# A phenomenological study of helicity amplitudes of high energy exclusive leptoproduction of the $\rho$ meson

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# Outline

- A phenomenological model of the helicity amplitudes of high energy exclusive leptoproduction of the $\rho$ meson

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  in collaboration with
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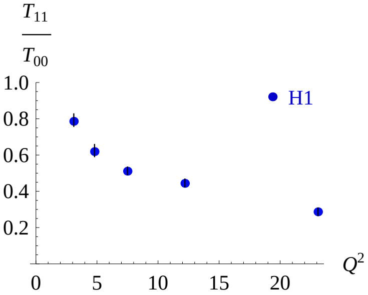
- Impact factor $\gamma^* \to \rho$ up to twist 3 - link to colour dipole model

  in collaboration with
  L. Szymanowski (SINS, Warsaw) and S. Wallon (LPT, Orsay)

# Introduction

## Experimental data of helicity amplitudes at high energy

- Helicity amplitudes $T_{\lambda_\rho \lambda_\gamma}$ : $\gamma^*_{\lambda_\gamma} + p \to \rho_{\lambda_\rho} + p$
- H1 and ZEUS data for Helicity Amplitudes at HERA:




S. Chekanov et al. (2007), F.D Aaron et al. (2010)

- Kinematics
  - High energy in the center of mass $30\,GeV < W < 180\,GeV$
  - Photon Virtuality $2.5\,GeV^2 < Q^2 < 60\,GeV^2$
  - $|t| < 1\,GeV^2$

$$\Rightarrow s_{\gamma^* p} = W^2 >> Q^2 >> \Lambda^2_{QCD}$$

# Introduction

## A Theoretical approach within $k_T$ factorisation

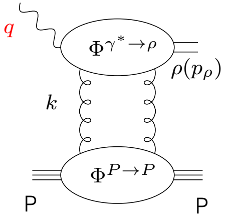
- Perturbative Regge Limit :

  Regge Limit : $s = W^2 >> Q^2, |t|, M^2_{\text{hadron}}$

  Hard scale : $Q >> \Lambda_{QCD}$

- $k_T$ factorisation

  Amplitudes with gluons exchange in t-channel dominate at large $s$ ($s = W^2$)




Born order: 2 $t$-channel gluons

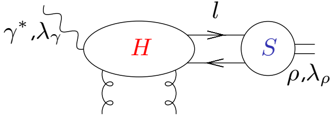
$$T_{\lambda_\rho \lambda_\gamma} = is \int \frac{d^2 \underline{k}}{(2\pi)^2 (\underline{k}^2)^2} \Phi^{\gamma^*(\lambda_\gamma) \to \rho(\lambda_\rho)}(\underline{k}) \; \Phi^{P \to P}(-\underline{k})$$

# Introduction
## A Theoretical approach

### Impact factors $\Phi^{\gamma^* \to \rho}$ and $\Phi^{P \to P}$

- $\Phi^{\gamma^* \to \rho}$ Light-Cone Collinear factorisation

  $Q^2 >> \Lambda^2_{QCD}$




  - Twist $t$ : Impact factor behaves as $1/Q^{t-1}$
  - $T_{00} \equiv \gamma^*_L \to \rho_L$ impact factor : Dominant term at twist 2 $\equiv 1/Q$
  - $T_{11} \equiv \gamma^*_T \to \rho_T$ impact factor : Dominant term at twist 3 $\equiv 1/Q^2$

  Recently computed at $t = t_{min} \approx 0$

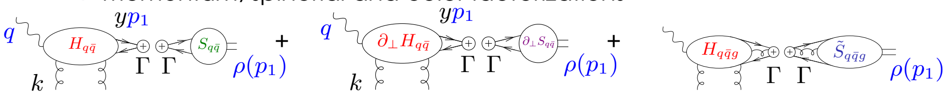
  Nucl. Phys. B **828** (2010) 1-68. by Anikin et al.

- Phenomenological model for $\Phi^{P \to P}$

# Collinear factorization
## Light-Cone Collinear approach

Collinear factorization of 2-body and 3-body contributions

- Momentum, spinorial and color factorizations




- Parametrization of Soft parts $S_{q\bar{q}}, \partial_\perp S_{q\bar{q}}, S_{q\bar{q}g}$
  - $\Rightarrow$ 5 2-body DAs $\{\varphi_1, \varphi_A, \varphi_3, \varphi_{1T}, \varphi_{AT}\}$
  - $\Rightarrow$ 2 3-body DAs $\{B(y_1, y_2), D(y_1, y_2)\}$
  - Relations between DAs : Equation of motion and n-independence $\Rightarrow$

    3 independent DAs : $\{\varphi_1, B(y_1, y_2), D(y_1, y_2)\}$

# Collinear factorization
## Wandzura-Wilczek and Genuine contributions

- Solution in the Wandzura-Wilczek Approximation (WW) $\equiv$ Only 2-body contributions

$$\varphi_1 \Rightarrow \{\varphi_3^{WW}(y), \varphi_A^{WW}(y), \varphi_{1T}^{WW}(y), \varphi_{AT}^{WW}(y)\}$$

- Genuine solutions

$$\{B(y_1, y_2), D(y_1, y_2)\} \Rightarrow \{\varphi_3^{gen}(y), \varphi_A^{gen}(y), \varphi_{1T}^{gen}(y), \varphi_{AT}^{gen}(y)\}$$

- Evolution of the DAs P. Ball, V.M Braun, Y. Koike, K. Tanaka

$$\varphi_1(y,\mu^2) = 6y\bar{y}(1 + a_2(\mu^2)\frac{3}{2}(5(y-\bar{y})^2 - 1)) \overset{\mu^2\to\infty}{\longrightarrow} 6y\bar{y}$$

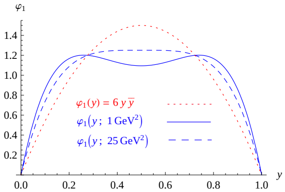
$$B(y_1, y_2; \mu^2) = -5040 y_1 \bar{y_2}(y_1 - \bar{y_2})(y_2 - y_1)$$

$$D(y_1, y_2; \mu^2) = -360 y_1 \bar{y_2}(y_2 - y_1)(1 + \frac{\omega^A_{\{1,0\}}(\mu^2)}{2}(7(y_2 - y_1) - 3))$$

with $\mu^2 \approx Q^2$ the collinear factorisation scale

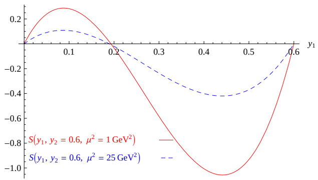
# Collinear factorisation
## DAs dependence on $\mu^2$

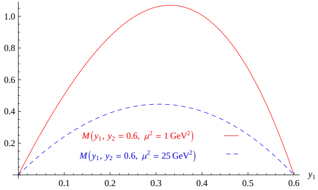
- $\varphi_1(y,\mu^2)$




- $M(y_1,y_2) = \zeta_\rho^V(\mu^2)B(y_1,y_2;\mu^2) - \zeta_\rho^A(\mu^2)D(y_1,y_2;\mu^2) \overset{\mu^2\to\infty}{\longrightarrow} 0$

$S(y_1,y_2) = \zeta_\rho^V(\mu^2)B(y_1,y_2;\mu^2) + \zeta_\rho^A(\mu^2)D(y_1,y_2;\mu^2) \overset{\mu^2\to\infty}{\longrightarrow} 0$



# Ratios of Helicity Amplitudes

## A model for the proton impact factor

- $T_{\lambda_\rho \lambda_\gamma}(Q,M) = is \int \frac{d^2\underline{k}}{(2\pi)^2} \frac{1}{(\underline{k}^2)^2} \Phi^{P\to P}(\underline{k}; M^2) \Phi^{\gamma^*(\lambda_\gamma)\to\rho(\lambda_\rho)}(\underline{k}; Q^2)$

- Phenomenological Model for $\Phi^{P\to P}$

$$\Phi^{P\to P}(\underline{k}; M^2) \propto \left[\frac{1}{M^2} - \frac{1}{M^2+\underline{k}^2}\right]$$

J.F Gunion, D.E Soper

- $\gamma^*_L \to \rho_L$ helicity amplitude:

$$T_{00} \propto \frac{is}{(2\pi)} \int_{\lambda^2}^{\infty} d\underline{k}^2 \frac{1}{(\underline{k}^2)^2} \left(\frac{1}{M^2} - \frac{1}{\underline{k}^2 + M^2}\right) \times \frac{1}{Q} \int_0^1 dy\, \varphi_1(y,\mu^2) \frac{\underline{k}^2}{\underline{k}^2 + y\bar{y}Q^2}$$

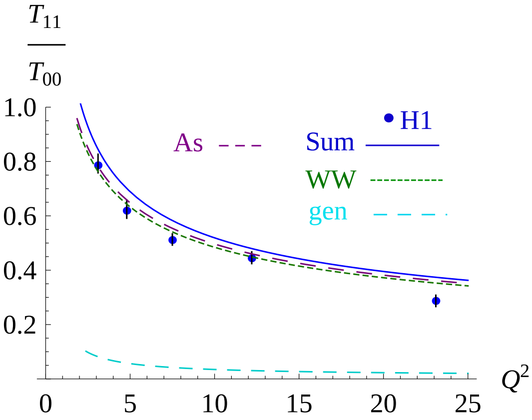
- The WW contribution:

$$T_{11}^{WW} \propto \frac{is}{2\pi} \int_{\lambda^2}^{\infty} d(\underline{k}^2) \frac{1}{(\underline{k}^2)^2} \left(\frac{1}{M^2} - \frac{1}{\underline{k}^2 + M^2}\right) \times \left(\frac{(\epsilon_\gamma . \epsilon^*_\rho) m_\rho}{Q^2} \int_0^1 du \frac{\varphi_1(u;\mu^2)}{u} \int_0^u dy \frac{\underline{k}^2(\underline{k}^2 + 2y\bar{y}Q^2)}{(\underline{k}^2 + y\bar{y}Q^2)^2}\right)$$

# Ratios of Helicity Amplitudes

## Comparison with H1 data : $T_{11}/T_{00}$

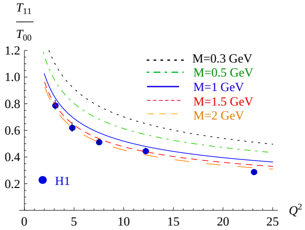
- Genuine and Wandzura-Wilczek Contributions at $M = 1\, GeV$



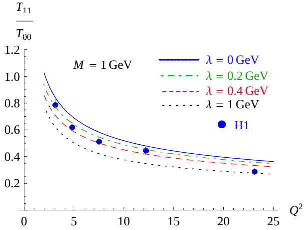
# Ratios of Helicity Amplitudes

## Dependence on parameters $M$ and $\lambda$

- $M$ dependence of the ratio $T_{11}/T_{00}$



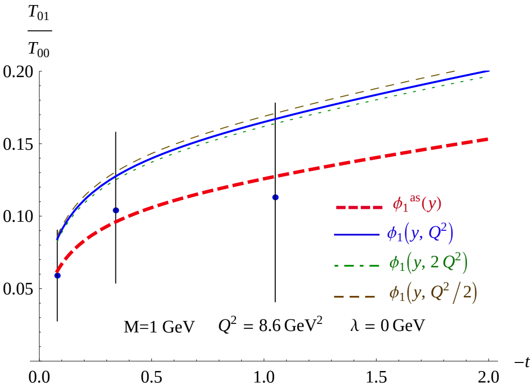

- Soft gluon effect : $\lambda$ IR cut-off on $k_T$ integrals



# Ratios of Helicity Amplitudes

## Comparison with H1 data : $T_{01}/T_{00}$

- $T_{01}/T_{00}$ at $M = 1\,GeV$, Dependence on $\mu^2$ at $M = 1\,GeV$ :



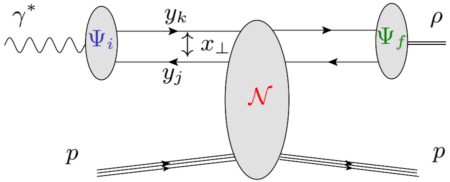
# Conclusion I : Perspectives for $\Phi^{\gamma_T^* \to \rho_T}$

- Good agreement with Experimental data
  - reasonable values of $M \approx M_p$ and $\lambda \approx 0\, GeV$
  - weak sensitivity in the parameters $M$ and $\lambda$

- Perspectives :
  - Extend the kinematic to $t \neq t_{min}$ $\Rightarrow$ access to all spin density matrix elements.
  - Link with the Dipole model and implementation of saturation effects

# Dipole Models
## Dipole model picture

- Factorization of a high energy scattering amplitude into:



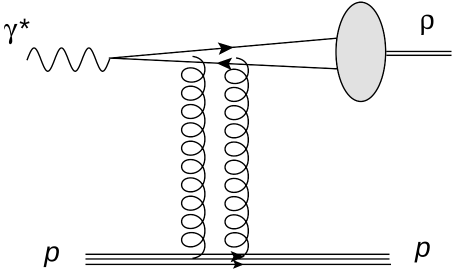

  - Initial $\Psi_i$ and final $\Psi_f$ states wave functions of projectiles
  - Universal scattering amplitude $\mathcal{N} \equiv \mathcal{N}_{\text{dipole-target}}$

- Dipole models are consistent with LO Collinear approximation but are they still consistent with collinear factorization at higher twist order?

# Dipole Models

## The $\gamma^* \to \rho$ impact factor in a dipole model

- Dipole representation at lowest Fock state ($q\bar{q}$ pair)




- In the dipole model representation, the amplitude for high energy electroproduction of the $\rho$ meson at $t = 0$ reads:

$$\mathcal{A} = is \int d^2\underline{x} \int dy \bar{\Psi}^{\rho\lambda_\rho}(y, \underline{x}) \mathcal{N}(x_{Bj}, \underline{x}) \Psi^{\gamma^* \lambda_\gamma}(y, \underline{x})$$

(from Bartels, Golec-Biernat, Peters ) with

$$\mathcal{N}(x_{Bj}, \underline{x}) \propto \alpha_s \frac{\delta^{ab}}{N_c} \int \frac{d^2\underline{k}}{(\underline{k}^2)^2} (1 - e^{i\underline{k}\cdot\underline{x}})(1 - e^{-i\underline{k}\cdot\underline{x}}) f(x_{Bj}, \underline{x})$$

# Dipole Models

## Fourrier transform of the $\gamma^* \to \rho$ impact factor

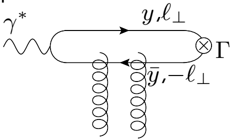
- Impact factor in transverse coordinate space:

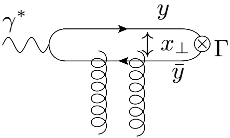
$$\Phi^{\gamma^* \to \rho}_{2body} = \int d^4\ell \, H(\ell) S(\ell) = -\frac{1}{4} \int d^4\ell H^{\Gamma}(\ell) S_{\Gamma}(\ell)$$

$$= -\frac{1}{4} \int dy \int d^2\ell_\perp \int \frac{d^2 x_\perp}{2\pi} e^{-i\ell_\perp \cdot x_\perp} \tilde{H}^{\Gamma}(y, x_\perp)$$

$$\int \frac{d^2 z_\perp}{2\pi} e^{-i\ell_\perp \cdot z_\perp} \int \frac{d\lambda}{2\pi} e^{-i\lambda y} \langle \rho | \bar{\psi}(\lambda n + z_\perp) \Gamma \, \psi(0) | 0 \rangle$$

- Hard parts in transverse coordinate space :

$H^{\Gamma}(y, \ell_\perp) =$ 

$\longleftrightarrow$ $\tilde{H}^{\Gamma}(y, x_\perp) =$ 

# Dipole Models

## Fourrier transform of the $\gamma^* \to \rho$ impact factor

Collinear Approximation up to twist 3 $\Rightarrow e^{-i\ell_\perp \cdot x_\perp} \approx 1 - i\ell_\perp \cdot x_\perp$

- "1" $\Rightarrow$

$$-\frac{1}{4}\int dy \int \frac{d^2 x_\perp}{2\pi} \tilde{H}^\Gamma(y, x_\perp) \int \frac{d\lambda}{2\pi} e^{-i\lambda y} \langle \rho | \bar{\psi}(\lambda n) \Gamma \psi(0) | 0 \rangle$$

- "$-i\ell_\perp \cdot x_\perp$" $\Rightarrow$

$$-\frac{1}{4}\int dy \int \frac{d^2 x_\perp}{2\pi} x_\perp^\alpha \tilde{H}^\Gamma(y, x_\perp) \int \frac{d\lambda}{2\pi} e^{-i\lambda y} \langle \rho | \bar{\psi}(\lambda n) \overleftrightarrow{\partial}_{\alpha\perp} \Gamma \psi(0) | 0 \rangle$$

# Dipole Models

## Fourrier transform of the $\gamma^* \to \rho$ impact factor

Hard parts in coordinate space:

- $\Gamma \equiv \gamma^\mu$

$$\tilde{H}^{\gamma^\mu}(y,\underline{x}) = -4\frac{2\pi e g^2}{\sqrt{2}s}\frac{\delta^{ab}}{2N_c}\{y\bar{y}sK_0(\mu|\underline{x}|)e^\mu_{\gamma_T} - (y-\bar{y})i\mu\frac{e_{\gamma_T}\cdot x}{|\underline{x}|}K_1(\mu|\underline{x}|)\left((1-e^{i\underline{k}\cdot\underline{x}})(1-e^{-i\underline{k}\cdot\underline{x}})-1\right)p_2^\mu\}$$

- Hard part $\Gamma \equiv \gamma_5\gamma^\mu$

$$\tilde{H}^{\gamma_5\gamma^\mu}(y,\underline{x}) = 4i\frac{2\pi e g^2}{\sqrt{2}s}\frac{\delta^{ab}}{2N_c}\varepsilon^{\mu\nu\rho\sigma}\{-y\bar{y}K_0(\mu|\underline{x}|)(e_{\gamma_T\nu}p_{1\rho}p_{2\sigma}+p_{2\nu}p_{1\rho}e_{\gamma_T\sigma}) - i\mu K_1(\mu|\underline{x}|)\left((1-e^{i\underline{k}\cdot\underline{x}})(1-e^{-i\underline{k}\cdot\underline{x}})-1\right)(y e_{\gamma_T\nu}\frac{x_{\perp\rho}}{|\underline{x}|}p_{2\sigma}-\bar{y}p_{2\nu}\frac{x_{\perp\rho}}{|\underline{x}|}p_{1\sigma})\}$$

- Equations of motion:

Termes $\times$ $(1-e^{i\underline{k}\cdot\underline{x}})(1-e^{-i\underline{k}\cdot\underline{x}})$ + Termes $\times$ $\underbrace{2y\bar{y}\varphi_3(y)+(y-\bar{y})\varphi_{1T}(y)+\varphi_{AT}(y)}_{=0}$

# Dipole Models

## Fourrier transform of the $\gamma^* \to \rho$ impact factor

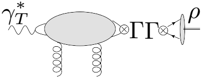
Twist 3, 2-body impact factors:

- Non-flip part:

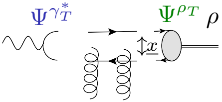
$$\Phi_{nf} = \frac{1}{4}\int dy \int \frac{d^2\underline{x}}{2\pi}\,\frac{e}{\sqrt{2}}\mu\,|\underline{x}|\,K_1(\mu\,|\underline{x}|)g^2\delta^{ab}(1-e^{i\underline{k}\cdot\underline{x}})(1-e^{-i\underline{k}\cdot\underline{x}})\frac{m_\rho f_\rho}{2N_c}\{(y-\bar{y})\varphi_{1T}(y)+\varphi_{AT}(y)\}$$

- Flip part:

$$\Phi_{f} = \frac{1}{2}\int dy \int \frac{d^2\underline{x}}{2\pi}\,\frac{e}{\sqrt{2}}\mu\,|\underline{x}|\,K_1(\mu\,|\underline{x}|)g^2\delta^{ab}(1-e^{i\underline{k}\cdot\underline{x}})(1-e^{-i\underline{k}\cdot\underline{x}})\frac{m_\rho f_\rho}{2N_c}\{(y-\bar{y})\varphi_{1T}(y)-\varphi_{AT}(y)\}$$



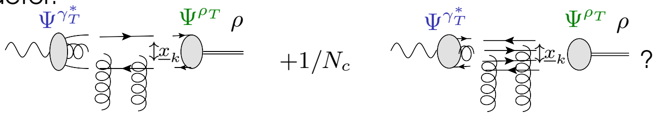

$+$ Equations of motion $\equiv$



# Conclusion II : Perspectives for $\Phi^{\gamma_T^* \to \rho_T}$

- Agreement between the higher twist computation in the Wandzura-Wilczek approximation and the dipole representation.

- Dipole factors appear in the large $N_c$ limit for the 3-body impact factor:




- Improvement of the phenomenological model by taking into account saturation effects in the previous phenomenological model