

NUCLEAR PARTON DISTRIBUTION FUNCTIONS

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Phys.Rev.Lett.106 122301 (2011)

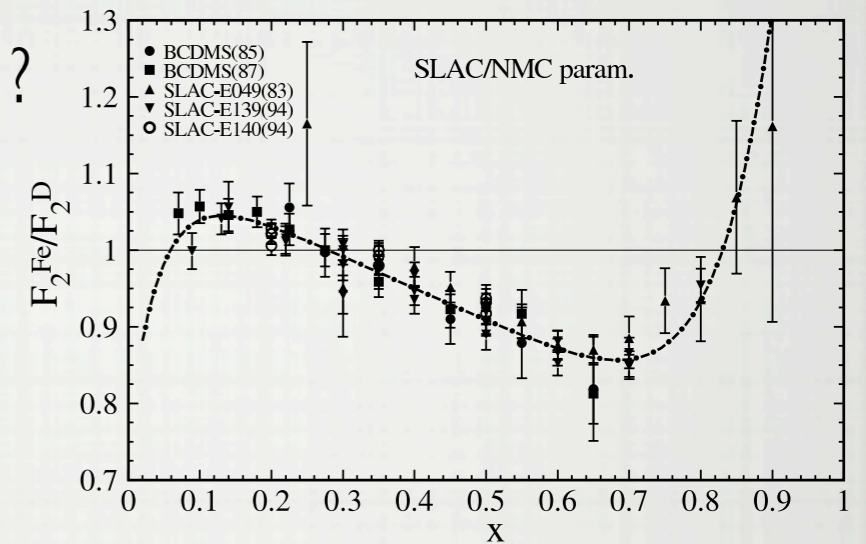
OUTLINE

1. Nuclear effects in PDFs
2. Overview of global nPDF analysis
3. Generalized CTEQ framework
4. Neutrino DIS data
5. Outlook

NUCLEAR EFFECTS

● What are nuclear parton density functions (nPDF) ?

- parton densities for partons in bound proton & neutron

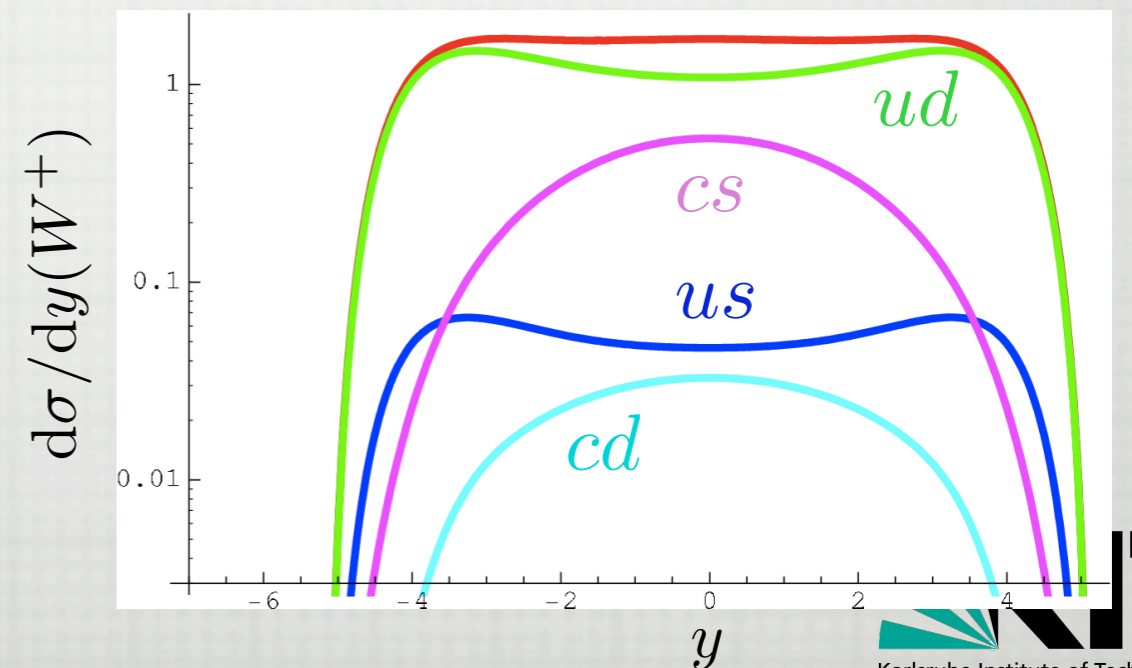


● Where are nuclear parton density functions useful ?

I. Strange quark content of the proton

strange PDF from neutrino DIS with heavy nuclei - nuclear effects very important

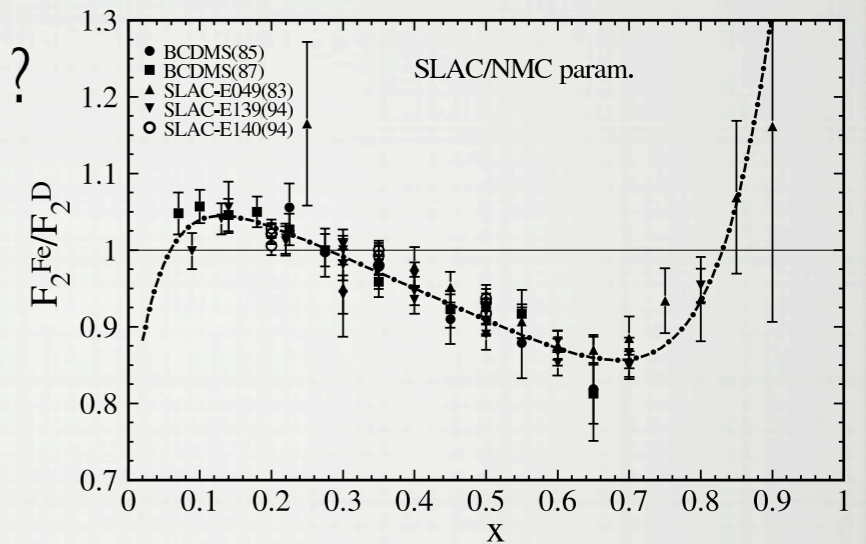
crucial for: W-boson production at the LHC
(standard candle process)



NUCLEAR EFFECTS

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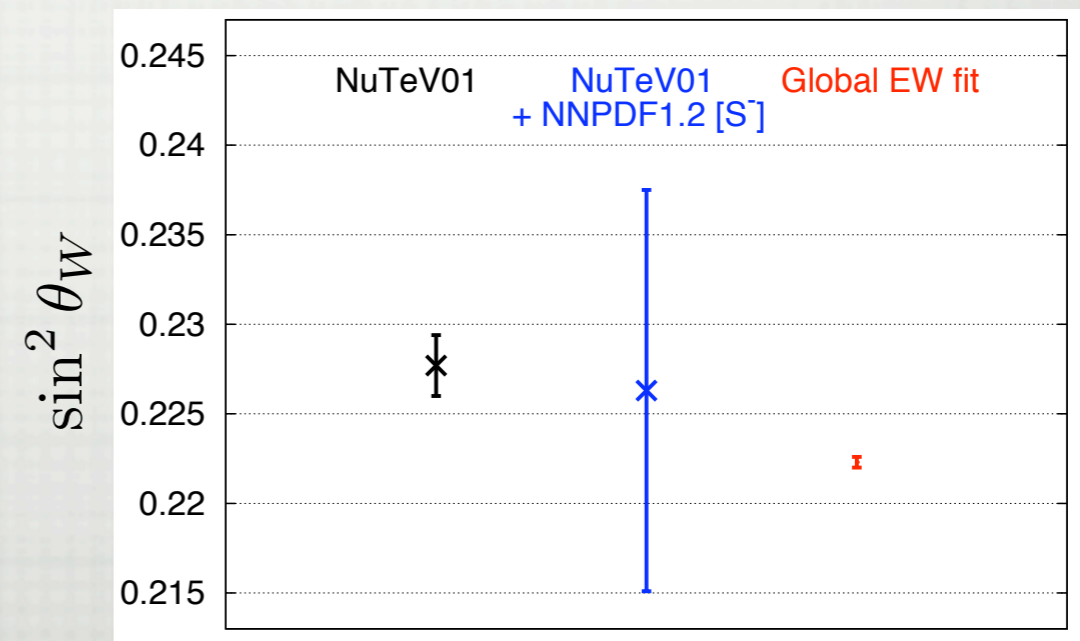


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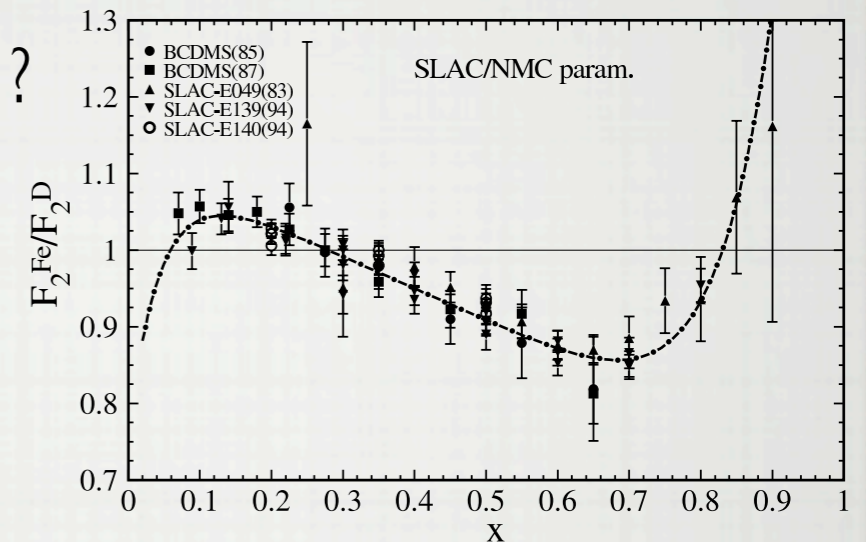
crucial for: determining weak mixing angle
from NuTeV experiment



NUCLEAR EFFECTS

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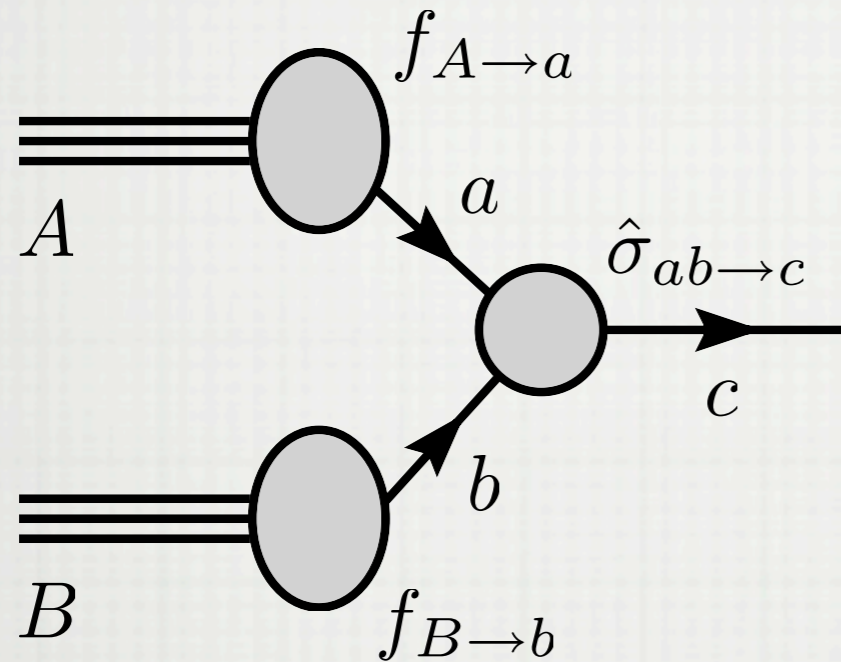


● Where are nuclear parton density functions useful ?

1. Strange quark content of the proton
2. Collisions of protons and nuclei at RHIC, ALICE & CMS
3. Neutrino scattering experiments e.g. MINERvA
4. Neutrino oscillations experiments e.g. MINOS

OVERVIEW NPDF

Factorization & PDFs



Parton distribution functions (PDFs)

$$f_{A \rightarrow a}(x, \mu_F)$$

- universal, non-perturbative objects
- describe the structure of hadrons (in terms of partons - quarks & gluons)
- obey DGLAP evolution equations

The hard cross-section $\hat{\sigma}_{ab \rightarrow c}$

- free of long distance effects
- calculable in pQCD
- process dependant

$$\sigma = f_{A \rightarrow a} \otimes f_{B \rightarrow b} \otimes \hat{\sigma}_{ab \rightarrow c}$$

\uparrow \uparrow \uparrow
 from experiment from pQCD

OVERVIEW NPDF

- Universality of PDFs - same parton distribution functions for all processes

- Deep Inelastic Scattering (DIS)

$$F_2^A(x, \mu^2) = \sum_i [f_i^A \otimes C_{2,i}](x, \mu^2)$$

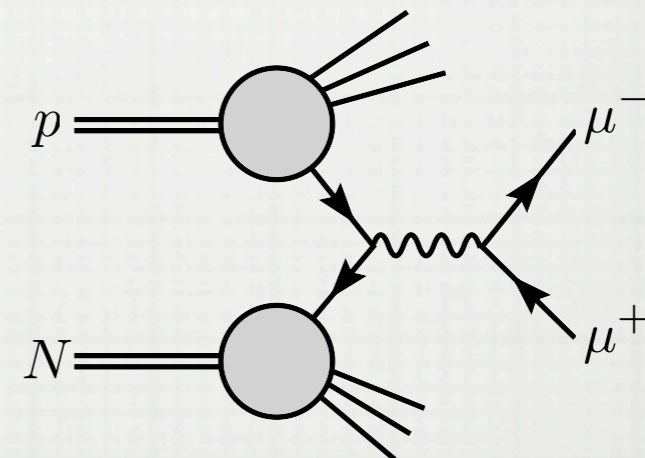
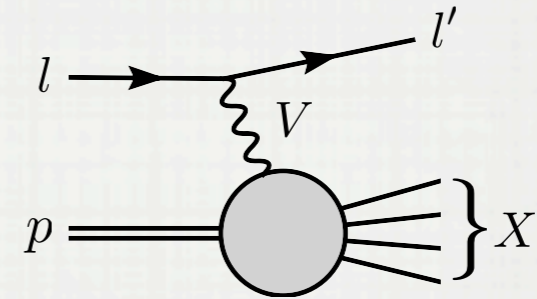
- Drell-Yan processes (DY)

$$\sigma_{A+B \rightarrow l^+ + l^- + X} = \sum_{i,j} f_i^A \otimes f_j^B \otimes \hat{\sigma}^{i+j \rightarrow l^+ + l^- + X}$$

- hadron production

$$\sigma_{A+B \rightarrow H+X} = \sum_{i,j,k} f_i^A \otimes f_j^B \otimes \hat{\sigma}^{i+j \rightarrow k+X} \otimes D_k^H$$

- PDFs give predictions for unexplored kinematic regions and for new physics at the LHC



OVERVIEW NPDF

● CTEQ framework to fit PDFs from experimental data

CTEQ6M [hep-ph/0201195](https://arxiv.org/abs/hep-ph/0201195)

- the input scale set to $\mu_0 = Q_0 = 1.3 \text{ GeV}$
- parameterization of the PDFs in x

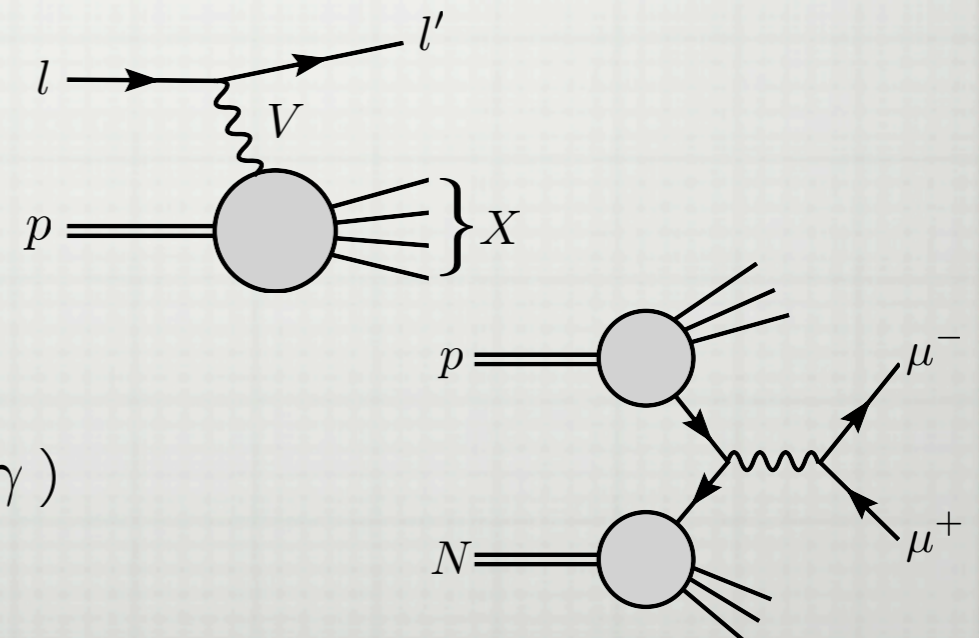
$$x f_k(x, Q_0) = c_0 x^{c_1} (1-x)^{c_2} e^{c_3 x} (1 + e^{c_4 x})^{c_5} \quad k = u_v, d_v, g, \bar{u} + \bar{d}, s, \bar{s}$$

$$\bar{d}(x, Q_0)/\bar{u}(x, Q_0) = c_0 x^{c_1} (1-x)^{c_2} + (1 + c_3 x)(1-x)^{c_4}$$

- make sure # of free parameters not too high - CTEQ approx. 20 free params
- carefully choose data sets & kinematic cuts to constrain free parameters
- perform χ^2 fit to data

● Which data sets are included ?

- Deep Inelastic Scattering ($l^\pm p, l^- d, \nu N, \bar{\nu} N$)
- Neutrino DIS di-muon production
- Drell-Yan & vector boson production (W^\pm, Z^0, γ)
- hadronic jet data



OVERVIEW NPDF

- Review of existing global analyses of nuclear PDF

DE FLORIAN, SASSOT'04 [PRD69(2004)074028]
LO, NLO

$\chi^2/\text{dof} = 0.76$

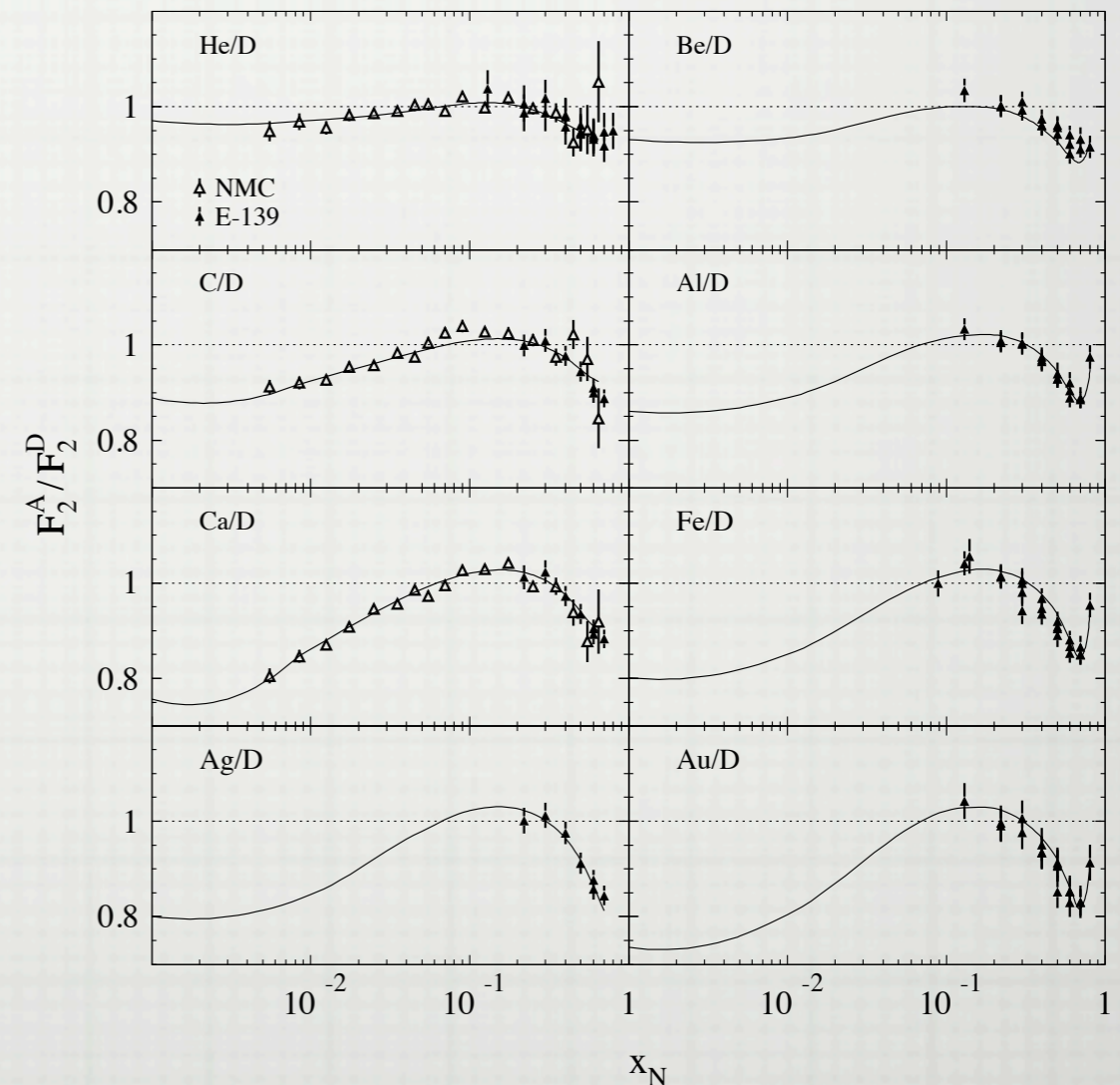
- first NLO analysis of nuclear data
- the only group using convolution relation

$$f_i^A(x_N, Q_0^2) = \int_{x_N}^A \frac{dy}{y} W_i(y, A, Z) f_i(x_N/y, Q_0^2)$$

- typical nucleon density for valence quarks

$$W_v(y, A, Z) = A[a_v \delta(1 - \epsilon_v - y) + (1 - a_v) \delta(1 - \epsilon_{v'} - y)] \\ + n_v \left(\frac{y}{A}\right)^{\alpha_v} \left(1 - \frac{y}{A}\right)^{\beta_v} + n_s \left(\frac{y}{A}\right)^{\alpha_s} \left(1 - \frac{y}{A}\right)^{\beta_s}$$

- the only framework using evolution in Mellin space & have PDFs also for $x_N > 1$
- only standard DIS data sets (semi-global)
- no error analysis



OVERVIEW NPDF

- Review of existing global analyses of nuclear PDF

$$\chi^2/\text{dof} = 1.2$$

HIRAI, KUMANO, NAGAI'07 [PRC76(2007)065207] LO, NLO, ERROR PDFS

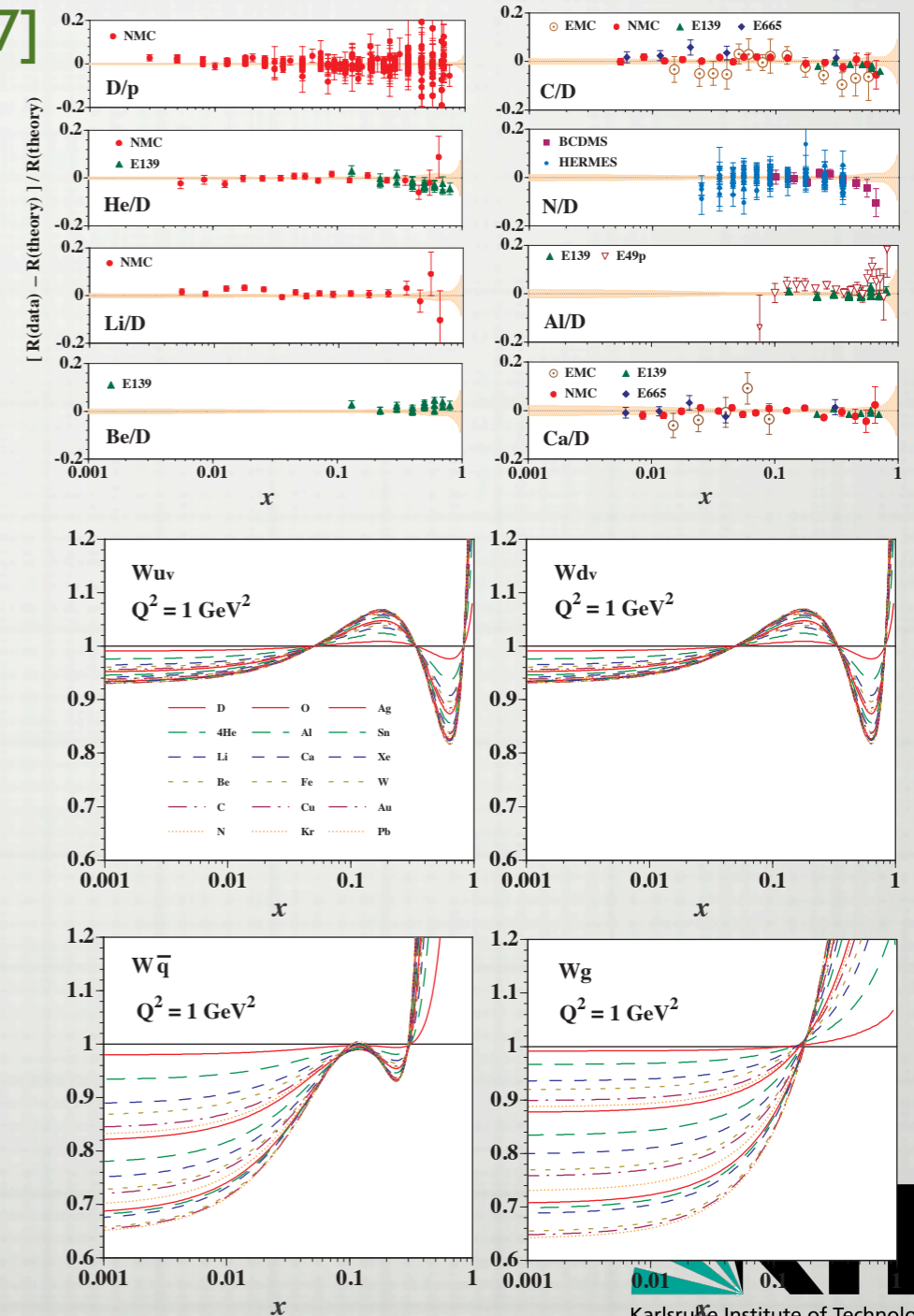
- uses multiplicative factor

$$f_i^A(x_N, Q_0^2) = R_i(x_N, Q_0, A, Z) f_i(x_N, Q_0^2)$$

where proton PDF in MRST 1998 and factor

$$R_i(x, A, Z) = 1 + \left(1 - \frac{1}{A^\alpha}\right) \frac{a_i + b_i x + c_i x^2 + d_i x^3}{(1-x)^{\beta_i}}$$

- neglects region $x > 1$
- includes all current DIS & DY data set (same as our analysis - discussed later)
- use Hessian method to produce error PDFs



OVERVIEW NPDF

- Review of existing global analyses of nuclear PDF

ESKOLA, PAUKKUNEN, SALGADO'09 [JHEP0904(2009)065]
 LO, NLO, ERROR PDFS

$\chi^2/\text{dof} = 0.8$

- uses multiplicative factor

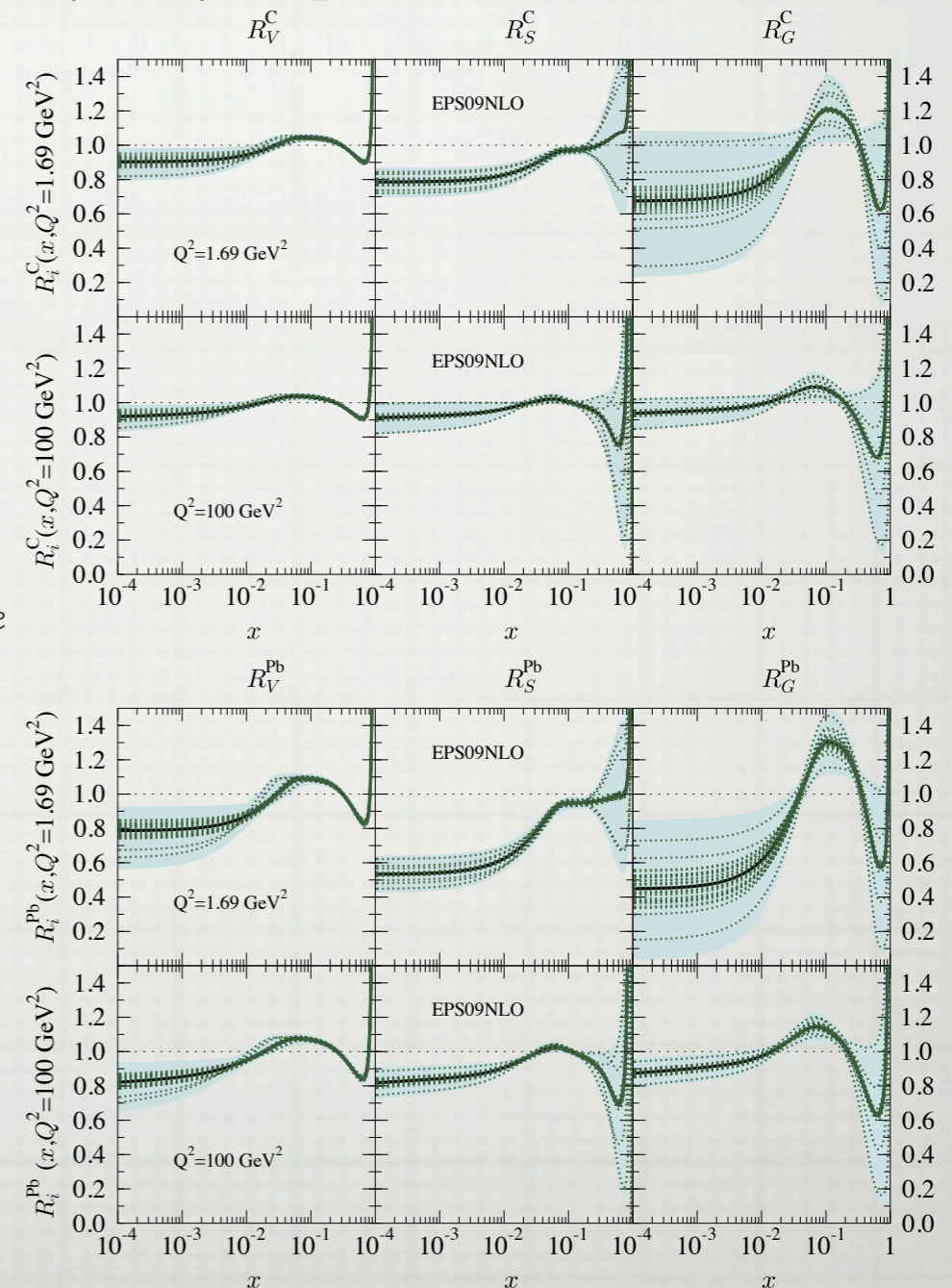
$$f_i^A(x_N, Q_0^2) = R_i(x_N, Q_0, A, Z) f_i(x_N, Q_0^2)$$

where proton PDF in CTEQ6.1M and factor is a complicated piecewise defined function

$$R_i(x, A, Z) = \begin{cases} a_0 + (a_1 + a_2 x)(e^{-x} - e^{-x_a}) & x \leq x_a \\ b_0 + b_1 x + b_2 x^2 + b_3 x^3 & x_a \leq x \leq x_e \\ c_0 + (c_1 - c_2 x)(1 - x)^{-\beta} & x_e \leq x \leq 1 \end{cases}$$

with A-dependent parameters

- neglects region $x > 1$
- includes all current DIS & DY data set & π^0 RHIC data to constrain gluon
- use Hessian method to produce error PDFs



OVERVIEW NPDF

● Why another set of NPDFs ?

- nuclear PDFs different from proton PDFs - PDF parameters contain more information but the fit has less data to constrain them
- big source of uncertainty - systematic uncertainty connected to assumptions made about PDF parameters & the parametric form of the PDF at Q_0

OVERVIEW NPDF

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- nuclear PDFs different from proton PDFs - PDF parameters contain more information but the fit has less data to constrain them
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1. nCTEQ framework entirely different from previous nPDF frameworks - useful to study parameterization dependence & estimate systematic uncertainty

2. nCTEQ analysis is in a close relation to the existing CTEQ proton analysis - allows to calculate nuclear correction factors in a flexible way (Q dependent & based on global analysis)

3. Our analysis aims at including also neutrino DIS data

NUCLEAR CTEQ

- nCTEQ framework for nuclear PDF - based on CTEQ6M proton fit

- functional form for bound protons same as for free proton PDF (restrict x to $0 < x < 1$)

$$x f_k(x, Q_0) = c_0 x^{c_1} (1 - x)^{c_2} e^{c_3 x} (1 + e^{c_4 x})^{c_5} \quad k = u_v, d_v, g, \bar{u} + \bar{d}, s, \bar{s}$$

$$\bar{d}(x, Q_0) / \bar{u}(x, Q_0) = c_0 x^{c_1} (1 - x)^{c_2} + (1 + c_3 x)(1 - x)^{c_4}$$

- coefficients with A-dependance (reduces to proton for $A=1$)

$$c_k \rightarrow c_k(A) \equiv c_{k,0} + c_{k,1} (1 - A^{-c_{k,2}}), \quad k = \{1, \dots, 5\}$$

- PDF for a nucleus with A-nucleons out of which Z-protons

$$f_i^{(A,Z)}(x, Q) = \frac{Z}{A} f_i^{p/A}(x, Q) + \frac{A - Z}{A} f_i^{n/A}(x, Q)$$

Note: PDF of neutron are related to the proton by isospin symmetry

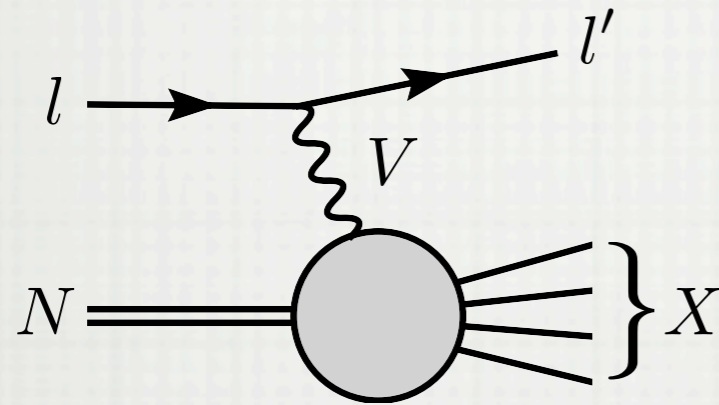
- Input scale and other input parameters as in CTEQ6M proton analysis

$$Q_0 = m_c = 1.3 \text{ GeV} \quad m_b = 4.5 \text{ GeV} \quad \alpha_s(m_Z) = 0.118$$

NUCLEAR CTEQ

- Experiments included in the analysis:

Deep Inelastic Scattering



$$l + N \rightarrow l' + X$$

CERN BCDMS & EMC & NMC

$N = (\text{D, Al, Be, C, Ca, Cu, Fe, Li, Pb, Sn, W})$

FNAL E-665

$N = (\text{D, C, Ca, Pb, Xe})$

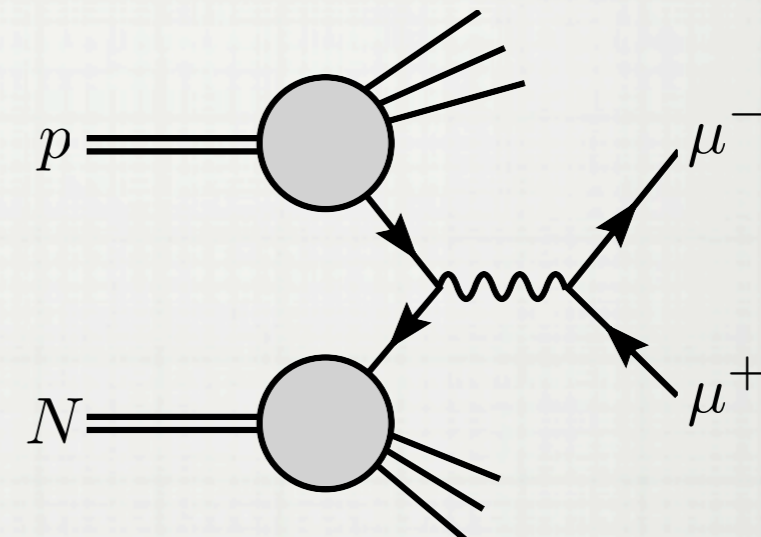
DESY Hermes

$N = (\text{D, He, N, Kr})$

SLAC E-139 & E-049

$N = (\text{D, Ag, Al, Au, Be, C, Ca, Fe, He})$

Drell-Yan process



$$p + N \rightarrow \mu^+ \mu^- + X$$

FNAL E-772 & E-886

$N = (\text{D, C, Ca, Fe, W})$

NUCLEAR CTEQ

NPDF fit properties:

- we fit nuclear data with NLO QCD predictions & include heavy quark effects (ACOT)
- added nuclear observables to CTEQ fitting routines (need to treat 2 nuclei at once)

$$\text{DIS: } F_2^A / F_2^{A'} \quad \text{Drell-Yan: } \sigma_{DY}^{pA} / \sigma_{DY}^{pA'}$$

- applied standard CTEQ kinematical cuts $Q > 2\text{GeV}$ & $W > 3.5\text{GeV}$

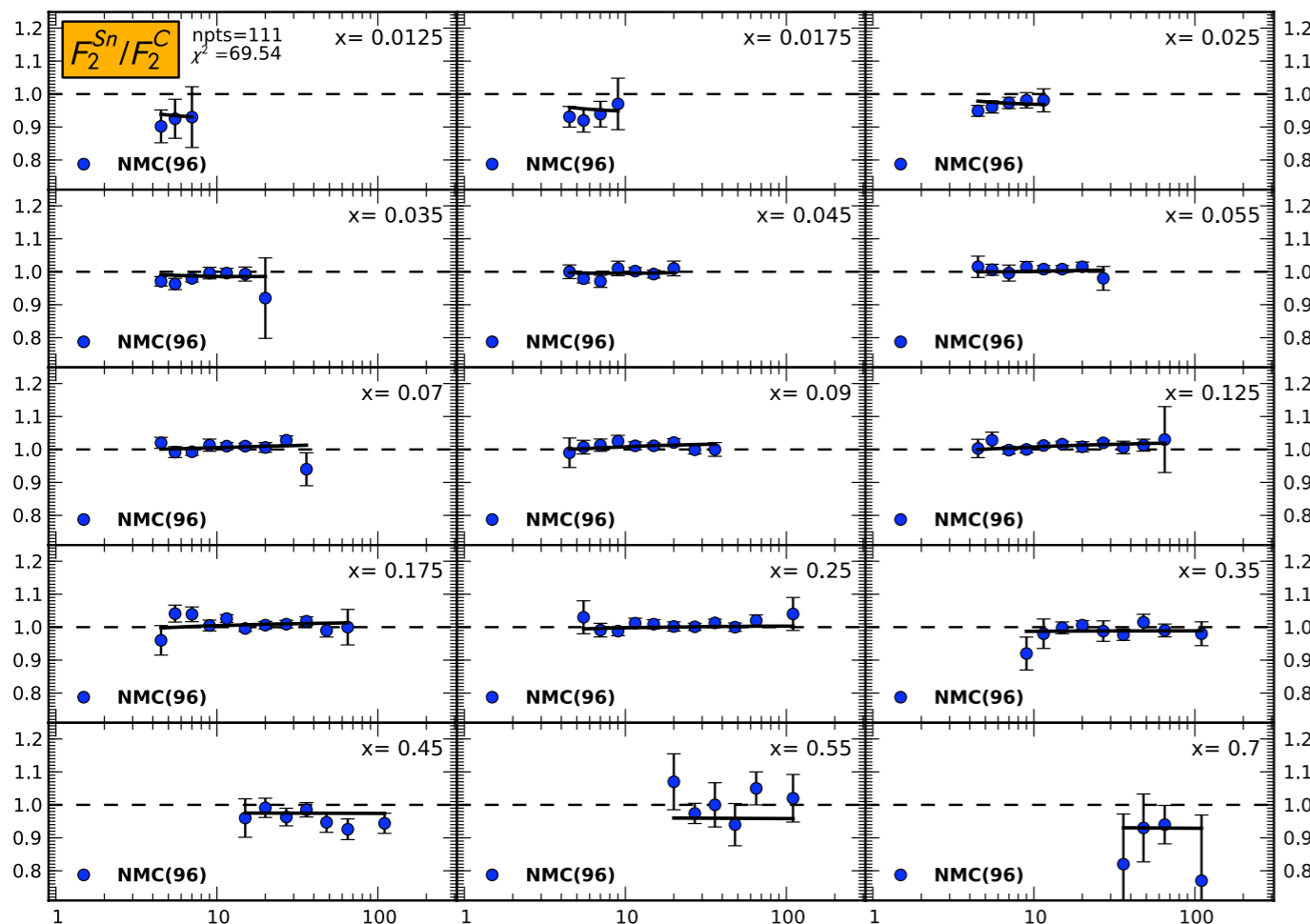
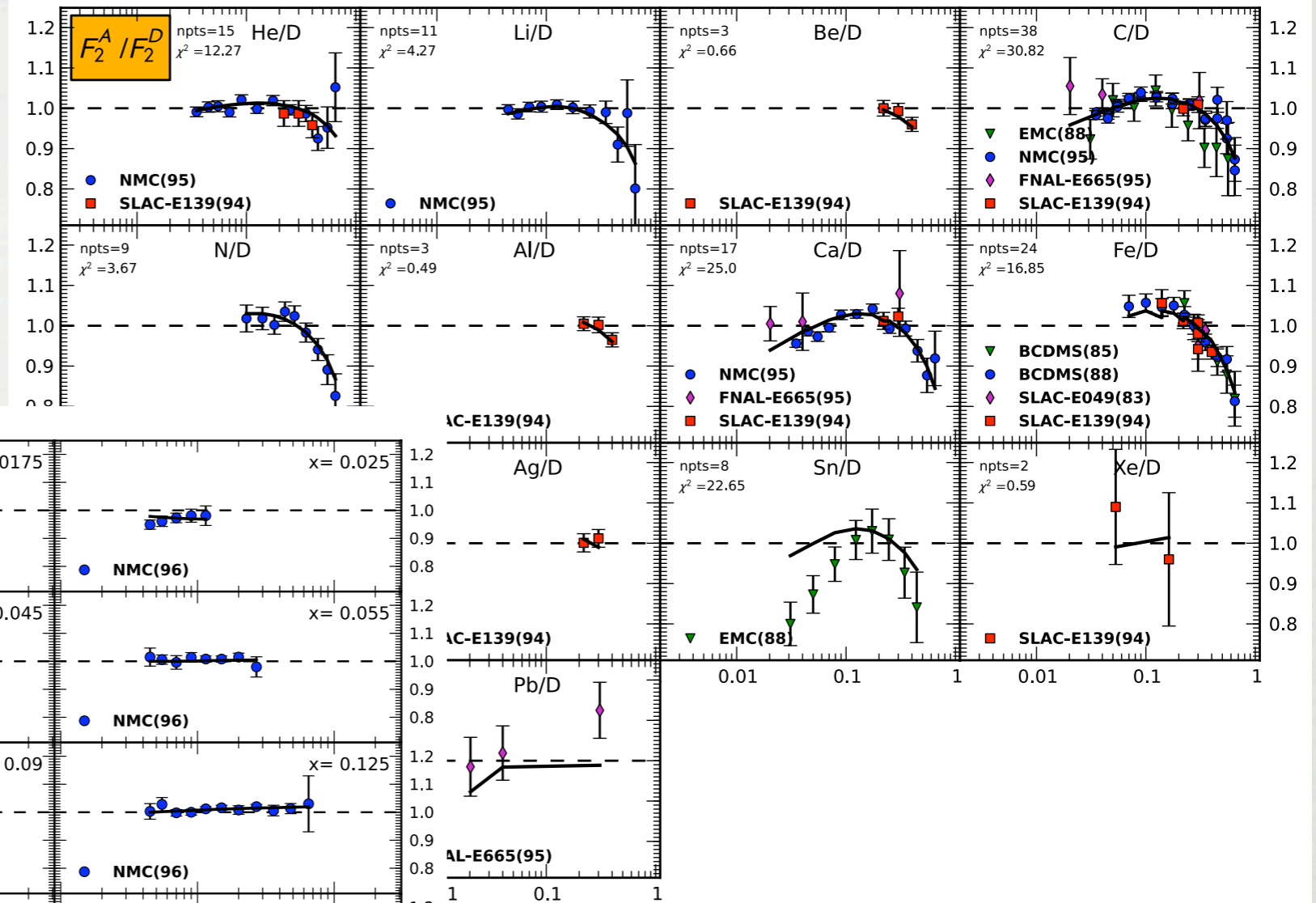
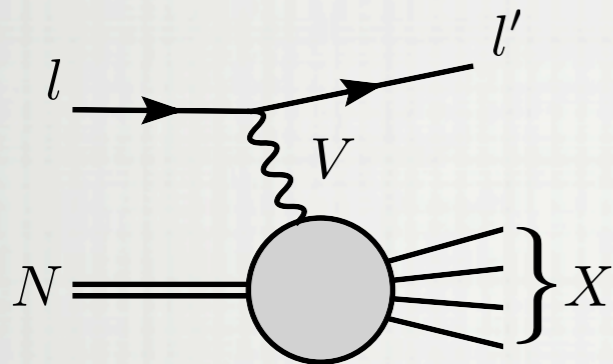
NPDF fit results:

- 708 (1233) data points after (before) cuts
- 32 free parameters - 675 degrees of freedom
- overall $\chi^2/\text{dof} = 0.95$
- individually for different data sets

- for F_2^A / F_2^D $\chi^2/\text{pt} = 0.92$
- for $F_2^A / F_2^{A'}$ $\chi^2/\text{pt} = 0.69$
- for $\sigma_{DY}^{pA} / \sigma_{DY}^{pA'}$ $\chi^2/\text{pt} = 1.08$

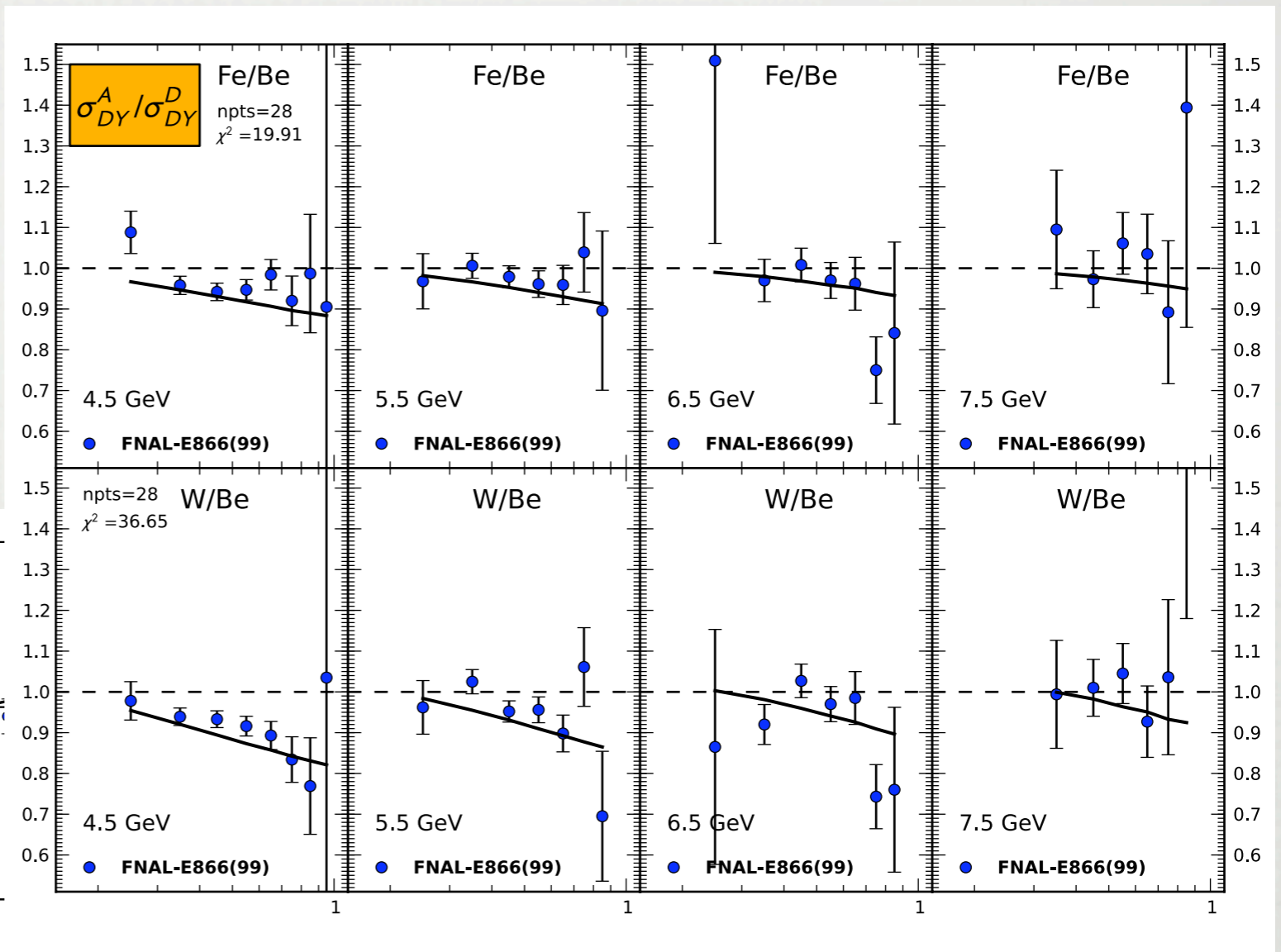
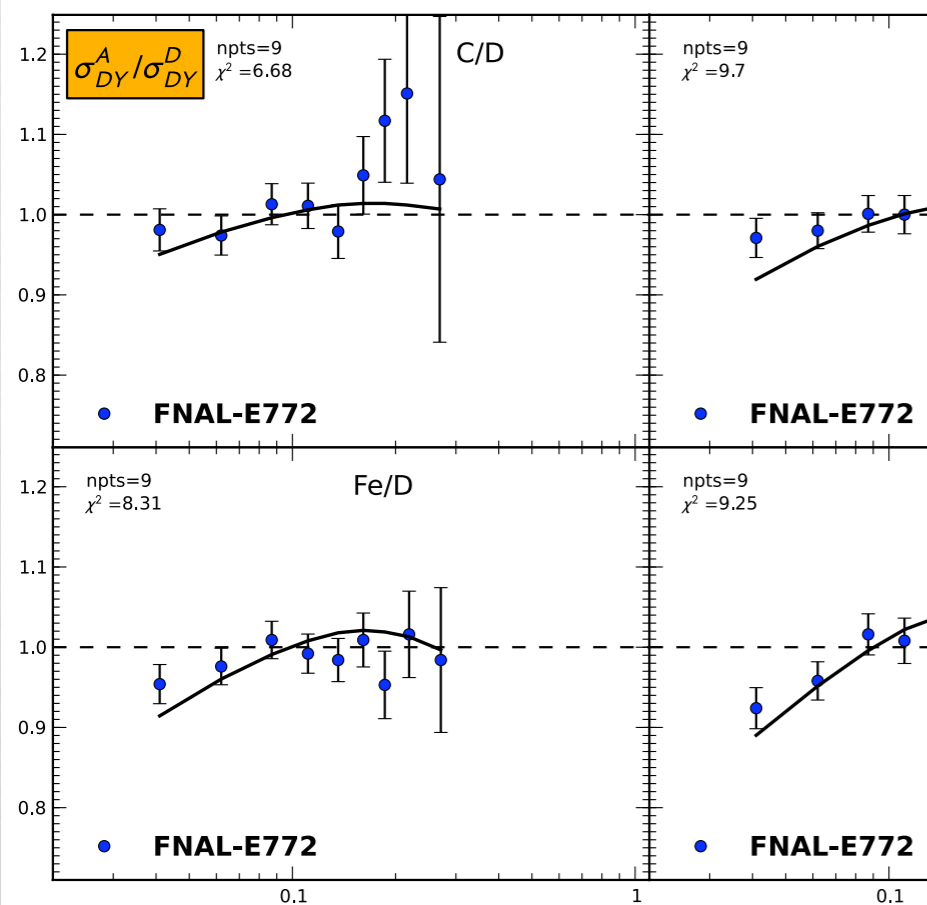
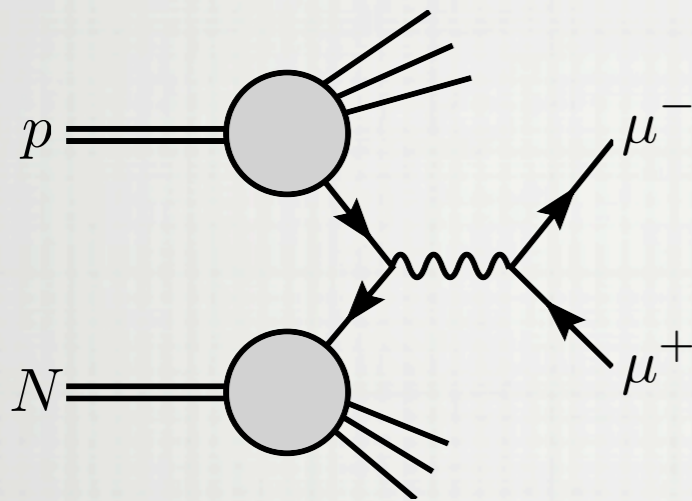
NUCLEAR CTEQ

Deep Inelastic Scattering



NUCLEAR CTEQ

Drell-Yan process

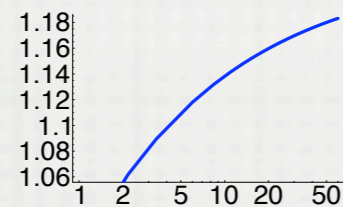
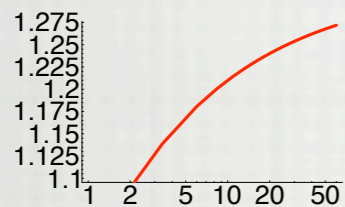


NUCLEAR CTEQ

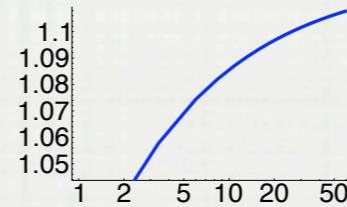
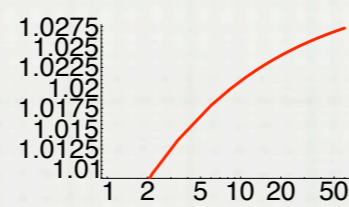
CTEQ parameters dependent on atomic number A - $c_k(A)/c_{k,0}$

for parton distributions $\begin{pmatrix} d_v & u_v \\ g & \bar{u} + \bar{d} \end{pmatrix}$

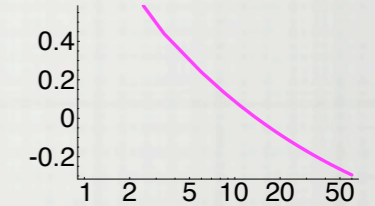
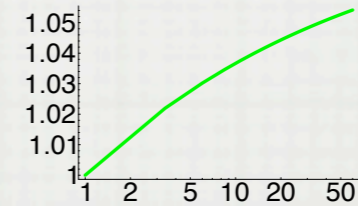
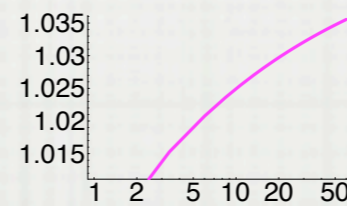
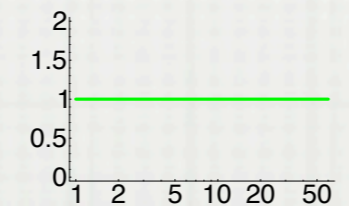
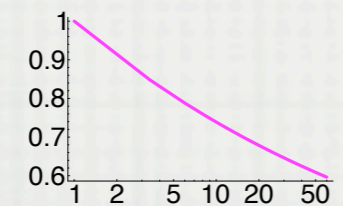
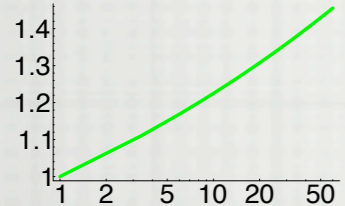
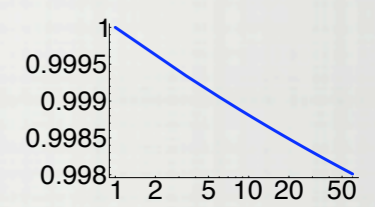
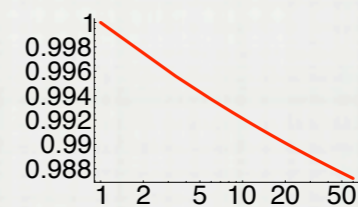
$c_1(A)/c_{k,0}$



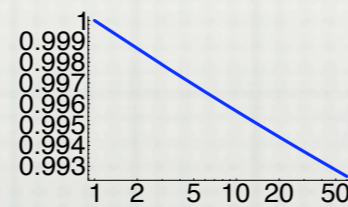
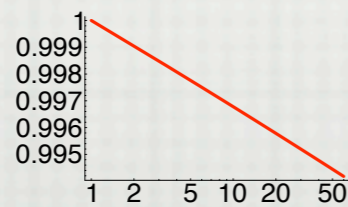
$c_2(A)/c_{k,0}$



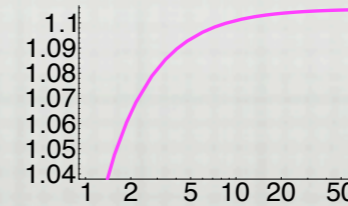
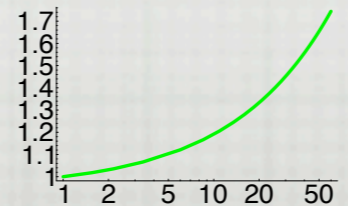
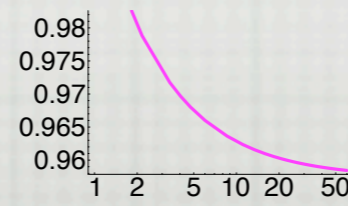
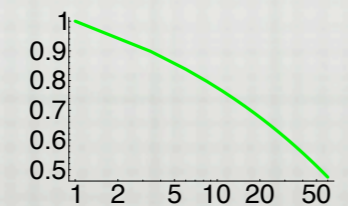
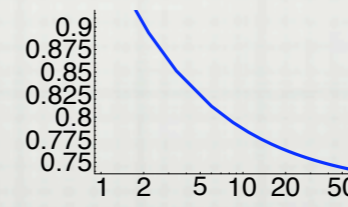
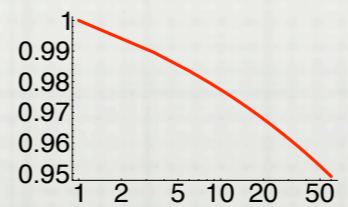
$c_3(A)/c_{k,0}$



$c_4(A)/c_{k,0}$



$c_5(A)/c_{k,0}$

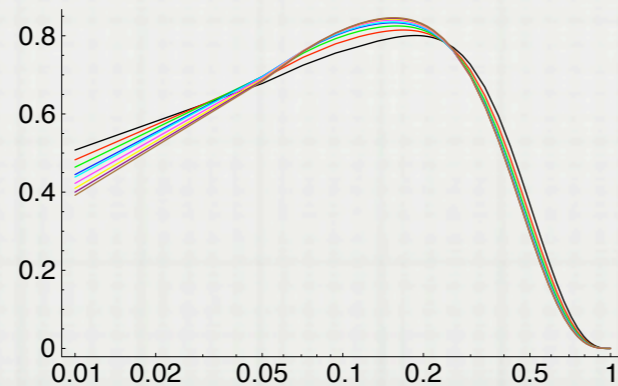


NUCLEAR CTEQ

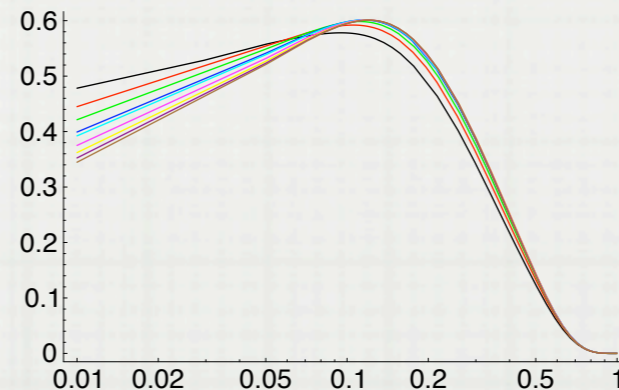
- Parton density functions for bound partons as a function of x

$x f_k^A(x, Q)$ for $A = (1, 2, 4, 9, 12, 27, 56, 108, 207)$
 black yellow brown
 red purple

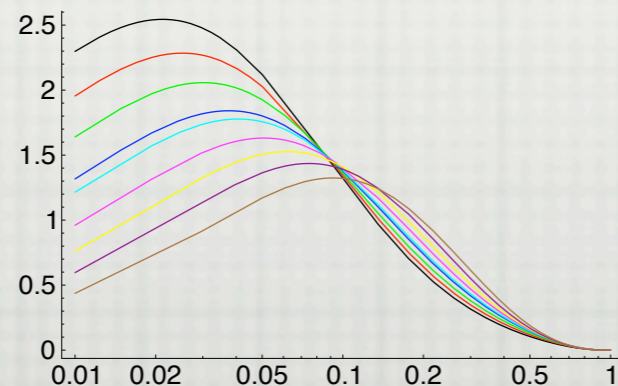
total up-quark



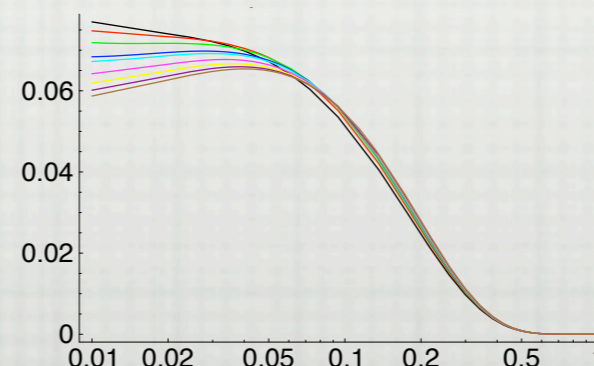
total down-quark



gluon



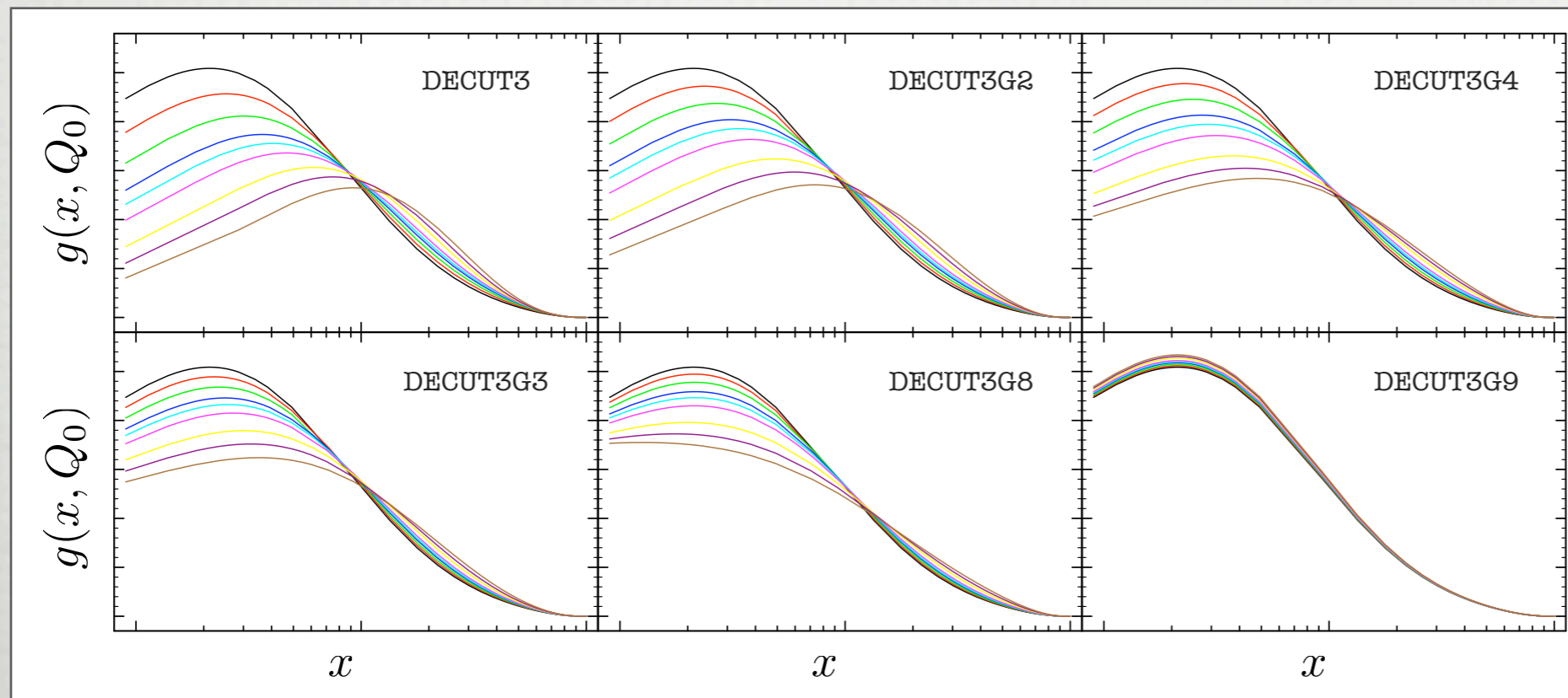
strange-quark



NUCLEAR CTEQ

- Example of different assumptions in nuclear gluon PDF
- nCTEQ estimate of gluon nPDF uncertainty
 - vary gluon nPDF assumptions & parametrizations
 - large uncertainty for low $x < 0.1$ in nCTEQ framework
 - need further data to constrain gluon nPDF

JHEP 1101 (2011) 152



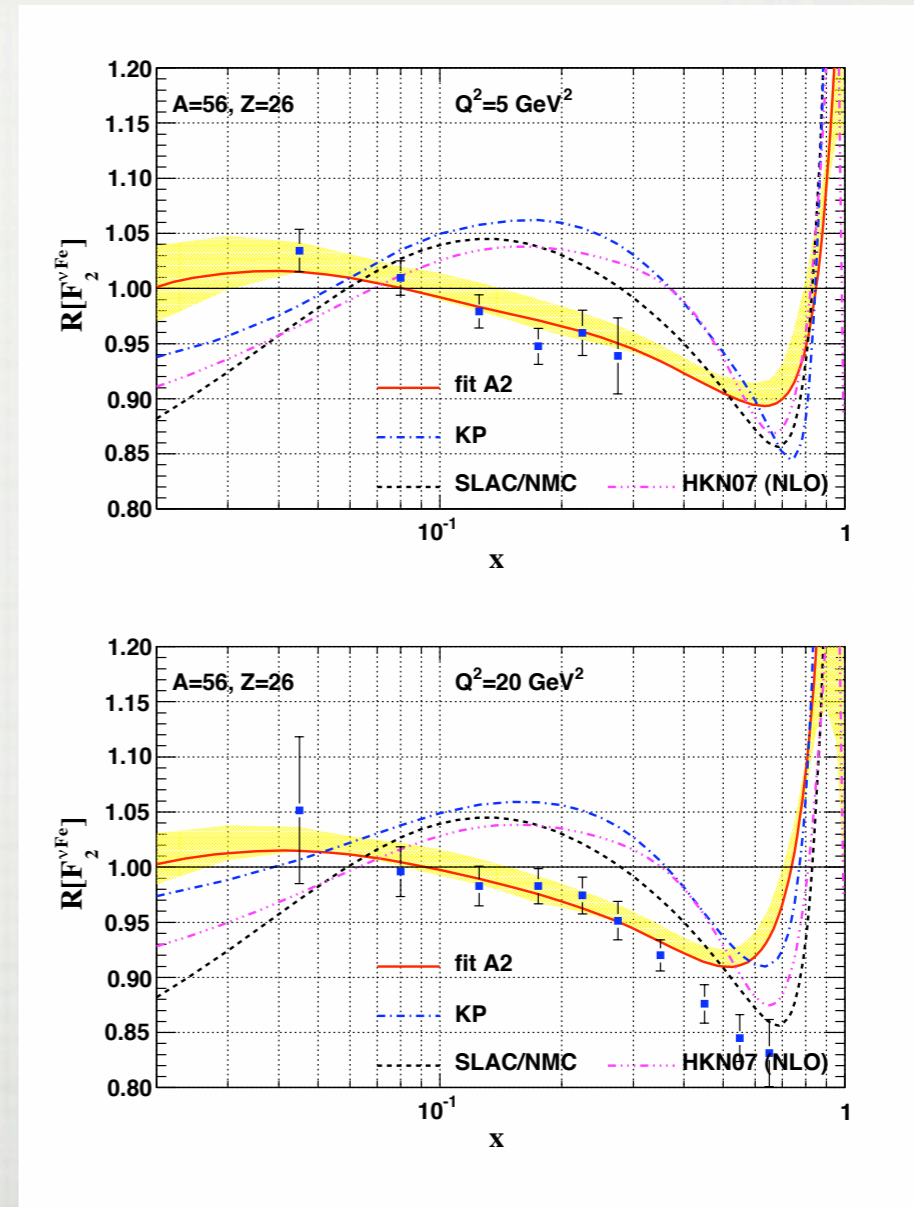
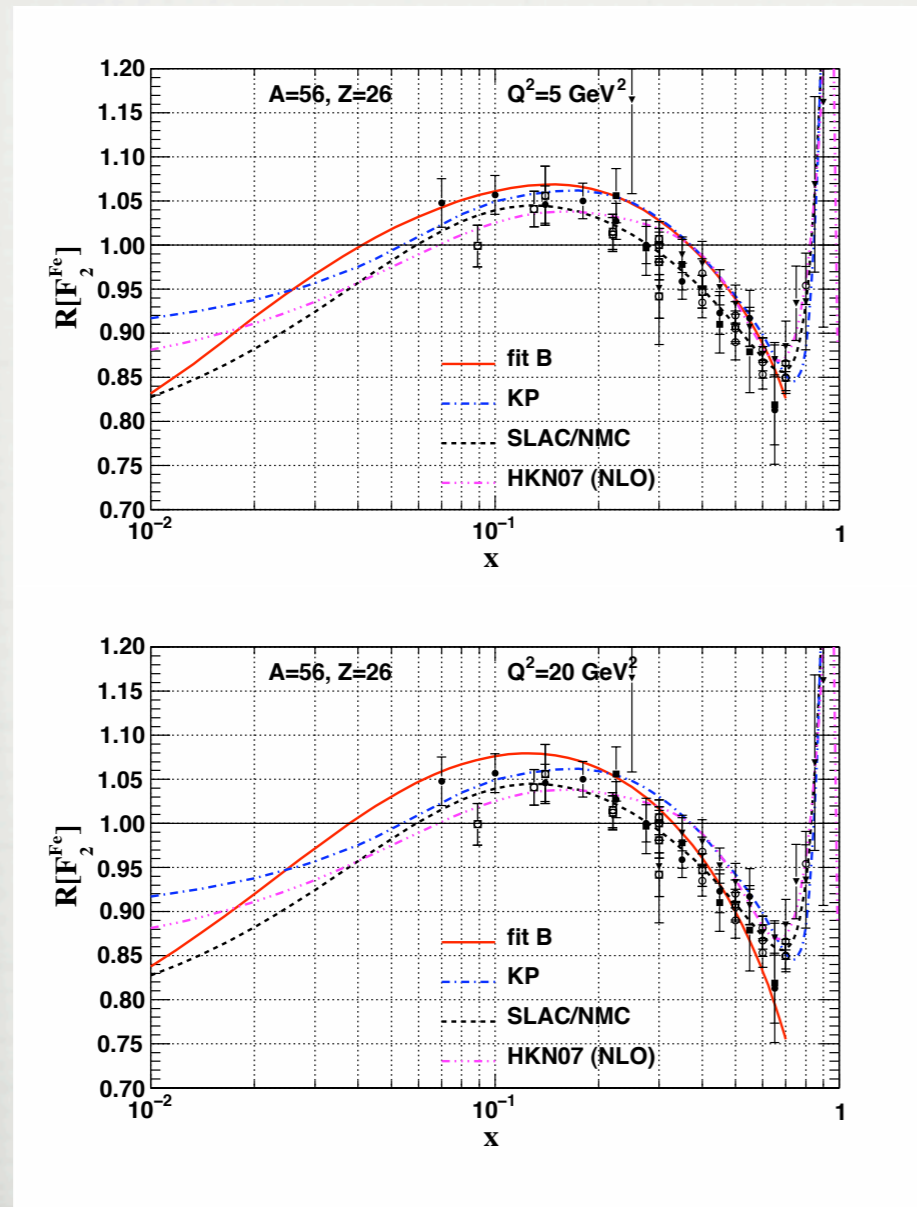
NEUTRINO DIS

Comparison of iron F_2 from neutrino and charged lepton DIS

$$R[F_2^{Fe}] = F_2^{Fe} / F_2^D$$

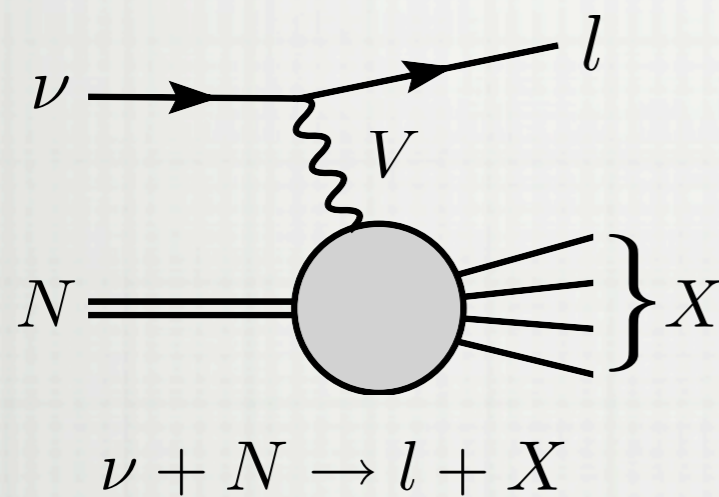
Phys.Rev.D80 094004, 2009

Phys.Rev.D77 054013, 2008



NEUTRINO DIS

- Re-analyze neutrino data within the same framework as for charged lepton
- Neutrino DIS cross-section data



NuTeV & di-muon

$$N = \text{Fe}$$

→ 2310 data points

CHORUS

$$N = \text{Pb}$$

→ 824 data points

All charged lepton DIS & Drell-Yan data

→ 708 data points

- Challenges in combining the neutrino & charged lepton data
 - deal with the disparity of number of data points - assigning weights to neutrino data
 - neutrino DIS data only with 2 heavy nuclei - insufficient to get a reliable A -dependence
 - do all neutrino data show the different behavior or only NuTeV ?

NEUTRINO DIS

○ Properties of neutrino fits

- CHORUS data are in good agreement with the charged lepton data

combined: $\chi^2/\text{pt}=1.03$

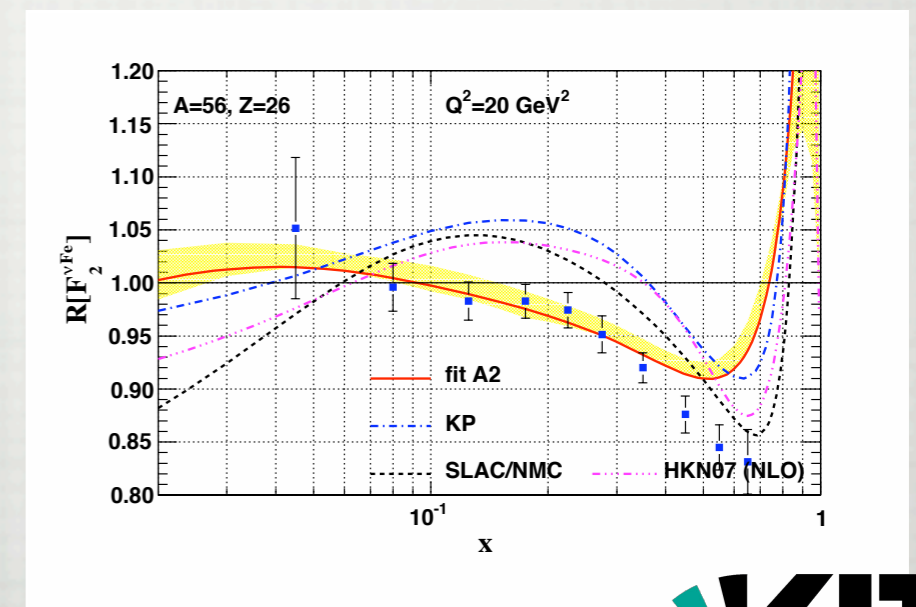
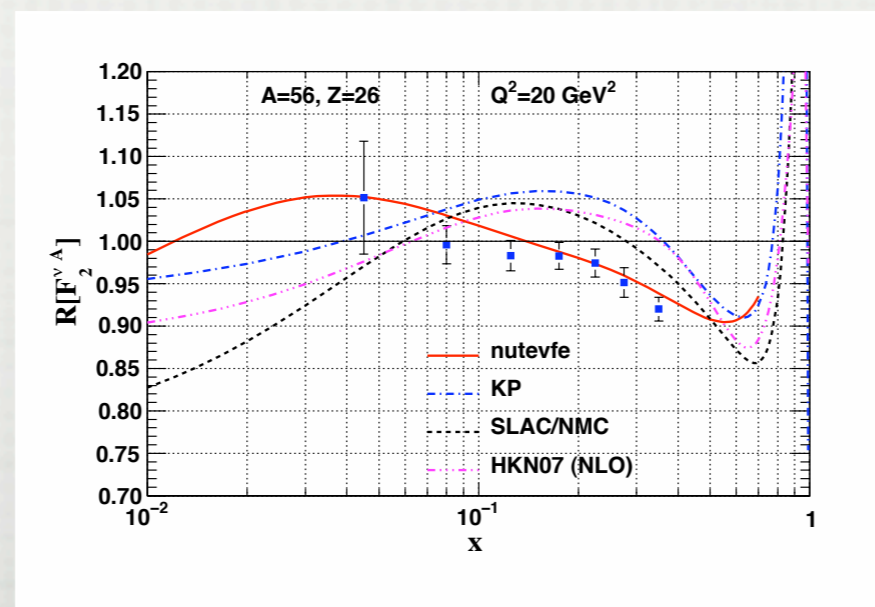
- NuTeV data (with correlated errors) difficult to fit alone or with the charged lepton data

alone: $\chi^2/\text{pt}=1.35$

combined: $\chi^2/\text{pt}=1.33$

- Neutrino data dominate the combined fit without re-weighting - final result depends from the weight chosen

Consistency check
with only NuTeV

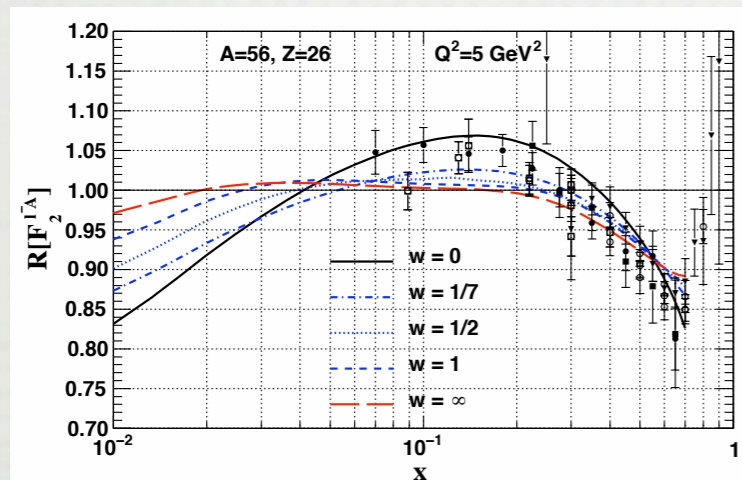


NEUTRINO DIS

Analysis of fits with different weights of neutrino DIS (corr. errors)

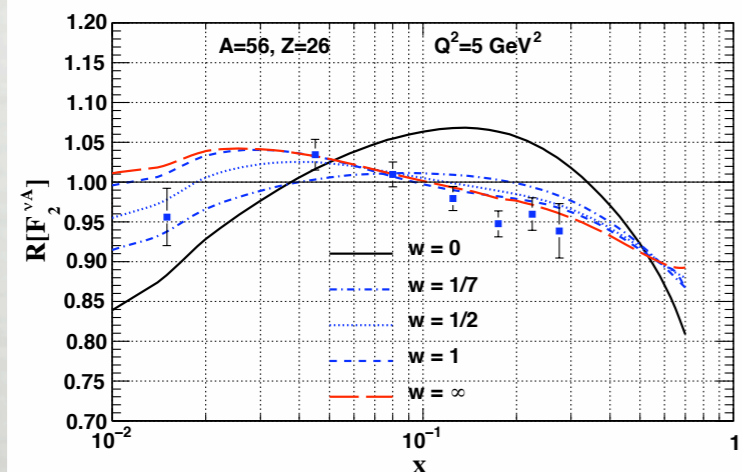
- Nuclear correction factors - $R = F_2^{Fe} / F_2^{Fe,0}$

w	$l^\pm A$	χ^2 (/pt)	νA	χ^2 (/pt)	total χ^2 (/pt)
0	708	638 (0.90)	-	-	638 (0.90)
1/7	708	645 (0.91)	3134	4710 (1.50)	5355 (1.39)
1/2	708	680 (0.96)	3134	4405 (1.40)	5085 (1.32)
1	708	736 (1.04)	3134	4277 (1.36)	5014 (1.30)
∞	-	-	3134	4192 (1.33)	4192 (1.33)



$$R[F_2^{l^\pm A}]$$

$Q^2 = 5 \text{ GeV}^2$



$$R[F_2^{\nu A}]$$

$Q^2 = 5 \text{ GeV}^2$

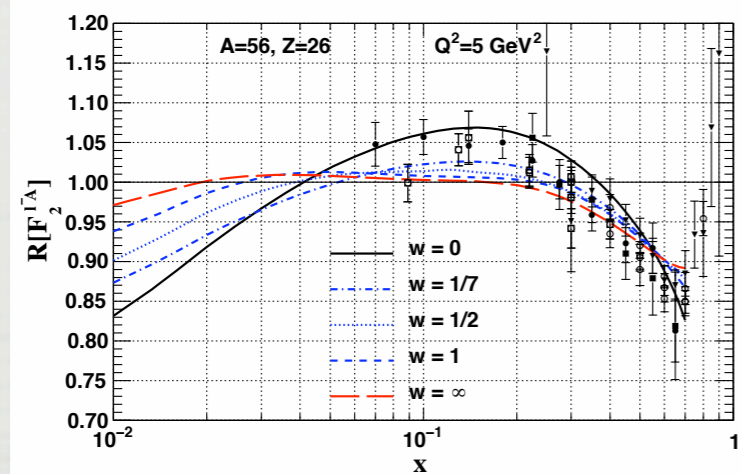


NEUTRINO DIS

Analysis of fits with different weights of neutrino DIS (corr. errors)

- Nuclear correction factors - $R = F_2^{Fe} / F_2^{Fe,0}$

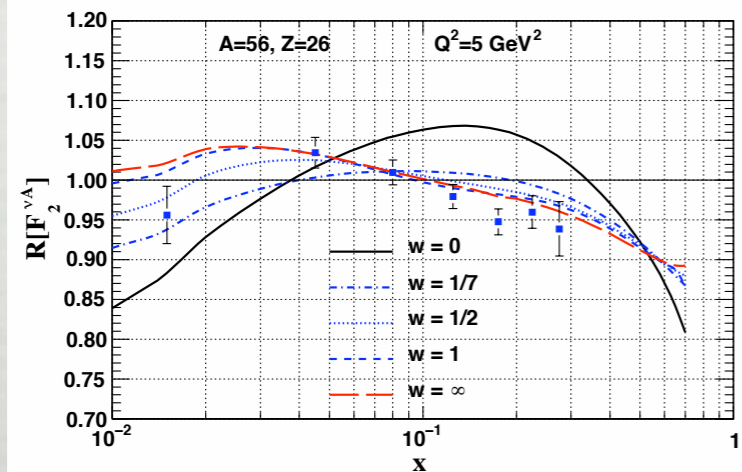
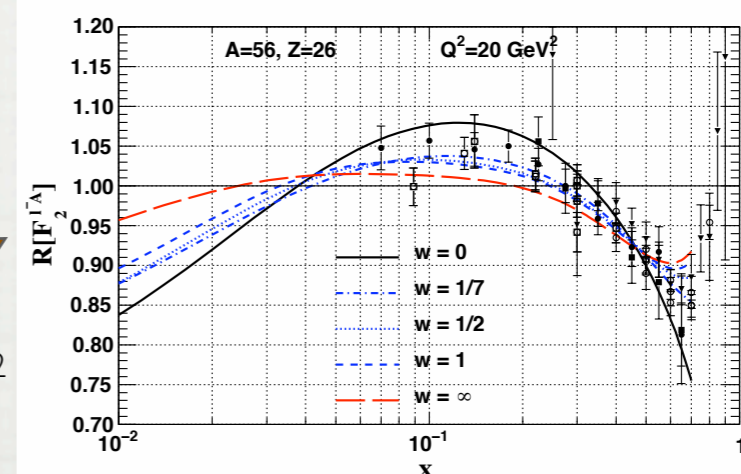
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∞	-	-	3134	4192 (1.33)	4192 (1.33)



$$R[F_2^{l^\pm A}]$$

$Q^2 = 5 \text{ GeV}^2$

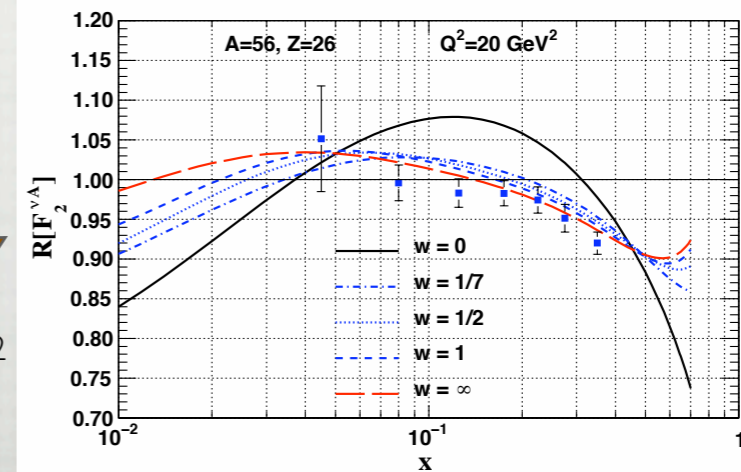
$Q^2 = 20 \text{ GeV}^2$



$$R[F_2^{\nu A}]$$

$Q^2 = 5 \text{ GeV}^2$

$Q^2 = 20 \text{ GeV}^2$



NEUTRINO DIS

● Analysis of fits with different weights of neutrino DIS (corr. errors)

- χ^2 - distribution criterion

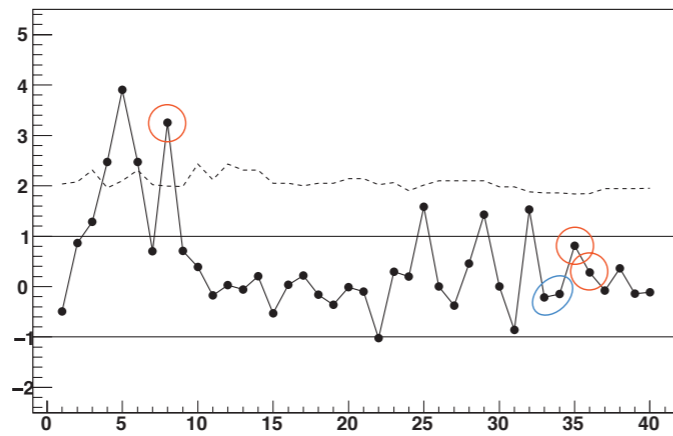
$$P(\chi^2, N) = \frac{(\chi^2)^{N/2-1} e^{-\chi^2/2}}{2^{N/2} \Gamma(N/2)}$$

CTEQ [hep-ph/0101051](#)

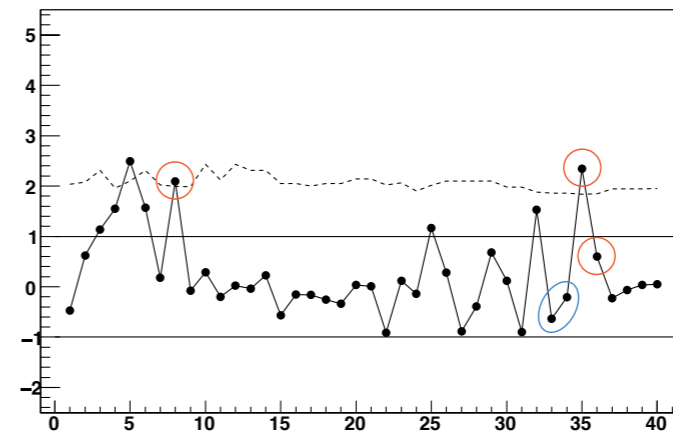
MSTW [arXiv:0901.0002 \[hep-ph\]](#)

error PDFs defined as 90% C.L. $\rightarrow \int_0^{\xi_{90}} P(\chi^2, N) d\chi^2 = 0.90$

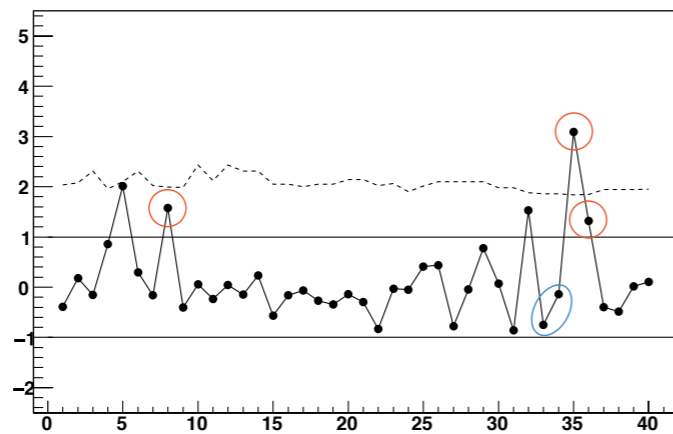
(w=1)



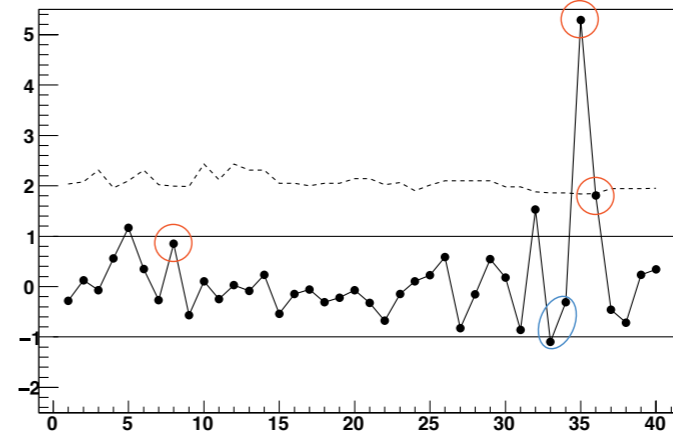
(w=1/2)



(w=1/4)



(w=1/7)



NEUTRINO DIS

● Analysis of fits with different weights of neutrino DIS (corr. errors)

- χ^2 - distribution criterion

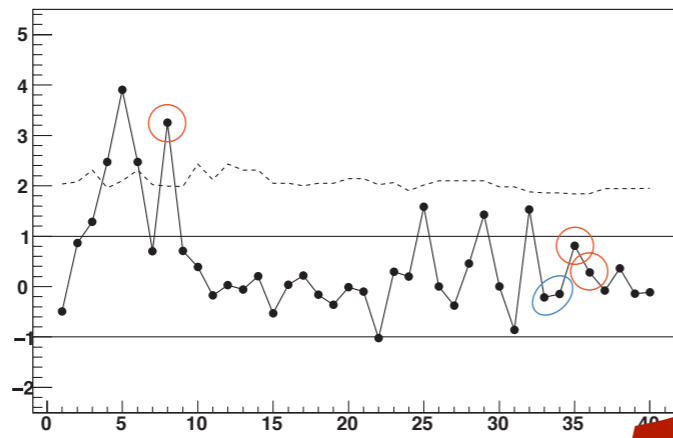
$$P(\chi^2, N) = \frac{(\chi^2)^{N/2-1} e^{-\chi^2/2}}{2^{N/2} \Gamma(N/2)}$$

CTEQ [hep-ph/0101051](https://arxiv.org/abs/hep-ph/0101051)

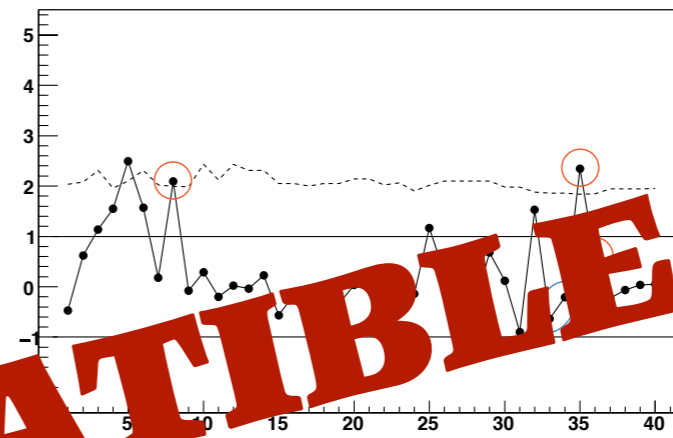
MSTW [arXiv:0901.0002 \[hep-ph\]](https://arxiv.org/abs/hep-ph/0901002)

error PDFs defined as 90% C.L. $\rightarrow \int_0^{\xi_{90}} P(\chi^2, N) d\chi^2 = 0.90$

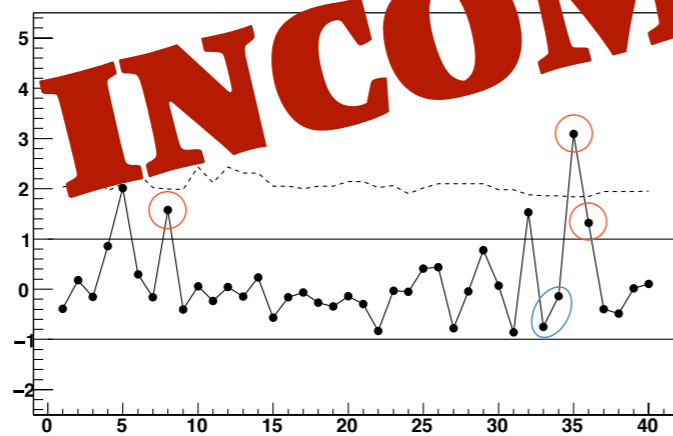
(w=1)



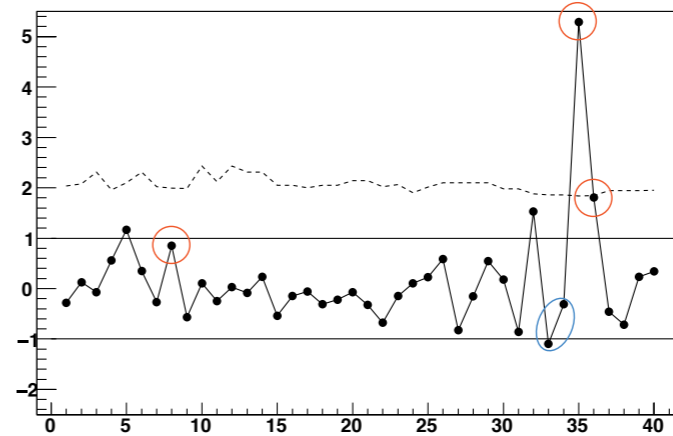
(w=1/2)



(w=1/4)



(w=1/7)



INCOMPATIBLE

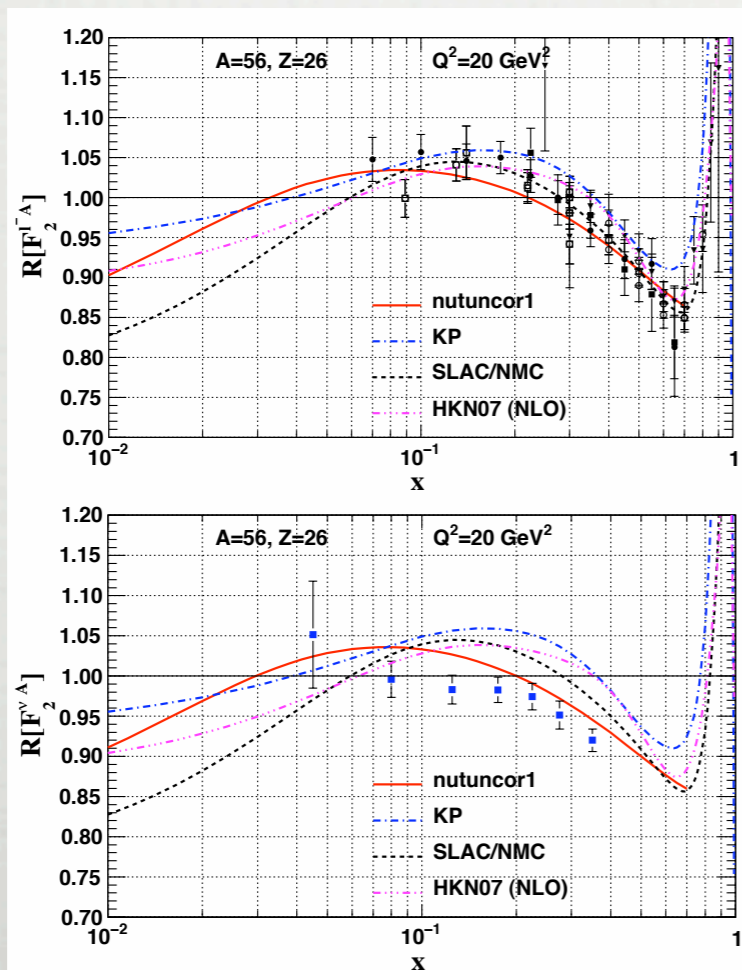
NEUTRINO DIS

Analysis of fits with neutrino DIS (uncorrelated errors)

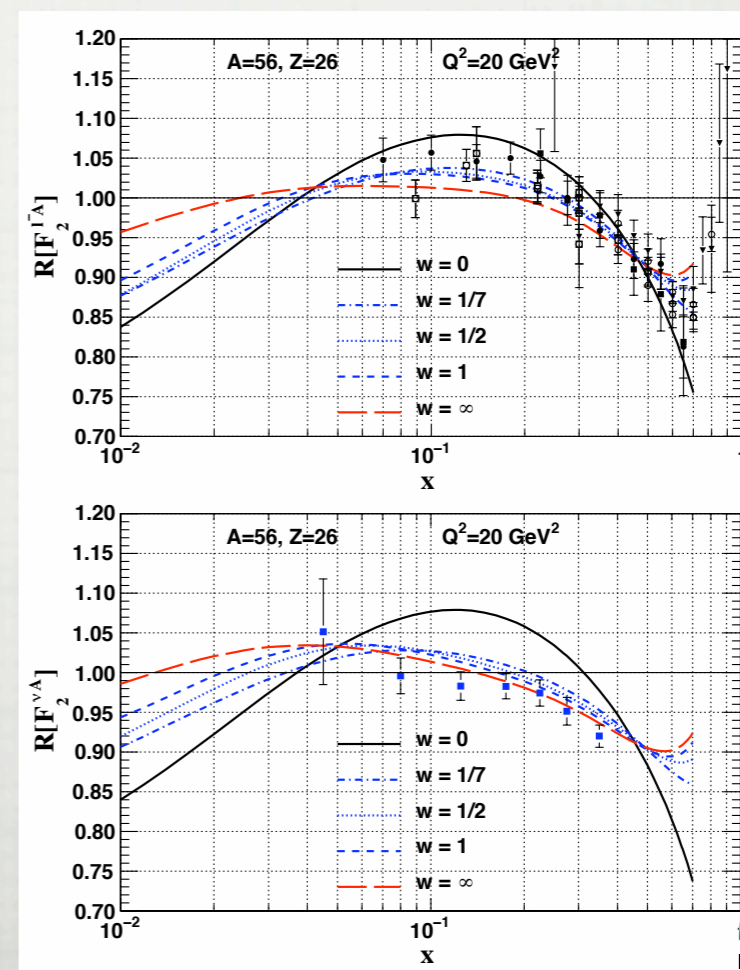
- Nuclear correction factors - $R = F_2^{Fe} / F_2^{Fe,0}$

w	$l^\pm A$	χ^2 (/pt)	νA	χ^2 (/pt)	total χ^2 (/pt)
1-uncorr	708	736 (1.04)	3134	4277 (1.36)	5014 (1.30)
1-corr	708	809 (1.14)	3110	3115 (1.00)	3924 (1.02)

uncorrelated errors



correlated errors



OUTLOOK

- Global nuclear CTEQ fit is able to describe the charged lepton data well
 - some challenges on the way to a comprehensive public nPDF release
 - relax kinematical cuts and fit Fermi motion peak in a natural way
 - error PDFs & realistic estimate of uncertainty
- Incompatibility of neutrino DIS with charged lepton DIS
 - incompatibility a "precision" effect - the result changes when using uncorrelated errors
 - tension in NuTeV data → high χ^2 of the fit to NuTeV alone → problem of NuTeV data ?
 - NOMAD data can help decide
- The impact of nuclear PDF from neutrino DIS on proton PDF
 - how does the incompatibility of neutrino DIS impact the uncertainty of strange quark PDF ?