Parton **D**istribution **F**unctions

Graeme Watt

CERN PH-TH

Fall meeting of the GDR PH-QCD: nucleon and nucleus structure studies with a LHC fixed-target experiment and electron–ion colliders

Institut de Physique Nucléaire (IPN), Orsay, France
19th October 2011

Introduction: **P**arton **D**istribution **F**unctions (**PDFs**)

- $f_{a/A}(x, Q^2)$ gives *number density* of partons a in hadron A with momentum fraction x at a hard scale $Q^2 \gg \Lambda^2_{\mathrm{QCD}}$.
- **Scope:** I will discuss only the plainest "*vanilla*" PDFs.

PDFs in this talk	PDFs in later talks
Proton ($A = p$)	Nuclear
Unpolarised	Polarised
Integrated over $\mathbf{k}_T$	Unintegrated/TMDs
Integrated over $\mathbf{b}_T$	GPDs
Perturbative heavy quarks	Intrinsic heavy quarks
Linear evolution	Non-linear evolution

- **Simplest case:** abundant data and precise calculations available.
- **Most important case (arguably):** goes well beyond simply understanding nucleon structure and QCD for its own sake.
- PDFs *necessary* to compute LHC cross sections and enable ~~discovery~~/exclusion of new physics beyond the Standard Model.

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Fixed-order collinear factorisation at hadron colliders

- Expand $\hat{\sigma}_{ab}$, $P_{aa'}$ and β as perturbative series in α_S.

$$\boxed{\sigma_{AB} = \sum_{a,b=q,g} \left[\hat{\sigma}_{ab}^{\rm LO} + \alpha_S(Q^2)\hat{\sigma}_{ab}^{\rm NLO} + \ldots\right] \otimes f_{a/A}(x_a, Q^2) \otimes f_{b/B}(x_b, Q^2)}$$

PDF evolution: $$\frac{\partial f_{a/A}}{\partial \ln Q^2} = \frac{\alpha_S}{2\pi} \sum_{a'=q,g} \left[P_{aa'}^{\rm LO} + \alpha_S P_{aa'}^{\rm NLO} + \ldots\right] \otimes f_{a'/A}$$

α_S evolution: $$\frac{\partial \alpha_S}{\partial \ln Q^2} = -\beta^{\rm LO}\alpha_S^2 - \beta^{\rm NLO}\alpha_S^3 - \ldots$$

- Need to extract input values $f_{a/A}(x, Q_0^2)$ and $\alpha_S(M_Z^2)$ from data.
- Structure functions in deep-inelastic scattering (DIS):

$$F_i\left(x_{\rm Bj}, Q^2\right) = \sum_{a=q,g} C_{i,a} \otimes f_{a/A}, \quad C_{i,a} = C_{i,a}^{\rm LO} + \alpha_S C_{i,a}^{\rm NLO} + \ldots$$

- The "standard" pQCD framework: holds up to formally power-suppressed ("higher-twist") terms $\mathcal{O}(\Lambda_{\rm QCD}^2/Q^2)$.

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Paradigm for PDF determination by “global analysis”

1. **Parameterise** the x dependence for each flavour $a = q, g$ at the input scale $Q_0^2 \sim 1\ \mathrm{GeV}^2$ in some flexible form, e.g.

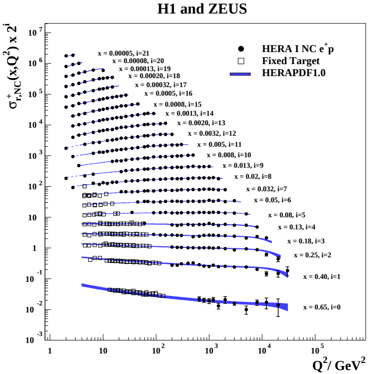
$$xf_{a/p}(x, Q_0^2) = A_a\, x^{\Delta_a}\, (1-x)^{\eta_a}\, (1 + \epsilon_a\, \sqrt{x} + \gamma_a\, x),$$

   subject to number- and momentum-sum rule constraints.
2. **Evolve** the PDFs to higher scales $Q^2 > Q_0^2$ using the **DGLAP** (**D**okshitzer–**G**ribov–**L**ipatov–**A**ltarelli–**P**arisi) evolution equations.
3. **Convolute** the evolved PDFs with $C_{i,a}$ and $\hat{\sigma}_{ab}$ to calculate theory predictions corresponding to a wide variety of data.
4. **Vary** the input parameters $\{A_a, \Delta_a, \eta_a, \epsilon_a, \gamma_a, \ldots\}$ to minimise

$$\chi^2 = \sum_{i=1}^{N_{\rm pts.}} \left(\frac{\mathrm{Data}_i - \mathrm{Theory}_i}{\mathrm{Error}_i} \right)^2$$

   or generalisations to account for *correlated* systematic errors.

Scaling violations of DIS structure functions



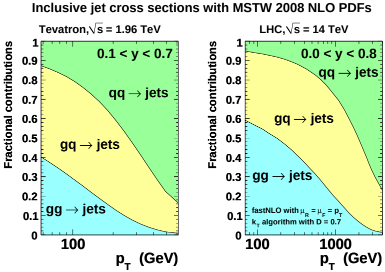
$$\boxed{F_2^{\mathrm{LO}}(x,Q^2) = \sum_q e_q^2\; x f_{q/p}(x,Q^2)}$$

Differentiate and insert DGLAP:

$$\Rightarrow \frac{\partial F_2^{\mathrm{LO}}(x,Q^2)}{\partial \ln Q^2} = \frac{\alpha_S}{2\pi}\left[P^{\mathrm{LO}}_{q\leftarrow q} \otimes F_2^{\mathrm{LO}} + \sum_q e_q^2\, P^{\mathrm{LO}}_{q\leftarrow g} \otimes f_{g/p}\right]$$

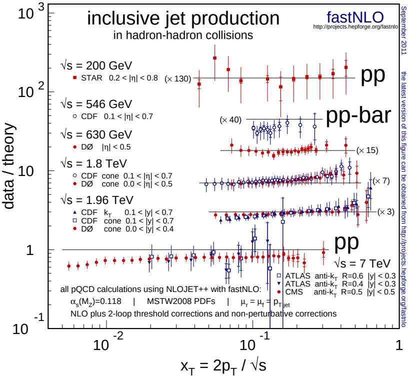
- F_2 falls with Q^2 at large x and rises with Q^2 at small x.
- $\Rightarrow$ Measure *scaling violations* to constrain small-x gluon.
- **But** no direct constraint on large-x gluon from inclusive DIS.

Inclusive jet production at the Tevatron and LHC



[MSTW, `arXiv:0905.3531`]

- Quarks constrained by other data ⇒ jets constrain gluon.



[M. Wobisch *et al.*, `arXiv:1109.1310`]

- **LHC jets**: generally lower x_T, no correlated systematics.
- Current best constraint on high-x gluon from **Tevatron jets**.
- Possible alternative: isolated prompt photons ($gq \to \gamma q$).

Flavour decomposition from structure functions

- Assume isospin symmetry ($u \equiv u^p = d^n$, $d \equiv d^p = u^n$) and isoscalar target ($N = (p+n)/2$), then structure functions are:

Neutral-current DIS in charged-lepton–nucleon scattering

$$F_2^{\ell^\pm p} = x\left[\frac{4}{9}\left(u+\bar{u}+\ldots\right)+\frac{1}{9}\left(d+\bar{d}+\ldots\right)\right]$$

$$F_2^{\ell^\pm d} = (F_2^{\ell^\pm p}+F_2^{\ell^\pm n})/2 = \frac{5}{18}x\left(u+\bar{u}+d+\bar{d}\right)+\ldots$$

Charged-current DIS in neutrino–nucleus scattering

$$F_2 \equiv (F_2^{\nu N}+F_2^{\bar{\nu} N})/2 = x\left(u+\bar{u}+d+\bar{d}+\ldots\right)$$

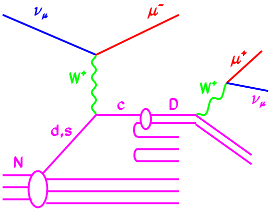
$$xF_3 \equiv (xF_3^{\nu N}+xF_3^{\bar{\nu} N})/2 = x\left(u-\bar{u}+d-\bar{d}+\ldots\right)$$

- $\nu N \to \mu\mu X$ constrains s; $\bar{\nu} N \to \mu\mu X$ constrains $\bar{s}$.

Deuterium and neutrino DIS data provide **flavour separation**.

(**But** need *nuclear corrections* to extract underlying proton PDFs.)

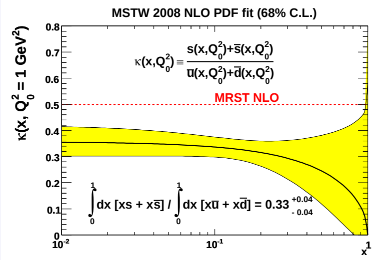
NuTeV/CCFR dimuon cross sections and strangeness



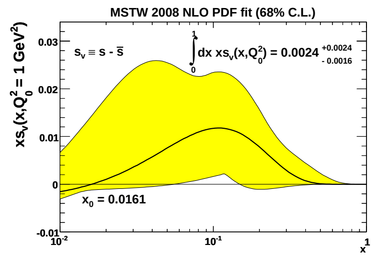
$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\mathrm{d}y}(\nu_\mu N \to \mu^+\mu^- X) = B_c\,\mathcal{A}\,\frac{\mathrm{d}\sigma}{\mathrm{d}x\mathrm{d}y}(\nu_\mu N \to \mu^- c\,X)$$
$$\propto |V_{cs}|^2\xi s(\xi, Q^2) + |V_{cd}|^2 \ldots$$

- ν_μ and $\bar{\nu}_\mu$ cross sections constrain s and $\bar{s}$, respectively, for $0.01 \lesssim x \lesssim 0.2$.

$(s+\bar{s})/(\bar{u}+\bar{d})$ at $Q_0^2 = 1$ GeV^2

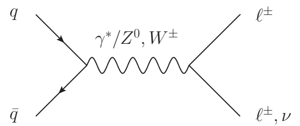


Strange asymmetry, $xs - x\bar{s}$



Flavour decomposition from Drell–Yan processes

Z and W production at Tevatron $p\bar{p}$ collider ($\sqrt{s} = 1.96$ TeV)



$$\frac{\mathrm{d}\sigma(Z)}{\mathrm{d}y_Z} \sim 0.29\,u(x_1)u(x_2) + 0.37\,d(x_1)d(x_2)$$

$$A_W(y_W) = \frac{\mathrm{d}\sigma(W^+)/\mathrm{d}y_W - \mathrm{d}\sigma(W^-)/\mathrm{d}y_W}{\mathrm{d}\sigma(W^+)/\mathrm{d}y_W + \mathrm{d}\sigma(W^-)/\mathrm{d}y_W} \approx \frac{d/u(x_2) - d/u(x_1)}{d/u(x_2) + d/u(x_1)}$$

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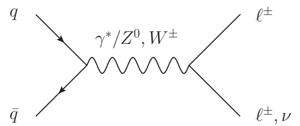

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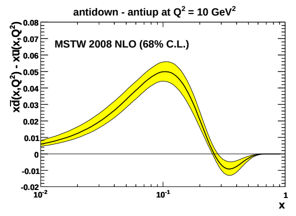
γ^* production at FNAL E866/NuSea ($\sqrt{s} = 38.8$ GeV)

- p beam (800 GeV) on p/d target.

$$\sigma^{pp} \sim \frac{4}{9}u(x_1)\bar{u}(x_2) + \frac{1}{9}d(x_1)\bar{d}(x_2)$$

$$\frac{\sigma^{pd}}{\sigma^{pp}} \sim 1 + \frac{\bar{d}(x_2)}{\bar{u}(x_2)} \quad \text{for } x_2 \in [0.015, 0.35]$$



Which processes constrain different PDFs?

- Processes included in **MSTW 2008** analysis [`arXiv:0901.0002`]:

Process	Subprocess	Partons	x range
$\ell^\pm\,\{p,n\} \to \ell^\pm\,X$	$\gamma^* q \to q$	$q,\bar{q},g$	$x \gtrsim 0.01$
$\ell^\pm\,n/p \to \ell^\pm\,X$	$\gamma^*\,d/u \to d/u$	d/u	$x \gtrsim 0.01$
$pp \to \mu^+\mu^-\,X$	$u\bar{u}, d\bar{d} \to \gamma^*$	$\bar{q}$	$0.015 \lesssim x \lesssim 0.35$
$pn/pp \to \mu^+\mu^-\,X$	$(u\bar{d})/(u\bar{u}) \to \gamma^*$	$\bar{d}/\bar{u}$	$0.015 \lesssim x \lesssim 0.35$
$\nu(\bar{\nu})\,N \to \mu^-(\mu^+)\,X$	$W^* q \to q'$	$q,\bar{q}$	$0.01 \lesssim x \lesssim 0.5$
$\nu\,N \to \mu^-\mu^+\,X$	$W^* s \to c$	s	$0.01 \lesssim x \lesssim 0.2$
$\bar{\nu}\,N \to \mu^+\mu^-\,X$	$W^* \bar{s} \to \bar{c}$	$\bar{s}$	$0.01 \lesssim x \lesssim 0.2$
$e^\pm\,p \to e^\pm\,X$	$\gamma^* q \to q$	$g,q,\bar{q}$	$0.0001 \lesssim x \lesssim 0.1$
$e^+\,p \to \bar{\nu}\,X$	$W^+\,\{d,s\} \to \{u,c\}$	d,s	$x \gtrsim 0.01$
$e^\pm p \to e^\pm\,c\bar{c}\,X$	$\gamma^* c \to c,\ \gamma^* g \to c\bar{c}$	$c,\ g$	$0.0001 \lesssim x \lesssim 0.01$
$e^\pm p \to \text{jet} + X$	$\gamma^* g \to q\bar{q}$	g	$0.01 \lesssim x \lesssim 0.1$
$p\bar{p} \to \text{jet} + X$	$gg, qg, qq \to 2j$	g,q	$0.01 \lesssim x \lesssim 0.5$
$p\bar{p} \to (W^\pm \to \ell^\pm\nu)\,X$	$ud \to W, \bar{u}\bar{d} \to W$	$u,d,\bar{u},\bar{d}$	$x \gtrsim 0.05$
$p\bar{p} \to (Z \to \ell^+\ell^-)\,X$	$uu, dd \to Z$	d	$x \gtrsim 0.05$

- **Heavy quarks** ($h = c, b$) generated perturbatively from $g \to h\bar{h}$.

Data sets fitted in MSTW 2008 NLO analysis [arXiv:0901.0002]

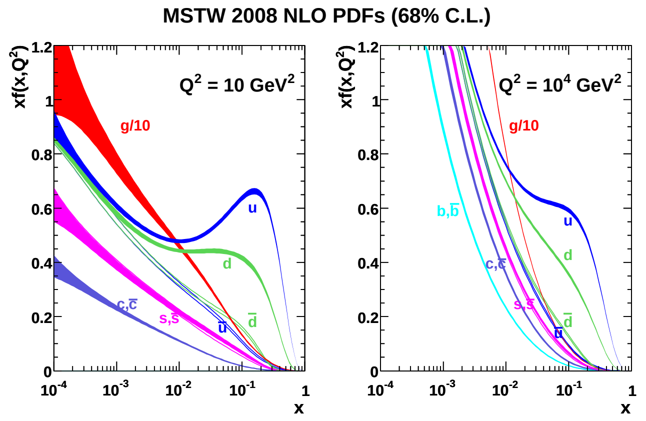
Data set	χ^2 / $N_{\rm pts.}$
H1 MB 99 e^+p NC	9 / 8
H1 MB 97 e^+p NC	42 / 64
H1 low Q^2 96–97 e^+p NC	44 / 80
H1 high Q^2 98–99 e^-p NC	122 / 126
H1 high Q^2 99–00 e^+p NC	131 / 147
ZEUS SVX 95 e^+p NC	35 / 30
ZEUS 96–97 e^+p NC	86 / 144
ZEUS 98–99 e^-p NC	54 / 92
ZEUS 99–00 e^+p NC	63 / 90
H1 99–00 e^+p CC	29 / 28
ZEUS 99–00 e^+p CC	38 / 30
H1/ZEUS $e^\pm p$ $F_2^{\rm charm}$	107 / 83
H1 99–00 e^+p incl. jets	19 / 24
ZEUS 96–97 e^+p incl. jets	30 / 30
ZEUS 98–00 $e^\pm p$ incl. jets	17 / 30
DØ II $p\bar{p}$ incl. jets	114 / 110
CDF II $p\bar{p}$ incl. jets	56 / 76
CDF II $W \to l\nu$ asym.	29 / 22
DØ II $W \to l\nu$ asym.	25 / 10
DØ II Z rap.	19 / 28
CDF II Z rap.	49 / 29

Data set	χ^2 / $N_{\rm pts.}$
BCDMS μp F_2	182 / 163
BCDMS μd F_2	190 / 151
NMC μp F_2	121 / 123
NMC μd F_2	102 / 123
NMC $\mu n/\mu p$	130 / 148
E665 μp F_2	57 / 53
E665 μd F_2	53 / 53
SLAC ep F_2	30 / 37
SLAC ed F_2	30 / 38
NMC/BCDMS/SLAC F_L	38 / 31
E866/NuSea pp DY	228 / 184
E866/NuSea pd/pp DY	14 / 15
NuTeV νN F_2	49 / 53
CHORUS νN F_2	26 / 42
NuTeV νN xF_3	40 / 45
CHORUS νN xF_3	31 / 33
CCFR $\nu N \to \mu\mu X$	66 / 86
NuTeV $\nu N \to \mu\mu X$	39 / 40
All data sets	**2543 / 2699**

- Red = New w.r.t. MRST 2006 fit.

MSTW 2008 PDFs [http://projects.hepforge.org/mstwpdf/]

A. D. **M**artin, W. J. **S**tirling, R. S. **T**horne, G. **W**att



- **Error bands** shown are obtained from propagation of experimental uncertainties on the fitted data points.

Status of PDFs from different groups in October 2011

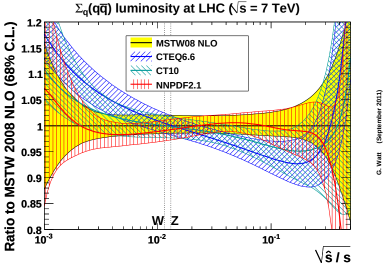
- Various fitting groups currently produce PDF sets: **MSTW**, CTEQ-TEA, NNPDF, HERAPDF, AB(K)M, (G)JR.
- Quantifying and understanding differences *between* groups is as (or more) important as continued improvements *within* groups.
- Highlight major differences in classes of data fitted:

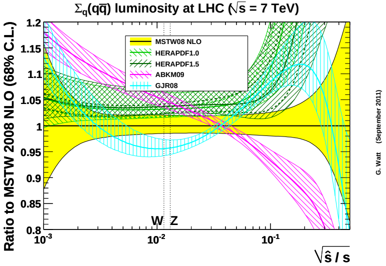
	MSTW08	**CT10**	**NNPDF2.1**	HERAPDF1.5	ABKM09	GJR08/JR09
HERA DIS	✔	✔	✔	✔	✔	✔
Fixed-target DIS	✔	✔	✔	✗	✔	✔
Fixed-target DY	✔	✔	✔	✗	✔	✔
Tevatron W,Z	✔	✔	✔	✗	✗	✗
Tevatron jets	✔	✔	✔	✗	✗	✔

- "Global" $\equiv$ includes all five main categories of data.
- GJR08 *almost* global but restrictive "dynamical" parameterisation.
- **NN**PDF group use a **N**eural **N**etwork parameterisation.
- **N**ext-to-**n**ext-to-**l**eading **o**rder (**NNLO**) PDFs now available from all groups other than CTEQ [work in progress].

Ratio of NLO quark–antiquark luminosity functions

$$\frac{\partial \mathcal{L}_{\Sigma_q(q\bar{q})}}{\partial \hat{s}} = \frac{1}{s}\int_\tau^1 \frac{\mathrm{d}x}{x} \sum_{q=d,u,s,c,b} \left[q(x,\hat{s})\,\bar{q}(\tau/x,\hat{s}) + (q \leftrightarrow \bar{q})\right], \qquad \tau \equiv \frac{\hat{s}}{s} = x_1\,x_2$$

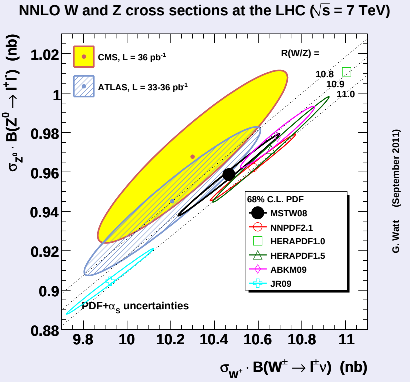


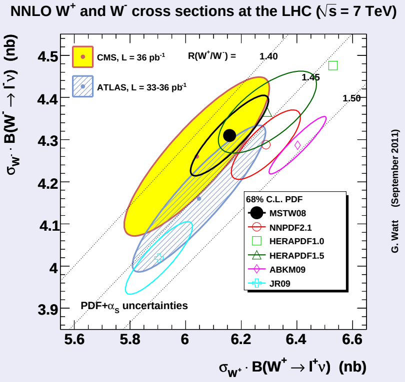


- Relevant values of $\sqrt{\hat{s}} = M_{W,Z}$ are indicated: good agreement for global fits (left), but more variation for other sets (right).

NNLO $W^\pm$ vs. Z^0 and W^+ vs. W^- total cross sections

- Correlation of ellipse ⇔ uncertainty in ratio of cross sections.





$$\frac{\sigma_{W^+}+\sigma_{W^-}}{\sigma_{Z^0}} \sim \frac{u(x_1)+d(x_1)}{0.29\,u(\tilde{x}_1)+0.37\,d(\tilde{x}_1)}$$

- Ratio insensitive to PDFs.

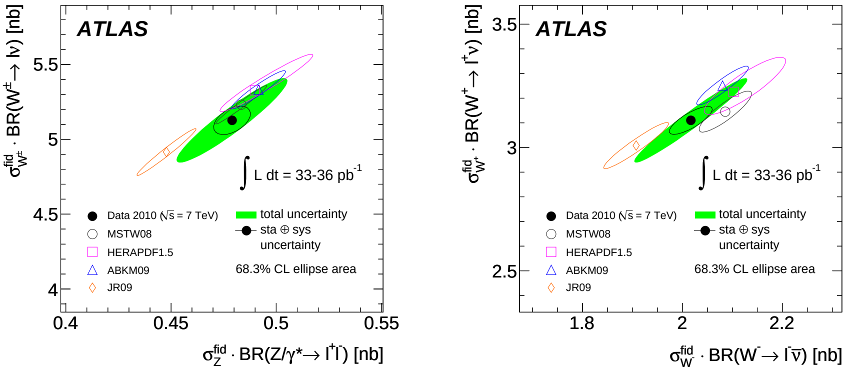
$$\frac{\sigma_{W^+}}{\sigma_{W^-}} \sim \frac{u(x_1)\bar{d}(x_2)}{d(x_1)\bar{u}(x_2)} \sim \frac{u(x_1)}{d(x_1)}$$

- Ratio sensitive to u/d.

- Luminosity uncertainty of 3.4% (ATLAS) or 4% (CMS).

NNLO $W^\pm$ vs. Z^0 and W^+ vs. W^- fiducial cross sections

[ATLAS Collaboration, `arXiv:1109.5141`]

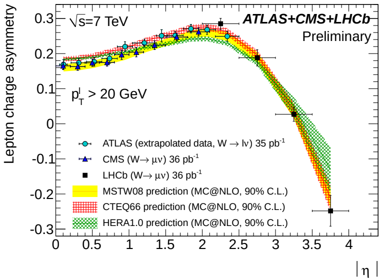


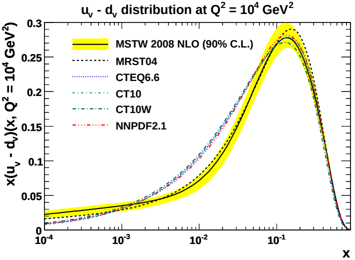
- Largest uncertainty in ATLAS/CMS total cross-section **ratios** from acceptance calculation ⇒ compare to theory **within** acceptance.
- **NNLO** comparisons now possible using FEWZ or DYNNLO codes.

$W^\pm \to \ell^\pm \nu$ charge asymmetry at the LHC

$$A_W(y_W) = \frac{\mathrm{d}\sigma(W^+)/\mathrm{d}y_W - \mathrm{d}\sigma(W^-)/\mathrm{d}y_W}{\mathrm{d}\sigma(W^+)/\mathrm{d}y_W + \mathrm{d}\sigma(W^-)/\mathrm{d}y_W} \approx \frac{u_v(x_1) - d_v(x_1)}{u(x_1) + d(x_1)}$$

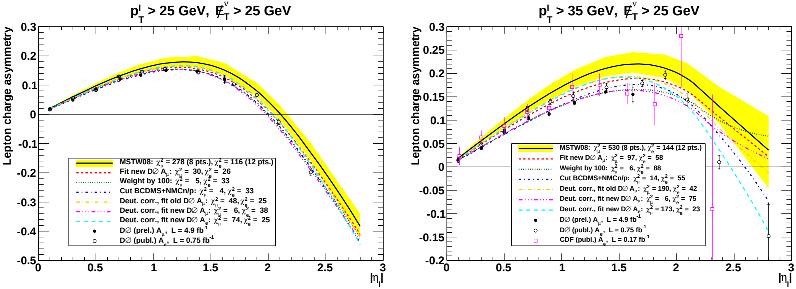
$$A_\ell(\eta_\ell) = \frac{\mathrm{d}\sigma(\ell^+)/\mathrm{d}\eta_\ell - \mathrm{d}\sigma(\ell^-)/\mathrm{d}\eta_\ell}{\mathrm{d}\sigma(\ell^+)/\mathrm{d}\eta_\ell + \mathrm{d}\sigma(\ell^-)/\mathrm{d}\eta_\ell} \equiv A_W(y_W) \otimes (W^\pm \to \ell^\pm \nu)$$





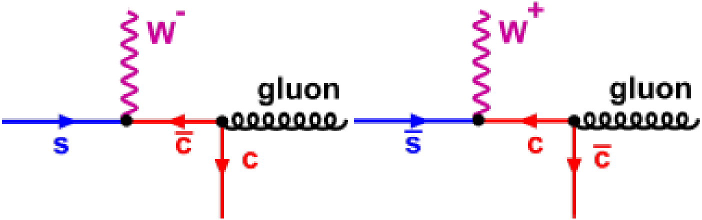
- First useful PDF constraint from LHC data (→ NNPDF2.2).

$W^\pm \to \ell^\pm \nu$ charge asymmetry at the Tevatron



- Outstanding issues to be resolved concerning Tevatron data, particularly when split up into p_T^ℓ bins [MSTW, `arXiv:1006.2753`].
- **Current plan:** consider only inclusive p_T^ℓ bin, try to fit nuclear effects in deuteron structure functions simultaneously with PDFs.

W+charm as a probe of strangeness [CMS PAS EWK-11-013]

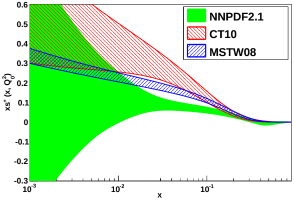


- Dominant $s\,\mathbf{g} \to W^-\,c$ and $\bar{s}\,\mathbf{g} \to W^+\,\bar{c}$.
- 15% from $d\,\mathbf{g} \to W^-\,c$, 5% from $\bar{d}\,\mathbf{g} \to W^+\,\bar{c}$.

$$R_c^\pm \equiv \frac{\sigma(W^+ + \bar{c})}{\sigma(W^- + c)} = 0.92 \pm 0.19(\mathrm{stat.}) \pm 0.04(\mathrm{syst.})$$

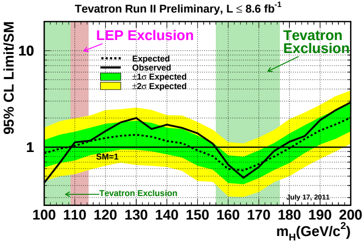
$$R_c \equiv \frac{\sigma(W + c)}{\sigma(W + \mathrm{jets})} = 0.143 \pm 0.015(\mathrm{stat.}) \pm 0.024(\mathrm{syst.})$$

$x(s+\bar{s})(x, Q^2 = 2\ \mathrm{GeV}^2)$:

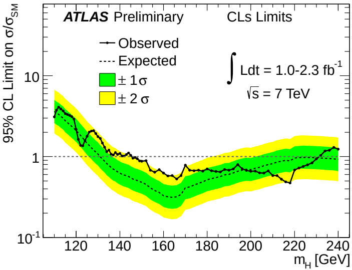


Ratio	MCFM (MSTW08)	MCFM (CT10)	MCFM (NNPDF2.1)
$R_c^\pm$	$0.881^{+0.022}_{-0.032}$	$0.915^{+0.006}_{-0.006}$	0.902 ± 0.008
R_c	$0.118^{+0.002}_{-0.002}$	$0.125^{+0.013}_{-0.007}$	0.103 ± 0.005

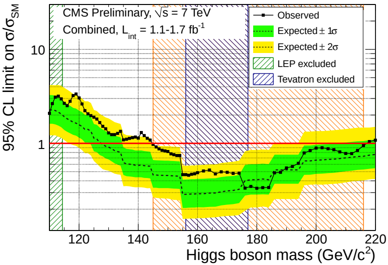
Exclusion limits at 95% C.L. for SM Higgs boson



[TEVNPHWG, `arXiv:1107.5518`]



	M_H (GeV)	$x \sim M_H/\sqrt{s}$
Tevatron	156 – 177	0.08 – 0.09
ATLAS	146 – 232	0.02 – 0.03
	256 – 282	0.04 – 0.04
	296 – 466	0.04 – 0.07
CMS	145 – 216	0.02 – 0.03
	226 – 288	0.03 – 0.04
	310 – 340	0.04 – 0.05

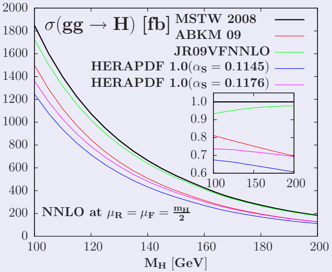


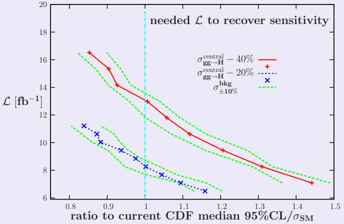
- $\sigma_{\rm SM}$ uses **MSTW 2008** PDFs.

Tevatron Higgs exclusion limits: a critical appraisal

[Baglio, Djouadi, Ferrag, Godbole, `arXiv:1101.1832`]

Phys. Lett. B **699** (2011) 368





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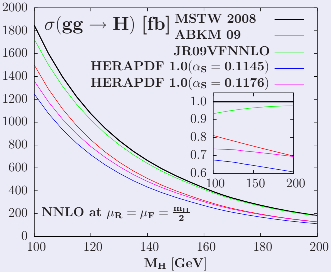
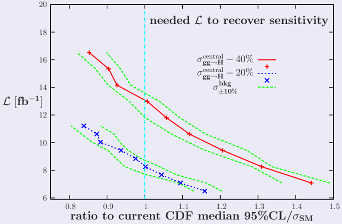
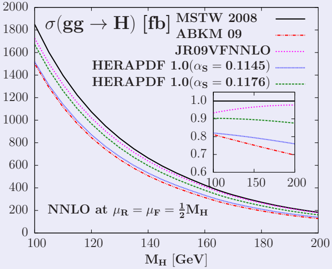
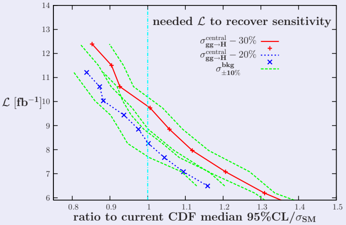




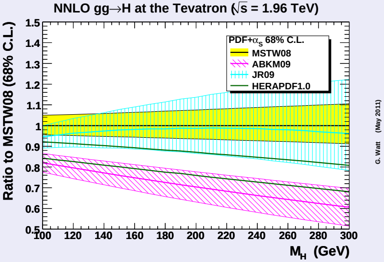
Erratum-ibid. B **702** (2011) 105



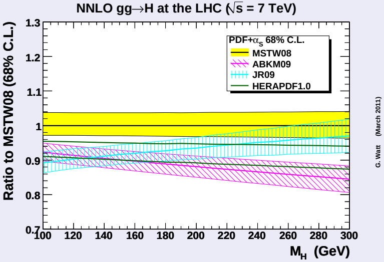


$gg \to H$ total cross sections vs. M_H (and gg luminosities)

Tevatron ($\sqrt{s} = 1.96$ TeV)



LHC ($\sqrt{s} = 7$ TeV)



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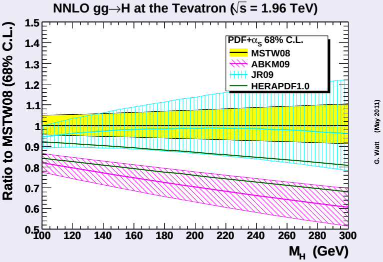
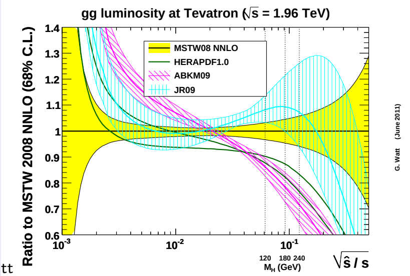

NNLO gg→H at the Tevatron ($\sqrt{s}$ = 1.96 TeV)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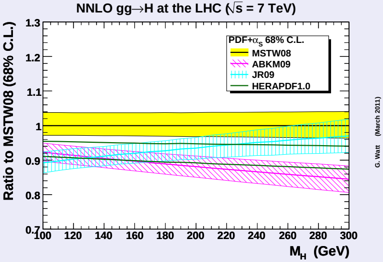
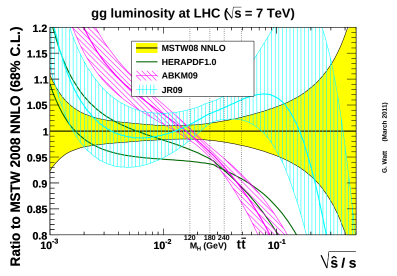
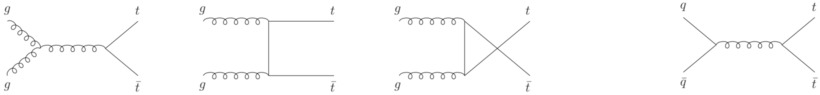
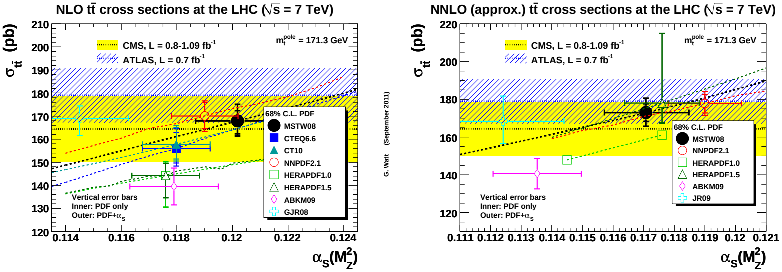

NNLO gg→H at the LHC ($\sqrt{s}$ = 7 TeV)


gg luminosity at LHC ($\sqrt{s}$ = 7 TeV)

$t\bar{t}$ total cross sections versus $\alpha_S(M_Z^2)$ at the LHC

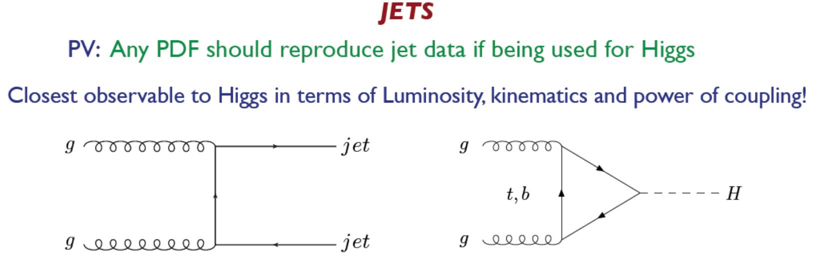


- $\sim$80% of $\sigma_{t\bar{t}}^{\mathrm{NLO}}$ from gg at LHC (7 TeV), cf. $\sim$15% at Tevatron.
- NNLO (approx.) using HATHOR code [Aliev *et al.*, `arXiv:1007.1327`].
- Compare to single most precise current LHC measurements:



- ABKM09 and some HERAPDF sets disfavoured by LHC data.

Jets as a discriminator of the high-x gluon distribution



JETS

PV: Any PDF should reproduce jet data if being used for Higgs

Closest observable to Higgs in terms of Luminosity, kinematics and power of coupling!

[D. de Florian, talk at "Higgs Hunting 2011", Orsay, 28th July 2011]

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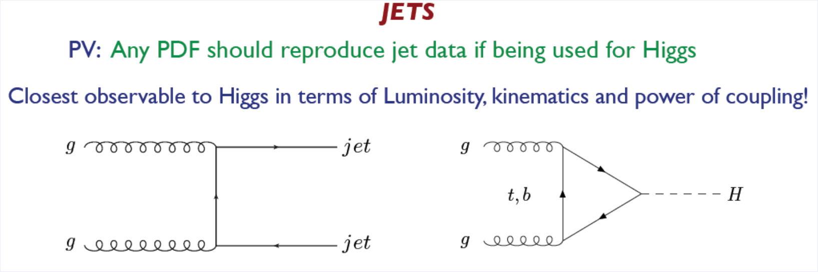

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[D. de Florian, talk at "Higgs Hunting 2011", Orsay, 28th July 2011]

- Values of $\chi^2/N_{\text{pts.}}$ for CDF II inclusive jet data [`hep-ex/0701051`]:

NNLO PDF	$\alpha_S(M_Z^2)$	$\mu = p_T/2$	$\mu = p_T$	$\mu = 2p_T$
MSTW08	0.1171	1.39	0.69	0.97
HERAPDF1.0	0.1145	2.64	2.15	2.20
HERAPDF1.0	0.1176	2.24	1.17	1.23
ABKM09	0.1135	2.55	2.76	3.41
JR09	0.1124	0.75	1.26	2.21

Input parameterisation in MSTW 2008 NLO fit

Input parameterisation ($Q_0^2 = 1$ GeV^2) in MSTW 2008 fit

$$\begin{aligned}
xu_v &= A_u\, x^{\eta_1}(1-x)^{\eta_2}(1+\epsilon_u\sqrt{x}+\gamma_u\, x)\\
xd_v &= A_d\, x^{\eta_3}(1-x)^{\eta_4}(1+\epsilon_d\sqrt{x}+\gamma_d\, x)\\
xS &= A_S\, x^{\delta_S}(1-x)^{\eta_S}(1+\epsilon_S\sqrt{x}+\gamma_S\, x)\\
x(\bar{d}-\bar{u}) &= A_\Delta\, x^{\eta_\Delta}(1-x)^{\eta_S+2}(1+\gamma_\Delta\, x+\delta_\Delta\, x^2)\\
xg &= A_g\, x^{\delta_g}(1-x)^{\eta_g}(1+\epsilon_g\sqrt{x}+\gamma_g\, x)+A_{g'}\, x^{\delta_{g'}}(1-x)^{\eta_{g'}}\\
x(s+\bar{s}) &= A_+\, x^{\delta_S}\,(1-x)^{\eta_+}(1+\epsilon_S\sqrt{x}+\gamma_S\, x)\\
x(s-\bar{s}) &= A_-\, x^{0.2}(1-x)^{\eta_-}(1-x/x_0)
\end{aligned}$$

- A_u, A_d, A_g and x_0 are determined from sum rules.
- 28 parameters allowed to go free to find best fit,
  20 parameters allowed to go free for error propagation.

Compare to input parameterisation in HERAPDF fits

Input parameterisation ($Q_0^2 = 1.9$ GeV^2) in HERAPDF1.0/1.5

$$
\begin{aligned}
xu_v &= A_{u_v}\, x^{B_{qv}}(1-x)^{C_{uv}}(1+E_{u_v}\, x^2)\\
xd_v &= A_{d_v}\, x^{B_{qv}}(1-x)^{C_{d_v}}\\
x\bar{u} &= A_{\bar{q}}\, x^{B_{\bar{q}}}(1-x)^{C_{\bar{u}}}\\
x\bar{d} &= A_{\bar{q}}\, x^{B_{\bar{q}}}(1-x)^{C_{\bar{d}}}\\
x\bar{s} &= 0.45\, x\bar{d}\\
xs &= x\bar{s}\\
xg &= A_g\, x^{B_g}(1-x)^{C_g}
\end{aligned}
$$

- 10 parameters for central fit and "experimental" uncertainties, additional "model" and "parameterisation" uncertainties.
- 4 more params. for HERAPDF1.5 NNLO (2 for g, 1 each for u_v, d_v).
- cf. **N**eural **N**etwork group ($7 \times 37 = 259$ free parameters).

Experimental error propagation in global PDF fits

Hessian method (used by MSTW and CTEQ groups)

- Based on *covariance* (inverse *Hessian*) matrix of ∼20 fit parameters: diagonalise to produce *eigenvector* PDF sets.
- Varying values of $\Delta\chi^2$ used to accommodate minor data inconsistencies (average value $\Delta\chi^2 \sim 10$ in MSTW08 fit).

Monte Carlo sampling using data replicas (used by NNPDF)

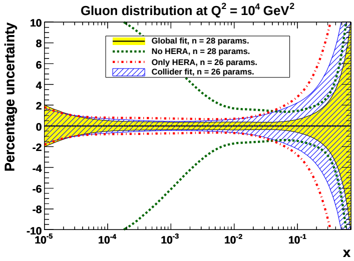
- Generate replica data sets with shifted central values:

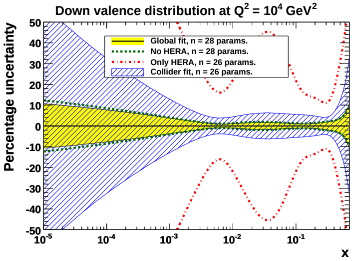
$$D_{m,i} \to \left(D_{m,i} + R_{m,i}^{\mathrm{uncorr.}}\,\sigma_{m,i}^{\mathrm{uncorr.}} + \sum_{k=1}^{N_{\mathrm{corr.}}} R_{m,k}^{\mathrm{corr.}}\,\sigma_{m,k,i}^{\mathrm{corr.}} \right) \cdot \left(1 + R_m^{\mathcal{N}}\,\sigma_m^{\mathcal{N}}\right)$$

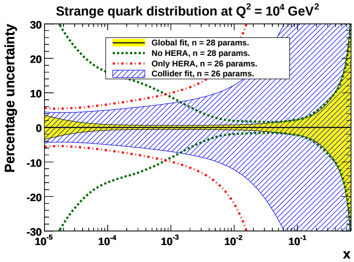
- Calculate average and s.d. over $N_{\mathrm{rep}} \sim \mathcal{O}(100)$ PDF sets.
- Equivalent to Hessian method with tolerance $\Delta\chi^2 = 1$.
- But more useful when fitting weakly-constrained parameters.

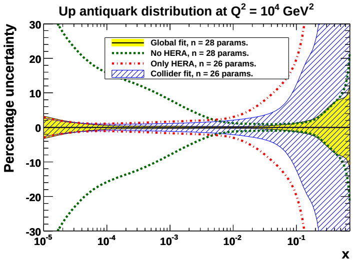
Constraints provided by subsets of data using MC sampling

- Fit subsets of global data in MSTW 2008 NLO analysis:









Bayesian reweighting to include new data sets

[Giele, Keller, `hep-ph/9803393`; NNPDF, `arXiv:1012.0836`, `arXiv:1108.1758`]

- Suppose we have an existing set of PDFs $\{f_k\}$. Compute χ_k^2 for new data set using each f_k, then calculate observables as:

$$\langle\mathcal{O}\rangle_{\rm old} = \frac{1}{N_{\rm rep}}\sum_{k=1}^{N_{\rm rep}}\mathcal{O}[f_k], \quad \langle\mathcal{O}\rangle_{\rm new} = \frac{1}{N_{\rm rep}}\sum_{k=1}^{N_{\rm rep}} w_k(\chi_k^2)\,\mathcal{O}[f_k].$$

- Valid also if PDFs $\{f_k\}$ generated randomly in parameter space (using covariance matrix from global fit) [work in progress].

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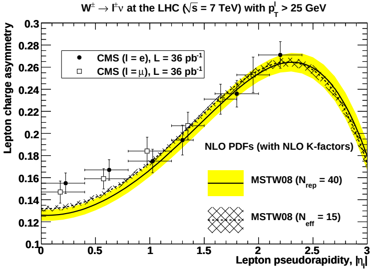
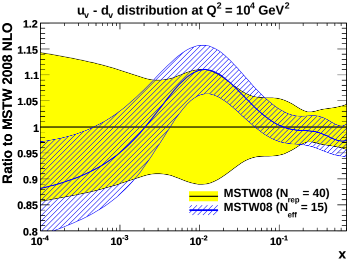


$\chi^2/N_{\rm pts.} = 2.43 \to 1.32$



Reweighting shifts $u_v - d_v$.

Summary

- Reviewed (*proton*, *unpolarised*, *integrated*) PDF determination. More complicated types of PDF ⇒ see later talks!
- The LHC is starting to provide useful PDF constraints, e.g.
  - $W^\pm$ charge asymmetry $\Rightarrow u_v - d_v$.
  - W+charm production ⇒ strangeness.
  - $t\bar{t}$, jets, isolated-γ ⇒ gluon (important for Higgs).
- *Non-global* PDF fits can be misleading or even dangerous: constrained by *assumed* parameterisation in absence of data.
- First look at *Monte Carlo sampling* and *Bayesian reweighting* in context of **MSTW** fit [work in progress]. Potentially useful techniques to study impact of new data/experiments on PDFs.