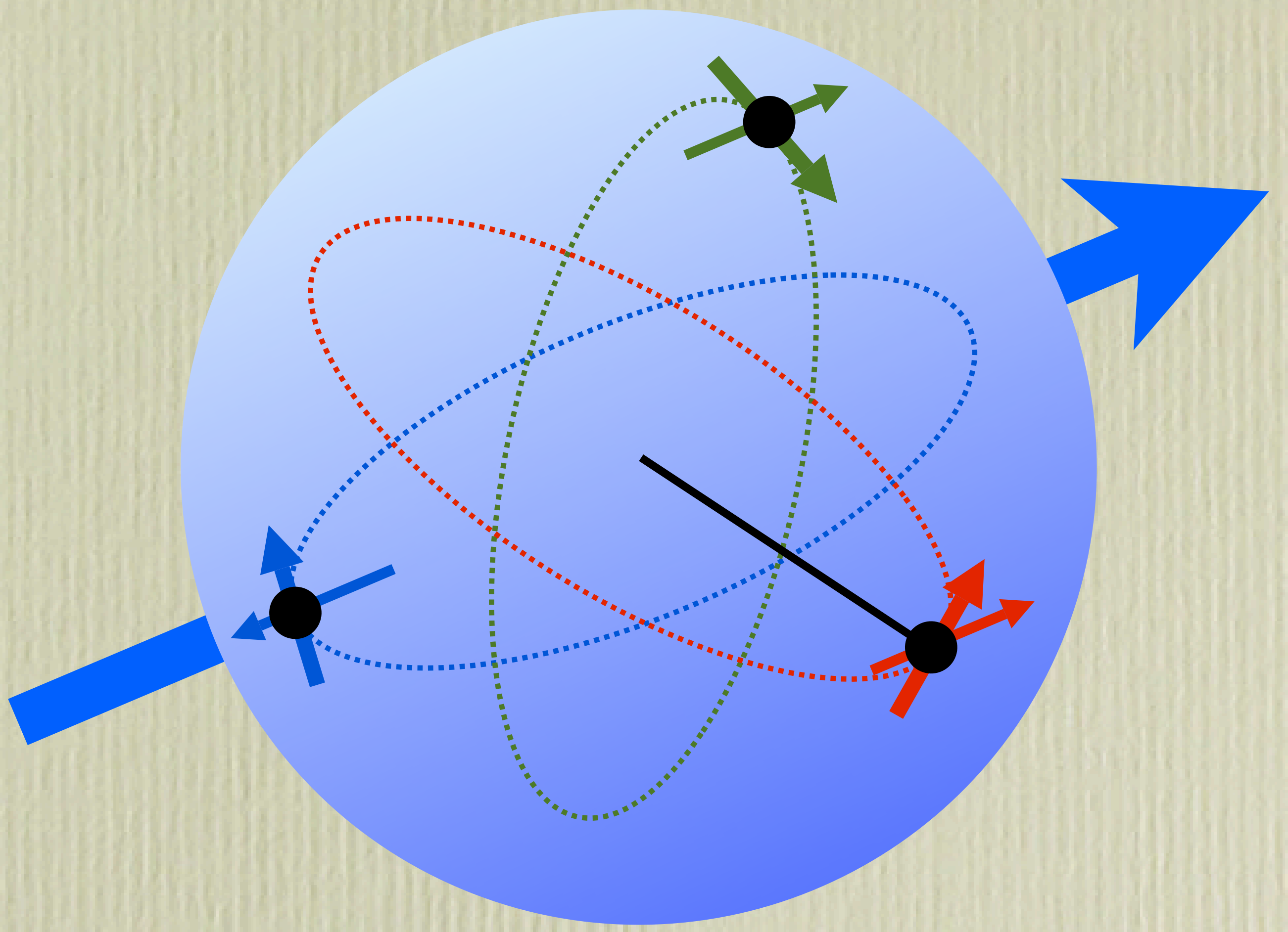
TMDs and the 3-dimensional momentum structure of the nucleon

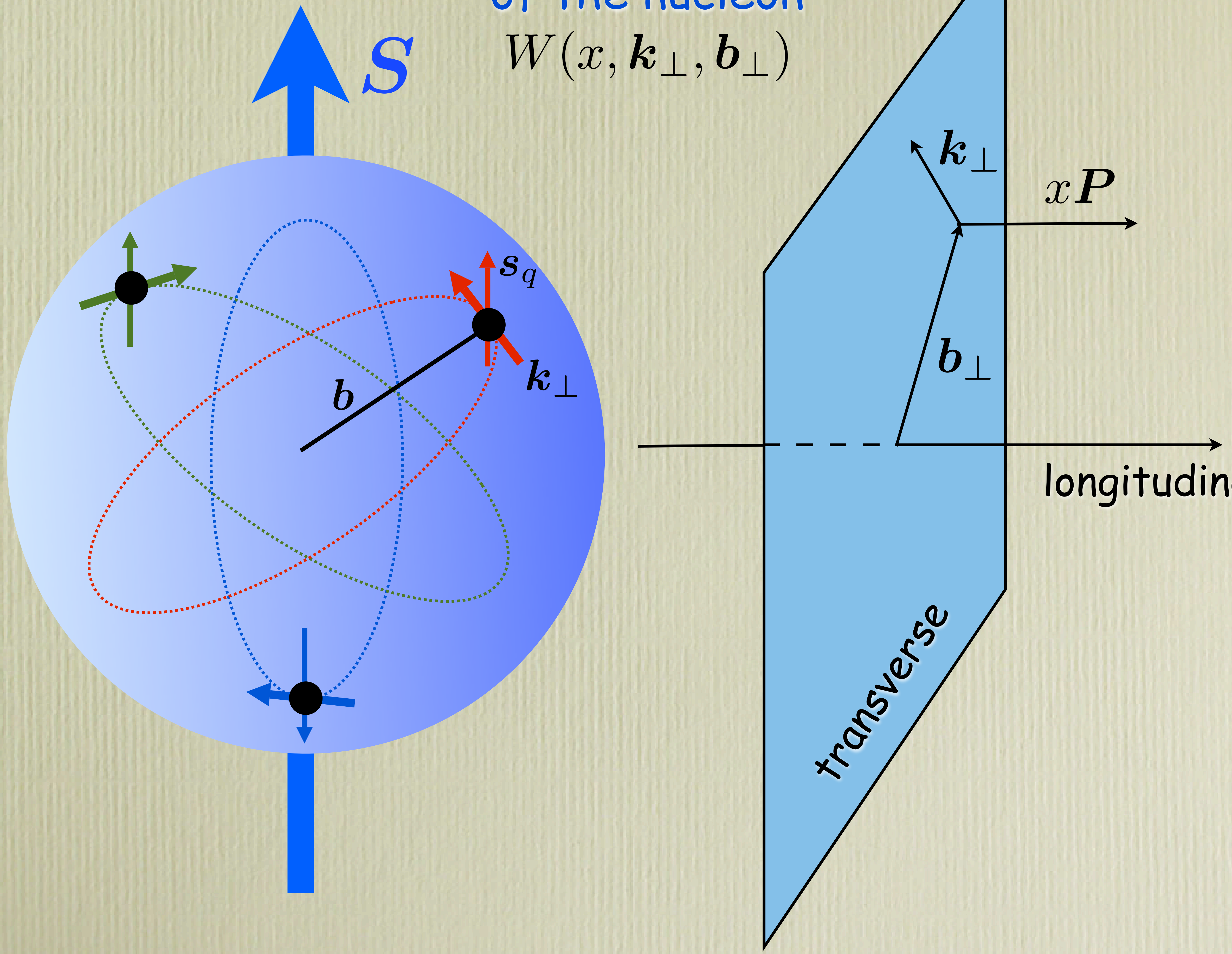


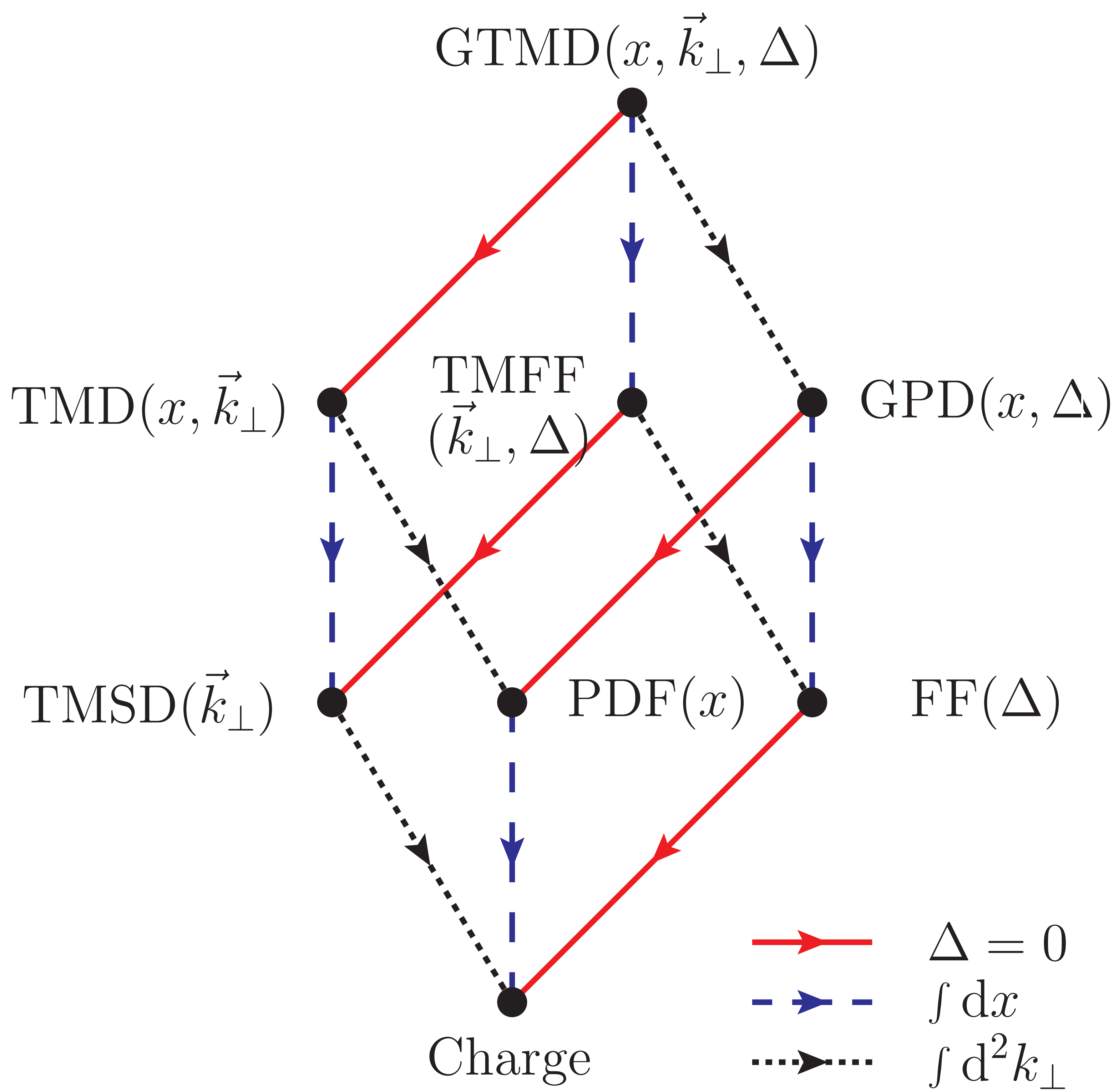
Fall meeting of the GDR PH-QCD: nucleon and nucleus structure studies with a LHC fixed-target experiment and electron-ion colliders

Mauro Anselmino, Torino University & INFN - Oct. 21, 2011

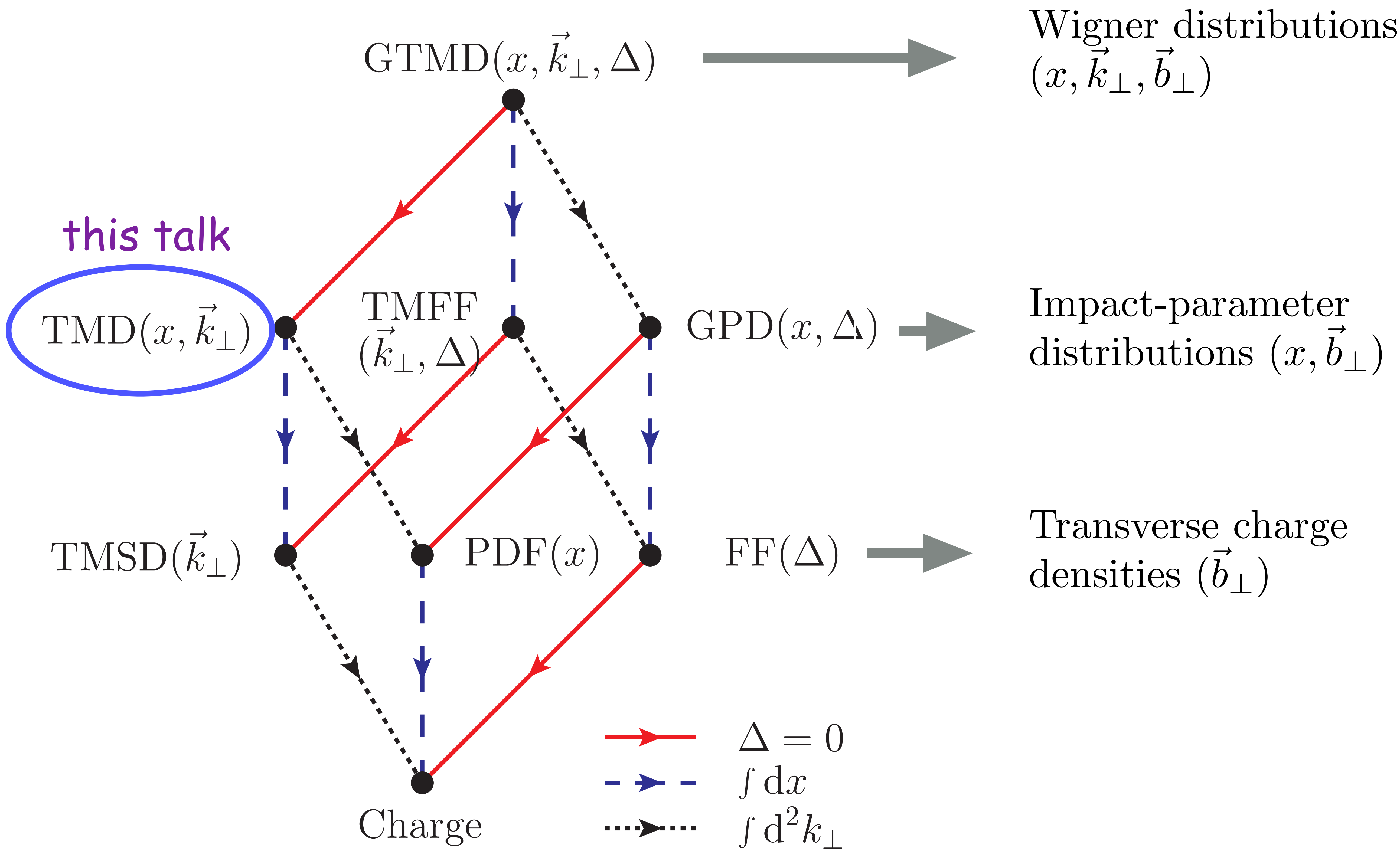
Exploring the 3-dimensional phase-space structure of the nucleon

$$W(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp)$$





Lorcé, Pasquini, Vanderhaeghen, Lorcé talk



courtesy of A. Bacchetta

information on TMDs

SIDIS:

$k_\perp$ dependence of unpolarized partonic distributions (Cahn effect)

Sivers distribution

Collins fragmentation and transversity

model (+ data) computation of J_q

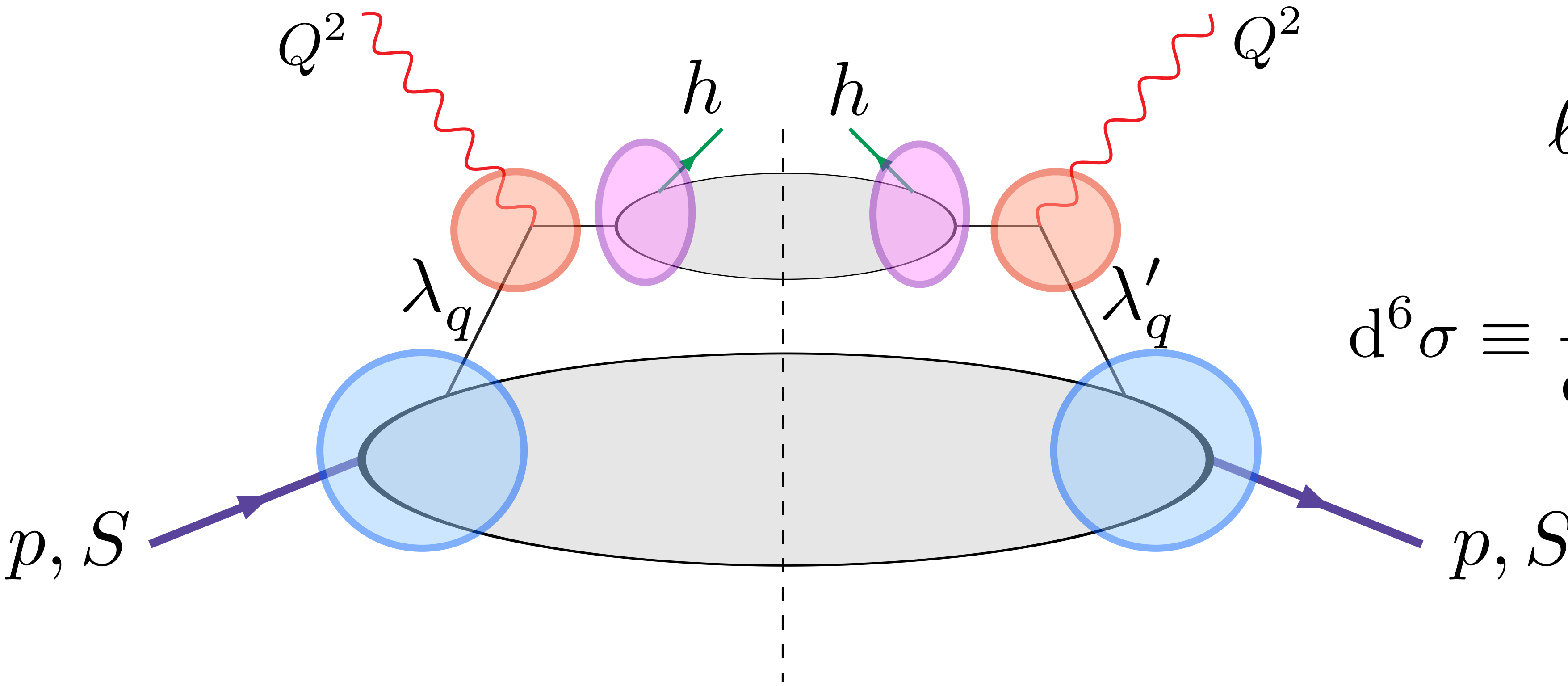
role of intrinsic motion in other processes:

D-Y processes

A_N in $pp \to h + X$

....

TMDs in SIDIS



$$\ell\, p^{\uparrow} \to \ell\, h\, X$$

$$\mathrm{d}^6\sigma \equiv \frac{\mathrm{d}^6\sigma^{\ell p^{\uparrow} \to \ell h X}}{\mathrm{d}x_{\!{}_B}\, \mathrm{d}Q^2\, \mathrm{d}z_h\, \mathrm{d}^2\boldsymbol{P}_T\, \mathrm{d}\phi_S}$$

TMD factorization holds at large Q^2, and $P_T \approx k_\perp \approx \Lambda_{\rm QCD}$

Two scales: $P_T \ll Q^2$

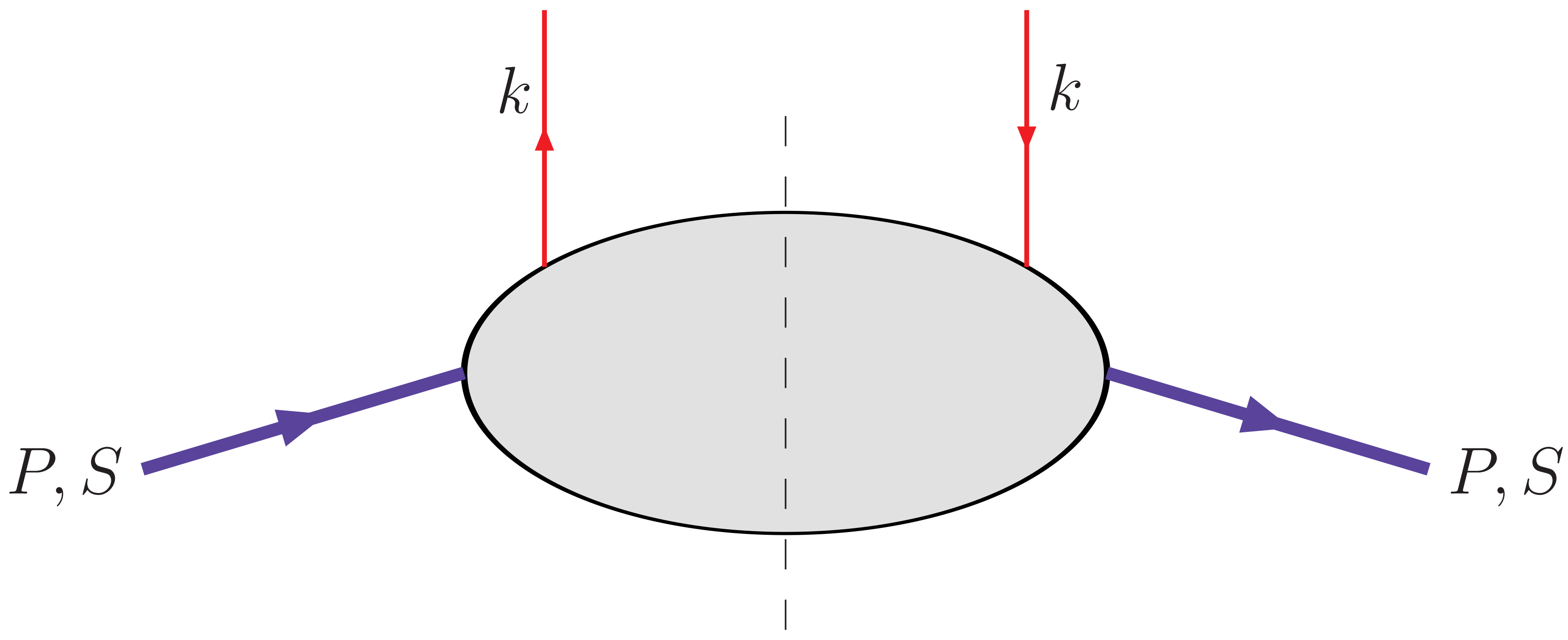
$$\boldsymbol{p}_\perp \simeq \boldsymbol{P}_T - z_h\, \boldsymbol{k}_\perp$$

$$\mathrm{d}\sigma^{\ell p \to \ell h X} = \sum_q f_q(x, \boldsymbol{k}_\perp; Q^2) \otimes \mathrm{d}\hat{\sigma}^{\ell q \to \ell q}(y, \boldsymbol{k}_\perp; Q^2) \otimes D_q^h(z, \boldsymbol{p}_\perp; Q^2)$$

(Collins, Soper, Ji, J.P. Ma, Yuan, Qiu, Vogelsang, Collins, Metz)

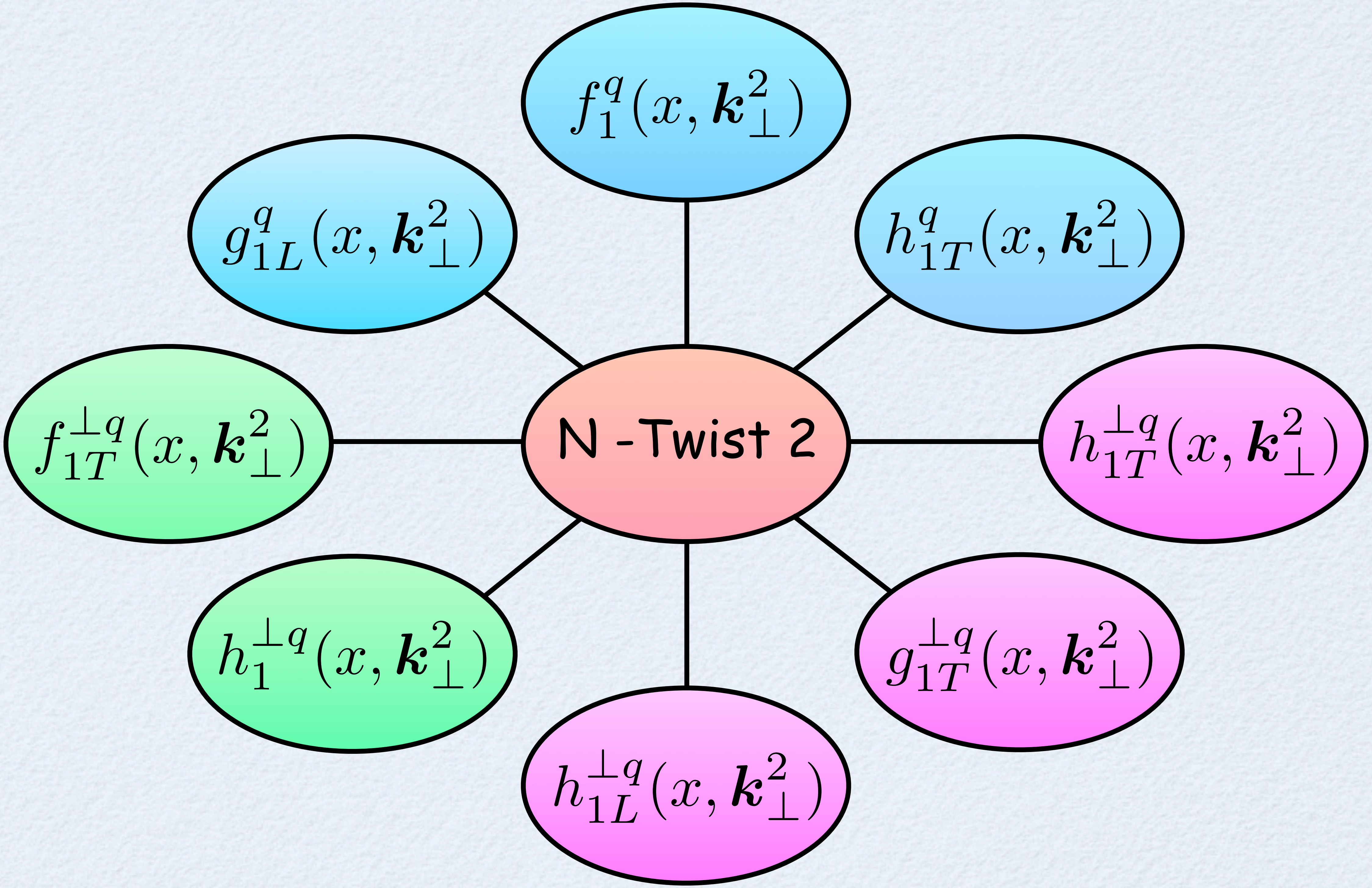
TMDs: the leading-twist correlator, with intrinsic $k_\perp$, contains 8 independent functions

$$\Phi(x, \boldsymbol{k}_\perp) = \frac{1}{2}\Big[f_1 \not{n}_+ + f_{1T}^\perp \frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu n_+^\nu k_\perp^\rho S_T^\sigma}{M} + \Big(S_L g_{1L} + \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_T}{M} g_{1T}^\perp \Big) \gamma^5 \not{n}_+$$
$$+ h_{1T} i\sigma_{\mu\nu}\gamma^5 n_+^\mu S_T^\nu + \Big(S_L h_{1L}^\perp + \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_T}{M} h_{1T}^\perp \Big) \frac{i\sigma_{\mu\nu}\gamma^5 n_+^\mu k_\perp^\nu}{M}$$
$$+ h_1^\perp \frac{\sigma_{\mu\nu} k_\perp^\mu n_+^\nu}{M} \Big]$$



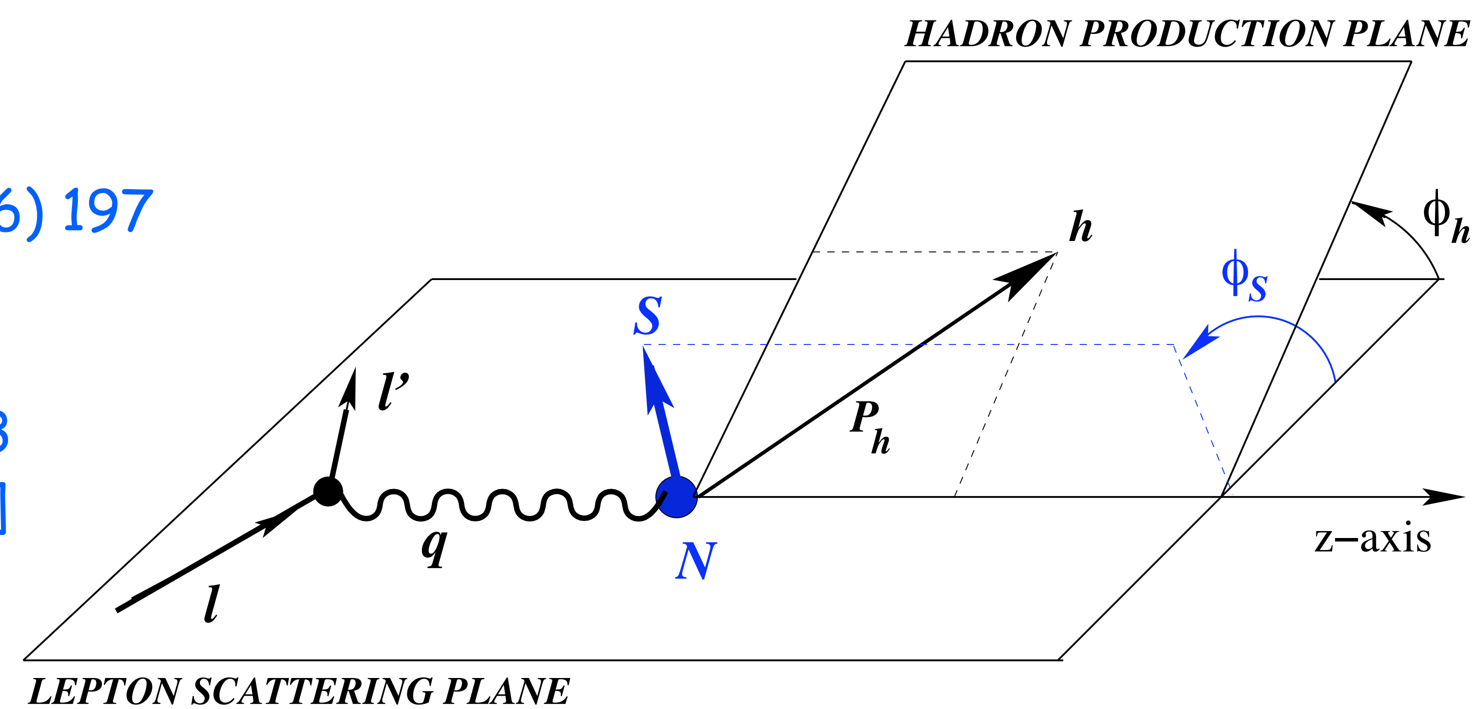
with partonic interpretation

The nucleon at twist-2

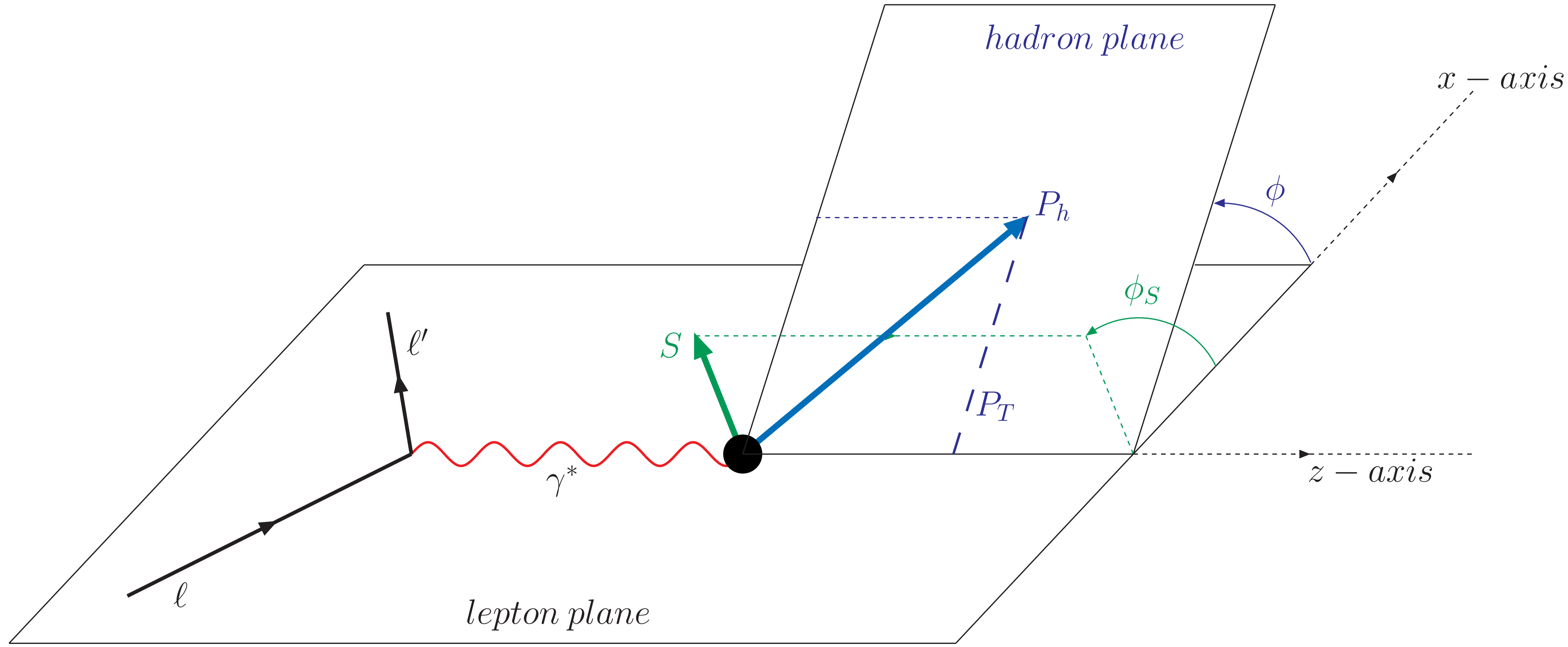


$$
\begin{aligned}
\frac{\mathrm{d}\sigma}{\mathrm{d}\phi} = \; & F_{UU} + \cos(2\phi)\, F_{UU}^{\cos(2\phi)} + \frac{1}{Q}\,\cos\phi\, F_{UU}^{\cos\phi} + \lambda\,\frac{1}{Q}\,\sin\phi\, F_{LU}^{\sin\phi} \\
& + S_L \left\{ \sin(2\phi)\, F_{UL}^{\sin(2\phi)} + \frac{1}{Q}\,\sin\phi\, F_{UL}^{\sin\phi} + \lambda \left[F_{LL} + \frac{1}{Q}\,\cos\phi\, F_{LL}^{\cos\phi} \right] \right\} \\
& + S_T \Big\{ \sin(\phi - \phi_S)\, F_{UT}^{\sin(\phi-\phi_S)} + \sin(\phi + \phi_S)\, F_{UT}^{\sin(\phi+\phi_S)} + \sin(3\phi - \phi_S)\, F_{UT}^{\sin(3\phi-\phi_S)} \\
& + \frac{1}{Q} \left[\sin(2\phi - \phi_S)\, F_{UT}^{\sin(2\phi-\phi_S)} + \sin\phi_S\, F_{UT}^{\sin\phi_S} \right] \\
& + \lambda \left[\cos(\phi - \phi_S)\, F_{LT}^{\cos(\phi-\phi_S)} + \frac{1}{Q} \left(\cos\phi_S\, F_{LT}^{\cos\phi_S} + \cos(2\phi - \phi_S)\, F_{LT}^{\cos(2\phi-\phi_S)} \right) \right] \Big\}
\end{aligned}
$$

Kotzinian, **NP B441** (1995) 234
Mulders and Tangermann, **NP B461** (1996) 197
Boer and Mulders, **PR D57** (1998) 5780
Bacchetta et al., **PL B595** (2004) 309
Bacchetta et al., **JHEP 0702** (2007) 093
Anselmino et al., arXiv:1101.1011 [hep-ph]



the $F^{(\dots)}_{S_B S_T}$ contain the TMDs



$$F_{UU} \sim \sum_a e_a^2 f_1^a \otimes D_1^a \qquad F_{LT}^{\cos(\phi-\phi_S)} \sim \sum_a e_a^2 g_{1T}^{\perp a} \otimes D_1^a$$

$$F_{LL} \sim \sum_a e_a^2 g_{1L}^a \otimes D_1^a \qquad F_{UT}^{\sin(\phi-\phi_S)} \sim \sum_a e_a^2 f_{1T}^{\perp a} \otimes D_1^a$$

chiral-even TMDs

$$F_{UU}^{\cos(2\phi)} \sim \sum_a e_a^2 h_1^{\perp a} \otimes H_1^{\perp a} \qquad F_{UT}^{\sin(\phi+\phi_S)} \sim \sum_a e_a^2 h_{1T}^a \otimes H_1^{\perp a}$$

$$F_{UL}^{\sin(2\phi)} \sim \sum_a e_a^2 h_{1L}^{\perp a} \otimes H_1^{\perp a} \qquad F_{UT}^{\sin(3\phi-\phi_S)} \sim \sum_a e_a^2 h_{1T}^{\perp a} \otimes H_1^{\perp a}$$

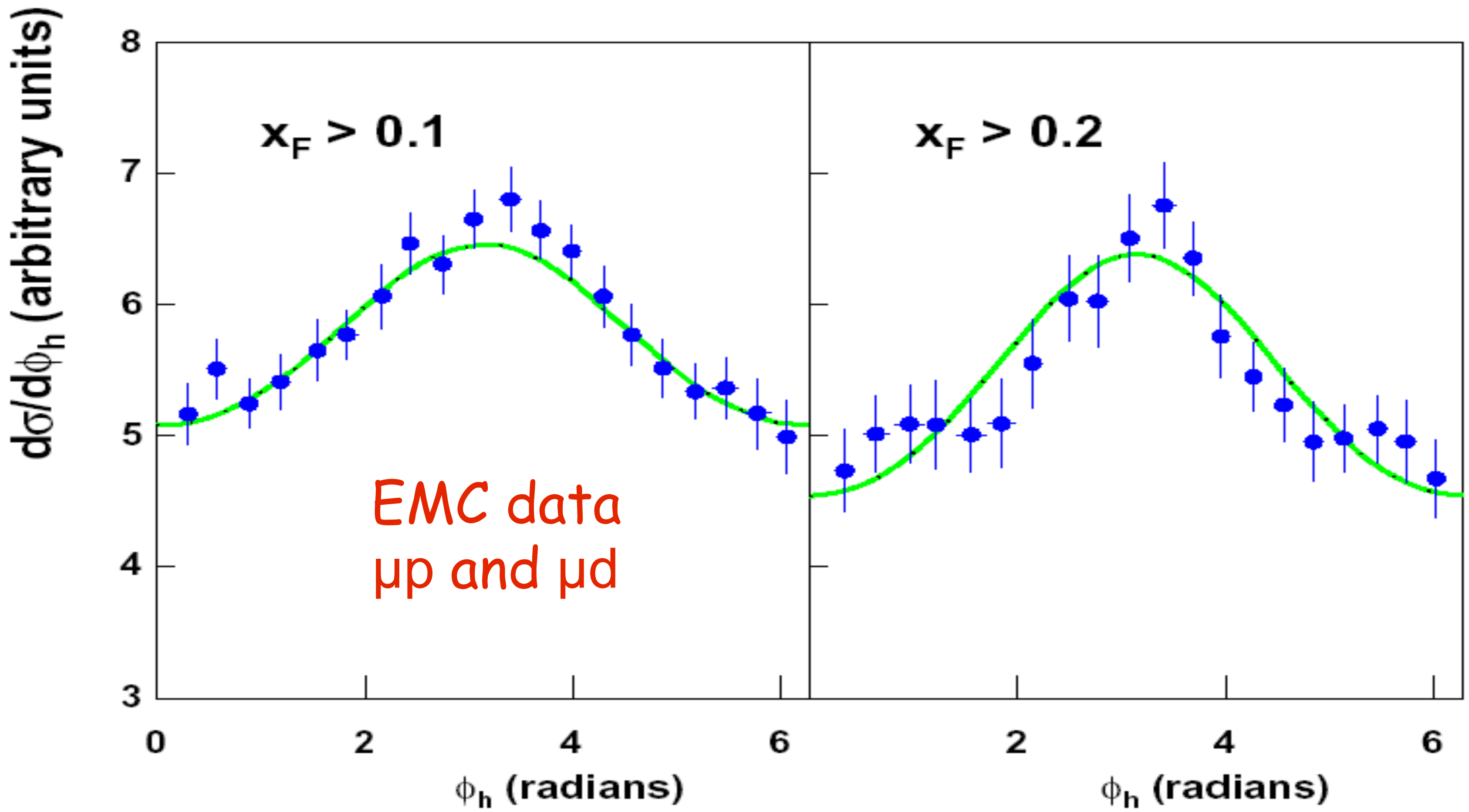
chiral-odd TMDs

$$f \otimes D \sim \int d^2\boldsymbol{k}_\perp \, d^2\boldsymbol{p}_\perp \, \delta^{(2)}(\boldsymbol{P}_T - z_h \boldsymbol{k}_\perp - \boldsymbol{p}_\perp) \, w(\boldsymbol{k}_\perp, \boldsymbol{P}_T) \, f(x_B, k_\perp) \, D(z_h, p_\perp)$$

TMDs in unpolarized SIDIS: "Cahn effect" at $\mathcal{O}(k_\perp/Q)$

$$\frac{1}{Q}\cos\phi\, F_{UU}^{\cos\phi} \sim \underbrace{f_1^q \otimes D_1^q \otimes \mathrm{d}\hat\sigma}_{} + \left(h_1^{q\perp} \otimes H_1^{q\perp} \otimes \mathrm{d}\Delta\hat\sigma\right)$$

$$d\hat\sigma^{\ell q\to\ell q} \propto \hat s^2 + \hat u^2 = \frac{Q^4}{y^2}\left[1+(1-y)^2 - 4\,\frac{k_\perp}{Q}\,(2-y)\,\sqrt{1-y}\,\cos\varphi\right]$$



assuming gaussian $k_\perp$ and $p_\perp$ dependences:

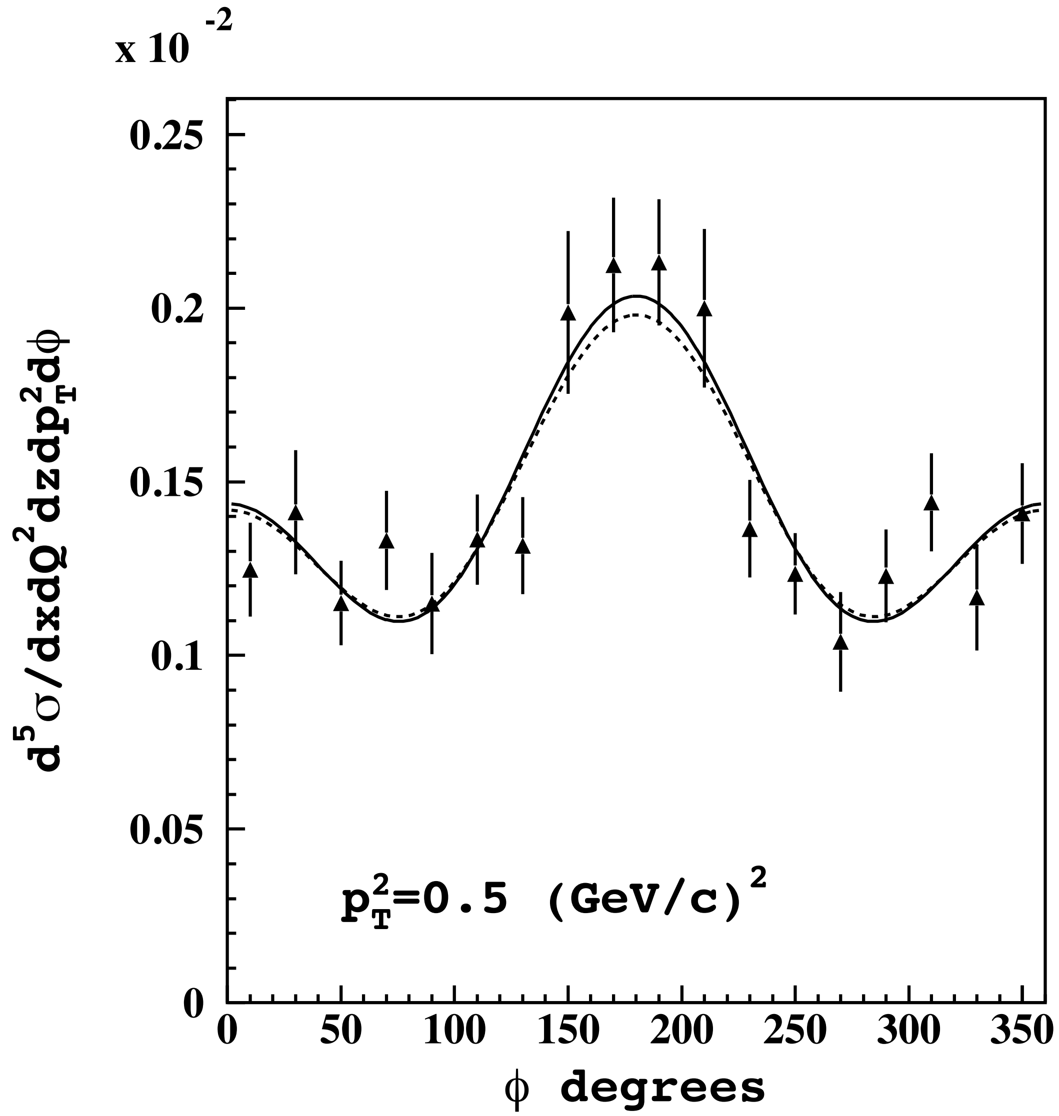
$$\langle k_\perp^2\rangle = 0.28\ (\mathrm{GeV})^2 \qquad \langle p_\perp^2\rangle = 0.25\ (\mathrm{GeV})^2$$

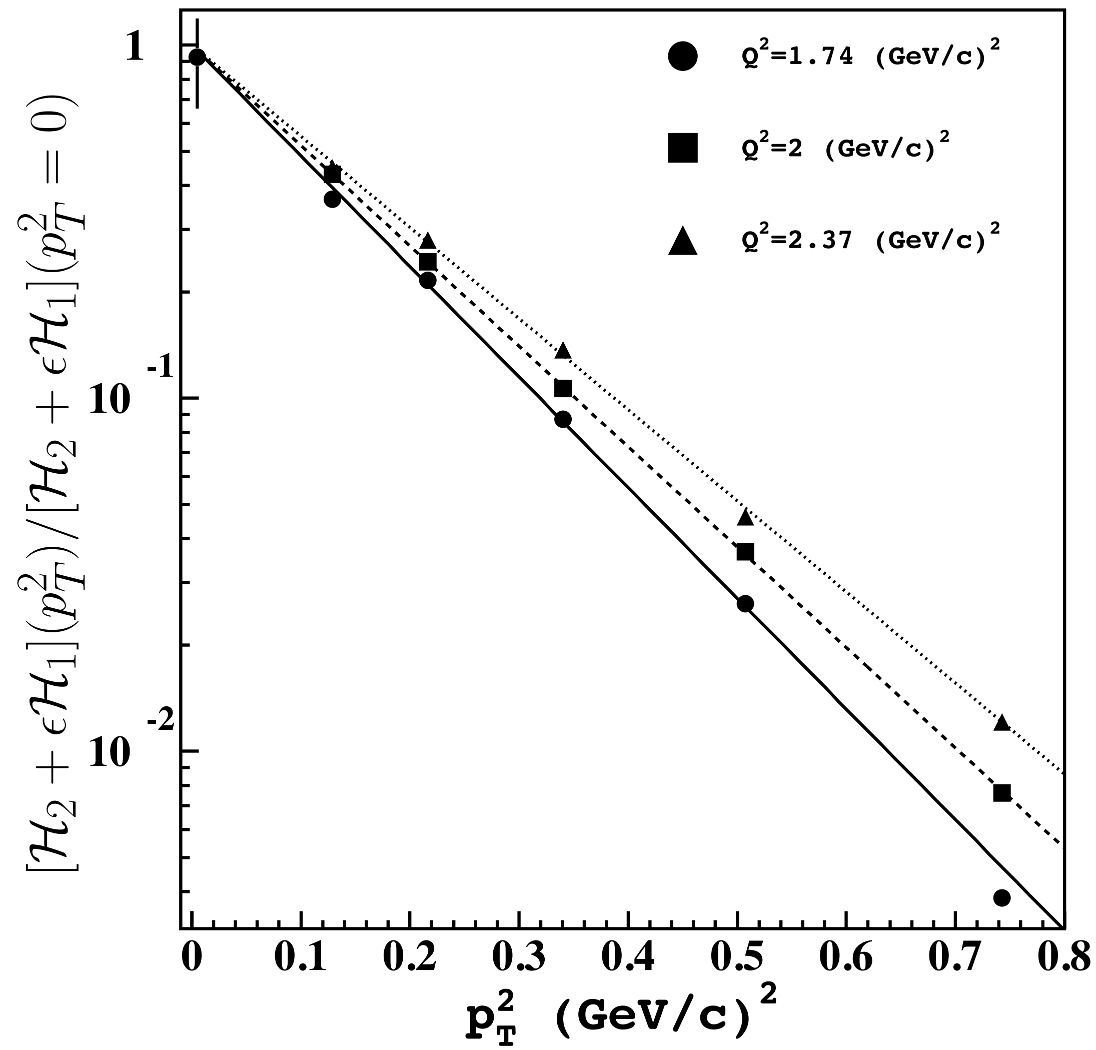
M.A., M. Boglione, U. D'Alesio, A. Kotzinian, F. Murgia and A. Prokudin

CLAS data

arXiv:. 0809.1153v5, PRD 80,032004 (2009)

$$\frac{\mathrm{d}^5\sigma}{\mathrm{d}x\,\mathrm{d}Q^2\,\mathrm{d}z\,\mathrm{d}P_T^2\,\mathrm{d}\phi} = C\left[\epsilon\mathcal{H}_1 + \mathcal{H}_2 + A\cos\phi + B\cos(2\phi)\right]$$

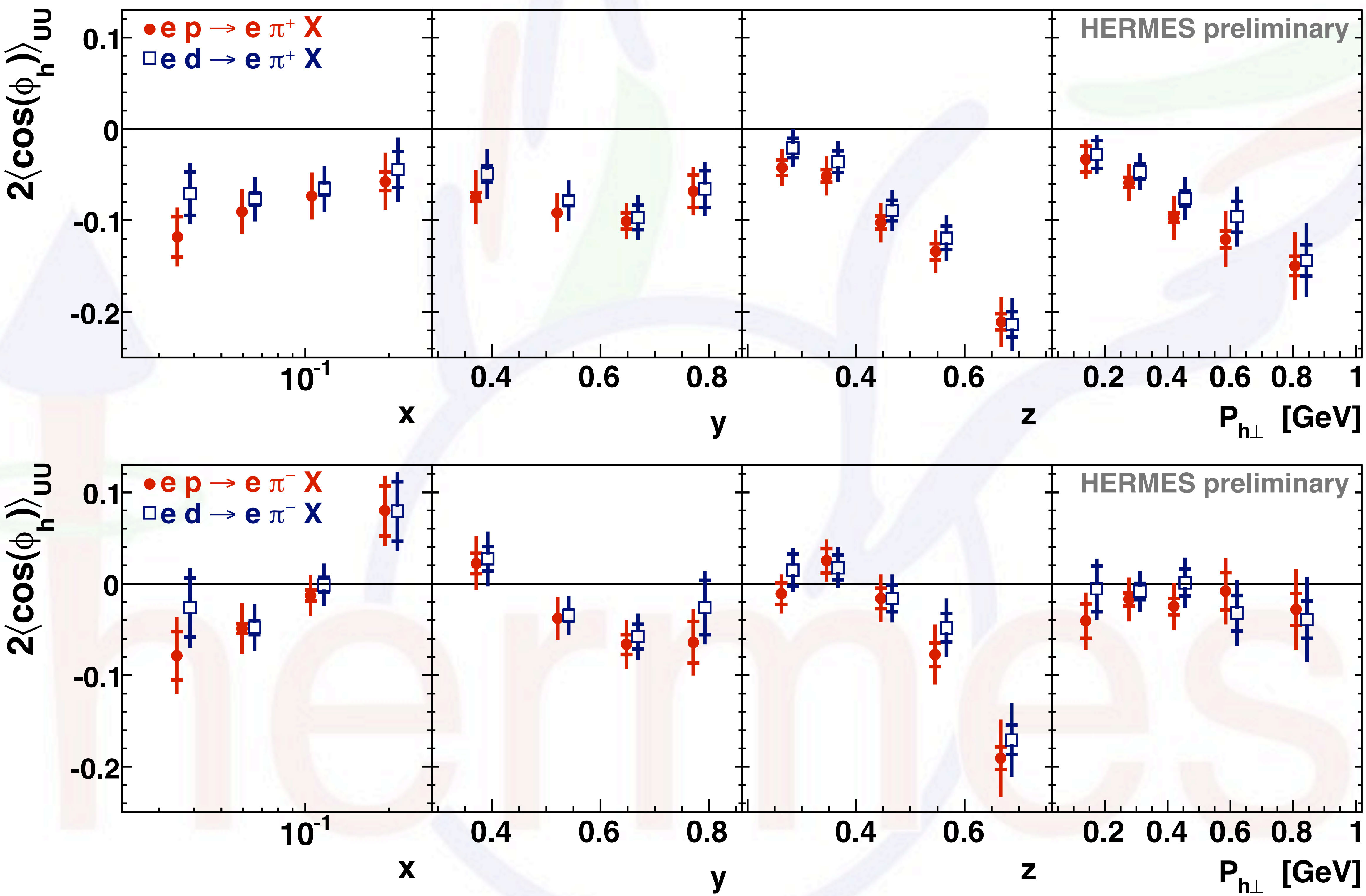


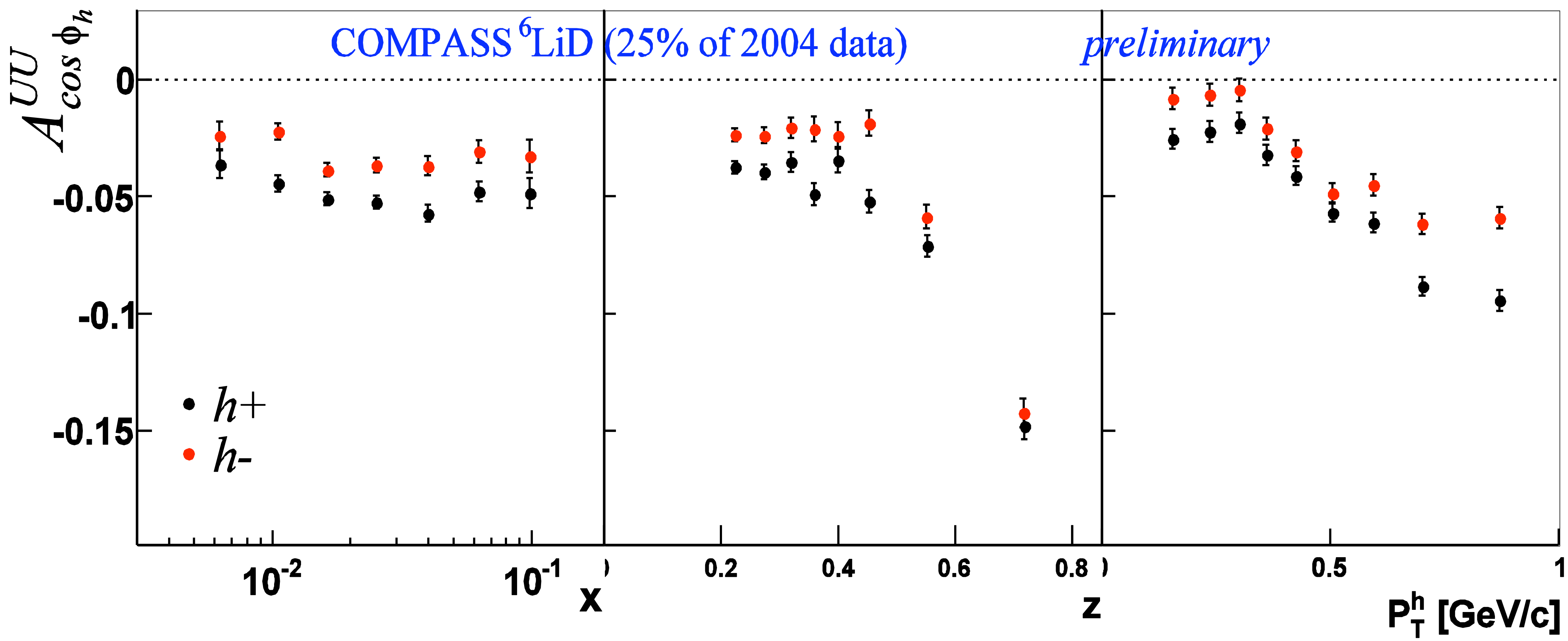


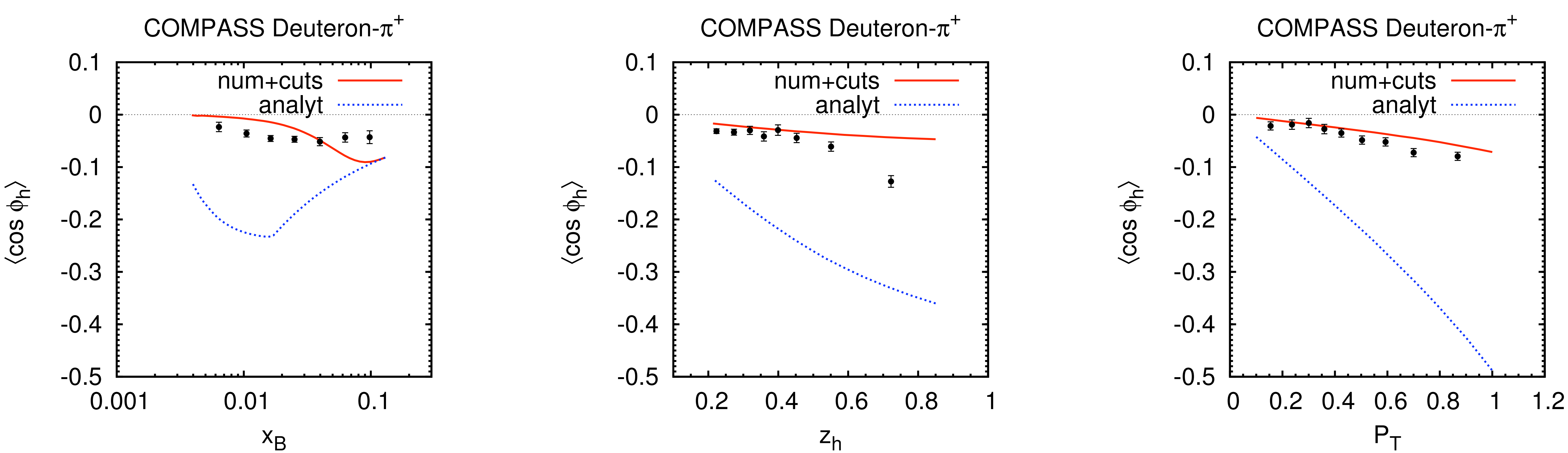
evidence in favor of gaussian dependence

Schweitzer, Teckentrup, Metz, arXiv:1003.2190

“Cahn modulation” - proton vs. deuteron







Boglione, Melis, Prokudin, PRD 84 (2011) 034033 the azimuthal dependence induced by intrinsic motion is clearly observed

phenomenolgical analysis and data need much improvement

Gaussian $k_\perp$ distribution of TMDs?

$$\langle k_\perp^2 \rangle (x, Q^2) \quad \langle p_\perp^2 \rangle (z, Q^2)$$

x, z dependence?

flavour dependence?

energy dependence?

$k_\perp$ dependence of Δq vs. q?

more data covering wider kinematical ranges

Siver function phenomenology in SIDIS

M.Anselmino, M.Boglione, J.C.Collins, U.D'Alesio, A.V.Efremov, K.Goeke, A.Kotzinian, S.Menzel, A.Metz, F.Murgia, A.Prokudin, P.Schweitzer, W.Vogelsang, F.Yuan, A. Bacchetta, M. Radici

$$2\,\langle \sin(\phi-\phi_S)\rangle = A_{UT}^{\sin(\phi-\phi_S)} \equiv 2\,\frac{\overbrace{\int \mathrm{d}\phi\,\mathrm{d}\phi_S\,(\mathrm{d}\sigma^{\uparrow}-\mathrm{d}\sigma^{\downarrow})\,\sin(\phi-\phi_S)}^{F_{UT}^{\sin(\phi-\phi_S)}}}{\int \mathrm{d}\phi\,\mathrm{d}\phi_S\,(\mathrm{d}\sigma^{\uparrow}+\mathrm{d}\sigma^{\downarrow})}$$

extraction of Sivers function based on very simple parameterization, with x and $\mathbf{k}_\perp$ factorization. Typically:

$$\Delta^N f_{q/p^{\uparrow}}(x,k_\perp) = -\frac{2k_\perp}{M} f_{1T}^{\perp q}(x,k_\perp) = N x^{\alpha}(1-x)^{\beta}\,h(k_\perp) f_{q/p}(x,k_\perp)$$

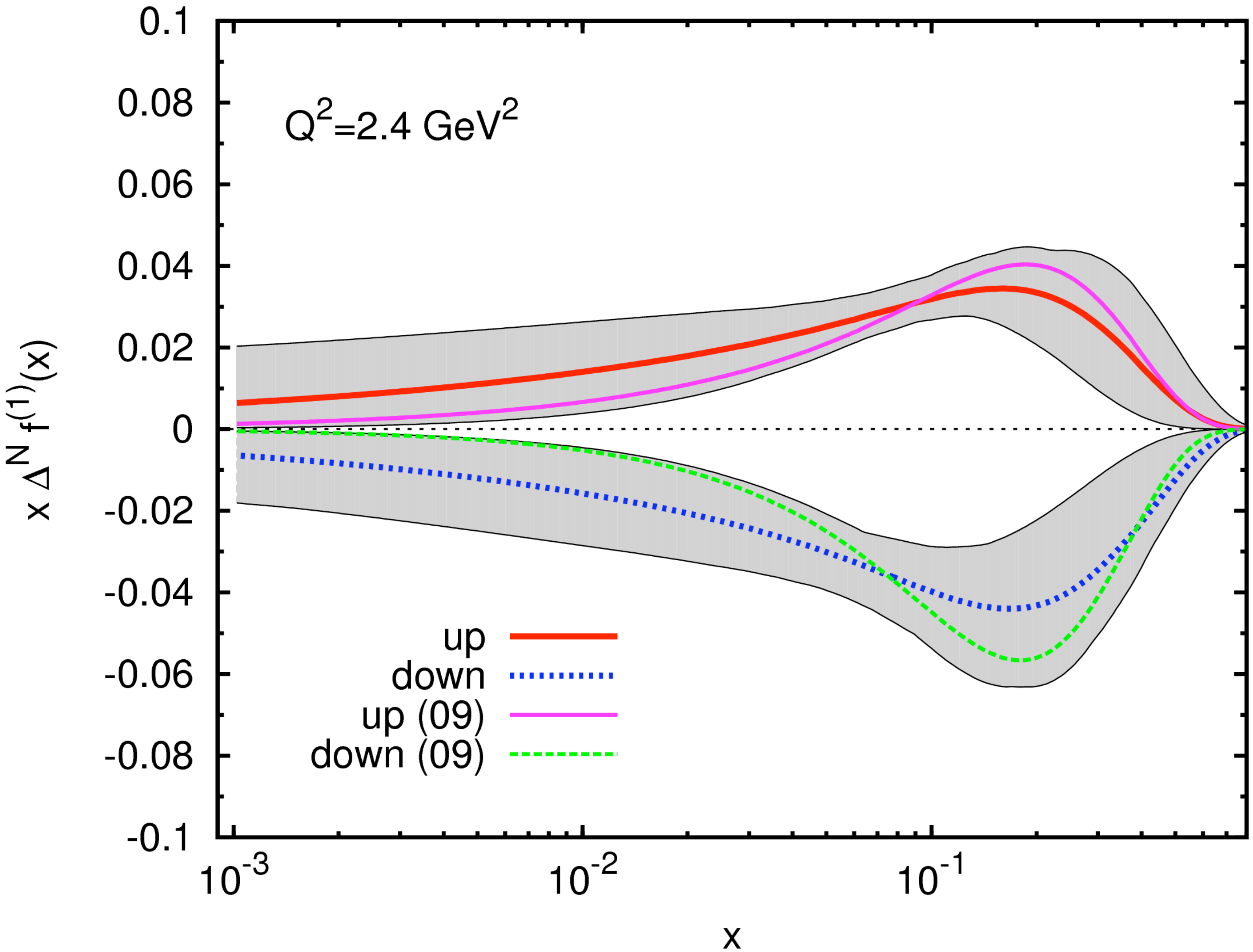
with

$$f_{q/p}(x,k_\perp) = f_q(x)\frac{1}{\pi\langle k_\perp^2\rangle}\,e^{-k_\perp^2/\langle k_\perp^2\rangle}$$

$\langle k_\perp^2\rangle$ constant and flavour independent

simple Sivers functions for u and d quarks are sufficient
to fit the available SIDIS data

large and very small x dependence not constrained by data



new and previous extraction of u and d Sivers functions

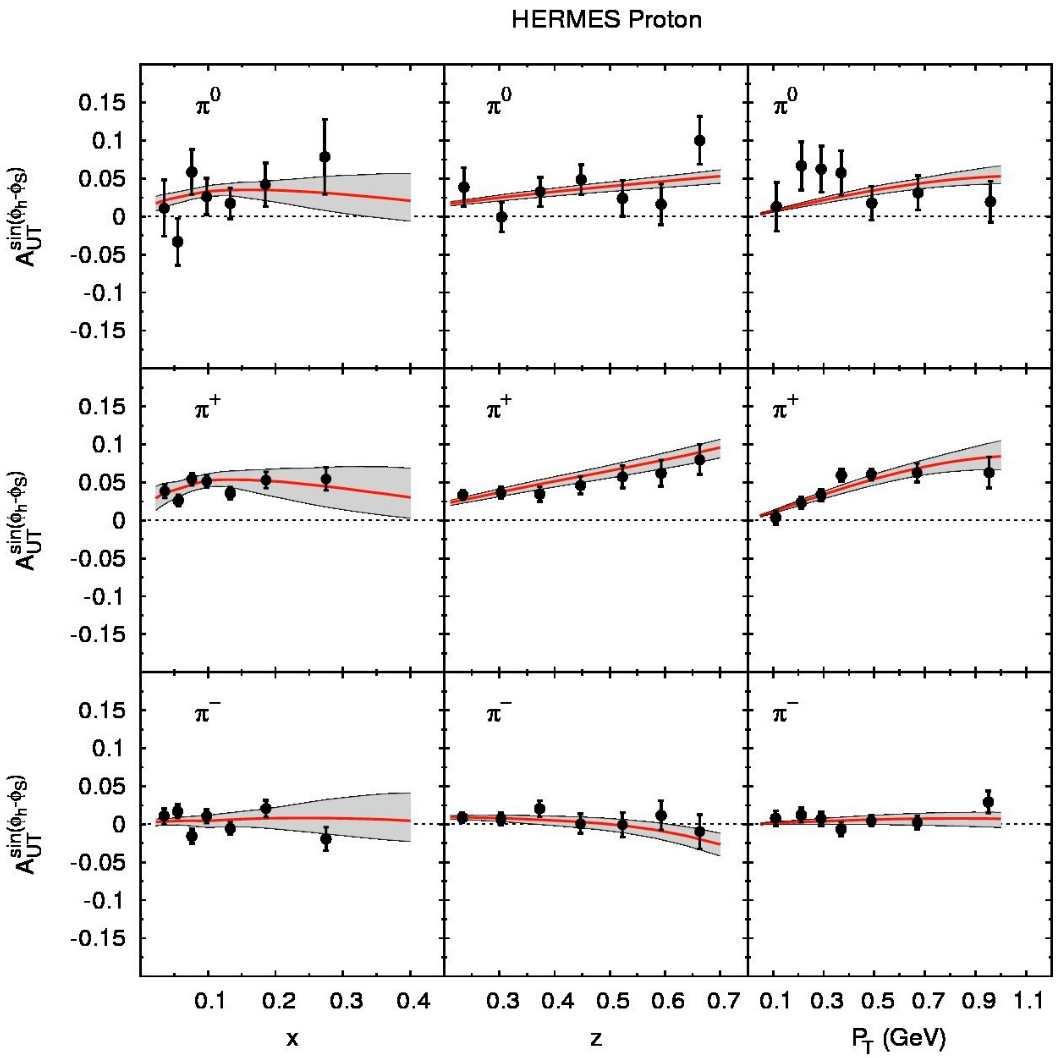
S. Melis and A. Prokudin, preliminary results

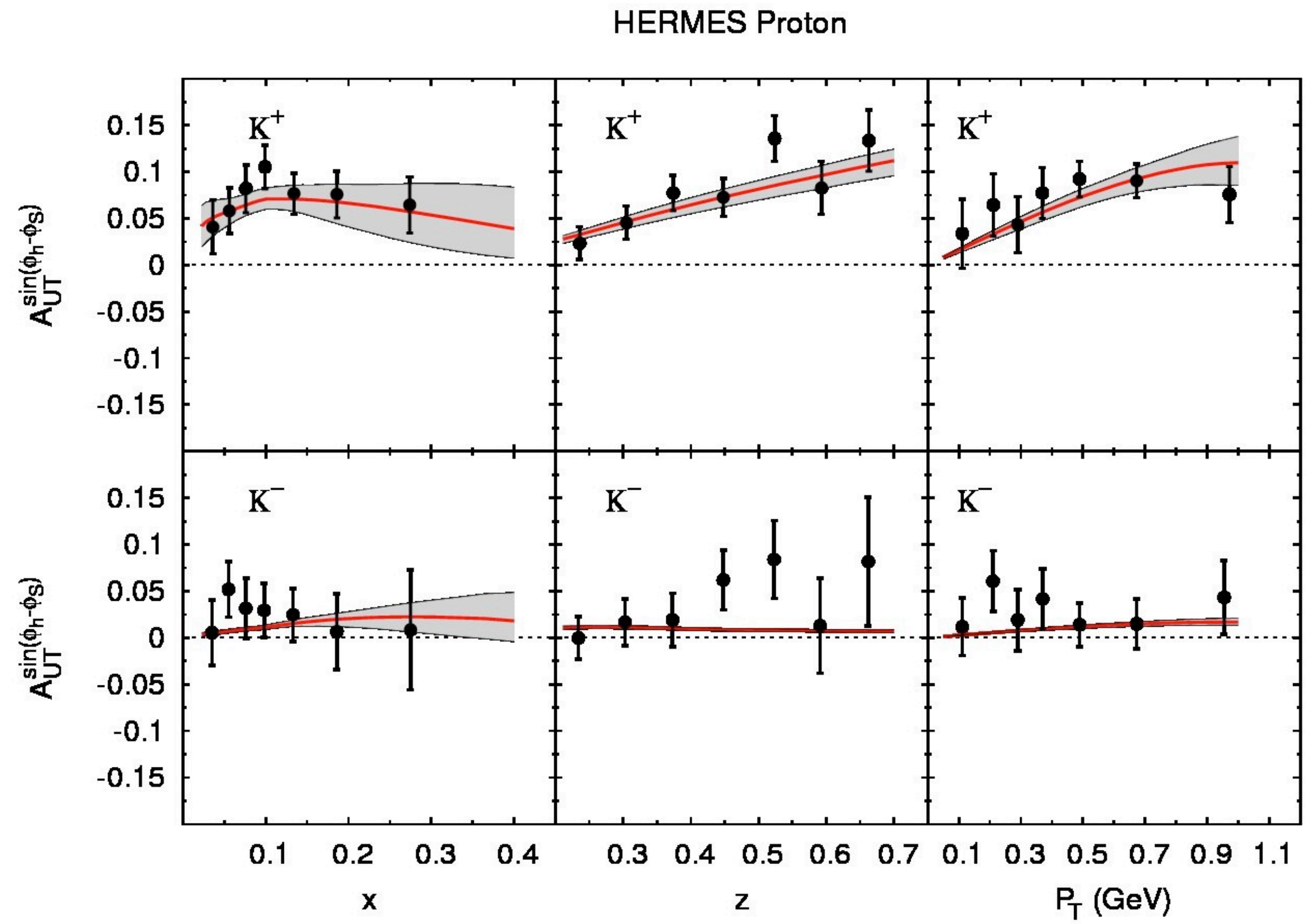
Anselmino et al. Eur. Phys. J. A39,89 (2009)

similar results from other groups

S. Melis, talk at Transversity 2011

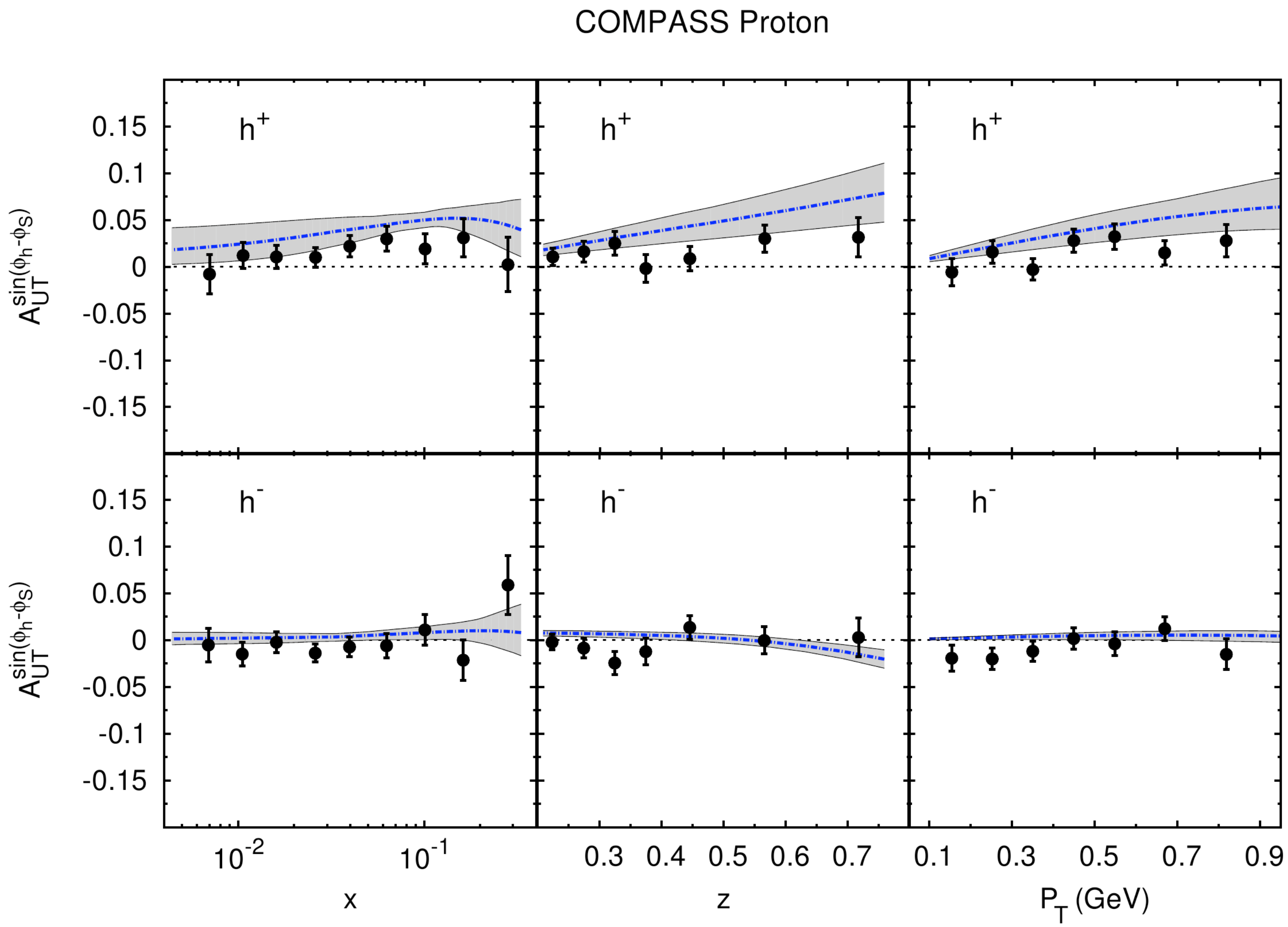
FIT u & d only



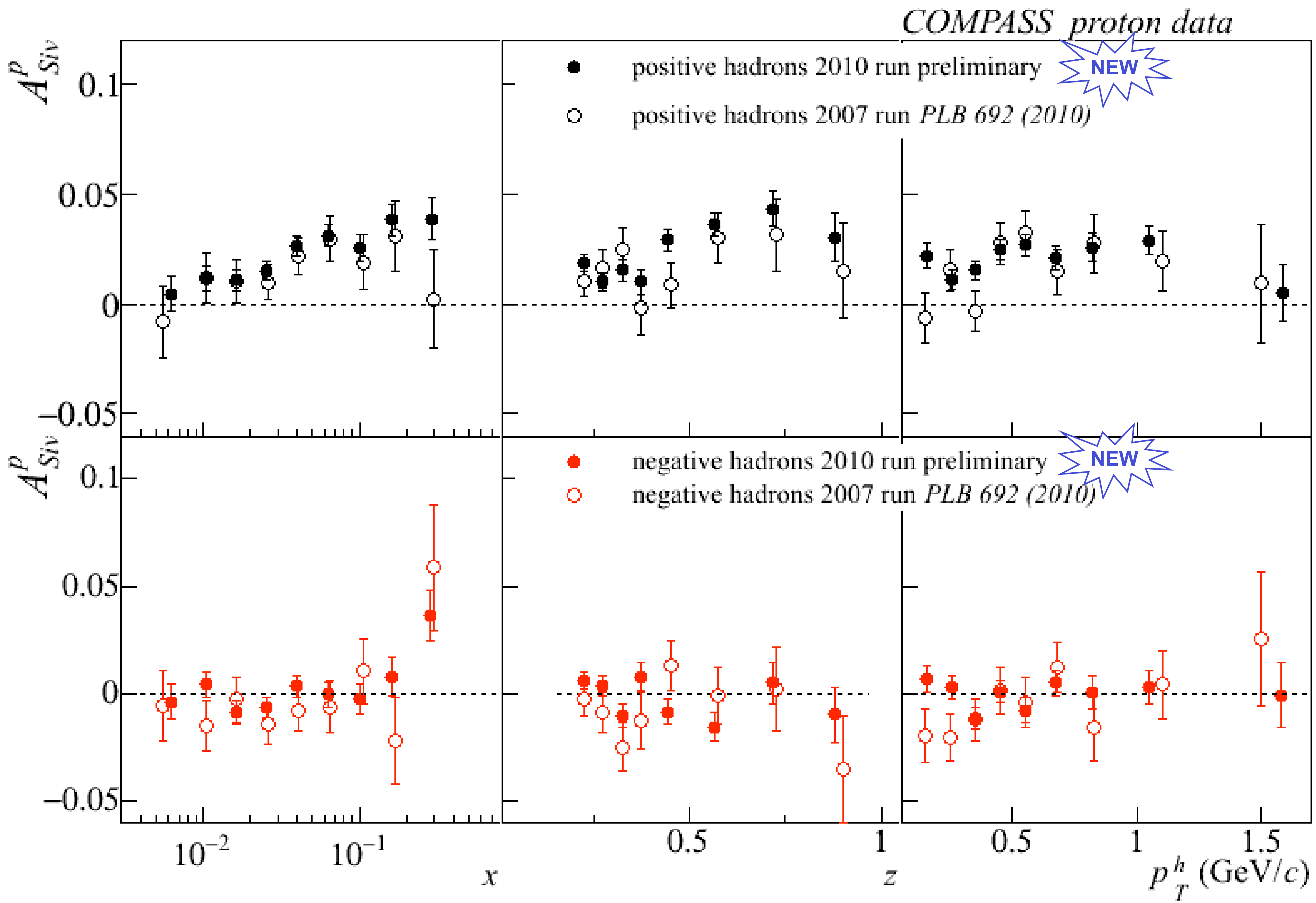


S. Melis, talk at Transversity 2011

FIT u & d only

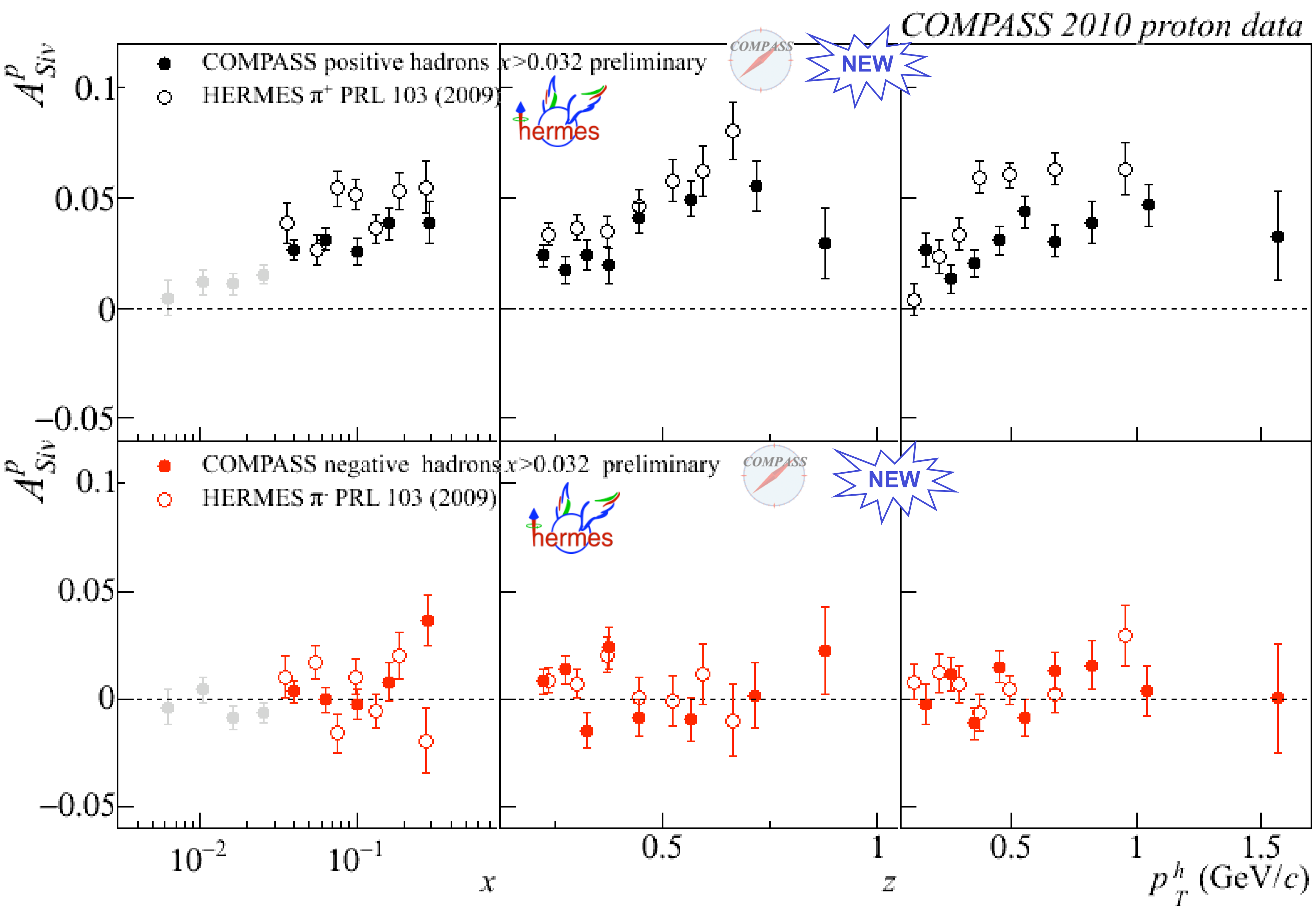


the Sivers asymmetry 2010 vs 2007 data



the Sivers asymmetry 2010 data

x > 0.032 region - comparison with HERMES results

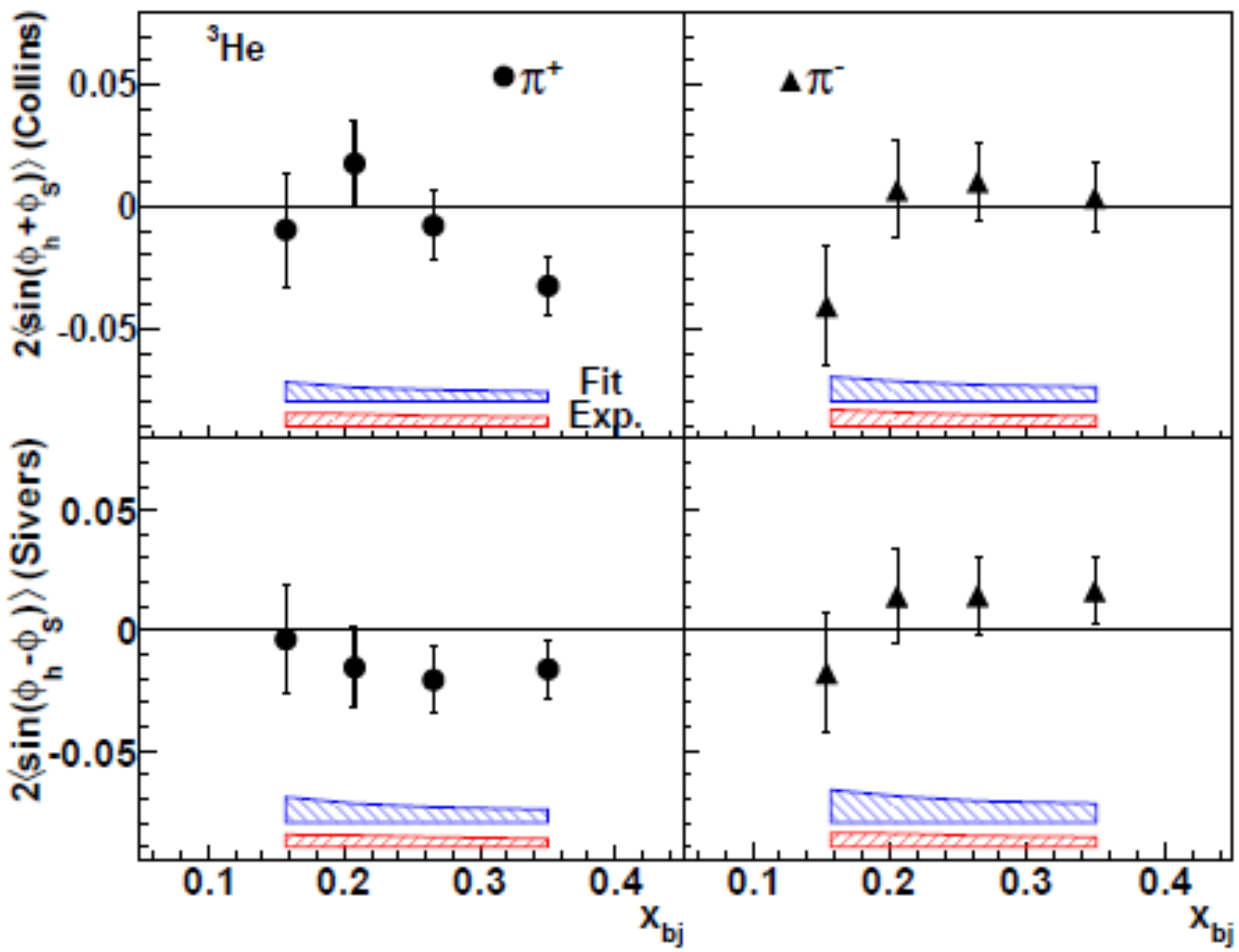


JLab - Hall A

^{3}He Target Single-Spin Asymmetry in SIDIS

arXiv: 1106.0363, submitted to PRL

$$^3\mathrm{He}^{\uparrow}(e,e'h),\ \ h=\pi^+,\pi^-$$

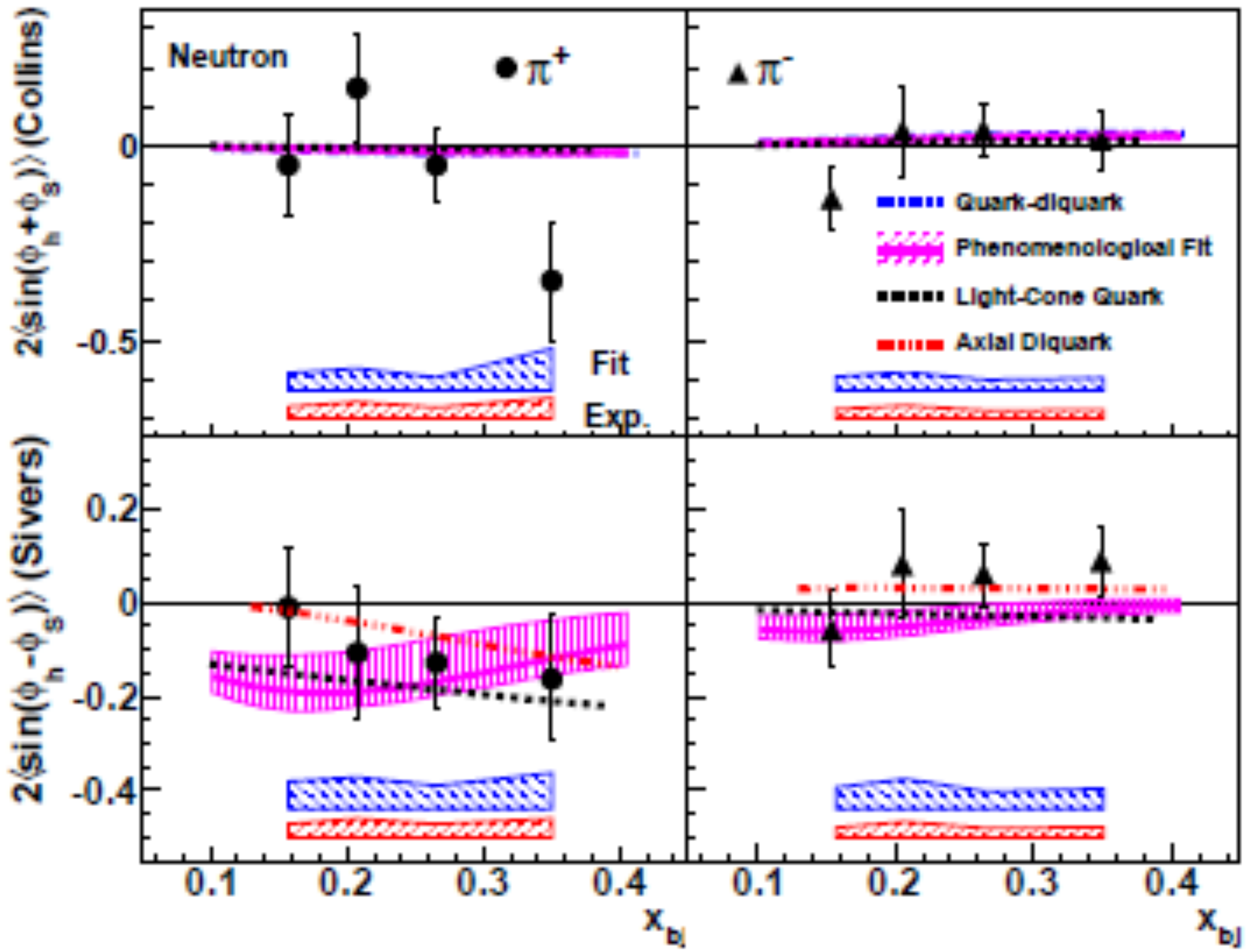


^{3}He Collins SSA small
Non-zero at highest x for π^+

^{3}He Sivers SSA:
negative for π^+,

Blue band: model (fitting) uncertainties
Red band: other systematic uncertainties

Results on Neutron



Collins asymmetries are not large, except at x=0.34

Sivers

$\pi^{+}\,(u\bar{d})$ negative

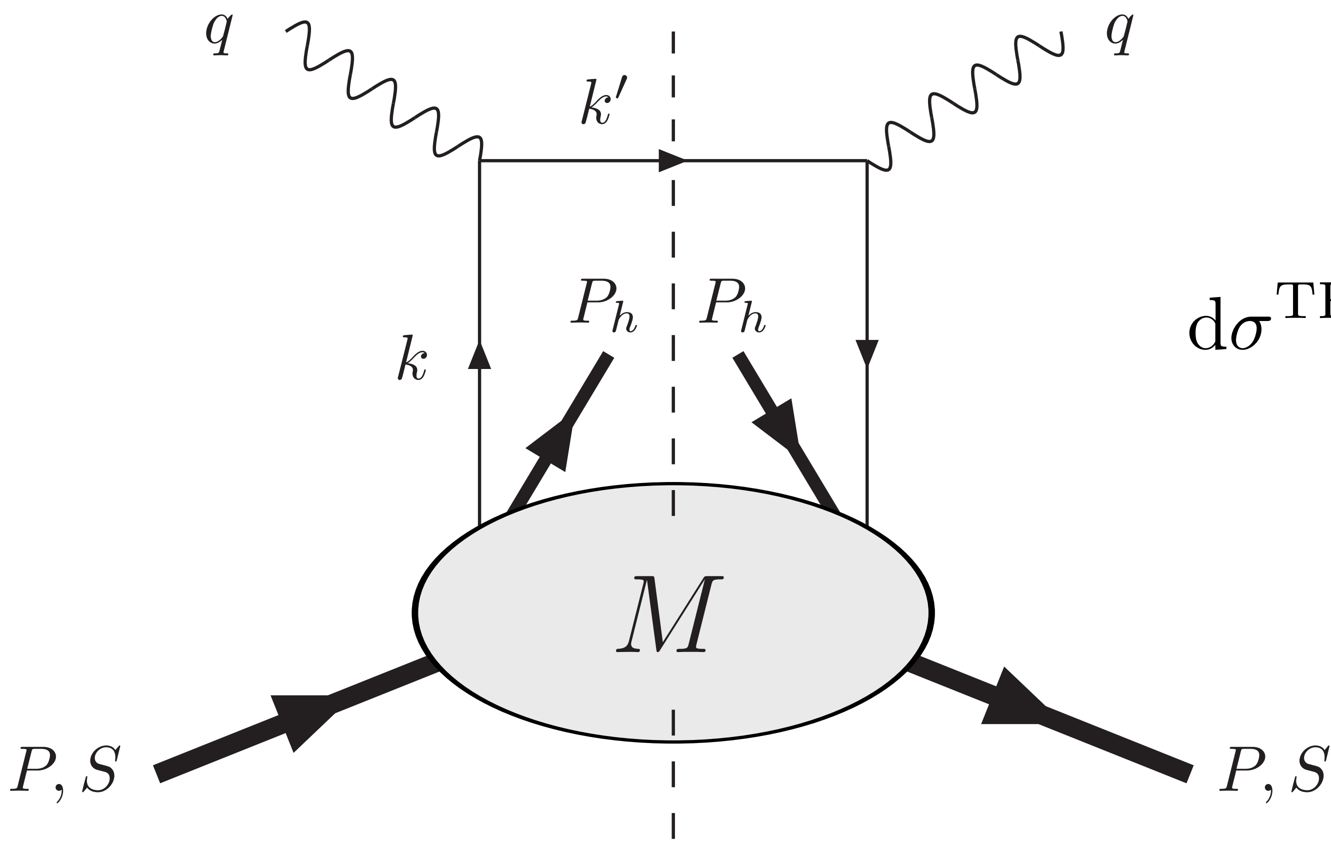
Blue band: model (fitting) uncertainties

Red band: other systematic uncertainties

Aghasyan talk at Transversity 2011

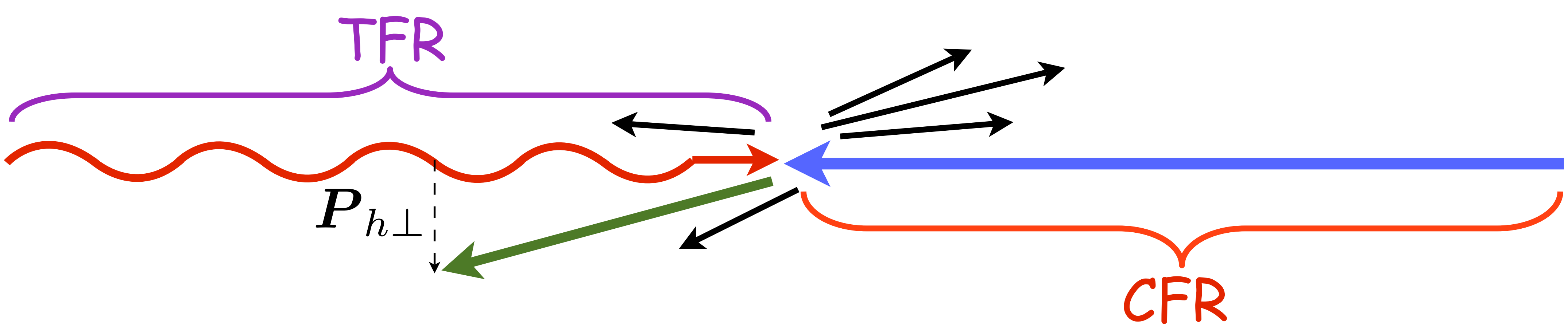
azimuthal dependences from target fragmentation region

(fracture functions)



$$d\sigma^{\mathrm{TFR}} = \sum_a \underbrace{M_a(x_B, \zeta, \boldsymbol{P}_{h\perp}^2)}_{\text{fracture functions}} \otimes d\hat{\sigma}(y)$$

$$\zeta \simeq \frac{E_h}{E} \simeq (1 - x_B)|x_F|$$



azimuthal modulations in TFR

(M.A, V. Barone, A. Kotzinian, PL B699 (2011) 108)

cross section for lepto-production of an unpolarized or spinless hadron in the TFR

$$\frac{\mathrm{d}\sigma^{\mathrm{TFR}}}{\mathrm{d}x_B\,\mathrm{d}y\,\mathrm{d}\zeta\,\mathrm{d}^2\boldsymbol{P}_{h\perp}\,\mathrm{d}\phi_S} = \frac{2\alpha_{\mathrm{em}}^2}{Q^2 y}\left\{\left(1-y+\frac{y^2}{2}\right)\right.$$
$$\times \sum_a e_a^2\left[M(x_B,\zeta,\boldsymbol{P}_{h\perp}^2) - |\boldsymbol{S}_\perp|\,\frac{|\boldsymbol{P}_{h\perp}|}{m_h}\,M_T^h(x_B,\zeta,\boldsymbol{P}_{h\perp}^2)\sin(\phi_h-\phi_S)\right]$$
$$+\lambda_l\,y\left(1-\frac{y}{2}\right)\sum_a e_a^2\left[S_\parallel\,\Delta M_L(x_B,\zeta,\boldsymbol{P}_{h\perp}^2)\right.$$
$$\left.\left. + \;|\boldsymbol{S}_\perp|\,\frac{|\boldsymbol{P}_{h\perp}|}{m_h}\,\Delta M_T^h(x_B,\zeta,\boldsymbol{P}_{h\perp}^2)\,\cos(\phi_h-\phi_S)\right]\right\}.$$

possible Sivers-like azimuthal dependence from target fragmentation region

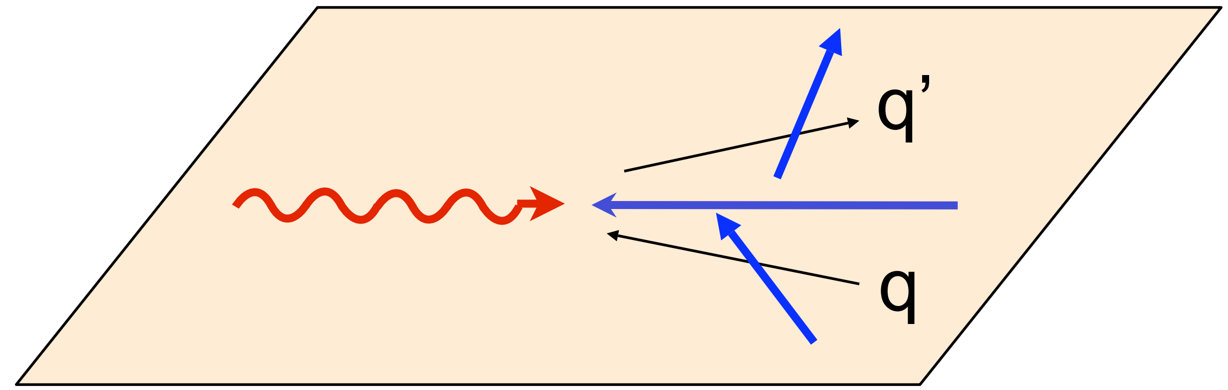
Sivers effect now observed by two experiments (+ Hall-A A_{UT} on neutrons), but needs further measurements

great improvement in study of QCD evolution
(Aybat, Rogers, arXiv:1101.5057)

Q^2 of data not so high, role of higher twists?
clear separation of TFR and CFR needed...
more sophisticated parameterization...
universality of Sivers function?...

Collins effect in SIDIS - $F_{UT}^{\sin(\phi+\phi_S)}$

$$D_{h/q,\boldsymbol{s}_q}(z,\boldsymbol{p}_\perp) = D_{h/p}(z,p_\perp) + \frac{1}{2}\Delta^N D_{h/q^\uparrow}(z,p_\perp)\,\boldsymbol{s}_q\cdot(\hat{\boldsymbol{p}}_q\times\hat{\boldsymbol{p}}_\perp)$$



$$\mathrm{d}\sigma^\uparrow - \mathrm{d}\sigma^\downarrow = \sum_q h_{1q}(x,k_\perp)\otimes \mathrm{d}\Delta\hat{\sigma}(y,\boldsymbol{k}_\perp)\otimes\Delta^N D_{h/q^\uparrow}(z,\boldsymbol{p}_\perp)$$

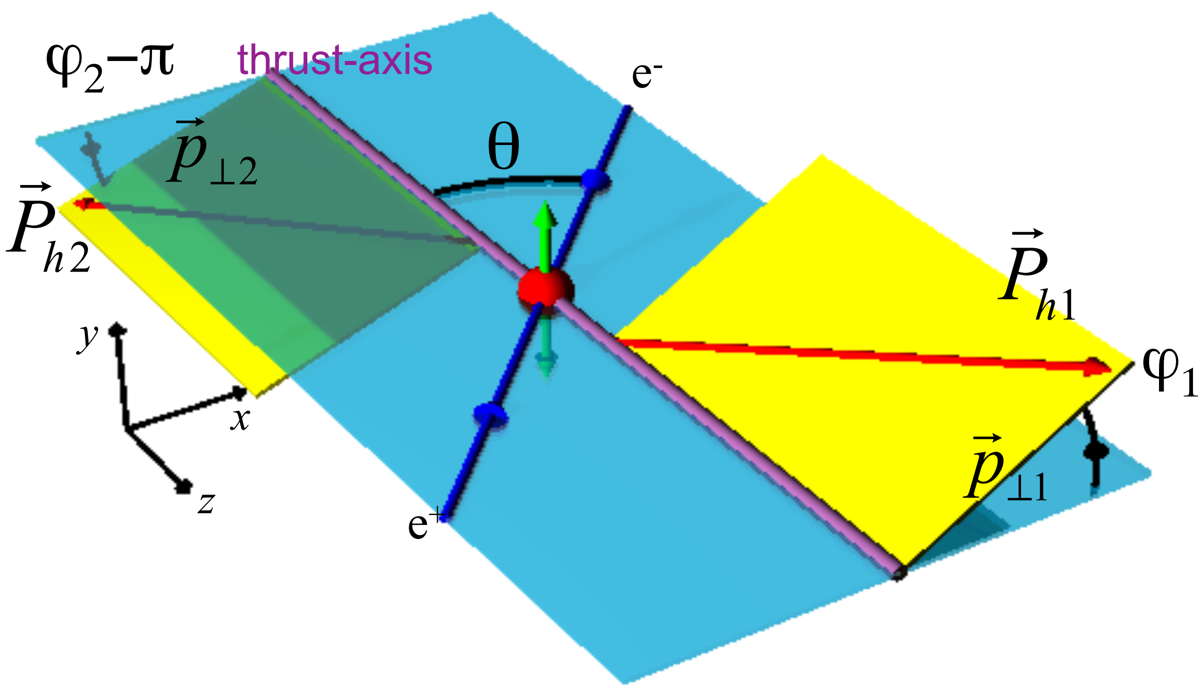
$$A_{UT}^{\sin(\phi+\phi_S)} \equiv 2\,\frac{\int \mathrm{d}\phi\,\mathrm{d}\phi_S\,[\mathrm{d}\sigma^\uparrow - \mathrm{d}\sigma^\downarrow]\,\sin(\phi+\phi_S)}{\int \mathrm{d}\phi\,\mathrm{d}\phi_S\,[\mathrm{d}\sigma^\uparrow + \mathrm{d}\sigma^\downarrow]}$$

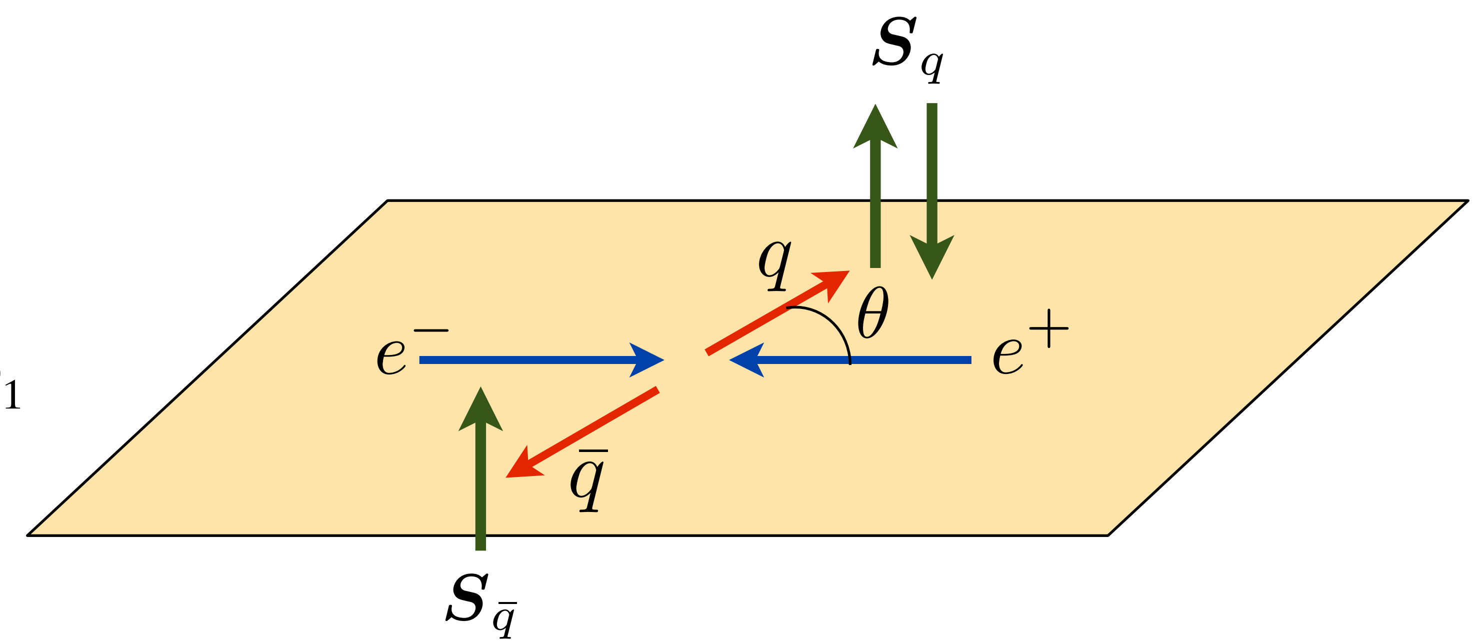
$$\mathrm{d}\Delta\hat{\sigma} = \mathrm{d}\hat{\sigma}^{\ell q^\uparrow\to\ell q^\uparrow} - \mathrm{d}\hat{\sigma}^{\ell q^\uparrow\to\ell q^\downarrow}$$

Collins effect in SIDIS couples to transversity

independent information on Collins function from e^+e^- processes

BELLE @ KEK

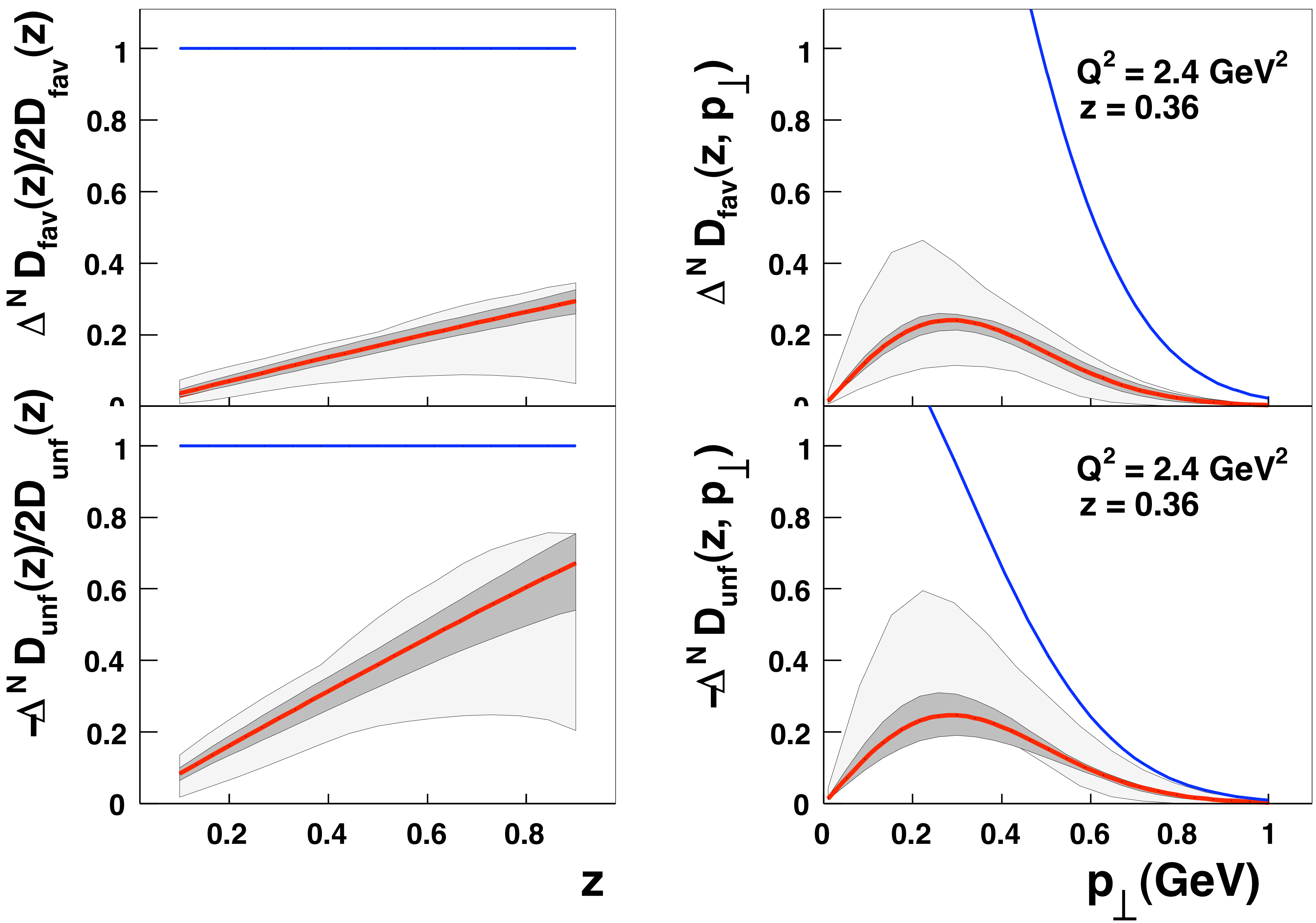




$$A_{12}(z_1,z_2,\theta,\varphi_1+\varphi_2) \equiv \frac{1}{\langle d\sigma\rangle}\,\frac{d\sigma^{e^+e^-\to h_1h_2X}}{dz_1\,dz_2\,d\cos\theta\,d(\varphi_1+\varphi_2)}$$

$$= 1+\frac{1}{4}\,\frac{\sin^2\theta}{1+\cos^2\theta}\,\cos(\varphi_1+\varphi_2)\times\frac{\sum_q e_q^2\,\Delta^N D_{h_1/q^\uparrow}(z_1)\,\Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)}{\sum_q e_q^2 D_{h_1/q}(z_1)\,D_{h_2/\bar{q}}(z_2)}$$

extracted Collins functions



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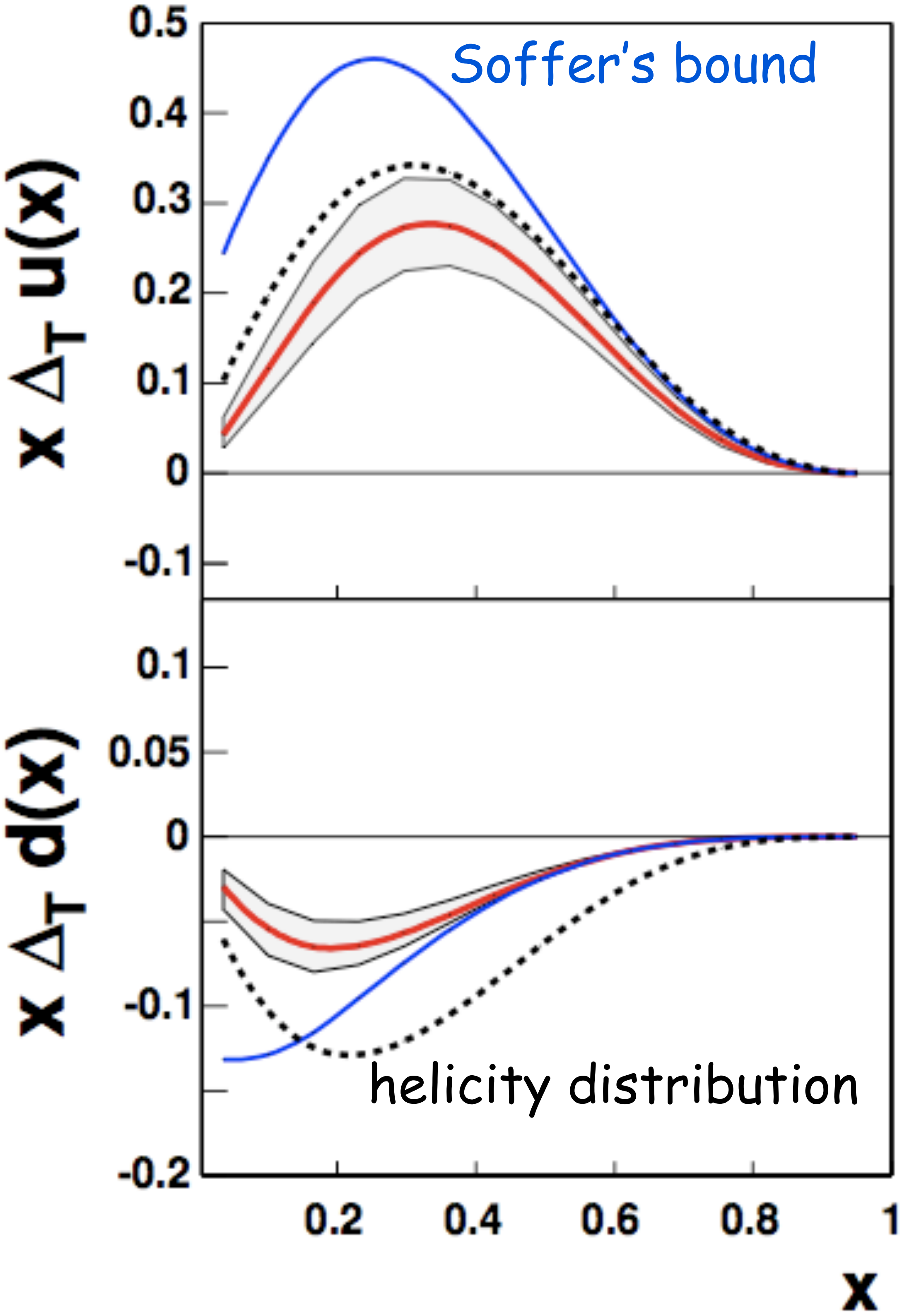
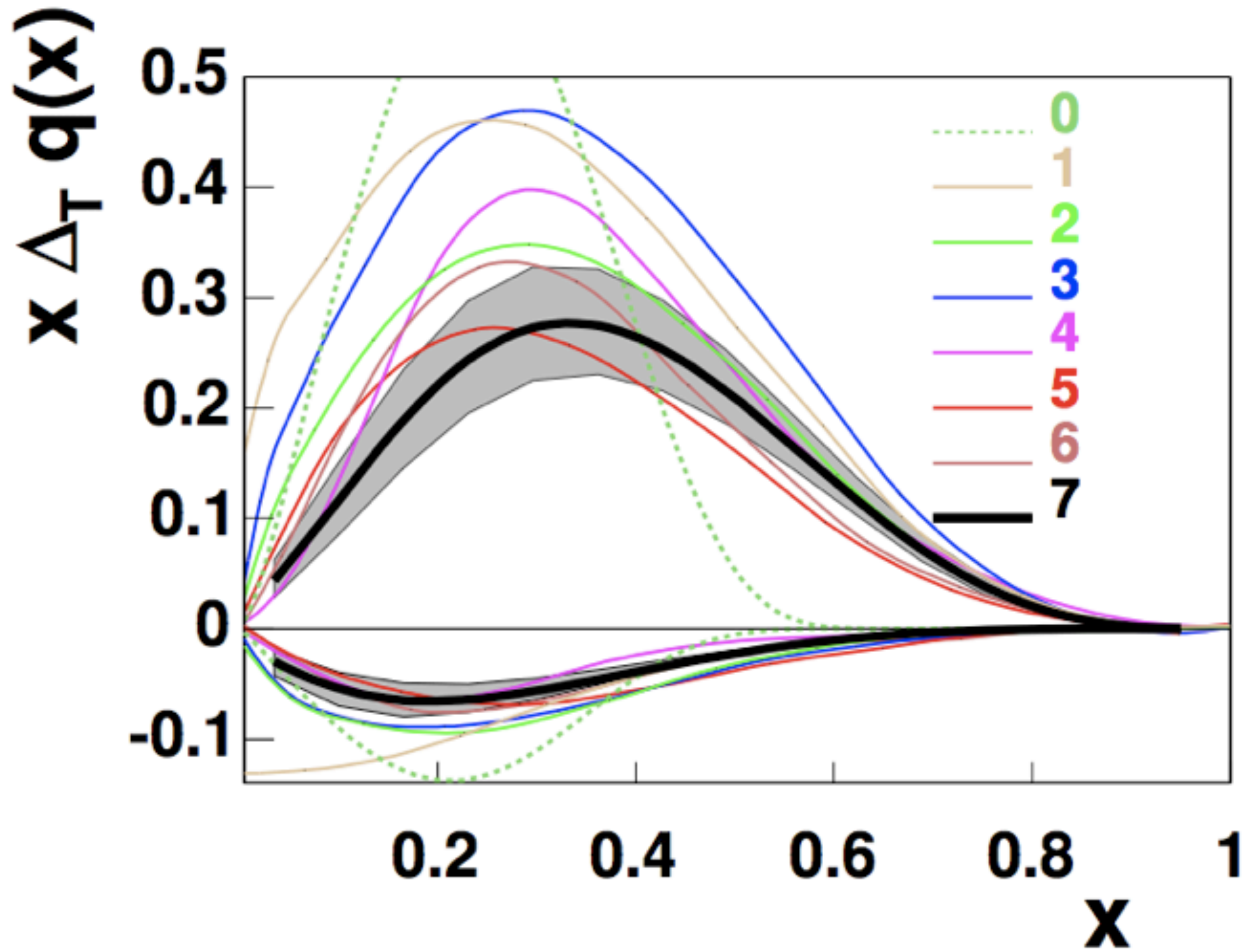
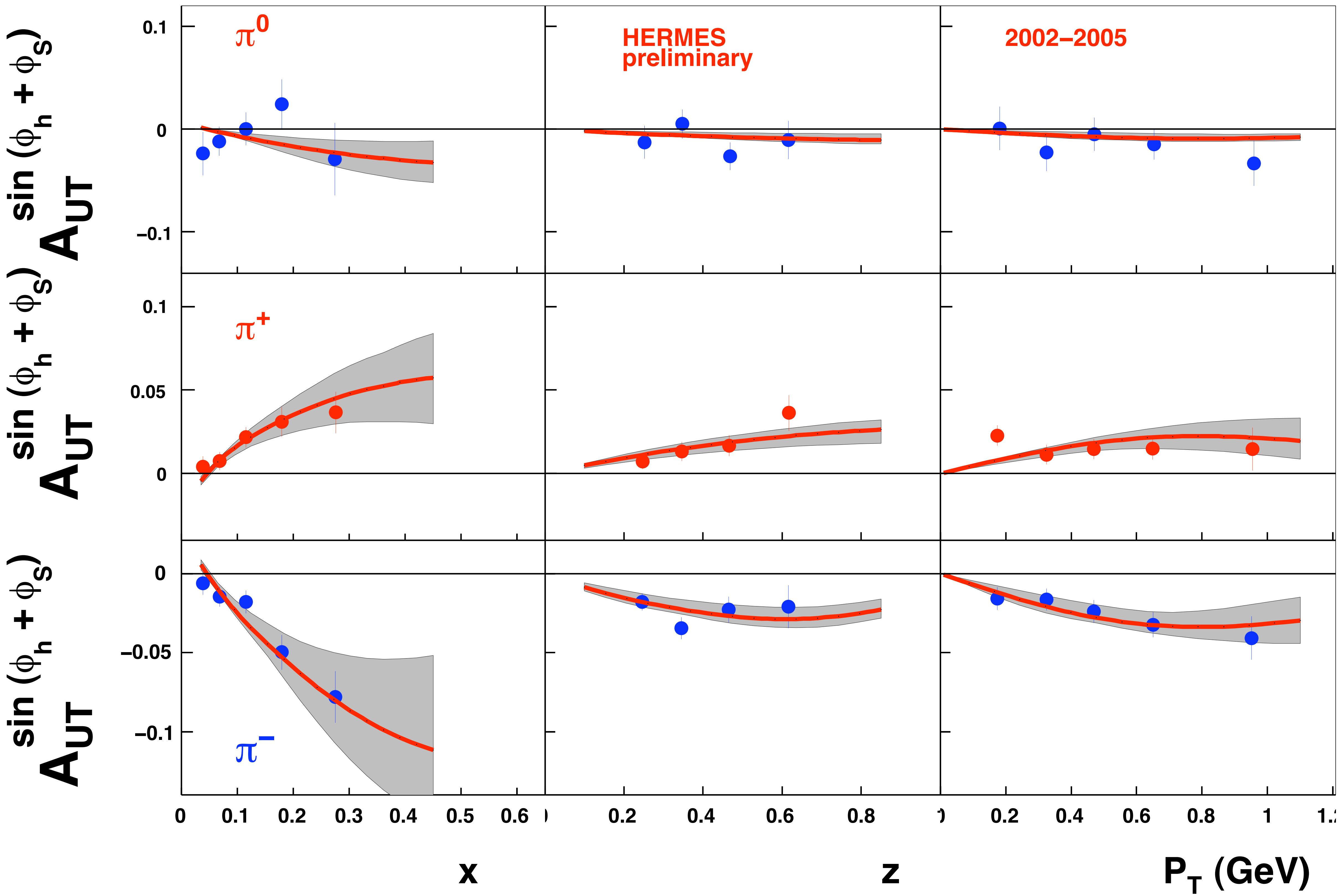


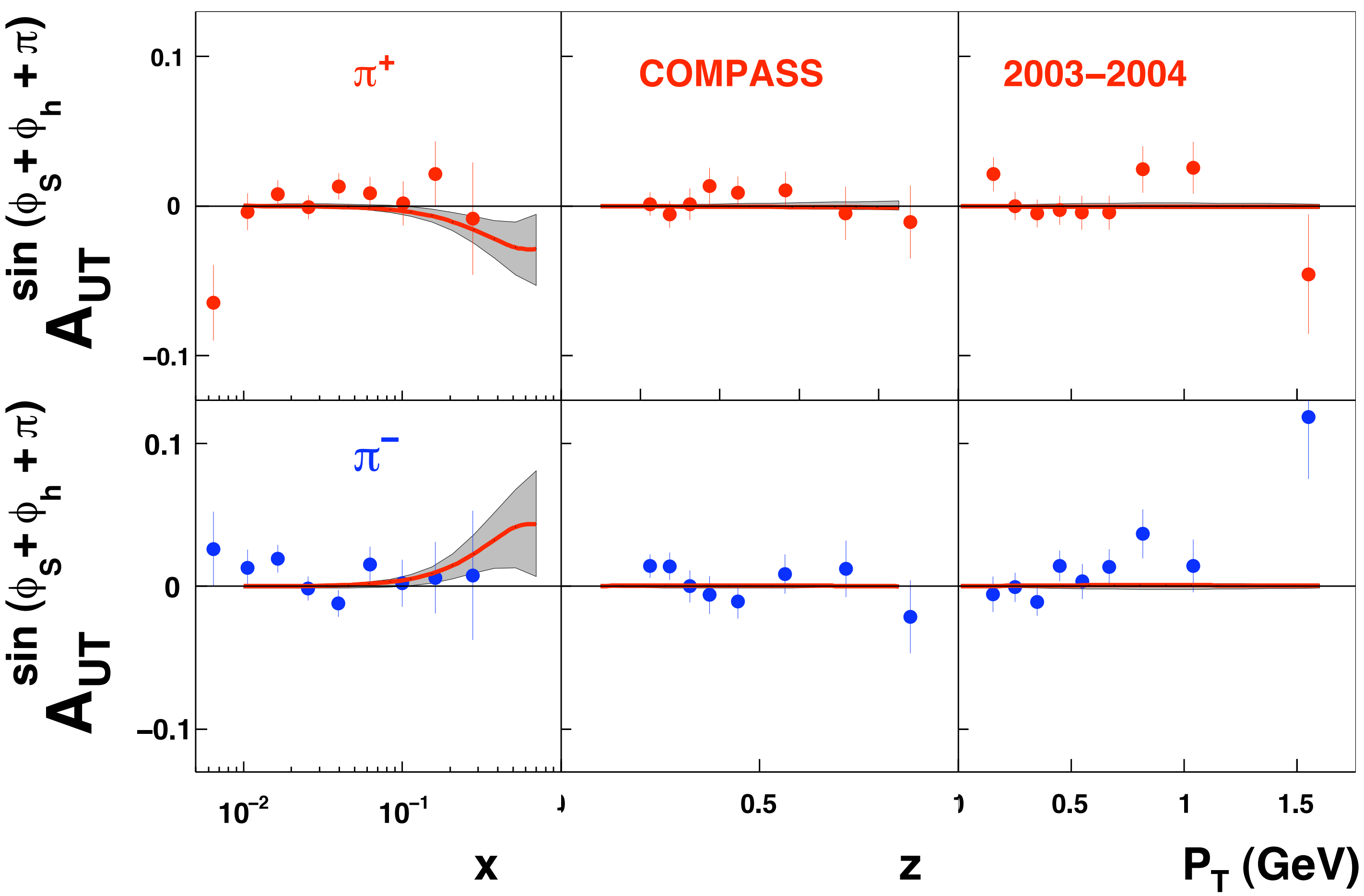
extracted transversity and comparison with models

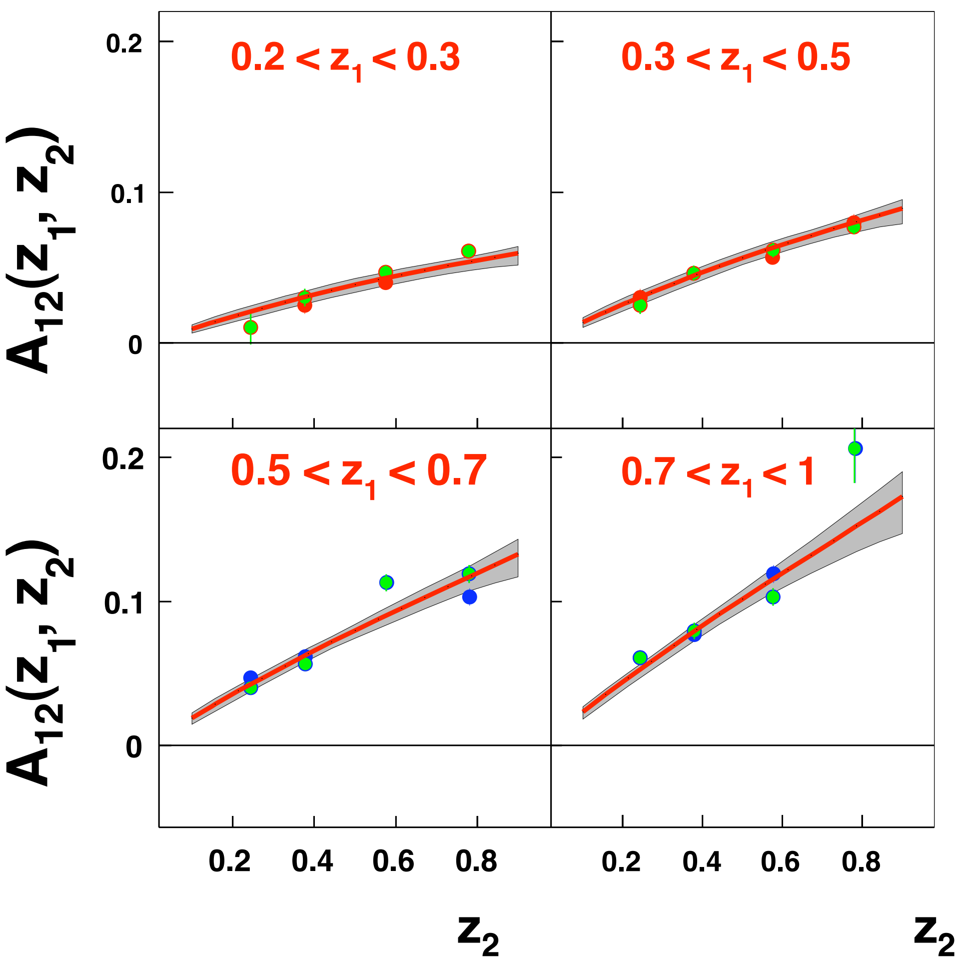


best fit of HERMES data

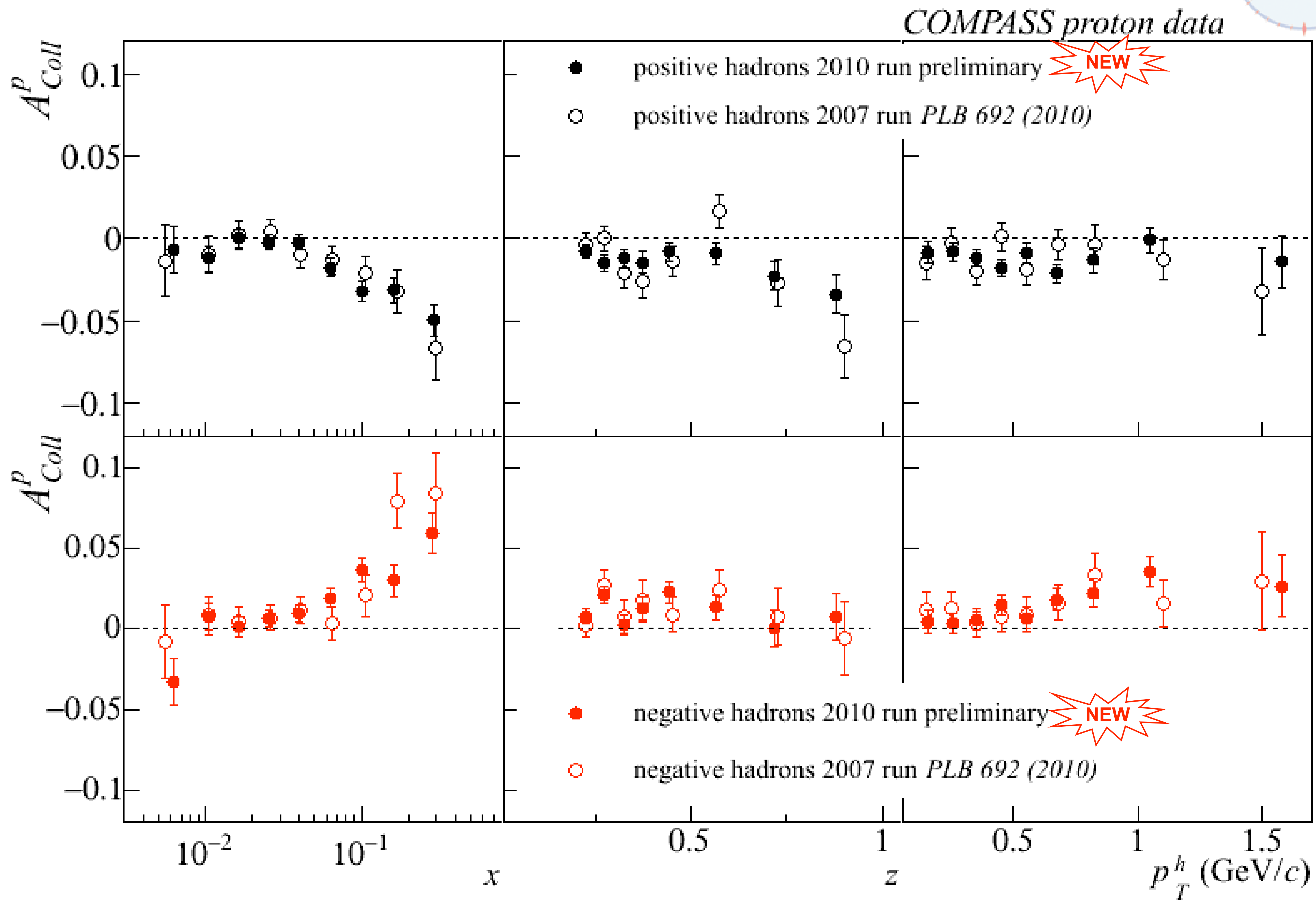


best fit of COMPASS and BELLE data





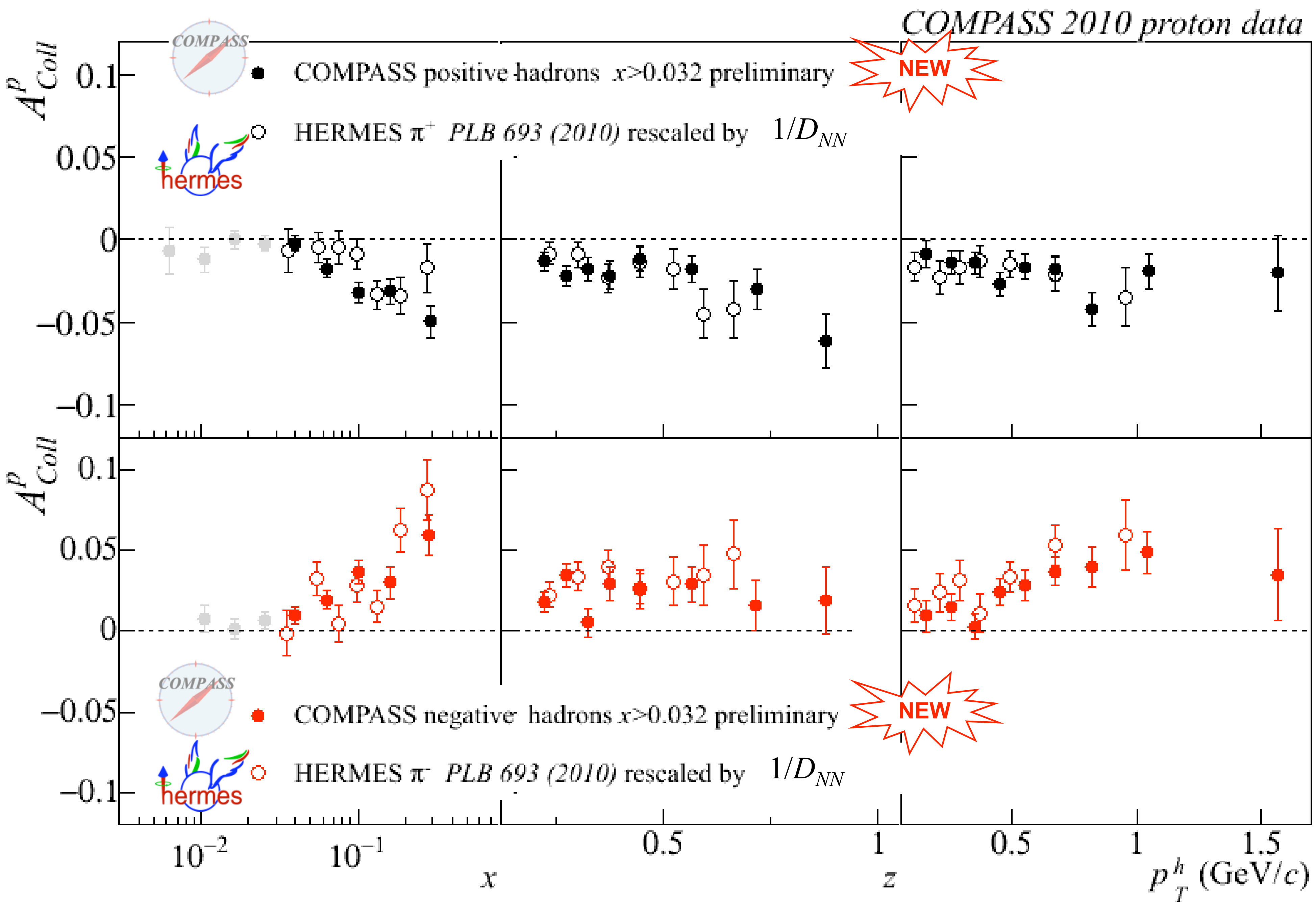
Collins asymmetry 2010 vs 2007 data



in agreement with predictions

Collins asymmetry   2010 data

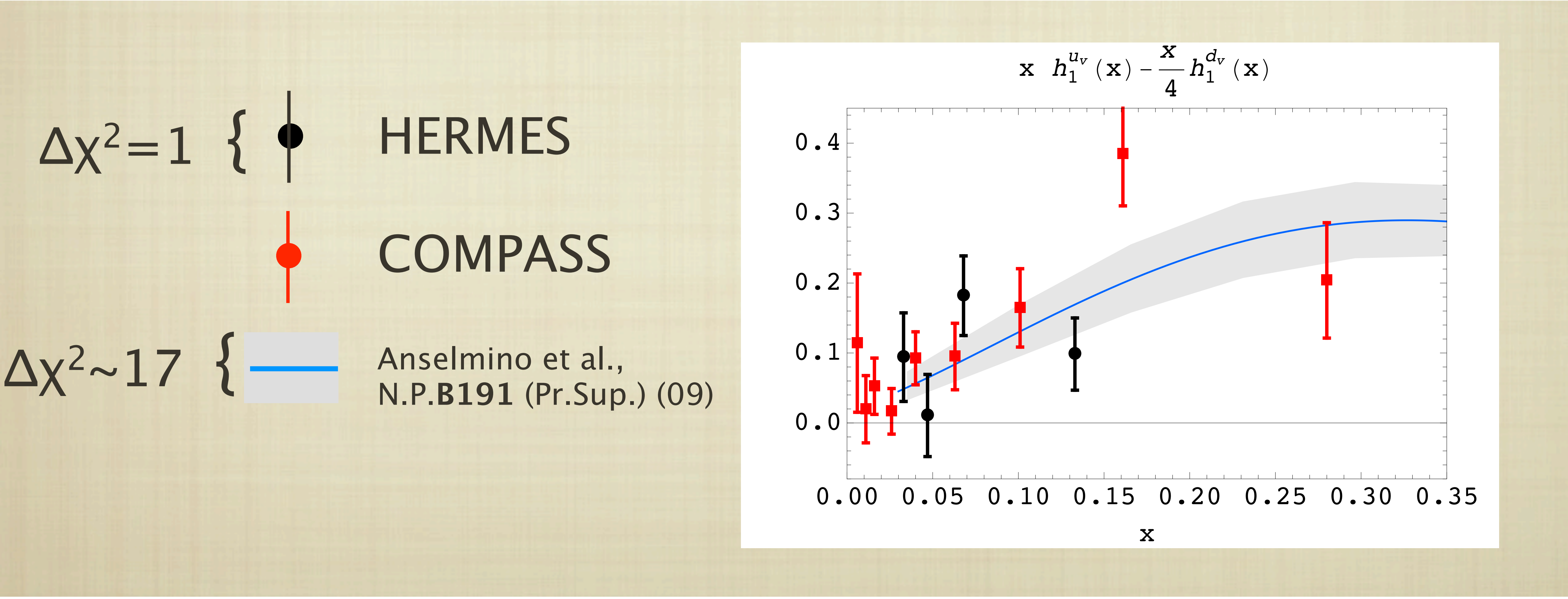
x > 0.032 region   - comparison with HERMES results



nice agreement

results recently confirmed by extraction based on coupling of transversity with di-hadron fragmentation function (SIDIS + BELLE data)

Bacchetta, Radici P.R.L. **107** (2011)



M. Radici, talk at Transversity 2011

is there a quantitative link between the Sivers distribution and orbital angular momentum?

Bacchetta, Radici, arXiv:1107.5755

use sum rule

$$J^a(Q^2) = \frac{1}{2}\int_0^1 dx\, x\left(\underbrace{H^a(x,0,0;Q^2)}_{\text{usual PDF}} + E^a(x,0,0;Q^2)\right)$$

with model assumption

$$f_{1T}^{\perp(0)a}(x;Q_L^2) = -L(x)\, E^a(x,0,0;Q_L^2)$$

fix E^q and $f_{1T}^{\perp}$ best fitting SIDIS data on Sivers asymmetry and the nucleon magnetic moments

$$\sum_q e_{q_v} \int_0^1 dx\, E^{q_v}(x,0,0) = \kappa$$

$$J^u = 0.266 \pm 0.002^{+0.009}_{-0.014}, \qquad J^{\bar{u}} = 0.014 \pm 0.004^{+0.001}_{-0.000},$$

$$J^d = -0.012 \pm 0.003^{+0.024}_{-0.006}, \qquad J^{\bar{d}} = 0.022 \pm 0.006^{+0.001}_{-0.000},$$

$$J^s = 0.005^{+0.000}_{-0.007}, \qquad J^{\bar{s}} = 0.004^{+0.000}_{-0.005}.$$

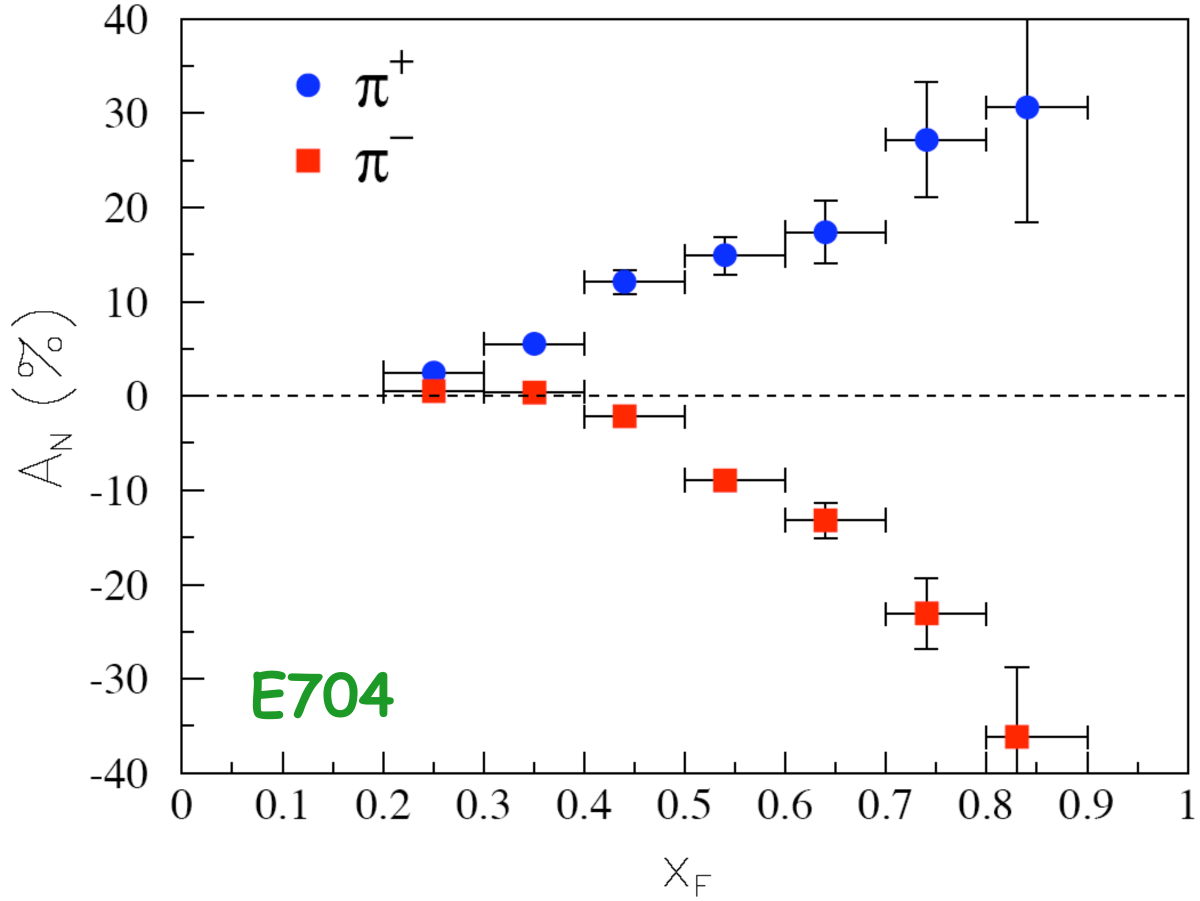
Transversity & Collins function phenomenology in SIDIS and e+e-

Same simple parametrization as for Sivers, but Collins effect has been clearly observed by three independent experiments: HERMES, COMPASS and BELLE

Collins function expected to be universal

QCD evolution important, as BELLE data are at a much higher energy than SIDIS data

A_N in p p → π X, the big challenge

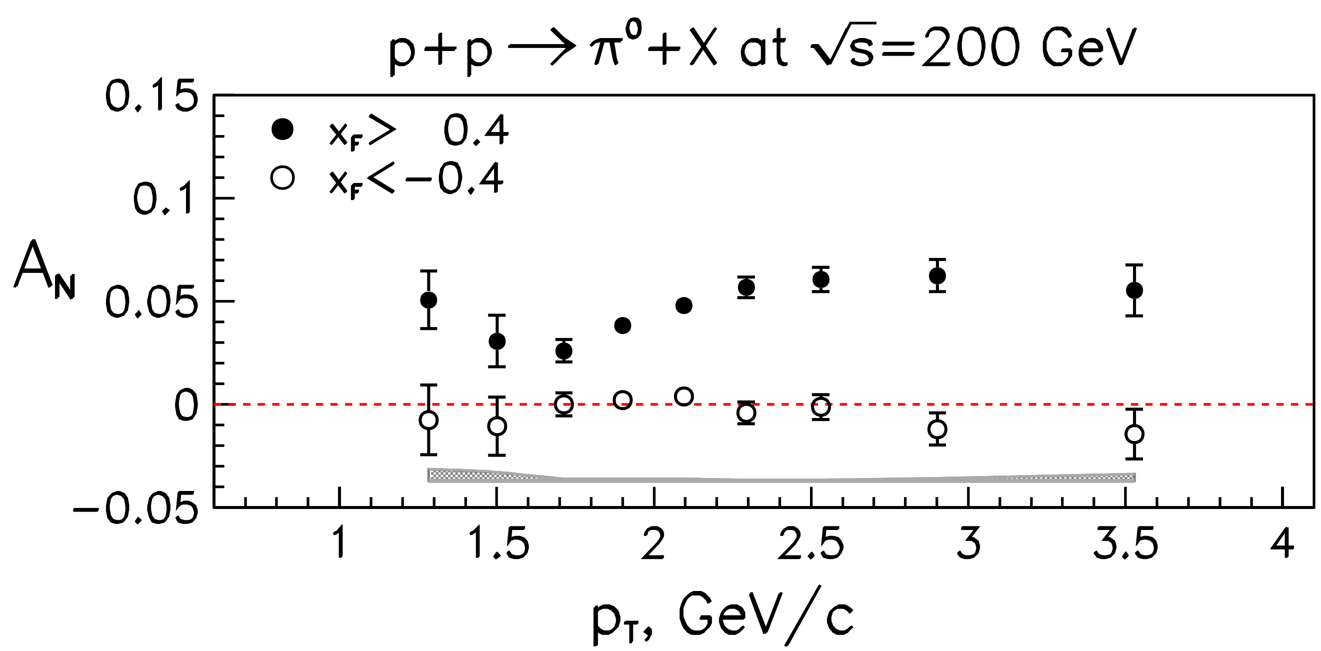


$$A_N \equiv \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$

E704 $\sqrt{s}$ = 20 GeV

0.7 < p_T < 2.0

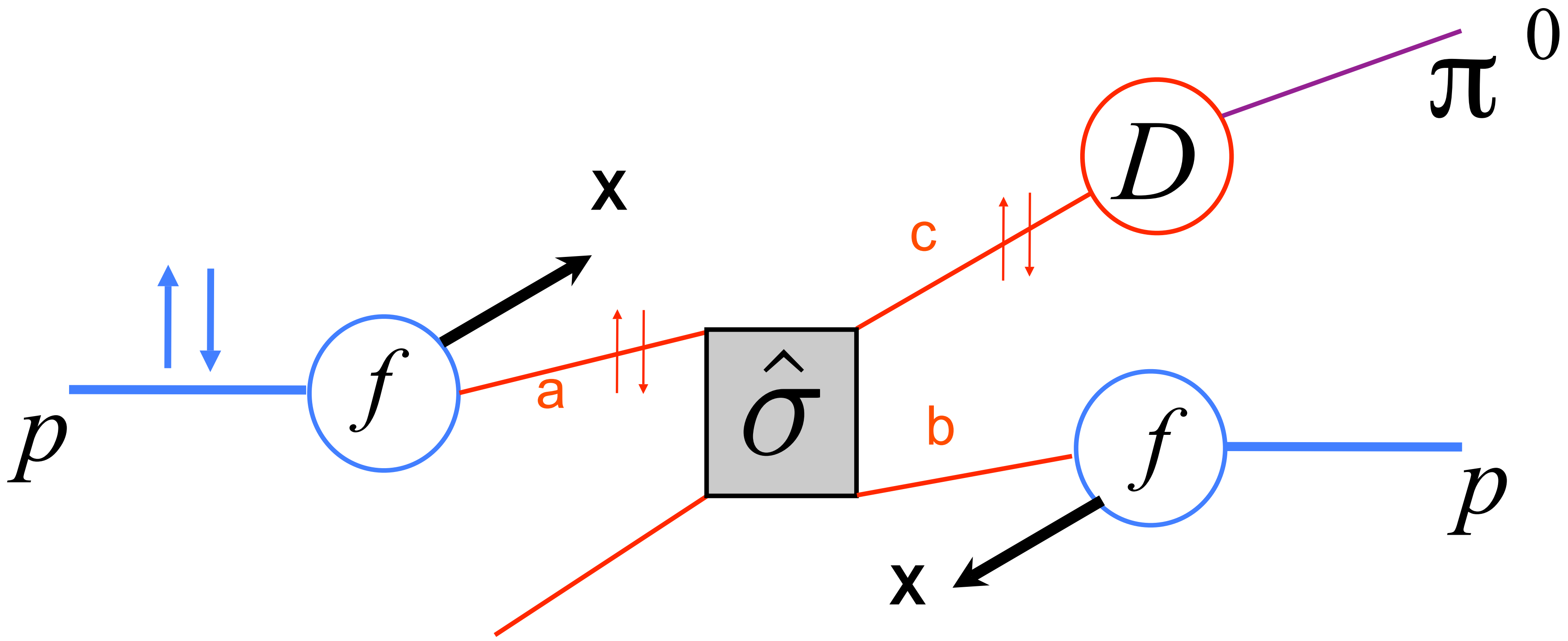
and all beautiful RHIC data, persisting at high energy...



Only one large scale, P_T. Any role for TMDs?

TMD factorization not proven

1. Generalization of collinear scheme (assuming factorization)



$$d\sigma^\uparrow = \sum_{a,b,c=q,\bar{q},g} \underbrace{f_{a/p^\uparrow}(x_a, \boldsymbol{k}_{\perp a})} \otimes \underbrace{f_{b/p}(x_b, \boldsymbol{k}_{\perp b})} \otimes d\hat{\sigma}^{ab\to cd}(\boldsymbol{k}_{\perp a}, \boldsymbol{k}_{\perp b}) \otimes \underbrace{D_{\pi/c}(z, \boldsymbol{p}_{\perp\pi})}$$

single spin effects in TMDs

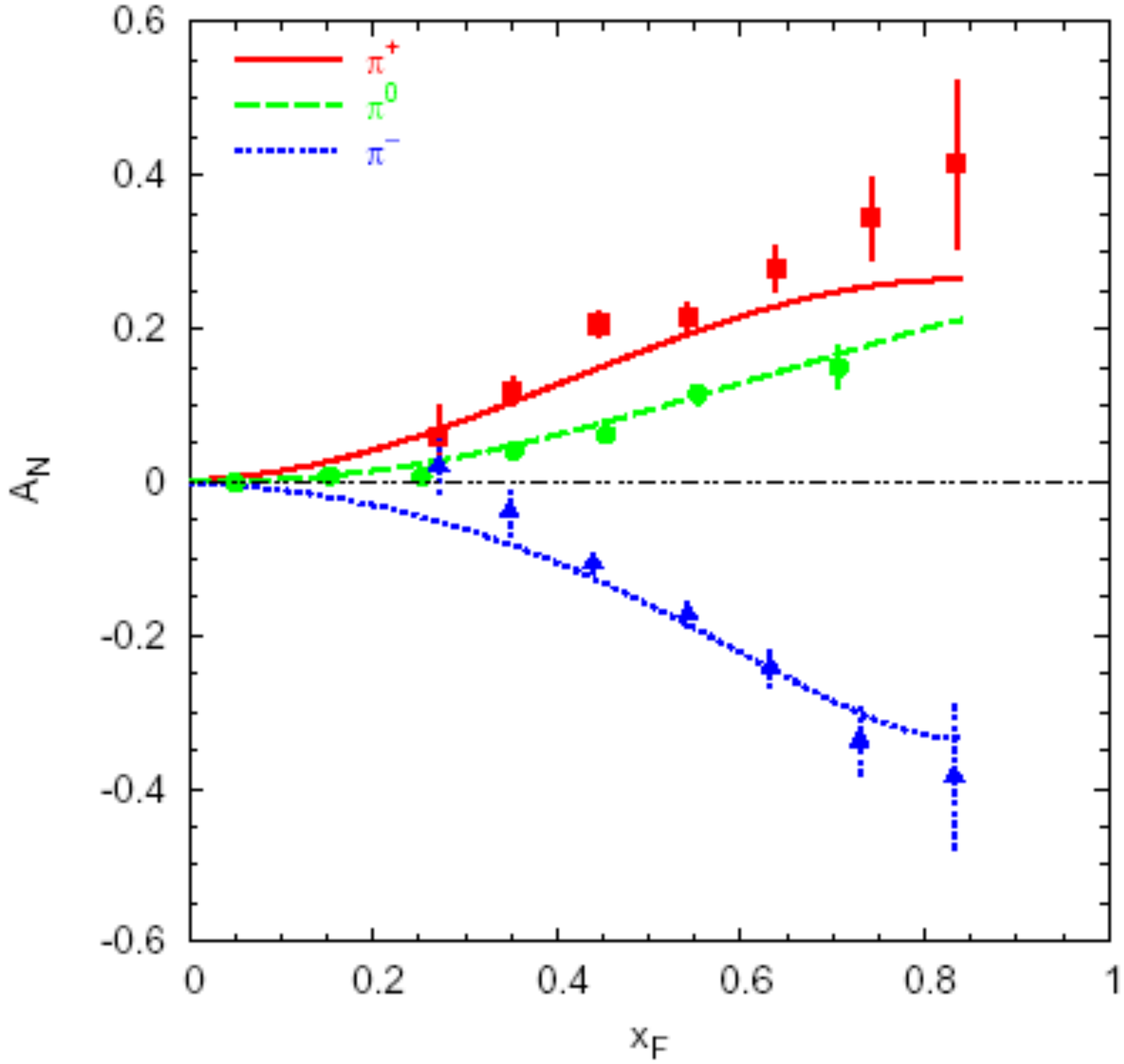
M.A., M. Boglione, U. D'Alesio, E. Leader, S. Melis, F. Murgia, A. Prokudin, ...

(Field-Feynman in unpolarized case)

TMD factorization at work ....

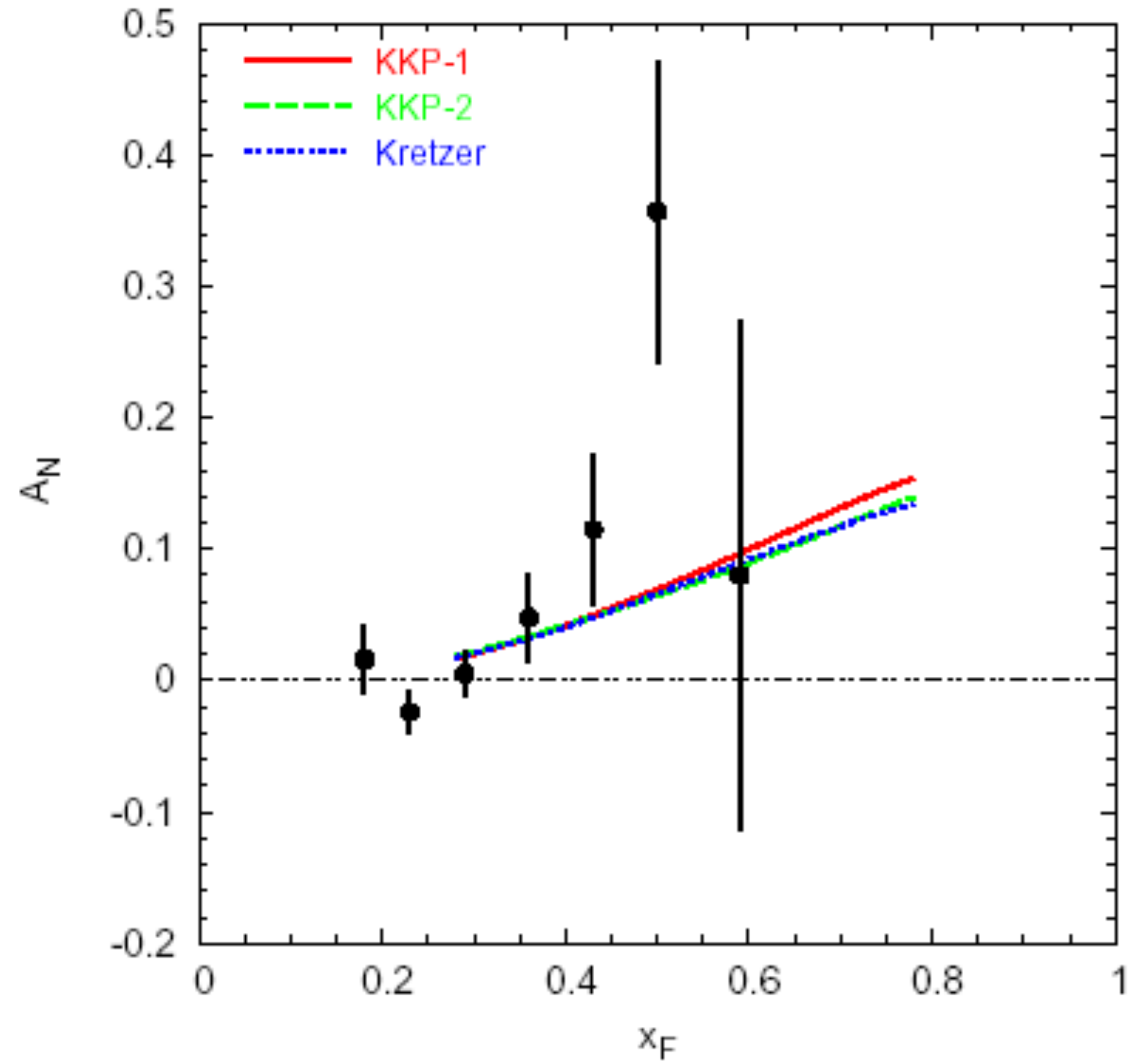
U. D'Alesio, F. Murgia

E704 data



fit

STAR data



prediction

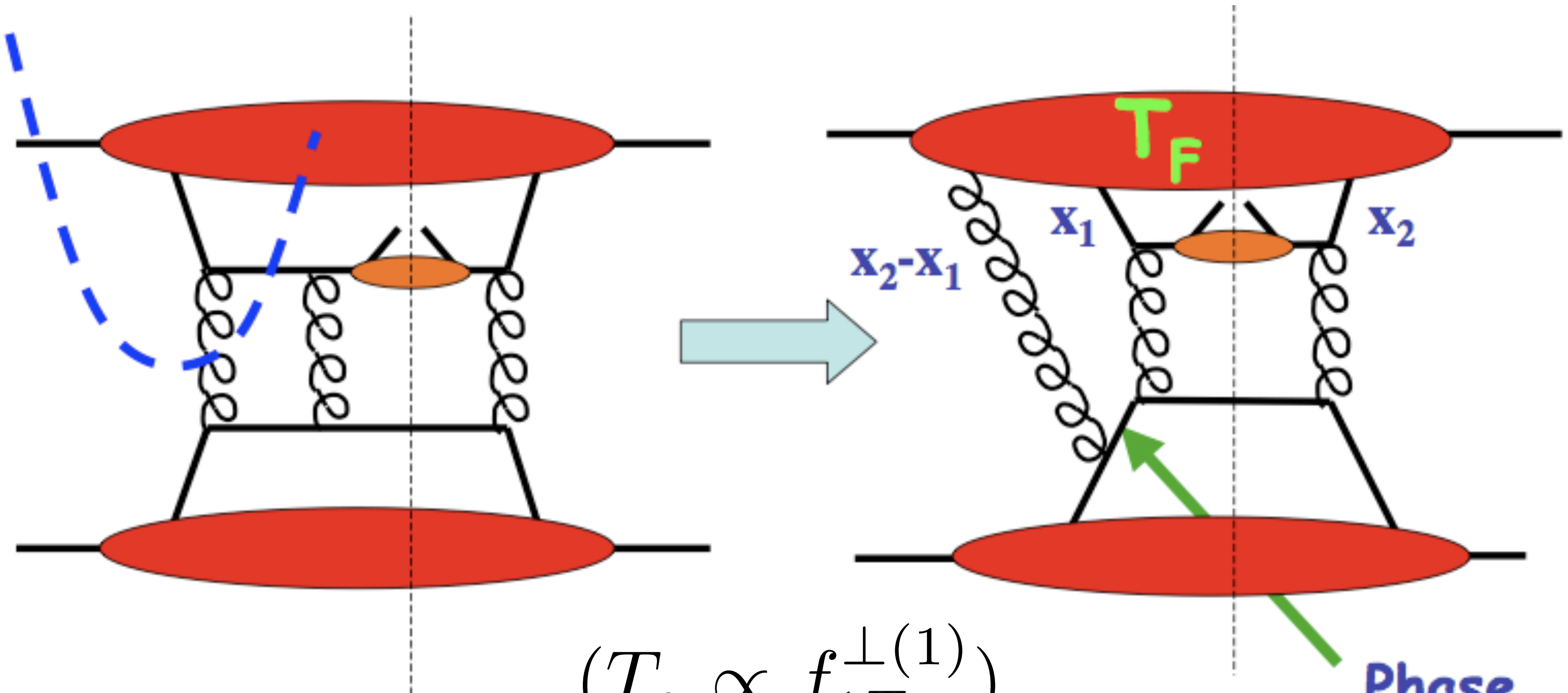
Sivers effect $p\,p \to \pi\,X$

2. Higher-twist partonic correlations

(Efremov, Teryaev; Qiu, Sterman; Kouvaris, Vogelsang, Yuan; Bacchetta, Bomhof, Mulders, Pijlman; Koike ...)

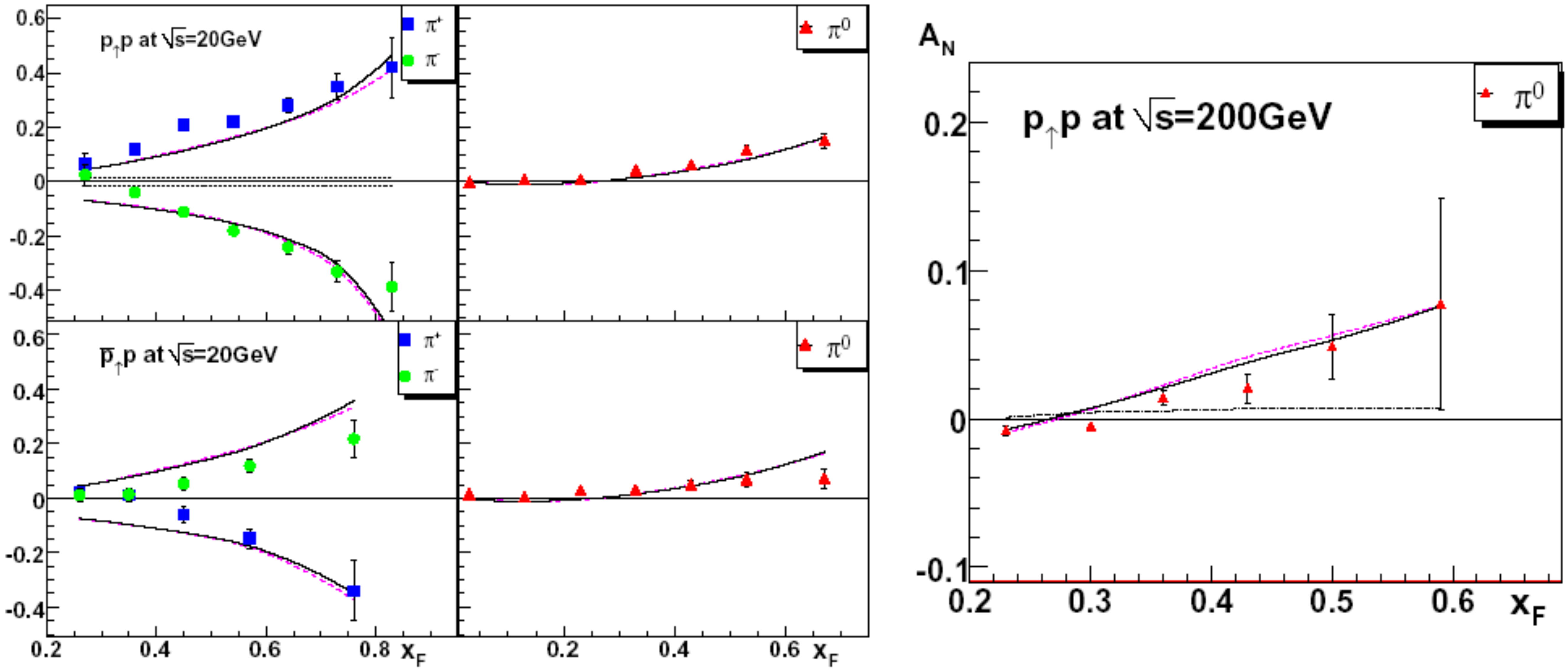
higher-twist partonic correlations - factorization OK

$$d\Delta\sigma \propto \sum_{a,b,c} \underbrace{T_a(k_1, k_2, \boldsymbol{S}_\perp)}_{\text{twist-3 functions}} \otimes f_{b/B}(x_b) \otimes \underbrace{H^{ab\to c}(k_1, k_2)}_{\text{hard interaction, not a cross section}} \otimes D_{h/c}(z)$$



$(T_a \propto f_{1T}^{\perp(1)})$

possible project: compute T_a using SIDIS extracted Sivers functions



fits of E704 and STAR data

Kouvaris, Qiu, Vogelsang, Yuan

sign mismatch

(Kang, Qiu, Vogelsang, Yuan)

compare

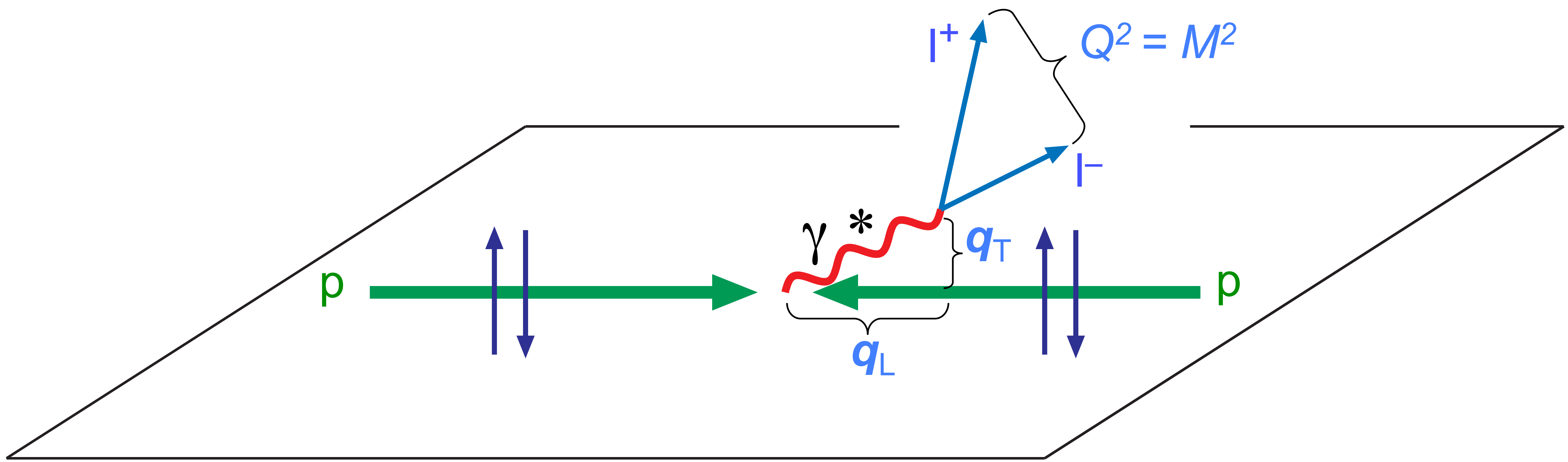
$$gT_{q,F}(x,x) = -\int d^2k_\perp \frac{|k_\perp|^2}{M} f_{1T}^{\perp q}(x,k_\perp^2)|_{\text{SIDIS}}$$

as extracted from fitting A_N data, with that obtained by inserting in the the above relation the SIDIS extracted Sivers functions

similar magnitude, but opposite sign!

the same mismatch does not occurr adopting TMD factorization; the reason is that the hard scattering part in higher-twist factorization is negative

TMDs in Drell-Yan processes



factorization holds, two scales, M^2, and $q_T \ll M$

$$d\sigma^{D-Y} = \sum_a f_q(x_1, \boldsymbol{k}_{\perp 1}; Q^2) \otimes f_{\bar{q}}(x_2, \boldsymbol{k}_{\perp 2}; Q^2)\, d\hat{\sigma}^{q\bar{q}\to\ell^+\ell^-}$$

direct product of TMDs, no fragmentation process

cross-section: most general pp leading-twist expression

$$\frac{d\sigma}{d^4q\,d\Omega} = \frac{\alpha_{em}^2}{F\,q^2}\times$$

S. Arnold, A. Metz and M. Schlegel, arXiv:0809.2262 [hep-ph]

$$\begin{aligned}
&\Big\{\Big((1+\cos^2\theta)\,F_{UU}^1 + (1-\cos^2\theta)\,F_{UU}^2 + \sin 2\theta\cos\phi\,F_{UU}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{UU}^{\cos 2\phi}\Big)\\
&+ S_{aL}\Big(\sin 2\theta\sin\phi\,F_{LU}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{LU}^{\sin 2\phi}\Big)\\
&+ S_{bL}\Big(\sin 2\theta\sin\phi\,F_{UL}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{UL}^{\sin 2\phi}\Big)\\
&+ |\vec{S}_{aT}|\Big[\sin\phi_a\Big((1+\cos^2\theta)\,F_{TU}^1 + (1-\cos^2\theta)\,F_{TU}^2 + \sin 2\theta\cos\phi\,F_{TU}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{TU}^{\cos 2\phi}\Big)\\
&\qquad + \cos\phi_a\Big(\sin 2\theta\sin\phi\,F_{TU}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{TU}^{\sin 2\phi}\Big)\Big]\\
&+ |\vec{S}_{bT}|\Big[\sin\phi_b\Big((1+\cos^2\theta)\,F_{UT}^1 + (1-\cos^2\theta)\,F_{UT}^2 + \sin 2\theta\cos\phi\,F_{UT}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{UT}^{\cos 2\phi}\Big)\\
&\qquad + \cos\phi_b\Big(\sin 2\theta\sin\phi\,F_{UT}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{UT}^{\sin 2\phi}\Big)\Big]\\
&+ S_{aL}\,S_{bL}\Big((1+\cos^2\theta)\,F_{LL}^1 + (1-\cos^2\theta)\,F_{LL}^2 + \sin 2\theta\cos\phi\,F_{LL}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{LL}^{\cos 2\phi}\Big)\\
&+ S_{aL}\,|\vec{S}_{bT}|\Big[\cos\phi_b\Big((1+\cos^2\theta)\,F_{LT}^1 + (1-\cos^2\theta)\,F_{LT}^2 + \sin 2\theta\cos\phi\,F_{LT}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{LT}^{\cos 2\phi}\Big)\\
&\qquad + \sin\phi_b\Big(\sin 2\theta\sin\phi\,F_{LT}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{LT}^{\sin 2\phi}\Big)\Big]\\
&+ |\vec{S}_{aT}|\,S_{bL}\Big[\cos\phi_a\Big((1+\cos^2\theta)\,F_{TL}^1 + (1-\cos^2\theta)\,F_{TL}^2 + \sin 2\theta\cos\phi\,F_{TL}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{TL}^{\cos 2\phi}\Big)\\
&\qquad + \sin\phi_a\Big(\sin 2\theta\sin\phi\,F_{TL}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{TL}^{\sin 2\phi}\Big)\Big]\\
&+ |\vec{S}_{aT}|\,|\vec{S}_{bT}|\Big[\cos(\phi_a+\phi_b)\Big((1+\cos^2\theta)\,F_{TT}^1 + (1-\cos^2\theta)\,F_{TT}^2 + \sin 2\theta\cos\phi\,F_{TT}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{TT}^{\cos 2\phi}\Big)\\
&\qquad + \cos(\phi_a-\phi_b)\Big((1+\cos^2\theta)\,\bar{F}_{TT}^1 + (1-\cos^2\theta)\,\bar{F}_{TT}^2 + \sin 2\theta\cos\phi\,\bar{F}_{TT}^{\cos\phi} + \sin^2\theta\cos 2\phi\,\bar{F}_{TT}^{\cos 2\phi}\Big)\\
&\qquad + \sin(\phi_a+\phi_b)\Big(\sin 2\theta\sin\phi\,F_{TT}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{TT}^{\sin 2\phi}\Big)\\
&\qquad + \sin(\phi_a-\phi_b)\Big(\sin 2\theta\sin\phi\,\bar{F}_{TT}^{\sin\phi} + \sin^2\theta\sin 2\phi\,\bar{F}_{TT}^{\sin 2\phi}\Big)\Big]\Big\}
\end{aligned}$$

Drell-Yan

48 structure functions ...

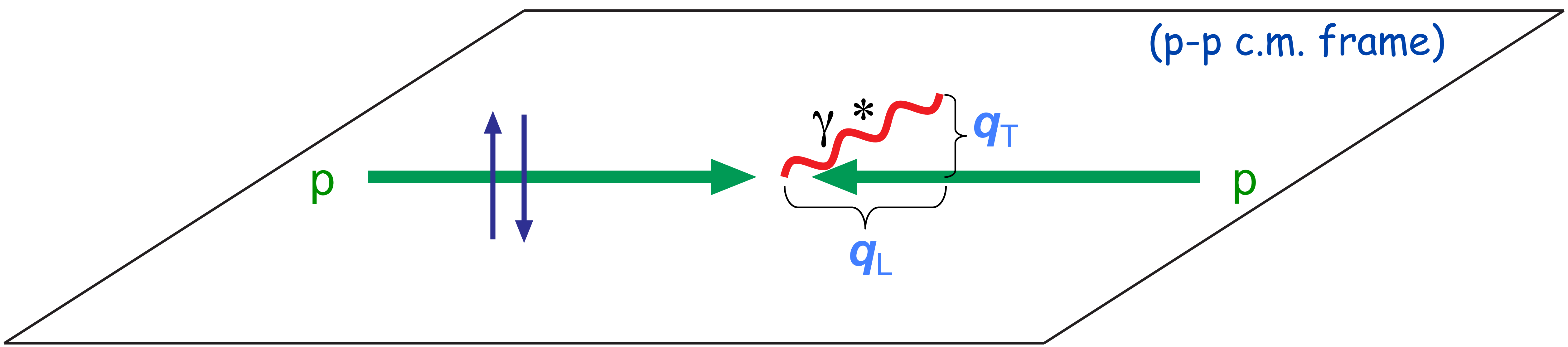
Sivers effect in D-Y processes

By looking at the $d^4\sigma/d^4q$ cross section one can single out the Sivers effect in D-Y processes

$$\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow} \propto \sum_q \Delta^N f_{q/p^\uparrow}(x_1, \boldsymbol{k}_\perp) \otimes f_{\bar{q}/p}(x_2) \otimes \mathrm{d}\hat{\sigma}$$

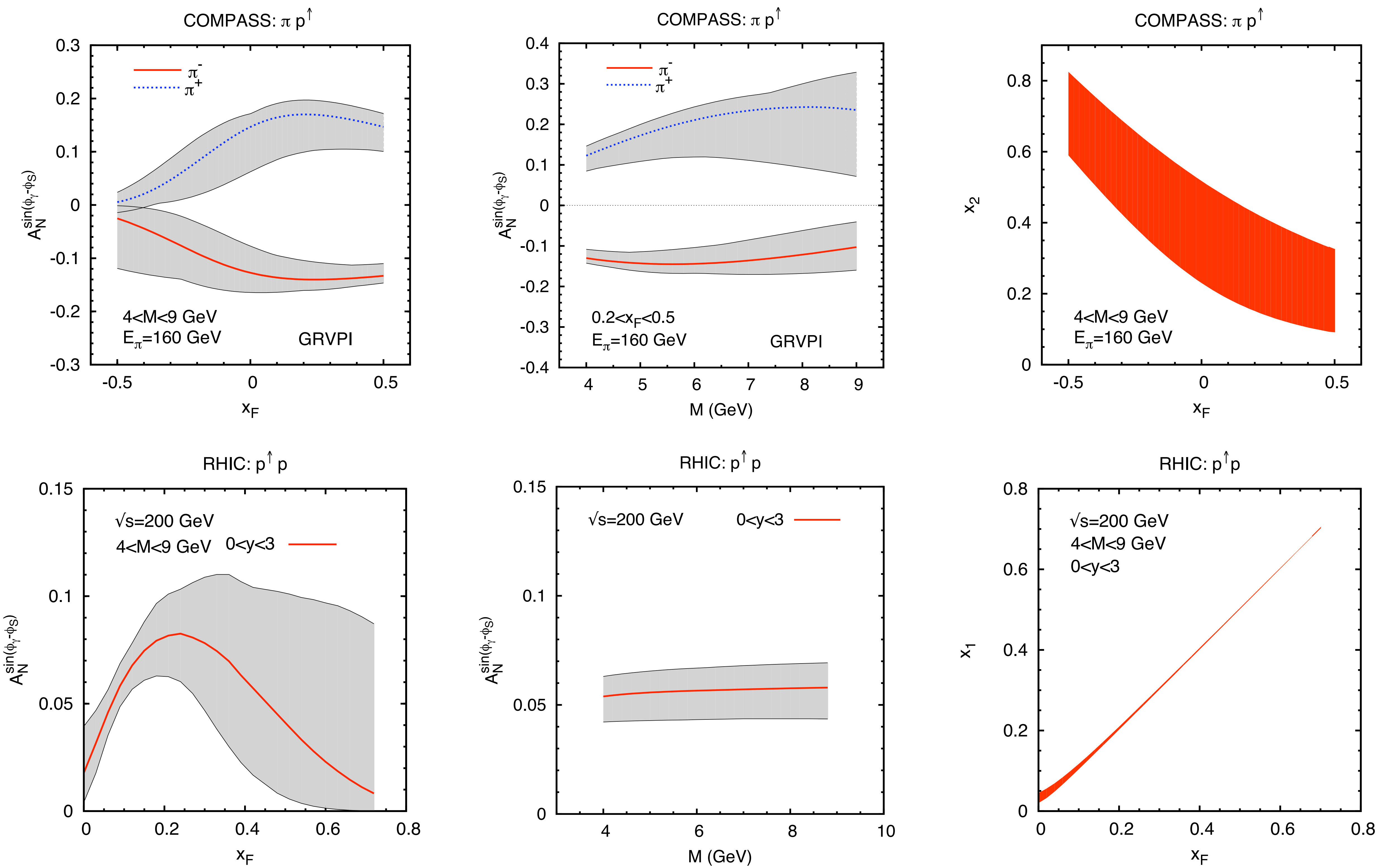
$q = u, \bar{u}, d, \bar{d}, s, \bar{s}$

$$A_N^{\sin(\phi_S - \phi_\gamma)} \equiv \frac{2\int_0^{2\pi} \mathrm{d}\phi_\gamma \, [\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow}] \, \sin(\phi_S - \phi_\gamma)}{\int_0^{2\pi} \mathrm{d}\phi_\gamma \, [\mathrm{d}\sigma^{\uparrow} + \mathrm{d}\sigma^{\downarrow}]}$$



Predictions for A_N

Sivers functions as extracted from SIDIS data, **with opposite sign**



M.A., M. Boglione, U. D'Alesio, S. Melis, F. Murgia, A. Prokudin, e-Print: arXiv:0901.3078

Conclusions

The3-dimensional exploration of the nucleon has just started: collect as much data as possible and try to reconstruct the nucleon phase-space structure

TMDs describe the momentum distribution; the actual knowledge covers limited kinematical regions, and assumes (too) simple functional forms

The properties of the Sivers function and its different role in different processes, have to be investigated

and much more to do .....