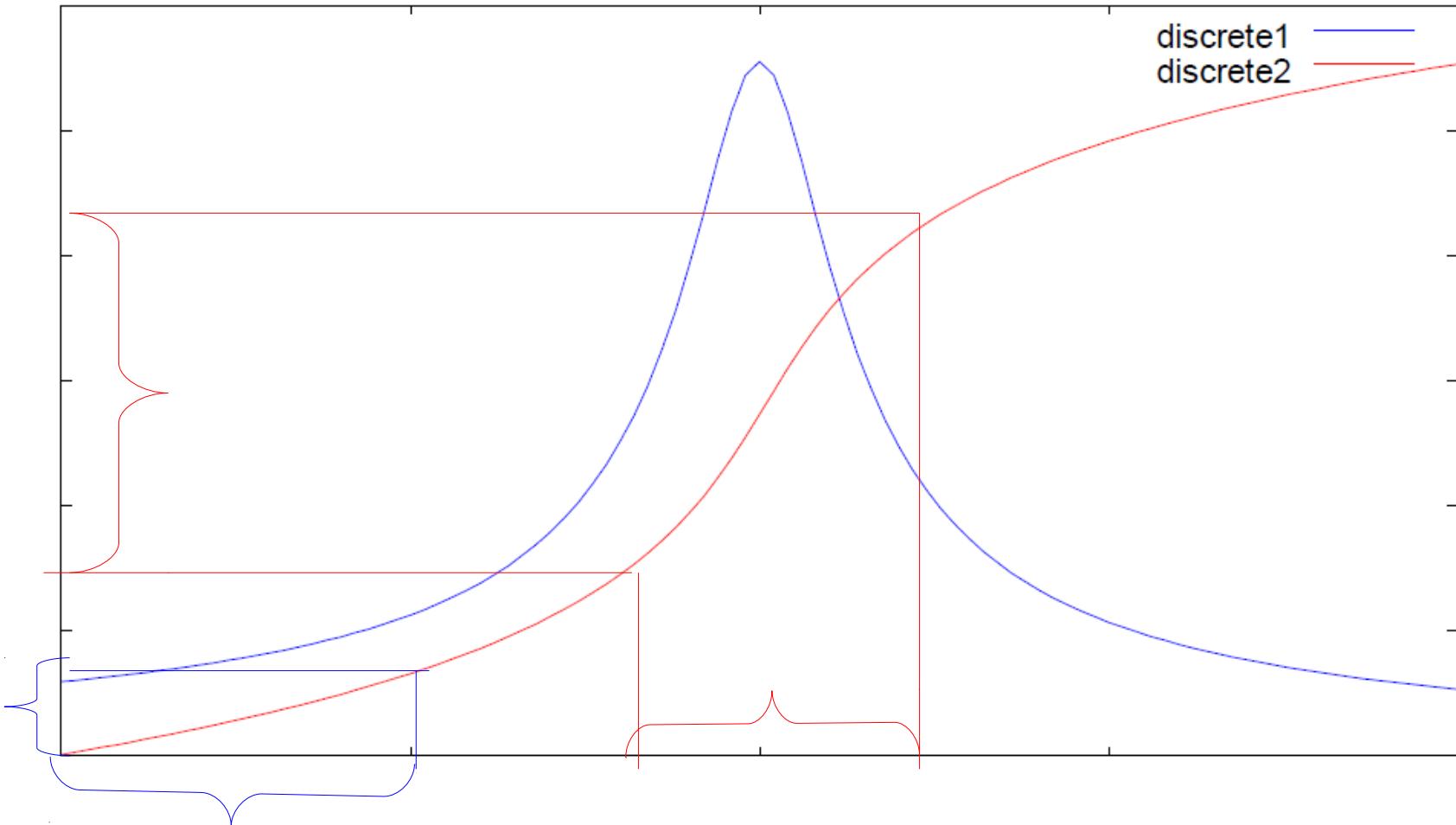
Flat random number



Distribution needed

Random # generation w/ arbitrary dist.

Flat random number



Random # generation w/ arbitrary dist.

Analytically integrable &
analytically invertible



directory convertible

otherwise

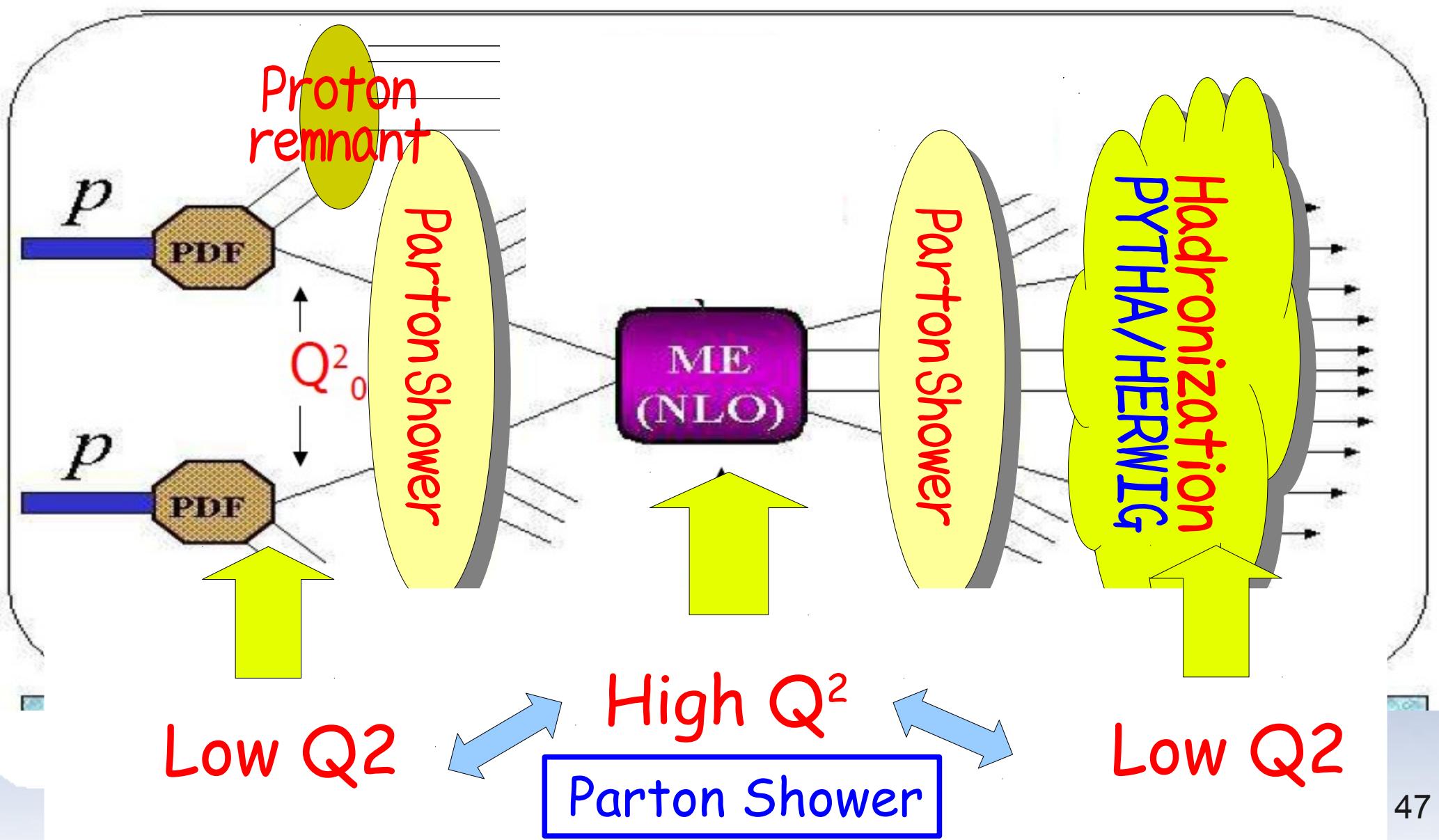


prepare numerical table

Exercise#1

EG Structure: Soft Part(1) Parton Shower

Overview of Event Generator



Parton Shower

DGLAP Equation

$$\frac{dD(x, Q^2)}{d\ln Q^2} = \frac{\alpha}{2\pi} \int_x^1 \frac{dy}{y} P_+(x/y) D(y, Q^2)$$

Splitting function \leftarrow pQCD



$$D(x, Q^2) = \Pi(Q^2, Q_s^2) D(x, Q_s^2) + \frac{\alpha}{2\pi} \int_{Q_s^2}^{Q^2} \frac{dK^2}{K^2} \Pi(Q^2, K^2) \int_x^{1-\epsilon} \frac{dy}{y} P(y) D(x/y, K^2)$$

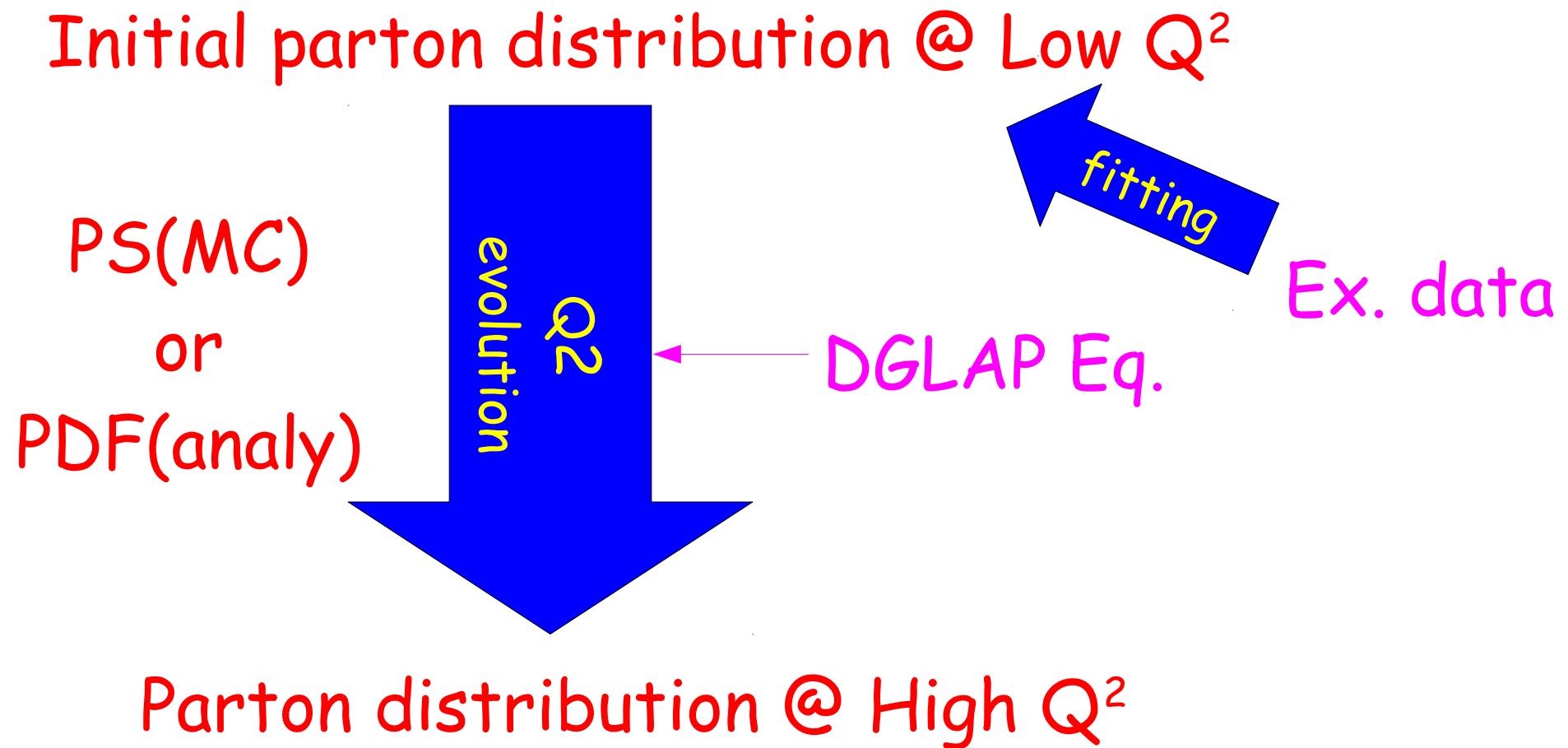
$$\Pi(Q^2, Q'^2) = \exp\left(-\frac{\alpha}{2\pi} \int_{Q^2}^{Q'^2} \frac{dK^2}{K^2} \int_0^{1-\epsilon} dx P(x)\right)$$

Sudakov Factor



non-branch probability

PS \leftrightarrow PDF



PS ↔ PDF

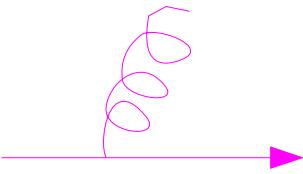
- PDF:
 - Initial distribution: data fitting
 - Q^2 evolution: Analytic solution of DGLAP Eq.
 - No kinematical information
- PS:
 - Initial distribution: from PDF
 - Q^2 evolution: MC method to solve DGLAP Eq.
 - generate Pt distribution

Parton Shower

DGLAP Eq. (Integral-differential equation)

$$\frac{d D(x, Q^2)}{d \log(Q^2)} = \frac{\alpha}{2\pi} \int_0^x \frac{dy}{y} P(x/y) D(y, Q^2)$$

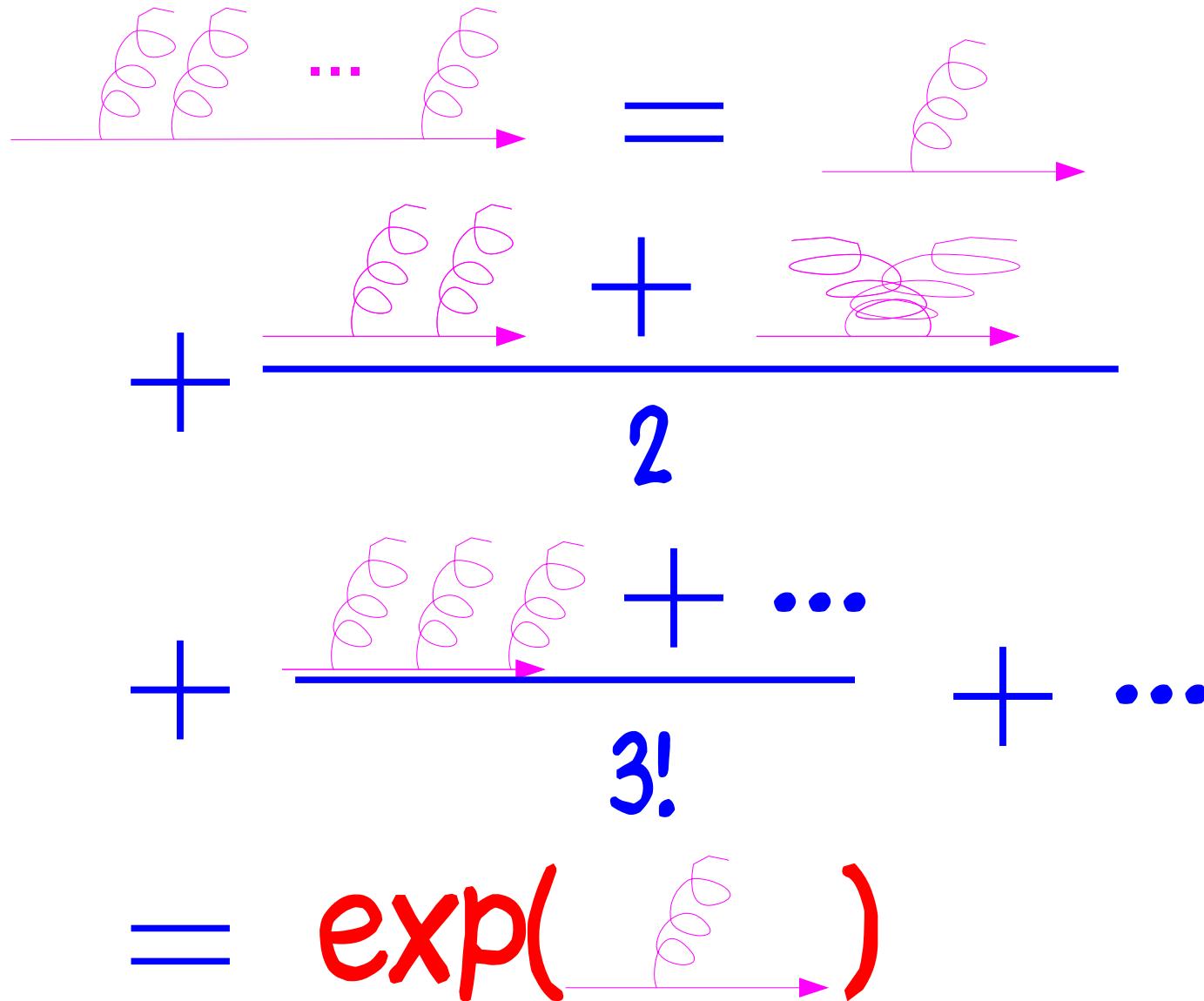
$$q \rightarrow q+g : P(x) = \left(\frac{1+x^2}{1-x} \right) \rightarrow \theta(1-\epsilon-x) - \delta(1-x) \int_0^{1-\epsilon} dy P(y)$$



$$\frac{d D}{d K^2} K^2 + \frac{\alpha}{2\pi} c(\epsilon) D = \frac{\alpha}{2\pi} P \otimes D$$

$$c(\epsilon) = \int_0^{1-\epsilon} dz p(z)$$

Parton Shower(exponensation)



Parton Shower

homogeneous Eq.

$$\frac{d \Pi}{d K^2} K^2 + \frac{\alpha}{2\pi} c \Pi = 0$$

Inhomogeneous Eq.

$$\frac{d D}{d K^2} K^2 + \frac{\alpha}{2\pi} c D = \frac{\alpha}{2\pi} P \otimes D$$

$$D(x, K^2) = \hat{D}(x, K^2) \Pi(K^2, Q^2)$$

Sudakov form factor
↓
non-branching prob.



$$\frac{d \hat{D}}{d K^2} K^2 = \frac{\alpha}{2\pi} P \otimes \hat{D}$$

$$\hat{D} = \frac{\alpha}{2\pi} \int P \otimes \hat{D} \frac{d K^2}{K^2} + c$$

Parton Shower

Integral equation

$$D(x, Q^2) = \Pi(Q^2, Q_s^2) D(x, Q_s^2) + \frac{\alpha}{2\pi} P \otimes D(x, K^2) \frac{dK^2}{K^2}$$
$$\Pi(Q_1^2, Q_0^2) = -\exp\left(\int_{Q_0^2}^{Q_1^2} \frac{\alpha}{2\pi} \frac{dK^2}{K^2} \int_0^{1-\epsilon} dx P(x)\right)$$

Method of successive substitution

Equation $\phi(x) = f(x) + \lambda \int_a^x K(x, y) \phi(y) dy$

0th approx.

$$\phi_0(x) = f(x)$$

1st

$$\phi_1(x) = f(x) + \lambda \int_a^x K(x, y) \phi_0(y) dy$$

2nd

$$\phi_2(x) = f(x) + \lambda \int_a^x K(x, y) \phi_1(y) dy$$

Parton Shower

Method of successive substitution

No emission

$$\phi_2 = f(x) + \lambda \int_a^x K(x, y) \phi(y) dy$$

$$+ \lambda^2 \int_{y_1}^x dy \int_a^{y_1} dy_1 K(x, y_1) \phi(y_1) K(y_1, y) \phi(y)$$

1 parton emission

2 parton emission

Parton Shower

Method of successive substitution

$$\phi(x) = f(x) + \lambda \int_a^x K(x, y) \phi(y) dy$$

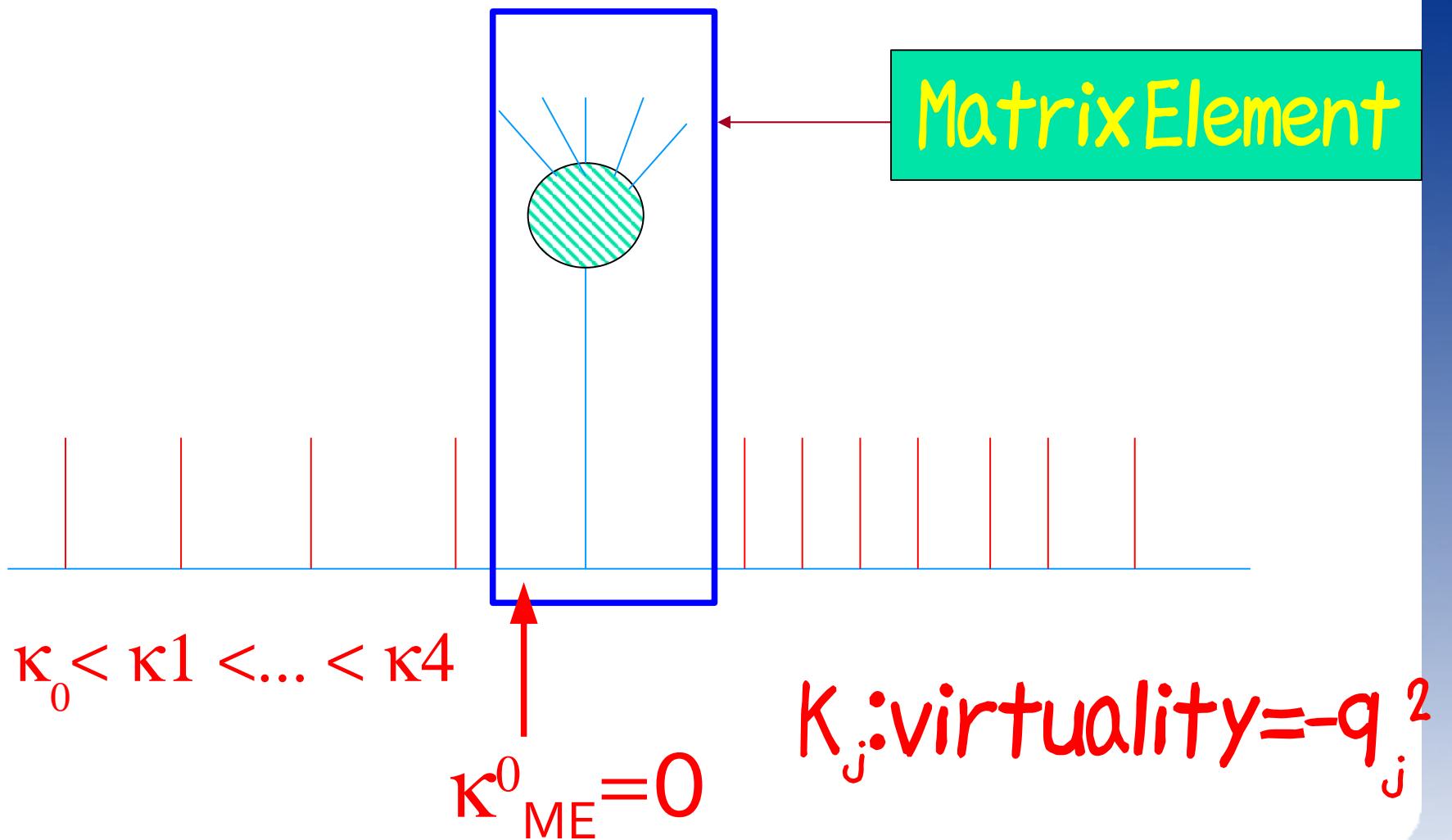
$$\begin{aligned}\phi_2 &= f(x) + \lambda \int_a^x K(x, y) \phi(y) dy \\ &\quad + \lambda^2 \int_{y_1}^x dy \int_a^{y_1} dy_1 K(x, y_1) \phi(y_1) K(y_1, y) \phi(y)\end{aligned}$$

$$\phi_n(x) = f(x) + \sum_{l=1}^n \lambda^l \int_a^x K_l(x, y) f(y) dy$$

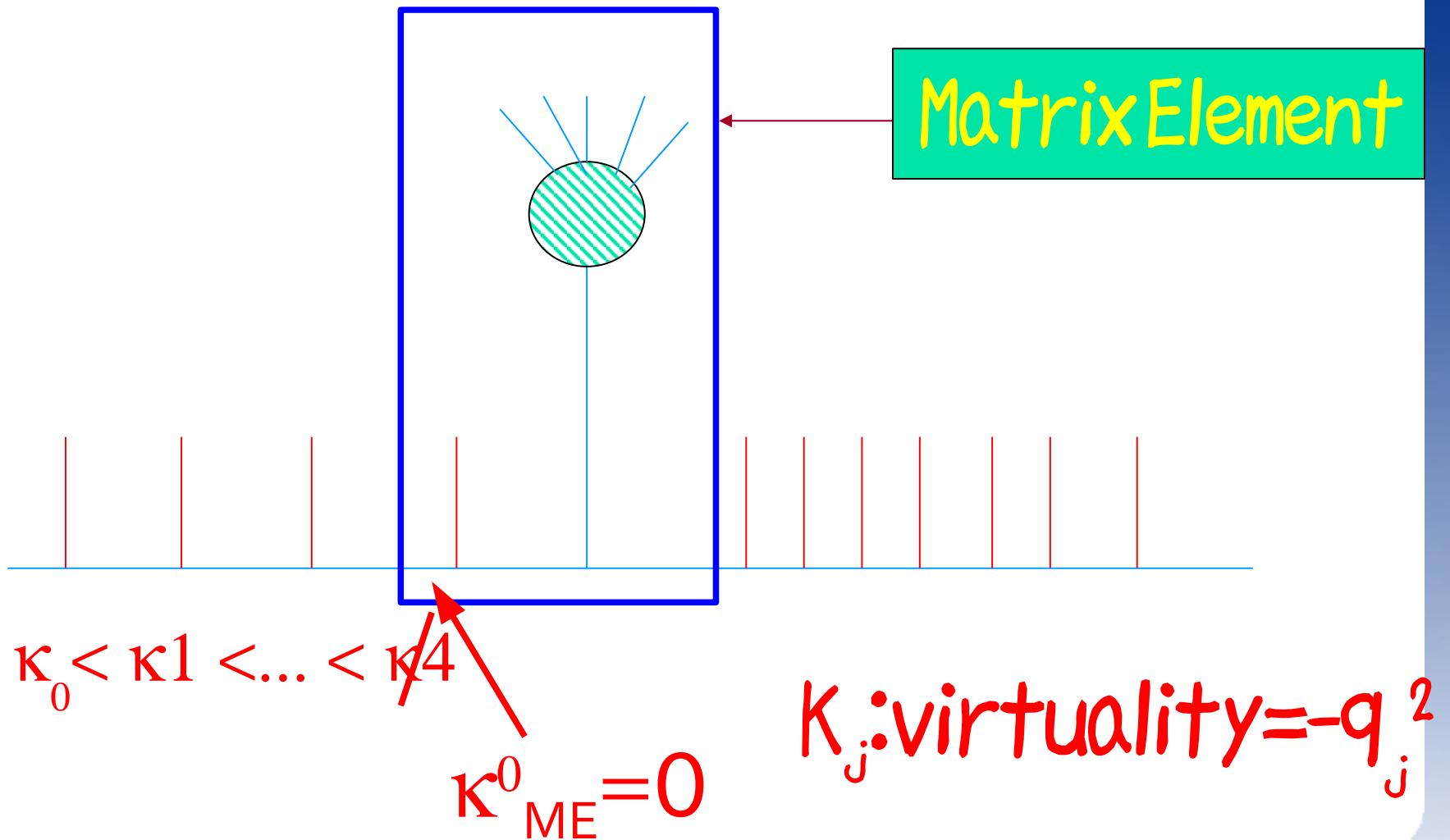
$$K_l(x, y) = \int \cdots \int dy_1 \cdots dy K(x, y_1) \cdots K(y_{l-1}, y)$$

ME/PS Matching

ME/PS Matching



ME/PS Matching

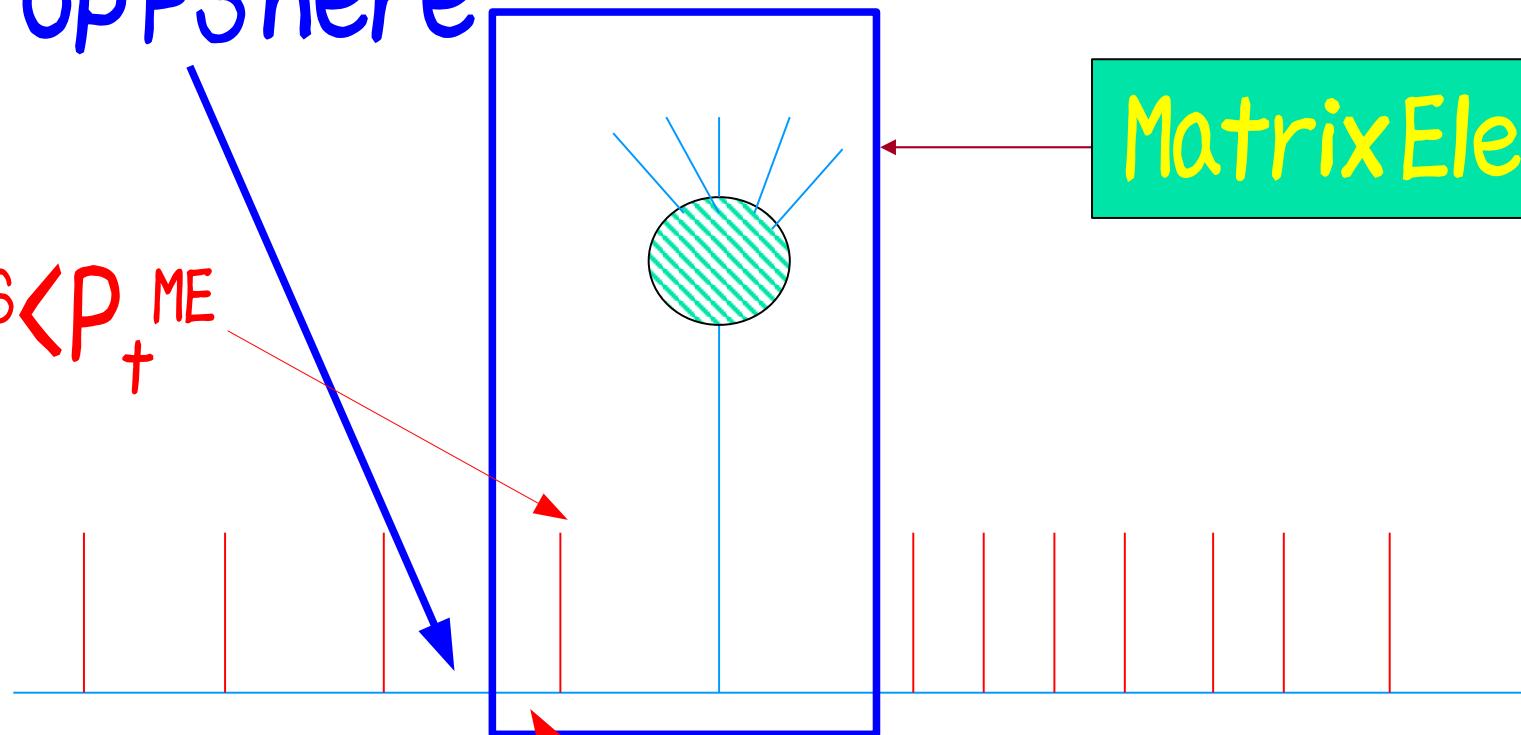


ME/PS Matching (phase space separation)

Stop PS here

$$P_+^{\text{PS}} < P_+^{\text{ME}}$$

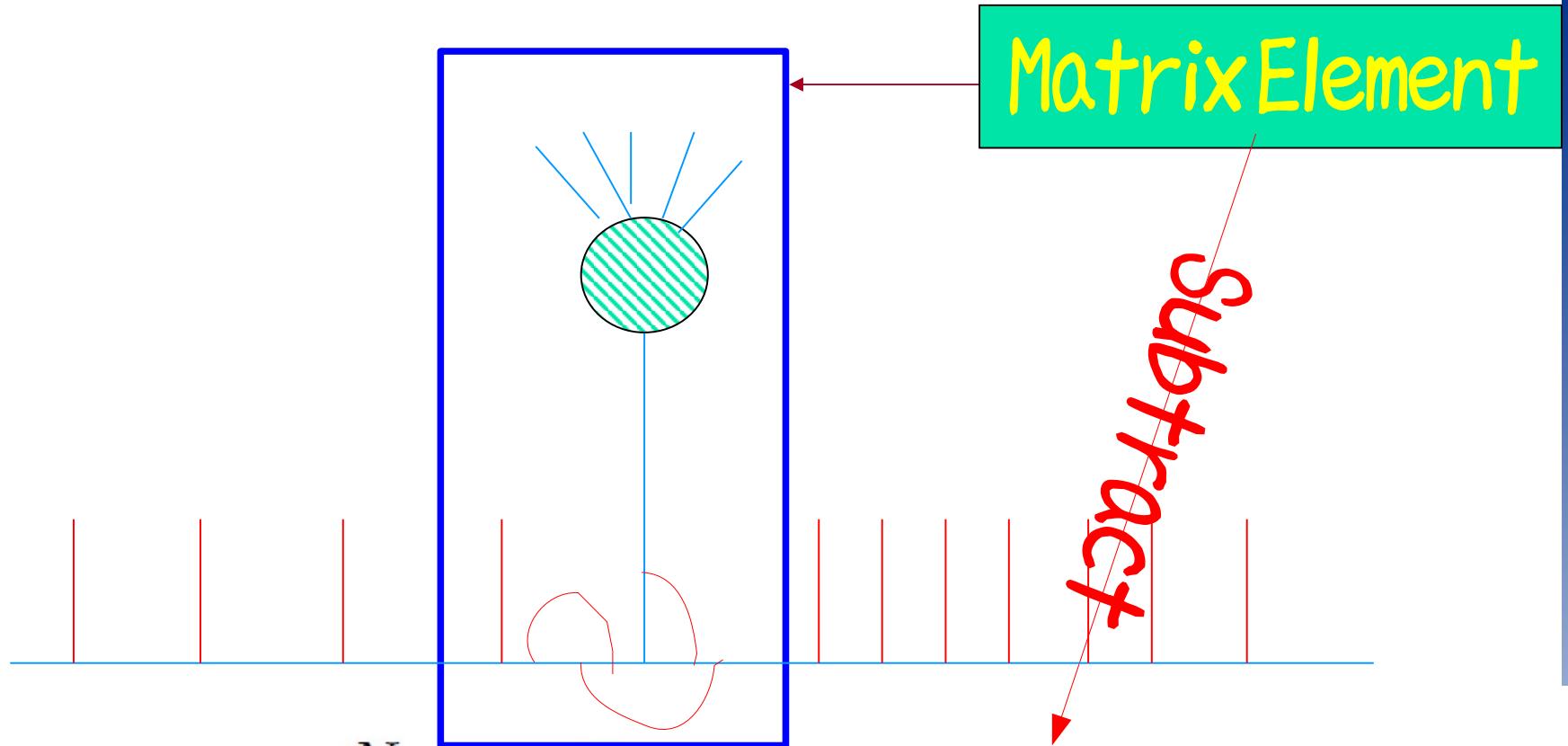
Matrix Element



$$\kappa_0 < \kappa_1 < \dots < \cancel{\kappa_4} \quad \kappa_{\text{ME}}^0 = 0$$

$$K_j \cdot \text{virtuality} = -q_j^2$$

ME/PS Matching(Subtraction)

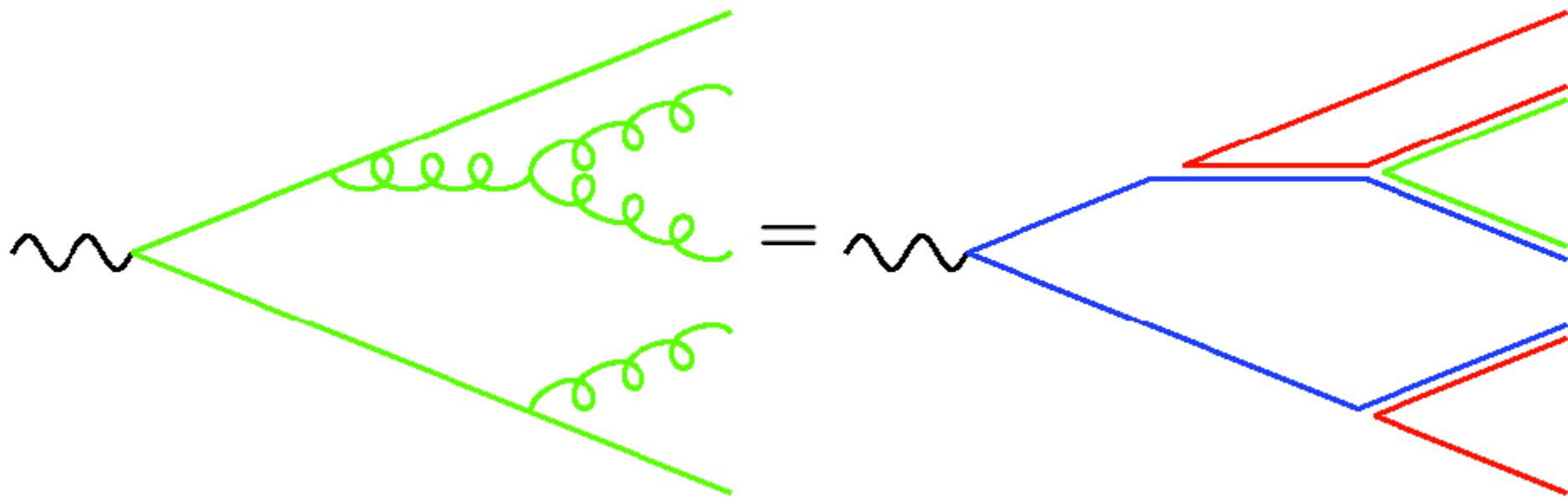


$$\left| \mathcal{M}_N^{(4)} \left(q \rightarrow \sum_{i=1}^N q_i \right) \right|^2 \frac{16\pi}{s\mu^{2\varepsilon_{IR}}} f_c \frac{\alpha_s}{2\pi} P(x) \frac{1}{k_T^2} \left(\frac{1-x}{x} \right)$$

EG Structure: Soft Part(2) Hadronization

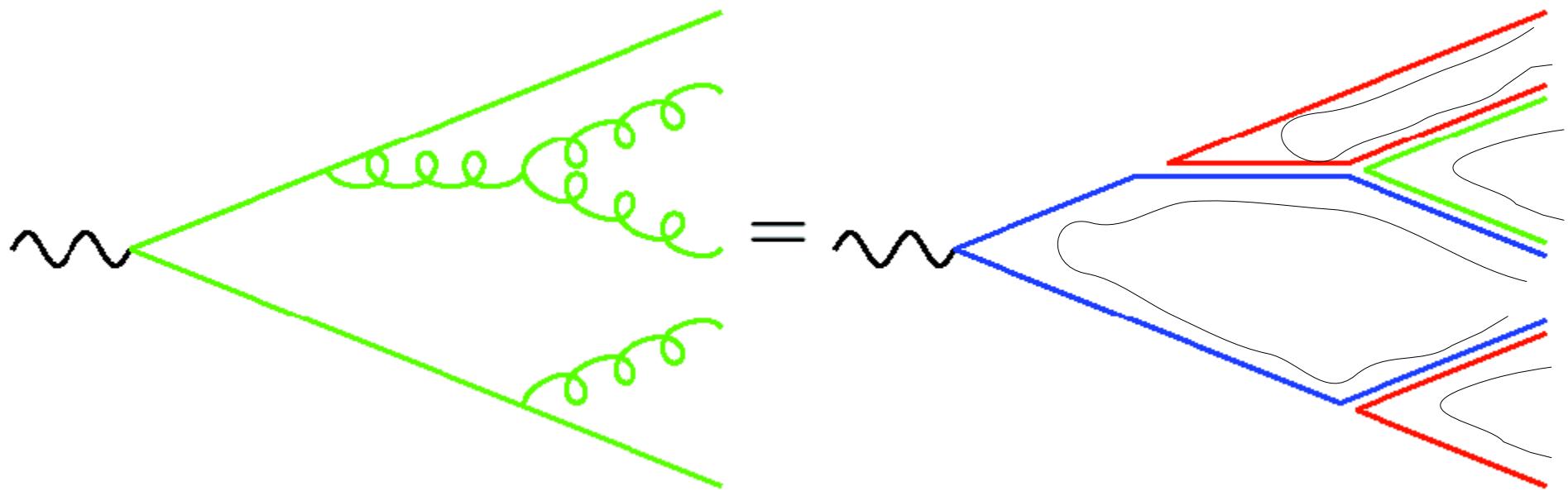
Hadronization (String Model)

quark, gluon \rightarrow color line



Hadronization (String Model)

quark, gluon \rightarrow color line



Hadronization (String Model)

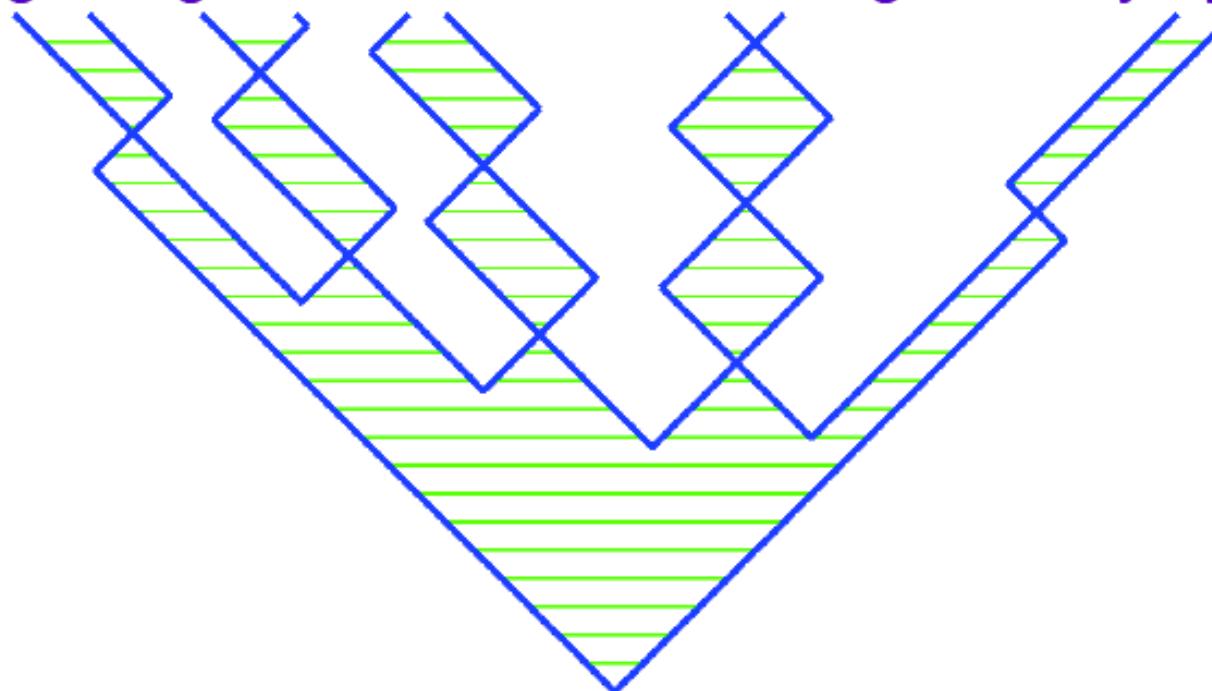
Start by ignoring gluon radiation:

e^+e^- annihilation = pointlike source of $q\bar{q}$ pairs

Intense chromomagnetic field within string $\rightarrow q\bar{q}$ pairs created by tunnelling. Analogy with QED:

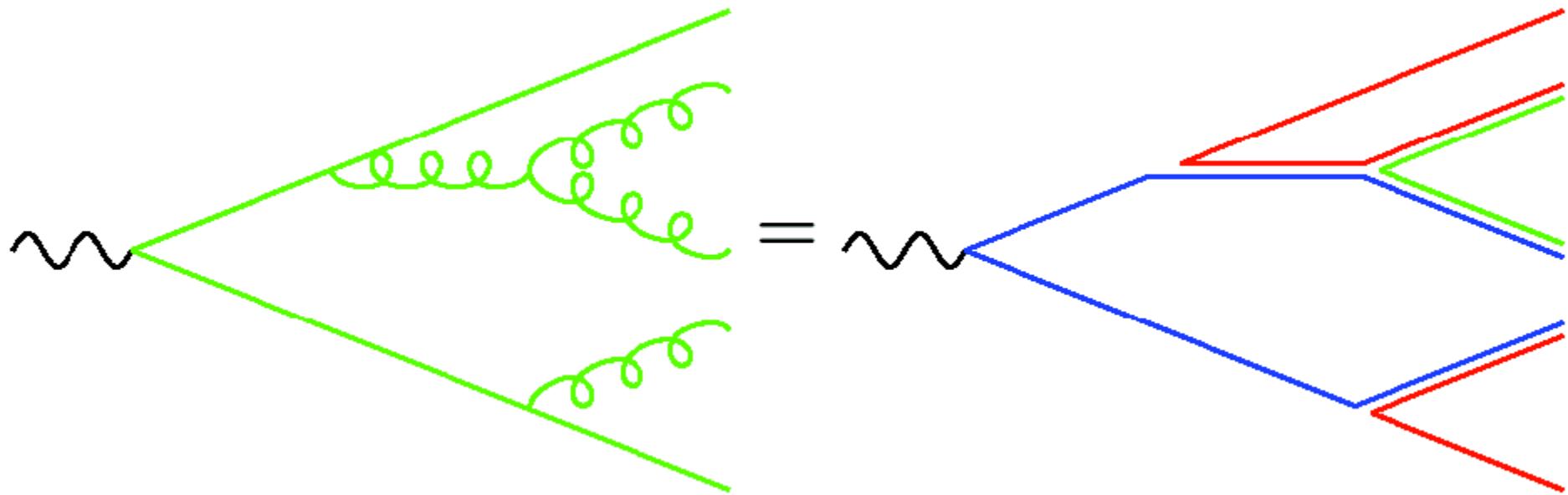
$$\frac{d(\text{Probability})}{dx \ dt} \propto \exp(-\pi m_q^2 / \kappa)$$

Expanding string breaks into mesons long before yo-yo point.



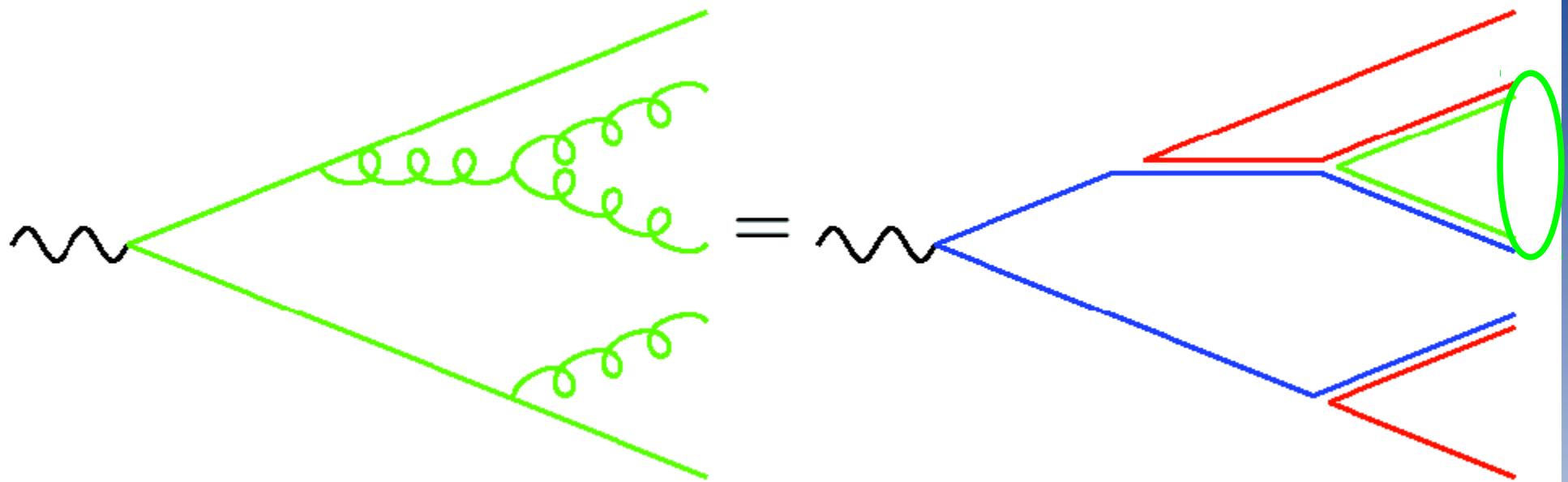
Hadronization (Cluster Model)

quark, gluon \rightarrow color line



Hadronization (Cluster Model)

quark, gluon \rightarrow color line

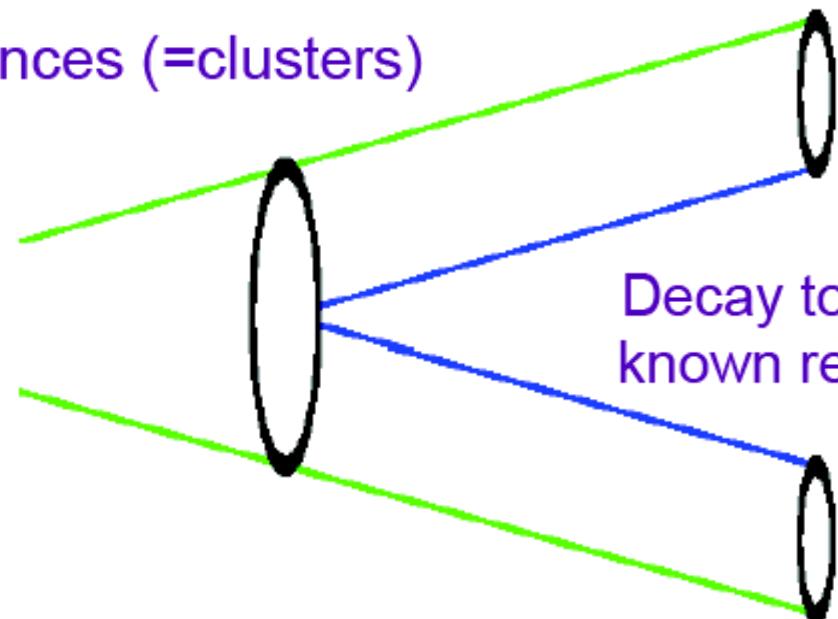


Hadronization (Cluster Model)

Although cluster mass spectrum peaked at small m , broad tail at high m .

“Small fraction of clusters too heavy for isotropic two-body decay to be a good approximation” → Longitudinal cluster fission:

mesonic resonances (=clusters)



Decay to lighter well-known resonances and stable hadrons.

Rather string-like.

Fission threshold becomes crucial parameter.

~15% of primary clusters get split but ~50% of hadrons come from them.

EG Structure: Summary & Home Work

Summary

- What is a "must" for EG at LHC-era?
 - $\sigma_{\text{tree}} \times (\text{K-factor}) \rightarrow \text{NLO ME}$
 - ME/PS Matching
 - Fast & Stable Numerical Calculation
 - High Generation Efficiency
 - Fully Exclusive
 - Non-p Effects if necessary
 - LHA-Compliant Interface

Exercise

- Exe. 1: Generate random numbers according to following distributions

$$(1) \quad f(x) = \frac{1}{1 - ax}, \quad a = 0.9, -1 < x < 1$$

$$(2) \quad f(x) = \frac{1}{x}, \quad \epsilon < x < 1 + \epsilon, \epsilon = 0.001$$

$$(3) \quad f(x) = \frac{1}{(x^2 - m^2)^2 + m^2 \Gamma^2}, \quad m = 90, \Gamma = 2, m - 5\Gamma < x < m + 5\Gamma$$

- Exe. 2: Solve following integral equation by a MC method

$$\phi(x) = f(x) + \lambda \int_a^x K(x, y) \phi(y) dy$$

$$f(x) = x(1 - x), \quad K(x, y) = \exp(-(x - y)), \lambda = 0.5$$