Exclusive processes beyond leading twist.
Some selected examples: ρ_T-meson production; chiral-odd GPDs

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I. V. Anikin, D .Yu. Ivanov, B. Pire, L. Szymanowski, S. W.
Phys. Lett. B **682** (2010) 413-418; Nucl. Phys. B **828** (2010) 1-68 (this talk)

I. V. Anikin, A. Besse, D .Yu. Ivanov, B. Pire, L. Szymanowski, S. W.
Phys. Rev. D **84** (2011) 054004 (see next talk)

Introduction
Why going beyond leading twist?

- Some processes may require an inclusion of higher twist corrections for finite values of Q^2. This might be the case of DVCS
- This might be a formal need: see for example the QED gauge invariance of DVCS amplitude, violated by terms $\sim \Delta_T$ ($\Delta =$ transfered momentum) 3-body t-channel exchange solves this problem at twist 3
  Anikin, Pire, Teryaev '00
- This might be an experimental requirement:
  e.g.: ρ_T-electroproduction which is copiously produced, while vanishing at twist 2!

Our aim is to construct a consistent and efficient tool to deal with subleading twist corrections

Introduction

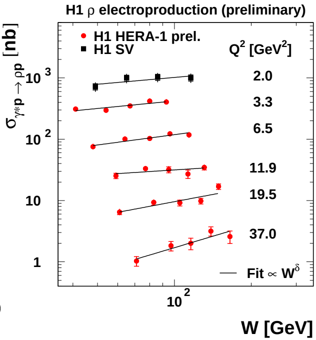
Exclusive ρ-production

Our studies attempt to describe exclusive processes involving the production of ρ-mesons in diffraction-type experiment. We choose $t = t_{min}$ for simplicity.

- $\gamma^*(q) + \gamma^*(q') \to \rho_T(p_1) + \rho(p_2)$ process in $e^+ e^- \to e^+ e^- \rho_T(p_1) + \rho(p_2)$ with double tagged lepton at ILC
- $\gamma^*(q) + P \to \rho_T(p_1) + P$ at HERA

This process was studied by H1 and ZEUS

- the total cross-section strongly decreases with Q^2
- dramatic increase with $W^2 = s_{\gamma^* P}$ (transition from soft to hard regime governed by Q^2)



(from X. Janssen (H1), DIS 2008)

Introduction
Exclusive ρ-production

Polarization effects in $\gamma^* P \to \rho\, P$ at HERA

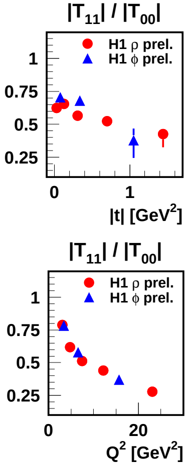
- one can experimentally measure all spin density matrix elements
- at $t = t_{min}$ one can experimentally distinguish

$$\left\{ \begin{array}{lll} \gamma^*_L \to \rho_L : & \text{dominates} & (\text{twist 2 dominance}) \\ \gamma^*_T \to \rho_T : & \text{sizable} & (\text{twist 3}) \end{array} \right.$$

- S-channel helicity conservation:

$$\left\{ \begin{array}{ll} \gamma^*_L \to \rho_L & (\equiv T_{00}) \\ \gamma^*_T \to \rho_T\,, & \end{array} \right.$$

Dominate with respect to all other transitions.
Experimentally, $\gamma^*_T \to \rho_T$ is dominated by $\gamma^*_{T(-)} \to \rho_{T(-)}$ and $\gamma^*_{T(+)} \to \rho_{T(+)}$ $(\equiv T_{11})$



(from X. Janssen (H1), DIS 2008)

Introduction

Exclusive ρ-production

The processes with vector particle such as $\rho-$meson probe deeper into the fine features of QCD.
It deserves theoretical developpement to describe HERA data in its special kinematical range:

- large $s_{\gamma^* P}$ $\Rightarrow$ small-x effects expected, within k_t-factorization
- large Q^2 $\Rightarrow$ hard scale $\Rightarrow$ perturbative approach and collinear factorization $\Rightarrow$ the ρ can be described through its chiral even Distribution Amplitudes

$$\begin{cases} \rho_L & \text{twist 2} \\ \rho_T & \text{twist 3} \end{cases}$$

The main ingredient is the $\gamma^* \to \rho$ impact factor

- For ρ_T, special care is needed: a pure 2-body description would violate gauge invariance.
- We show that:
  - Including in a consistent way all twist 3 contributions, i.e. 2-body and 3-body correlators, gives a gauge invariant impact factor
  - Our treatment is free of end-point singularities and does not violates the QCD factorization

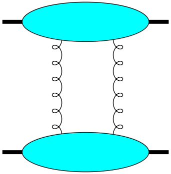
Impact factor for exclusive processes

Theoretical motivations

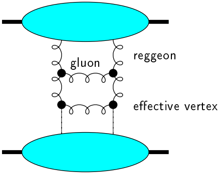
QCD in perturbative Regge limit

- In this limit, the dynamics is dominated by gluons (dominance of spin 1 exchange in t channel)
- BFKL (and extensions: NLL, saturations effects, ...) is expected to dominate with respect to Born order at large relative rapidity.

Born order:



BFKL ladder:



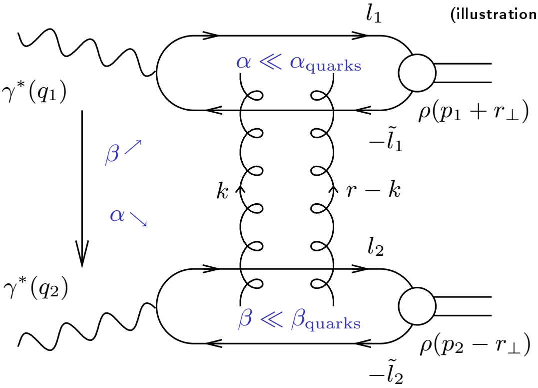
Impact factor for exclusive processes

k_T factorization

$\gamma^* \, \gamma^* \to \rho \, \rho$ as an example

- Use Sudakov decomposition $k = \alpha \, p_1 + \beta \, p_2 + k_\perp$ $(p_1^2 = p_2^2 = 0, \; 2 p_1 \cdot p_2 = s)$
- write $\quad d^4 k = \frac{s}{2} \, d\alpha \, d\beta \, d^2 k_\perp$
- $t-$channel gluons with non-sense polarizations $(\epsilon_{NS}^{up} = \frac{2}{s} \, p_2, \; \epsilon_{NS}^{down} = \frac{2}{s} \, p_1)$ dominate at large s

(illustration for 2-body case)



$\Rightarrow$ set $\alpha = 0$ and $\int d\beta$

$\int d^2 k_\perp$

$\Rightarrow$ set $\beta = 0$ and $\int d\alpha$

Impact factor for exclusive processes
k_T factorization

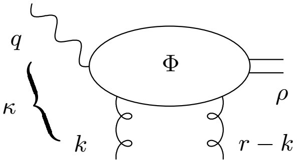
impact representation $\underline{k}$ = Eucl. $\leftrightarrow$ $k_\perp$ = Mink.

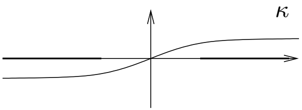
$$\mathcal{M} = is \int \frac{d^2\,\underline{k}}{(2\pi)^2 \underline{k}^2\,(\underline{r}-\underline{k})^2} \Phi^{\gamma^*(q_1)\to\rho(p_1^\rho)}(\underline{k}, \underline{r}-\underline{k})\; \Phi^{\gamma^*(q_2)\to\rho(p_2^\rho)}(-\underline{k}, -\underline{r}+\underline{k})$$

The $\gamma^*_{L,T}(q) g(k_1) \to \rho_{L,T}\, g(k_2)$ impact factor is normalized as

$$\Phi^{\gamma^*\to\rho}(\underline{k}^2) = e^{\gamma^*\mu} \frac{1}{2\,s} \int \frac{d\kappa}{2\pi} \,\mathrm{Disc}_\kappa\, \mathcal{S}_\mu^{\gamma^*\,g\to\rho\,g}(\underline{k}^2),$$

with $\kappa = (q+k)^2 = \beta\, s - Q^2 - \underline{k}^2$



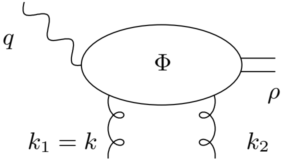


Impact factor for exclusive processes
Gauge invariance within subleading twists

Gauge invariance

- QCD gauge invariance (probes are colorless)
  $\Rightarrow$ impact factor should vanish when $\underline{k} \to 0$ or $\underline{r} - \underline{k} \to 0$

- In the following we will restrict ourselve to the case $t = t_{min}$, i.e. to $\underline{r} = 0$



$$k_1 = \frac{\kappa + Q^2 + \underline{k}^2}{s} p_2 + k_\perp$$

$$k_2 = \frac{\kappa + \underline{k}^2}{s} p_2 + k_\perp,$$

$$k_1^2 = k_2^2 = -\underline{k}^2$$

This kinematics takes into account skewedness effects along p_2
$t = t_{min}$ $\Rightarrow$ restriction to the transitions

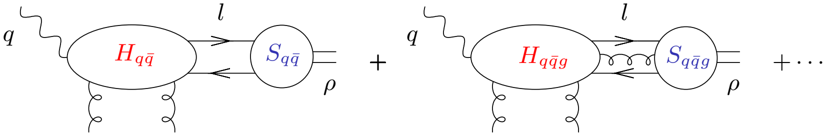
$$\begin{cases} 0 & \rightarrow & 0 & \text{(twist 2)} \\ (+ \text{ or } -) & \rightarrow & (+ \text{ or } -) & \text{(twist 3)} \end{cases}$$

- At twist 3 level (for $\gamma_T^* \to \rho_T$ transition), gauge invariance is a non trivial statement which requires 2 *and* 3 body correlators

Collinear factorization
Light-Cone Collinear approach

- The impact factor can be written as

$$\Phi = \int d^4 l \cdots \, \mathrm{tr}[\underbrace{H(l\cdots)}_{\text{hard part}} \quad \underbrace{S(l\cdots)}_{\text{soft part}}]$$



- At the 2-body level:

$$S_{q\bar{q}}(l) = \int d^4 z \, e^{-il\cdot z} \langle \rho(p) | \psi(0) \, \bar{\psi}(z) | 0 \rangle,$$

- H and S are related by $\int d^4 l$ and by the summation over spinor indices

Collinear factorization

Light-Cone Collinear approach: 2 steps of factorization (2-body case)

1 - Momentum factorization

- Use Sudakov decomposition in the form $(p = p_1,\, n = 2\,p_2/s \Rightarrow p \cdot n = 1)$

$$l_\mu \quad = \quad y\,p_\mu \quad + \quad l^\perp_\mu \quad + \quad (l \cdot p)\,n_\mu, \qquad y = l \cdot n$$

$$\text{scaling:} \qquad 1 \qquad 1/Q \qquad 1/Q^2$$

- Taylor expansion of the hard part $H(\ell)$ along the collinear direction p:

$$H(\ell) = H(yp) \quad + \quad \left.\frac{\partial H(\ell)}{\partial \ell_\alpha}\right|_{\ell=up} (\ell - y\,p)_\alpha + \ldots \quad \text{with} \quad (\ell - y\,p)_\alpha \approx \ell^\perp_\alpha$$

- $l^\perp_\alpha \xrightarrow{Fourier}$ derivative of the soft term: $\int d^4z\; e^{-i\ell\cdot z}\langle\rho(p)|\psi(0)\, i\; \overleftrightarrow{\partial}_{\alpha^\perp}\bar{\psi}(z)|0\rangle$

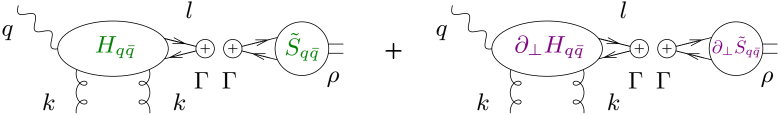
$\Longrightarrow \Phi = \sum$ "modified hard part (purely y-dependent)" $\otimes_y$ "modified soft terms"

Collinear factorization

Light-Cone Collinear approach: 2 steps of factorization (2-body case)

2 - Spinorial (and color) factorization

- Use Fierz decomposition of the Dirac (and color) matrices $\psi(0)\,\bar{\psi}(z)$ and $\psi(0)\, i\, \overleftrightarrow{\partial_\perp}\, \bar{\psi}(z)$:



- Φ has now the simple factorized form:

$$\Phi = \int dy \left\{ \mathrm{tr}\left[H_{q\bar{q}}(y\,p)\,\Gamma\right]\; S^{\Gamma}_{q\bar{q}}(y) + \mathrm{tr}\left[\partial_\perp H_{q\bar{q}}(y\,p)\,\Gamma\right]\; \partial_\perp S^{\Gamma}_{q\bar{q}}(y) \right\}$$

$\Gamma = \gamma^\mu$ and $\gamma^\mu\,\gamma^5$ matrices

$$\begin{aligned} S^{\Gamma}_{q\bar{q}}(y) &= \int \frac{d\lambda}{2\pi}\, e^{-i\lambda y} \langle \rho(p)|\bar{\psi}(\lambda n)\,\Gamma\,\psi(0)|0\rangle \\ \partial_\perp S^{\Gamma}_{q\bar{q}}(y) &= \int \frac{d\lambda}{2\pi}\, e^{-i\lambda y} \langle \rho(p)|\bar{\psi}(\lambda n)\,\Gamma\, i\, \overleftrightarrow{\partial_\perp}\,\psi(0)|0\rangle \end{aligned}$$

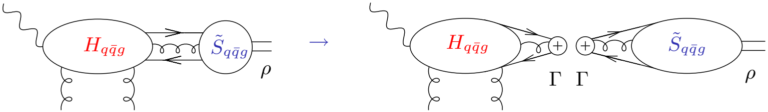
- choose axial gauge condition for gluons, *i.e.* $n \cdot A = 0 \Rightarrow$ no Wilson line

Collinear factorization

Light-Cone Collinear approach: 2 steps of factorization (3-body case)

Factorization of 3-body contributions

- 3-body contributions start at genuine twist 3
  $\Rightarrow$ no need for Taylor expansion
- Momentum factorization goes in the same way as for the 2-body case
- Spinorial (and color) factorization is similar:



Collinear factorization

Parametrization of vacuum–to–rho-meson matrix elements (DAs): 2-body correlators

2-body non-local correlators

ρ_L twist 2

ρ_T kinematical twist 3 (WW)
genuine twist 3
genuine + kinematical twist 3

- vector correlator

$$\langle \rho(p)|\bar{\psi}(z)\gamma_\mu\psi(0)|0\rangle \stackrel{\mathcal{F}}{=} m_\rho\, f_\rho\, \left[\varphi_1(y)\,(e^*\cdot n)p_\mu + \varphi_3(y)\, e^{*T}_\mu\right]$$

- axial correlator

$$\langle \rho(p)|\bar{\psi}(z)\gamma_5\gamma_\mu\psi(0)|0\rangle \stackrel{\mathcal{F}}{=} m_\rho\, f_\rho\, i\, \varphi_A(y)\, \varepsilon_{\mu\lambda\beta\delta}\, e^{*T}_\lambda\, p_\beta\, n_\delta$$

- vector correlator with transverse derivative

$$\langle \rho(p)|\bar{\psi}(z)\gamma_\mu\, i\, \overleftrightarrow{\partial^\perp_\alpha}\, \psi(0)|0\rangle \stackrel{\mathcal{F}}{=} m_\rho\, f_\rho\, \varphi_1^T(y)\, p_\mu e^{*T}_\alpha$$

- axial correlator with transverse derivative

$$\langle \rho(p)|\bar{\psi}(z)\gamma_5\gamma_\mu\, i\, \overleftrightarrow{\partial^\perp_\alpha}\, \psi(0)|0\rangle \stackrel{\mathcal{F}}{=} m_\rho\, f_\rho\, i\, \varphi_A^T(y)\, p_\mu\, \varepsilon_{\alpha\lambda\beta\delta}\, e^{*T}_\lambda\, p_\beta\, n_\delta,$$

where y ($\bar{y} \equiv 1-y$) = momentum fraction along $p \equiv p_1$ of the quark (antiquark) and
$\stackrel{\mathcal{F}}{=} \int_0^1 dy \exp\left[i\, y\, p\cdot z\right]$, with $z = \lambda n$

$\Rightarrow$ 5 2-body DAs

Collinear factorization

Parametrization of vacuum–to–rho-meson matrix elements: 3-body correlators

3-body non-local correlators **genuine twist 3**

- vector correlator

$$\langle\rho(p)|\bar{\psi}(z_1)\gamma_\mu g A_\alpha^T(z_2)\psi(0)|0\rangle \stackrel{\mathcal{F}_2}{=} m_\rho\, f_3^V\, B(y_1,y_2)\, p_\mu\, e_\alpha^{*T},$$

- axial correlator

$$\langle\rho(p)|\bar{\psi}(z_1)\gamma_5\gamma_\mu g A_\alpha^T(z_2)\psi(0)|0\rangle \stackrel{\mathcal{F}_2}{=} m_\rho\, f_3^A\, i\, D(y_1,y_2)\, p_\mu\, \varepsilon_{\alpha\lambda\beta\delta}\, e_\lambda^{*T}\, p_\beta\, n_\delta,$$

where y_1, $\bar{y}_2$, $y_2 - y_1$ = quark, antiquark, gluon momentum fraction

and $\stackrel{\mathcal{F}_2}{=} \int_0^1 dy_1 \int_0^1 dy_2 \; \exp\left[i\, y_1\, p\cdot z_1 + i(y_2-y_1)\, p\cdot z_2\right],$ with $z_{1,2} = \lambda n$

$\Rightarrow$ 2 3-body DAs

Collinear factorization

Equations of motion

Equations of motion

twist 2
kinematical twist 3 (WW)
genuine twist 3
genuine + kinematical twist 3

- Dirac equation leads to

$$\langle i(\overrightarrow{\not{D}}(0)\psi(0))_\alpha \, \bar{\psi}_\beta(z)\rangle = 0 \qquad (i\,\overrightarrow{D}_\mu = i\,\overrightarrow{\partial}_\mu + g\,A_\mu)$$

- Apply the Fierz decomposition to the above 2 and 3-body correlators

$$-\langle\psi(x)\,\bar{\psi}(z)\rangle = \frac{1}{4}\langle\bar{\psi}(z)\gamma_\mu\psi(x)\rangle\gamma_\mu + \frac{1}{4}\langle\bar{\psi}(z)\gamma_5\gamma_\mu\psi(x)\rangle\gamma_\mu\gamma_5.$$

- $\Rightarrow$ 2 Equations of motion:

$$\bar{y}_1\,\varphi_3(y_1) + \bar{y}_1\,\varphi_A(y_1) + \varphi_1^T(y_1) + \varphi_A^T(y_1) + \int dy_2\left[\zeta_3^V\,B(y_1,\,y_2) + \zeta_3^A\,D(y_1,\,y_2)\right] = 0 \qquad \text{and} \quad (\bar{y}_1 \leftrightarrow y_1)$$

- In WW approximation: genuine twist 3 = 0 i.e. $B = D = 0$

$$\begin{cases} \varphi_A^T(y) = \frac{1}{2}[(y-\bar{y})\,\varphi_A^{WW}(y) - \varphi_3^{WW}(y)] \\ \varphi_1^T(y) = \frac{1}{2}[(y-\bar{y})\,\varphi_3^{WW}(y) - \varphi_A^{WW}(y)] \end{cases}$$

Collinear factorization

$n-$independence

A minimal set of DAs

- The non-perturbative correlators cannot be obtained from perturbative QCD (!)
- one should reduce them to a minimal set before using any model
- this can be achieved by using an additional condition:
  independence of the full amplitude with respect to the light-cone vector n
  n enters 3 places:
  - light-cone direction of z: $z = \lambda\, n$
  - definition of ρ_T polarization: $e_T \cdot n = 0$
  - axial gauge: $n \cdot A = 0$

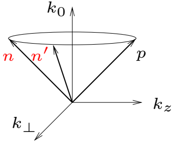
$\Rightarrow$ we prove that 3 independent Distribution Amplitudes are needed:
7 - 2 (=nb of equations of motion) - 2 (=nb of eq. from n-ind. cond.)

$\phi_1(y)$ $\leftarrow$ 2 body twist 2 correlator
$B(y_1,\, y_2)$ $\leftarrow$ 3 body genuine twist 3 vector correlator
$D(y_1,\, y_2)$ $\leftarrow$ 3 body genuine twist 3 axial correlator

Collinear factorization

$n-$independence

$n-$independence in practice

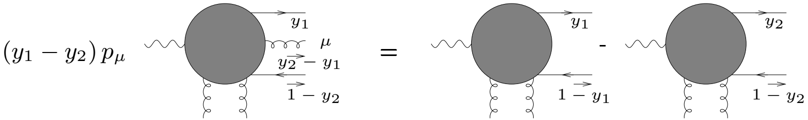


- ρ_T polarization: $e^{*T}_\mu = e^*_\mu - p_\mu\, e^* \cdot n$ keeping $n \cdot p = 1$
- for the full factorized amplitude:

$$\mathcal{A} = H \otimes S \qquad \frac{d\mathcal{A}}{dn_\perp{}^\mu} = 0\,,$$

- rewrite hard terms in one single form, of 2-body type: use Ward identities
  Example: hard 3-body $\longrightarrow$ hard 2-body

$$\mathrm{tr}\left[H_{3\rho}(y_1, y_2)\, p^\rho\, \not{p}\right]\; B(y_1,\, y_2) \;=\; \frac{1}{y_1 - y_2}\,(\mathrm{tr}\,[H_2(y_1)\,\not{p}] - \mathrm{tr}\,[H_2(y_2)\,\not{p}])B(y_1,\, y_2)\,,$$



- thus, symbolically,

$$\frac{dS}{dn_\perp{}^\mu} = 0$$

Collinear factorization

$n-$independence

Constraints from $n-$independence

twist 2
kinematical twist 3 (WW)
genuine twist 3
genuine + kinematical twist 3

- vector correlators

$$\frac{d}{dy_1}\varphi_1^T(y_1) = -\varphi_1(y_1) + \varphi_3(y_1) - \zeta_3^V \int_0^1 \frac{dy_2}{y_2 - y_1}\left(B(y_1, y_2) + B(y_2, y_1)\right)$$

- axial correlators

$$\frac{d}{dy_1}\varphi_A^T(y_1) = \varphi_A(y_1) - \zeta_3^A \int_0^1 \frac{dy_2}{y_2 - y_1}\left(D(y_1, y_2) + D(y_2, y_1)\right)$$

Collinear factorization

A set of independent non-perturbative correlators

Solution

twist 2
kinematical twist 3 (WW)
genuine twist 3
genuine + kinematical twist 3

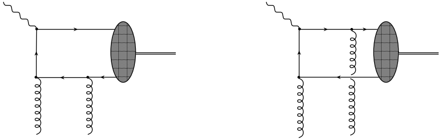
- the set of 4 equations (2 EOM + 2 n-independence relations) can be solved analytically
- $7 \longrightarrow$ 3 independent DAs

Computation and results

Computation of the hard part

2-body diagrams

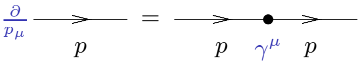
- without derivative

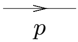


twist 2 $(\gamma^*_L \to \rho_L)$

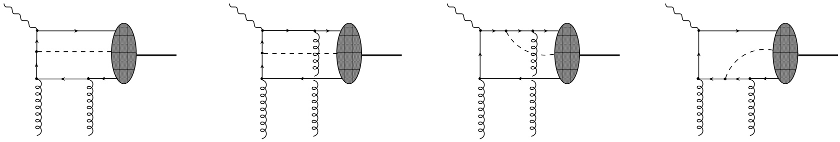
twist 3 $(\gamma^*_T \to \rho_T)$

- practical trick for computing $\partial_\perp H$: use the Ward identity



where 

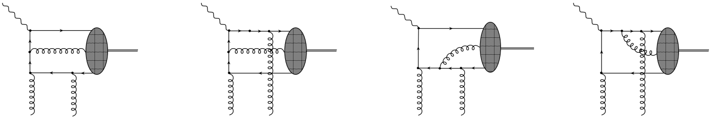
$= \frac{1}{m - \not{p} - i\epsilon}$



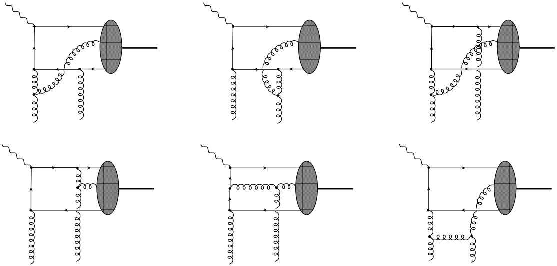
Computation and results
Computation of the hard part

3-body diagrams

- "abelian" type



- "non-abelian" type



Computation and results

Recall: $\gamma_L^* \to \rho_L$ impact factor

$\gamma_L^* \to \rho_L$ impact factor

$$\Phi^{\gamma_L^* \to \rho_L}(\underline{k}^2) = \frac{2\, e\, g^2\, f_\rho}{Q} \frac{\delta^{ab}}{2\, N_c} \int dy\, \varphi_1(y) \frac{\underline{k}^2}{y\, \bar{y}\, Q^2 + \underline{k}^2}$$

pure twist 2 scaling (from ρ-factorization point of view)

Computation and results

Results: $\gamma_T^* \to \rho_T$ impact factor

$\gamma_T^* \to \rho_T$ impact factor:

Spin Non-Flip/Flip separation appears

$$\Phi^{\gamma_T^* \to \rho_T}(\underline{k}^2) = \Phi_{n.f.}^{\gamma_T^* \to \rho_T}(\underline{k}^2)\, T_{n.f.} + \Phi_{f.}^{\gamma_T^* \to \rho_T}(\underline{k}^2)\, T_{f.}$$

where

$$T_{n.f.} = -(e_\gamma \cdot e^*) \quad \text{and} \quad T_{f.} = \frac{(e_\gamma \cdot k)(e^* k)}{\underline{k}^2} + \frac{(e_\gamma \cdot e^*)}{2}$$

non-flip transitions $\left\{ \begin{array}{l} + \to + \\ - \to - \end{array} \right.$ flip transitions $\left\{ \begin{array}{l} + \to - \\ - \to + \end{array} \right.$

Computation and results

Results: $\gamma^*_T \to \rho_T$ impact factor

pure twist 3 scaling (from ρ-factorization point of view)

$$\Phi^{\gamma^*_T \to \rho_T}_{n.f.}(\underline{k}^2)$$

$$= -\frac{e\,g^2 m_\rho f_\rho}{2\sqrt{2}\,Q^2}\,\frac{\delta^{ab}}{2\,N_c}\left\{-2\int dy_1\,\frac{\left(\underline{k}^2+2\,Q^2\,y_1\,(1-y_1)\right)\underline{k}^2}{y_1\,(1-y_1)\,(\underline{k}^2+Q^2\,y_1\,(1-y_1))^2}\left[(2y_1-1)\,\varphi_1^T(y_1)+\varphi_A^T(y_1)\right]\right.$$

$$+2\int dy_1\,dy_2\left[\zeta_3^V\,B\,(y_1,y_2)-\zeta_3^A\,D\,(y_1,y_2)\right]\frac{y_1\,(1-y_1)\,\underline{k}^2}{\underline{k}^2+Q^2\,y_1\,(1-y_1)}\left[\frac{(2-N_c/C_F)Q^2}{\underline{k}^2\,(y_1-y_2+1)+Q^2\,y_1\,(1-y_2)}\right.$$

$$\left.-\frac{N_c}{C_F}\,\frac{Q^2}{y_2\underline{k}^2+Q^2\,y_1\,(y_2-y_1)}\right]-2\int dy_1\,dy_2\left[\zeta_3^V\,B\,(y_1,y_2)+\zeta_3^A\,D\,(y_1,y_2)\right]\left[\frac{2+N_c/C_F}{1-y_1}\right.$$

$$+\frac{y_1\,Q^2}{\underline{k}^2+Q^2y_1\,(1-y_1)}\left(\frac{(2-N_c/C_F)\;y_1\,\underline{k}^2}{\underline{k}^2\,(y_1-y_2+1)+Q^2y_1\,(1-y_2)}-2\right)$$

$$\left.\left.+\frac{N_c}{C_F}\,\frac{(y_1-y_2)\,(1-y_2)}{1-y_1}\,\frac{Q^2}{\underline{k}^2\,(1-y_1)+Q^2\,(y_2-y_1)\,(1-y_2)}\right]\right\}$$

and

$$\Phi^{\gamma^*_T \to \rho_T}_{f.}(\underline{k}^2) = -\frac{e\,g^2 m_\rho f_\rho}{2\sqrt{2}\,Q^2}\,\frac{\delta^{ab}}{2\,N_c}\left\{4\int dy_1\,\frac{\underline{k}^2\,Q^2}{(\underline{k}^2+Q^2\,y_1\,(1-y_1))^2}\left[\varphi_A^T(y_1)-(2y_1-1)\,\varphi_1^T(y_1)\right]\right.$$

$$-4\int dy_1\,dy_2\,\frac{y_1\,\underline{k}^2}{\underline{k}^2+Q^2\,y_1\,(1-y_1)}\left[\zeta_3^A\,D\,(y_1,y_2)\,(-y_1+y_2-1)+\zeta_3^V\,B\,(y_1,y_2)\,(y_1+y_2-1)\right]$$

$$\left.\times\left[\frac{(2-N_c/C_F)Q^2}{\underline{k}^2\,(y_1-y_2+1)+Q^2\,y_1\,(1-y_2)}-\frac{N_c}{C_F}\,\frac{Q^2}{y_2\,\underline{k}^2+Q^2y_1\,(y_2-y_1)}\right]\right\}$$

Computation and results

Results: $\gamma^*_T \to \rho_T$ impact factor

WW limit

- WW limit: keep only twist 2 + kinematical twist 3 terms (i.e $B = D = 0$)
- The only remaining contributions come from the two-body correlators
- non-flip transition

$$\Phi^{\gamma^*_T \to \rho_T}_{n.f.}(\underline{k}^2) = \frac{-e\, m_\rho f_\rho}{2\sqrt{2}\,Q^2}\,\frac{\delta^{ab}}{2\,N_c}\int_0^1 dy \left\{ \frac{(y-\bar{y})\varphi_1^{T\,WW}(y) + 2\,y\,\bar{y}\,\varphi_3^{WW}(y) + \varphi_A^{T\,WW}(y)}{y\,\bar{y}} - \frac{2\,\underline{k}^2\left(\underline{k}^2 + 2\,Q^2\,y\,\bar{y}\right)\left((y-\bar{y})\,\varphi_1^{T\,WW}(y) + \varphi_A^{T\,WW}(y)\right)}{y\,\bar{y}\,(\underline{k}^2 + Q^2\,y\,(1-y))^2} \right\}$$

which simplifies, using equation of motion:

$$\int dy\,[(y-\bar{y})\varphi_1^{T\,WW}(y) + 2\,y\,\bar{y}\,\varphi_3^{WW}(y) + \varphi_A^{T\,WW}(y)] = 0$$

$$\Phi^{\gamma^*_T \to \rho_T}_{n.f.}(\underline{k}^2) = \frac{e\, m_\rho f_\rho}{\sqrt{2}\,Q^2}\,\frac{\delta^{ab}}{2\,N_c}\int_0^1 dy\,\frac{2\,\underline{k}^2\left(\underline{k}^2 + 2\,Q^2\,y\,\bar{y}\right)}{y\,\bar{y}\,(\underline{k}^2 + Q^2\,y\,\bar{y})^2}\left[(2\,y-1)\,\varphi_1^{T\,WW}(y) + \varphi_A^{T\,WW}(y)\right].$$

- flip transition:

$$\Phi^{\gamma^*_T \to \rho_T}_{n.f.}(\underline{k}^2) = -\frac{e\, m_\rho f_\rho}{\sqrt{2}\,Q^2}\,\frac{\delta^{ab}}{2\,N_c}\int_0^1 \frac{2\,\underline{k}^2\,Q^2}{(\underline{k}^2 + Q^2\,y\,\bar{y})^2}\left[(1-2\,y)\varphi_1^{T\,WW}(y) + \varphi_A^{T\,WW}(y)\right].$$

Computation and results

Discussion: gauge invariance

- The obtained results are gauge invariant:

$$\Phi^{\gamma_T^* \to \rho_T} \to 0 \quad \text{when} \quad \underline{k} \to 0$$

  - this is straightforward in the WW limit
  - at the full twist 3 order:
    - the C_F part of the abelian 3-body contribution cancels the 2-body contribution after using the equation of motion
    - the N_c part of the abelian 3-body contribution cancels the 3-body non-abelian contribution
    - thus $\gamma_T^* \to \rho_T$ impact factor is gauge-invariant only provided the 2 and 3-body contributions have been taken into account in a consistent way

Computation and results

Discussion: consistence with factorization

- Our results are free of end-point singularities, in both WW approximation and full twist-3 order calculation:
  - the flip contribution obviously does not have any end-point singularity because of the $\underline{k}^2$ which regulates them
  - the potential end-point singularity for the non-flip contribution is spurious since $\varphi_A^T(y)$, $\varphi_1^T(y)$ vanishes at $y = 0, 1$ as well as $B(y_1, y_2)$ and $D(y_1, y_2)$.

Transversity GPDs

Twist 2 GPDs

Classification of twist 2 GPDs

- For quarks, one should distinguish the exchanges
  - without helicity flip (chiral-even Γ' matrices): 4 chiral-even GPDs:
    $H^q \xrightarrow{\xi=0,t=0}$ PDF q, E^q, $\tilde{H}^q \xrightarrow{\xi=0,t=0}$ polarized PDFs Δq, $\tilde{E}^q$

$$
\begin{aligned}
F^q &= \frac{1}{2}\int \frac{dz^+}{2\pi}\, e^{ixP^-z^+} \langle p'|\,\bar{q}(-\tfrac{1}{2}z)\,\gamma^-\,q(\tfrac{1}{2}z)\,|p\rangle\Big|_{z^-=0,\, z_\perp=0} \\
&= \frac{1}{2P^-}\left[H^q(x,\xi,t)\,\bar{u}(p')\gamma^-u(p) + E^q(x,\xi,t)\,\bar{u}(p')\frac{i\,\sigma^{-\alpha}\Delta_\alpha}{2m}u(p)\right], \\
\tilde{F}^q &= \frac{1}{2}\int \frac{dz^+}{2\pi}\, e^{ixP^-z^+} \langle p'|\,\bar{q}(-\tfrac{1}{2}z)\,\gamma^-\gamma_5\,q(\tfrac{1}{2}z)\,|p\rangle\Big|_{z^-=0,\, z_\perp=0} \\
&= \frac{1}{2P^-}\left[\tilde{H}^q(x,\xi,t)\,\bar{u}(p')\gamma^-\gamma_5 u(p) + \tilde{E}^q(x,\xi,t)\,\bar{u}(p')\frac{\gamma_5\,\Delta^-}{2m}u(p)\right].
\end{aligned}
$$

  - with helicity flip (chiral-odd Γ' mat.): 4 chiral-odd GPDs:
    $H_T^q \xrightarrow{\xi=0,t=0}$ quark transversity PDFs $\Delta_T q$, E_T^q, $\tilde{H}_T^q$, $\tilde{E}_T^q$

$$
\frac{1}{2}\int \frac{dz^+}{2\pi}\, e^{ixP^-z^+} \langle p'|\,\bar{q}(-\tfrac{1}{2}z)\,i\,\sigma^{-i}\,q(\tfrac{1}{2}z)\,|p\rangle\Big|_{z^-=0,\, z_\perp=0}
$$

$$
= \frac{1}{2P^-}\bar{u}(p')\left[H_T^q\, i\sigma^{-i} + \tilde{H}_T^q\,\frac{P^-\Delta^i - \Delta^-P^i}{m^2} + E_T^q\,\frac{\gamma^-\Delta^i - \Delta^-\gamma^i}{2m} + \tilde{E}_T^q\,\frac{\gamma^-P^i - P^-\gamma^i}{m}\right]
$$

Transversity GPDs
Twist 2 GPDs

Classification of twist 2 GPDs

- analogously, for gluons:
  - 4 gluonic GPDs without helicity flip:
    $H^g \xrightarrow{\xi=0,t=0}$ PDF $x\,g$
    E^g
    $\tilde{H}^g \xrightarrow{\xi=0,t=0}$ polarized PDF $x\,\Delta g$
    $\tilde{E}^g$
  - 4 gluonic GPDs with helicity flip:
    H_T^g
    E_T^g
    $\tilde{H}_T^g$
    $\tilde{E}_T^g$
    (no forward limit reducing to gluons PDFs here: a change of 2 units of helicity cannot be compensated by a spin 1/2 target)

Transversity GPDs
Spin transversity in the nucleon

What is transversity?

- Tranverse spin content of the proton:

$$\begin{array}{ccc} |\uparrow\rangle_{(x)} & \sim & |\rightarrow\rangle + |\leftarrow\rangle \\ |\downarrow\rangle_{(x)} & \sim & |\rightarrow\rangle - |\leftarrow\rangle \\ \text{spin along } x & & \text{helicity state} \end{array}$$

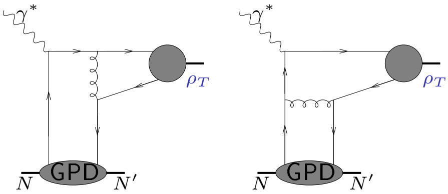
- An observable sensitive to helicity spin flip gives thus access to the transversity $\Delta_T q(x)$, which is very badly known (first data have recently been obtained by COMPASS)
- The transversity GPDs are completely unknown
- Chirality: $q_\pm(z) \equiv \frac{1}{2}(1 \pm \gamma^5) q(z)$ with $q(z) = q_+(z) + q_-(z)$
  Chiral-even: chirality conserving
  $\bar{q}_\pm(z)\gamma^\mu q_\pm(-z)$ et $\bar{q}_\pm(z)\gamma^\mu\gamma^5 q_\pm(-z)$
  Chiral-odd: chirality reversing
  $\bar{q}_\pm(z) \cdot 1 \cdot q_\mp(-z), \quad \bar{q}_\pm(z) \cdot \gamma^5 \cdot q_\mp(-z)$ et $\bar{q}_\pm(z)[\gamma^\mu, \gamma^\nu] q_\mp(-z)$
- For a massless (anti)particle, chirality = (-)helicity
- Transversity is thus a chiral-odd quantity
- QCD and QED are chiral even $\Rightarrow \mathcal{A} \sim (\text{Ch.-odd})_1 \otimes (\text{Ch.-even})_2$

Transversity GPDs

Spin transversity in the nucleon

How to get access to transversity?

- The dominant DA for ρ_T is of twist 2 and chiral-odd ($[\gamma^\mu, \gamma^\nu]$ coupling)
- Unfortunately $\gamma^* N^\uparrow \to \rho_T N' = 0$
  - this is true at any order in perturbation theory (i.e. corrections as powers of α_s), since this would require a transfer of 2 units of helicity from the proton: impossible! Collins, Diehl '00
  - diagrammatic argument at Born order:



vanishes: $\gamma^\alpha[\gamma^\mu, \gamma^\nu]\gamma_\alpha = 0$

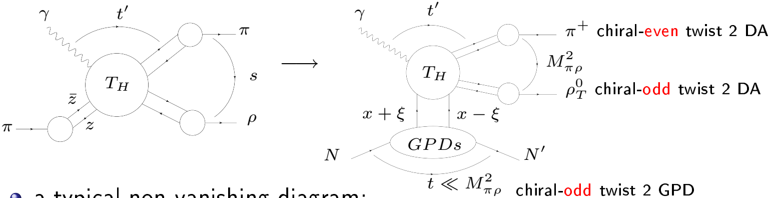
Diehl, Gousset, Pire '99

Transversity GPDs

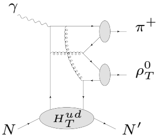
Spin transversity in the nucleon

$\gamma N \to \pi^+ \rho^0_T N'$ gives access to transversity

- Factorization à la Brodsky Lepage of $\gamma + \pi \to \pi + \rho$ at large s and fixed angle (i.e. fixed ratio $t'/s, u'/s$)
  $\Longrightarrow$ factorization of the amplitude for $\gamma + N \to \pi + \rho + N'$ at large $M^2_{\pi\rho}$



- a typical non-vanishing diagram:



M. El Beiyad, P. Pire, M. Segond, L. Szymanowski, S.W Phys.Lett.B688:154-167,2010

see also, at large s, with $\mathbb{P}$omeron exchange:

R. Ivanov, B. Pire, L. Symanowski, O. Teryaev '02

R. Enberg, B. Pire, L. Symanowski '06

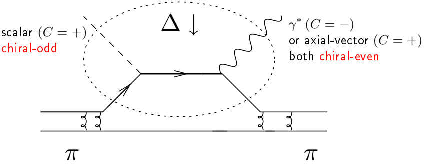
- These processes with 3 body final state can give access to all GPDs: $M^2_{\pi\rho}$ plays the role of the γ^* virtuality of usual DVCS (here in the time-like domain) JLab, COMPASS

Transversity GPDs

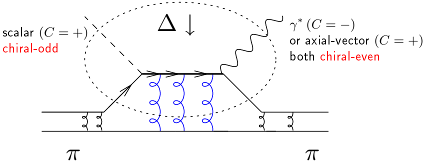
Chiral-odd GPDs beyond leading twist

Classification of higher twist chiral-odd GPDs (pion case)

Two sources of higher twist contributions for "DVCS":



$\Delta_\perp$ contributions
(analogous to the $\ell_\perp$ for DAs)



genuine higher twist contributions:
transversally polarized $t-$channel gluons

- Introduce a set of chiral-odd GPDs up to a given twist
- Write QCD equations of motion
- When factorizing long/short distance dynamics, one introduces an arbitrary light-cone vector n which enters
  - the gauge fixing
  - the $\perp$-space definition

  Write $n-$independency contraints

This should provide a set of independent GPDs, in a consistent way when truncating at a given twist

B. Pire, L. Szymanowski, S. W., in progress

Conclusions
1

- We have performed a full up to twist 3 computation of the $\gamma^* \to \rho$ impact factor, in the $t = t_{min}$ limit.
- Our result respects gauge invariance.
- It is free of end-point singularities
(this should be contrasted with standard collinear treatment, at moderate s, where k_T-factorization is NOT applicable: see Mankiewicz-Piller).
- In this talk we relied on the Light-Cone Collinear approach
(Ellis + Furmanski + Petronzio; Efremov + Teryaev; Anikin + Teryaev),
which is non-covariant, but very efficient for practical computations.

Conclusions
2

- Comparison with a fully covariant approach by Ball+Braun et al:
  We have established the dictionary between the two approaches within a full twist 3 treatment:

$$
\begin{aligned}
B(y_1,\,y_2) &= -\frac{V(y_1,\,1-y_2,\,y_2-y_1)}{y_2-y_1}\,,\\
D(y_1,\,y_2) &= -\frac{A(y_1,\,1-y_2,\,y_2-y_1)}{y_2-y_1}\\
\varphi_1(y) &= f_\rho\, m_\rho\, \phi_{||}(y)\\
\varphi_3(y) &= f_\rho\, m_\rho\, g^{(v)}(y)\,,\\
\varphi_A(y) &= -\frac{1}{4} f_\rho\, m_\rho \frac{\partial g^{(a)}(y)}{\partial y}
\end{aligned}
$$

- We also performed calculations of the same impact factor within the covariant approach by Ball+Braun et al: calculations proceed in quite different way : eg. no $\varphi^T_{1,A}-$DAs but Wilson line effects are important !!
  We got a full agreement with our approach

- The Light-Cone Collinear approach is systematic and simple. It can be extended to any process.
  e.g.: classification of chiral-odd GPDs beyond leading twist.