

MC Generation (II)Exercise

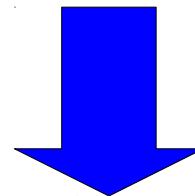
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Outline

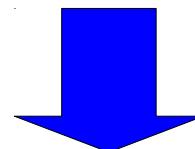
Tody's exercise

- Topics/Exercises
 - Generation of random numbers w/ arbitrary distribution



Event Generation

- Solve an integral equation by the MC method



Parton Shower

Exercise

- Exe. 1: Generate random numbers according to following distributions

$$(1) \quad f(x) = \frac{1}{1 - ax}, \quad a = 0.9, -1 < x < 1$$

$$(2) \quad f(x) = \frac{1}{x}, \quad \epsilon < x < 1 + \epsilon, \epsilon = 0.001$$

$$(3) \quad f(x) = \frac{1}{(x^2 - m^2)^2 + m^2 \Gamma^2}, \quad m = 90, \Gamma = 2, m - 5\Gamma < x < m + 5\Gamma$$

- Exe. 2: Solve following integral equation by a MC method

$$\phi(x) = f(x) + \lambda \int_a^x K(x, y) \phi(y) dy$$

$$f(x) = x(1 - x), \quad K(x, y) = \exp(-(x - y)), \lambda = 0.5$$

Random # generation w/ arbitrary dist.

Step 1: Prepare Probability density table

$f(x)$: Distribution function
 $P(x)$: Probability density

If $f(x)$ is integrable:

$$P(\xi) = \frac{\int_a^{\xi} f(x) dx}{A}, \quad A = \int_a^b f(x) dx$$

Random # generation w/ arbitrary dist.

Step 1: Prepare Probability density table

$f(x)$: Distribution function
 $P(x)$: Probability density

$$P(\xi) = \frac{\int_a^{\xi} f(x) dx}{A}, \quad A = \int_a^b f(x) dx$$

If $f(x)$ is not integrable:



$$P_i = \frac{\sum_{k=1}^i f(x_k)}{A}, \quad A = \sum_{i=1}^{N_{bin}} f(x_i)$$

Random # generation w/ arbitrary dist.

Step 2: Take inverse of PD

ξ_j : Random #(0~1)

- Generate one random number: ξ_j
- Find $P(j/N_{\text{bin}}) < \xi_j < P((j+1)/N_{\text{bin}})$
 $(P_j < \xi_j < P_{j+1})$
- $X_j = \xi_j / N_{\text{bin}}$

Repeat up to you need

Step 3: Draw a plot

Random # generation w/ arbitrary dist.

Step 2: Take inverse of PD

ξ_j : Random #(0~1)

If $f(x)$ is integrable and invertible:

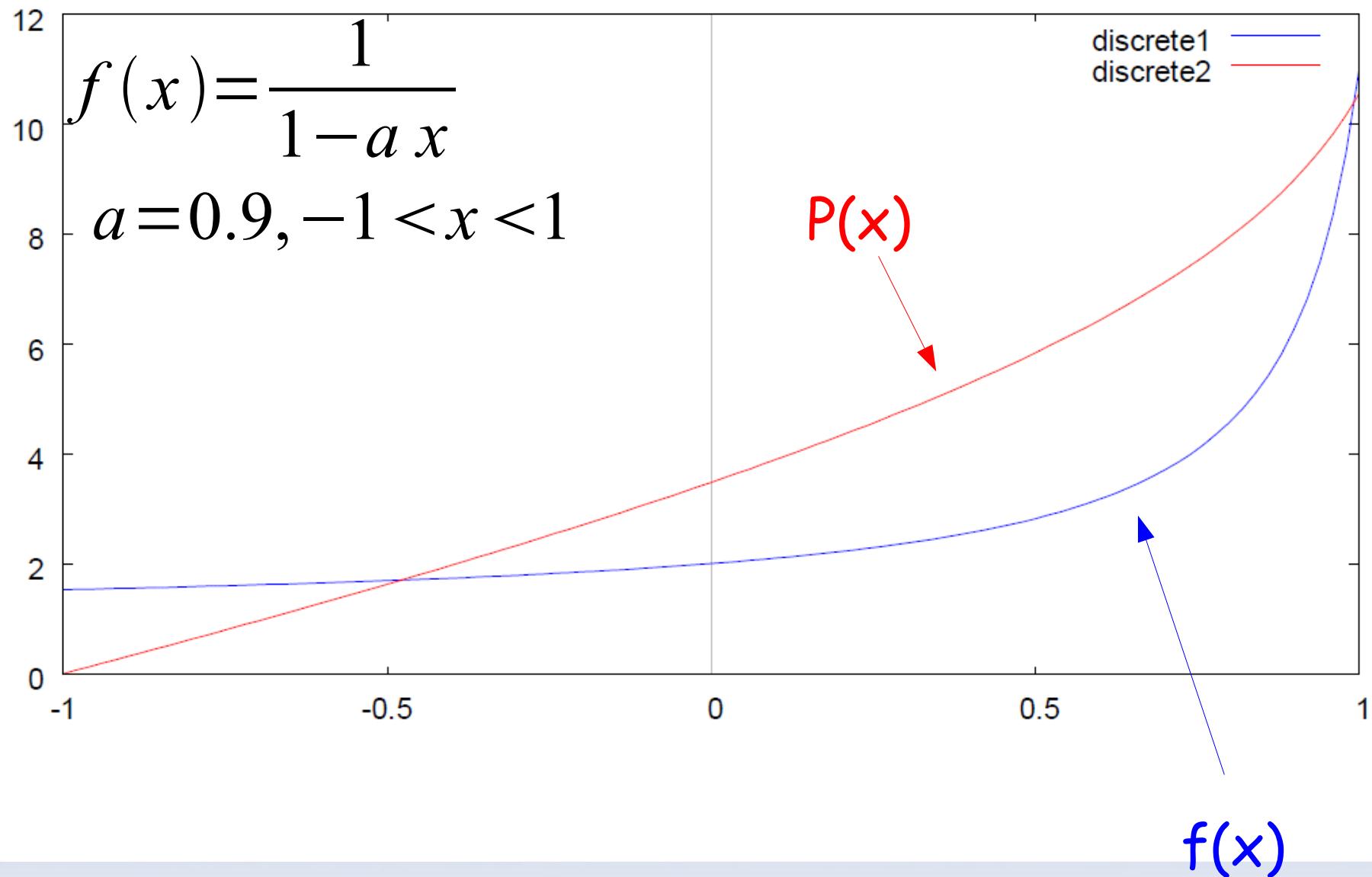
- Generate one random number: ξ_j

$$\bullet x_j = P^{-1}(\xi_j)$$

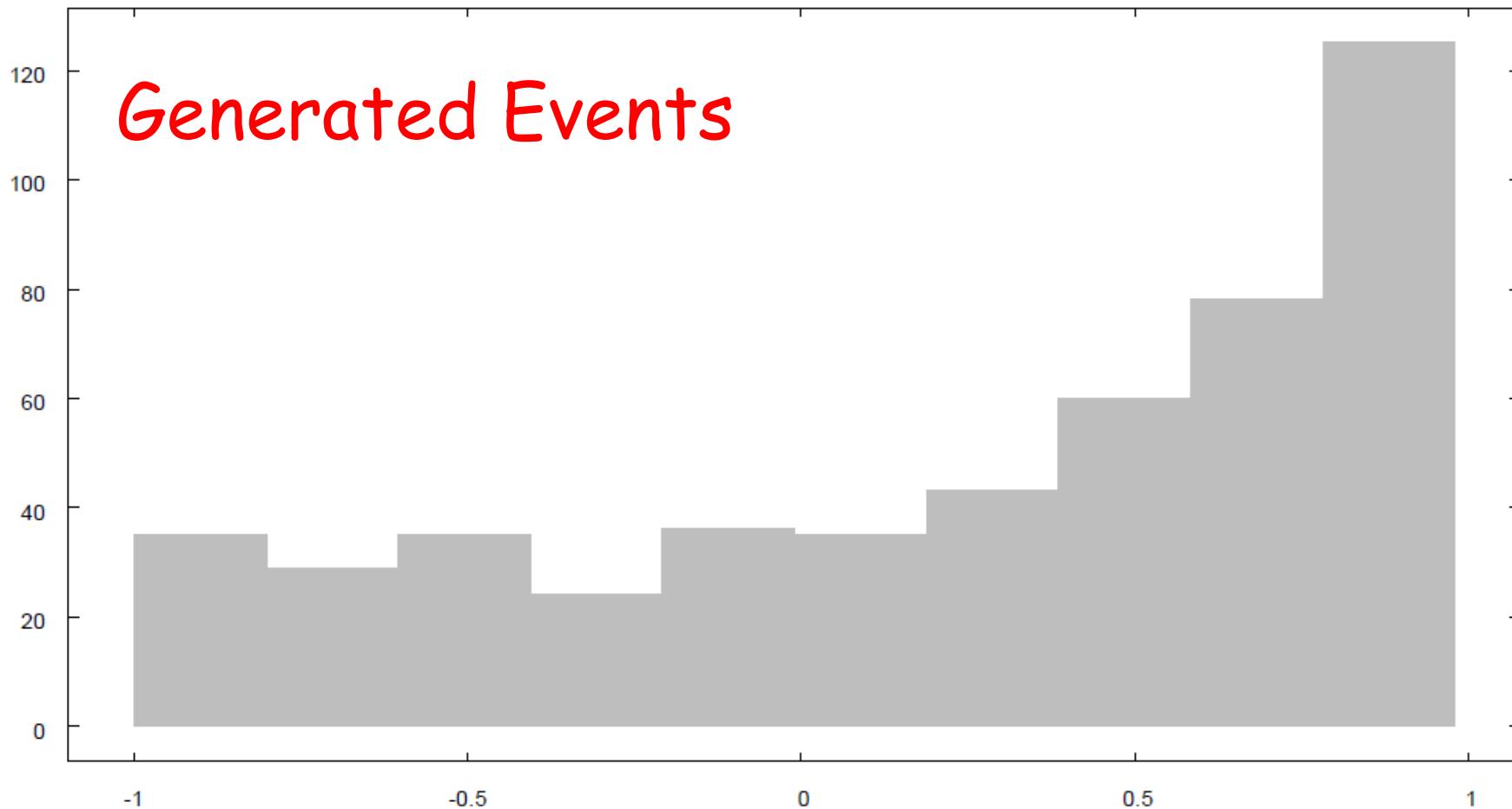
$$P(\xi) = \frac{\int_a^{\xi} f(x) dx}{\int_a^b f(x) dx}$$

Repeat up to you need

Random # generation w/ arbitrary dist.



Random # generation w/ arbitrary dist.



Solve integral equation (PS)

Integral Eq.

$$\phi(x) = f(x) + \lambda \int_a^x K(x, y) \phi(y) dy$$

$$f(x) = x(1-x), \quad K(x, y) = \exp(-(x-y)), \quad \lambda = 0.5$$

Step 1 : Prepare two probability density functions

$$P_f(\xi) = \frac{\int_0^\xi f(x) dx}{A}, \quad A = \int_0^1 f(x) dx$$

$$P_K(\xi, y) = \frac{\int_y^\xi K(x, y) dx}{A}, \quad A = \int_y^1 K(x, y) dx$$

Solve integral equation (PS)

Step 2 : Generate an initial value (x) according to $f(x)$

- Generate one random number : ξ_j
- Find $P_f(i / N_{bin}) < \xi_j < P_f((i+1) / N_{bin})$
- $x_i = i / N_{bin}$

Step 3 : Decide "Branch" happens or not

- Generate one random number: η_j
- Compare η_j with $K(x, y)$:

If $\eta_j < \lambda \int_{x_i}^1 K(x, x_i) dx$, branch happens.

Solve integral equation (PS)

Step 4-1 : If no-branch,

- Store x_i and goto 'Step 2'.

Step 4-2 : If branch,

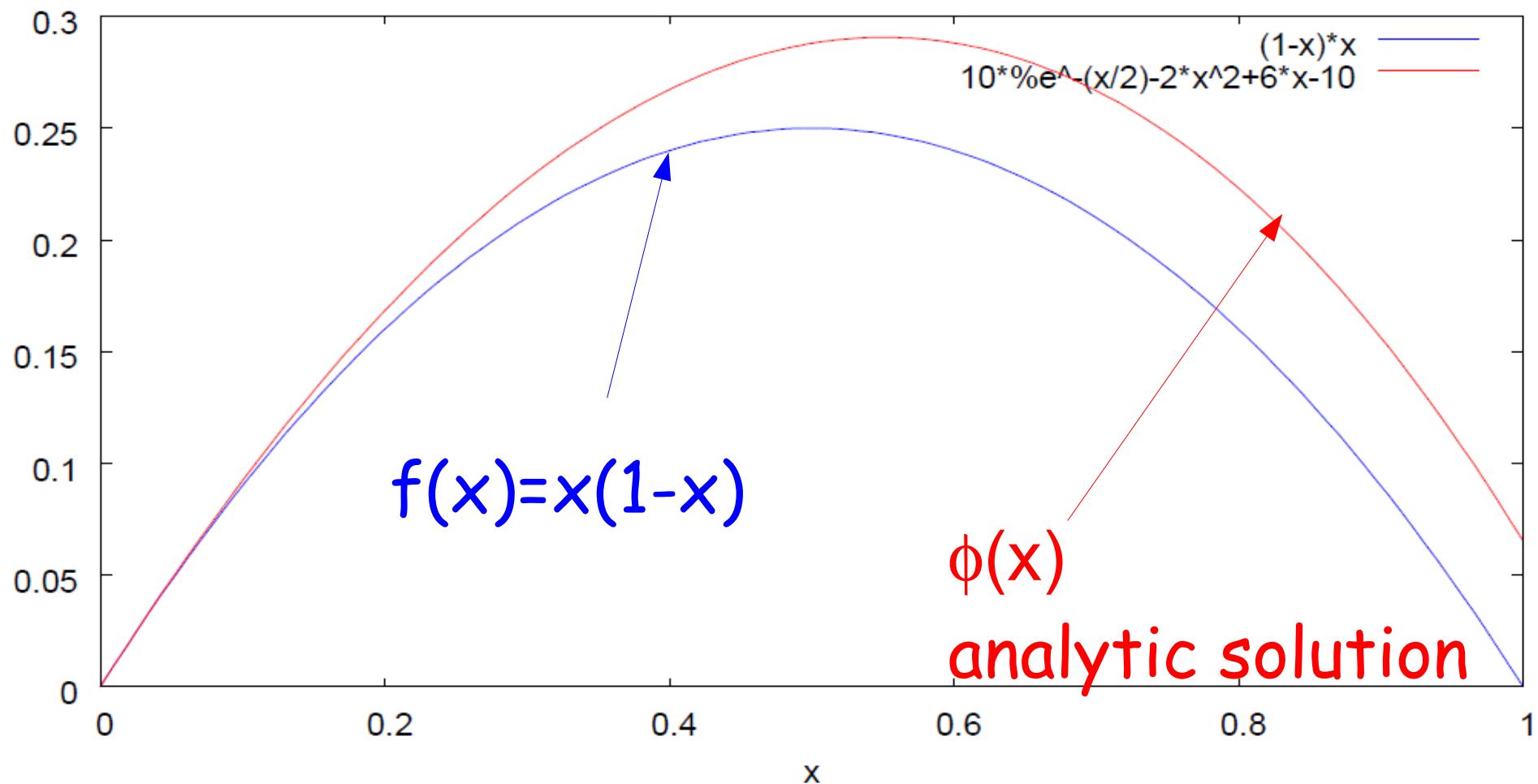
Generate a new value (x) according to $K(x,y)$

- Generate one random number: ξ
- Find $P_{K,y}(i/N_{\text{bin}}, x_i) < \xi_j < P_{K,y}((i+1)/N_{\text{bin}}, x_i)$
- $x_{i+1} = i/N_{\text{bin}}$
- Goto 'Step 3'.

Solve integral equation (PS)

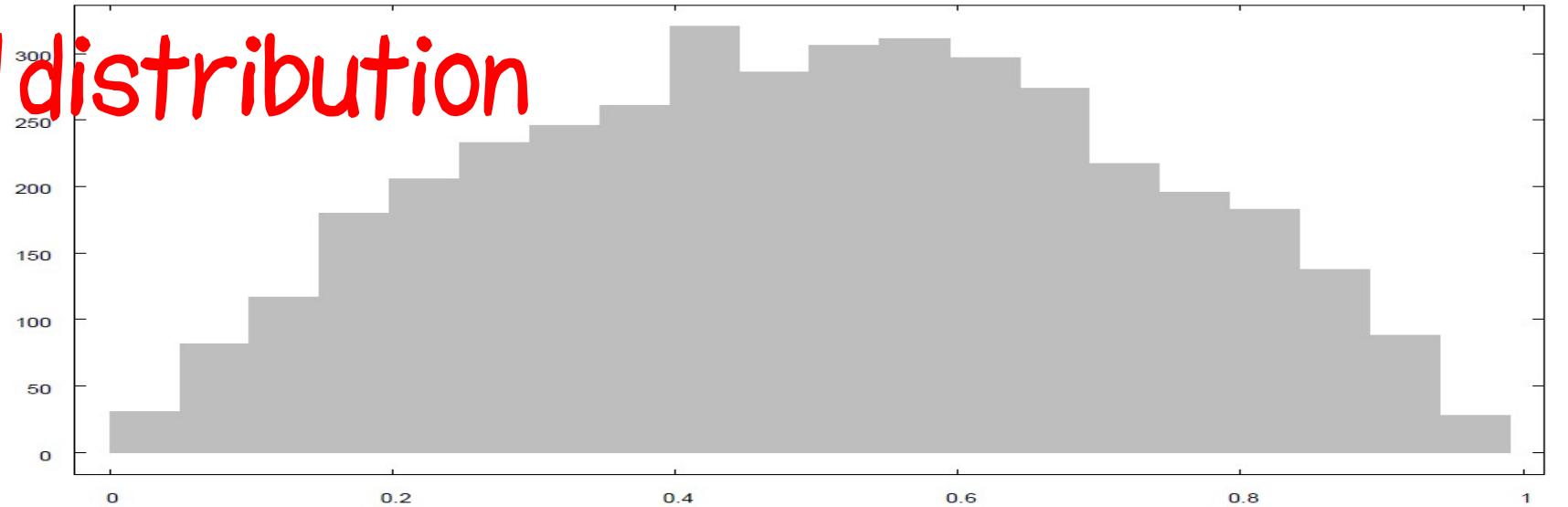
Step 5: Repeat 'Step 2 ~ Stop 4' while you want.

Step 6: Draw a plot



Solve integral equation (PS)

Initial distribution



After evolution

