Introducing πN transition distribution amplitudes: Spectral representation, symmetries and constraints from chiral dynamics

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Outline

1. Introduction
2. πN TDAs: definition, properties, support, spectral representation
3. Crossing and chiral constrains for πN TDAs
4. Factorized Ansatz for quadruple distributions; nucleon pole exchange contribution.
5. $\gamma^* p \to \pi^+ n$ cross section v.s. preliminary CLAS data
6. Summary and Outlook

B. Pire, K. S., L. Szymanowski Phys. Rev. D **82**, 094030 (2010)

B. Pire, K. S., L. Szymanowski, Phys. Rev. D **84**, 074014 (2011)

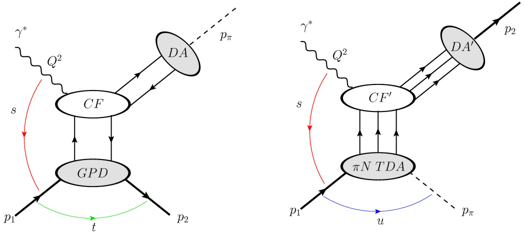
B. Pire, K. S., L. Szymanowski et al. in preparation

Introducing TDAs

$$\gamma^*(q) + N(p_1) \to N'(p_2) + \pi(p_\pi)$$

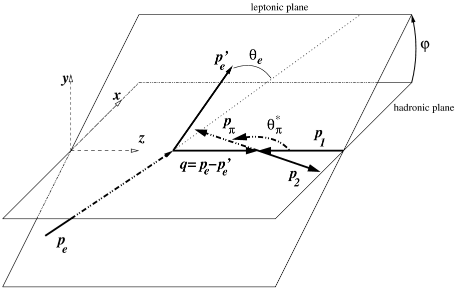
- Factorization theorem J.Collins, L.Frankfur, M.Strikman'96 $\Rightarrow$ GPD description;
- A factorization theorem claimed for the backward regime ($q^2 = -Q^2$ and $s \equiv (p_1 + q)^2$ - large; $x_{\rm Bj} \equiv \frac{Q^2}{2p_1 \cdot q}$ and $\xi \equiv -\frac{(p_\pi - p_1)\cdot n}{(p_\pi + p_1)\cdot n}$ -fixed; $u \equiv (p_\pi - p_1)^2 \sim 0$

L.Frankfurt, P.Pobylitsa, M.Polyakov and M.Strikman'99; L.Frankfurt, M.Polyakov, M.Strikman, D.Zhalov and M.Zhalov'02



Backward electroproduction of a pion on a proton

- Backward kinematics: $e\,N \to e'N'\pi$ in the center-of-mass of the γ^*-proton
$u = (p_\pi - p_1)^2 \equiv \Delta^2 \sim 0$ ($\cos\theta^*_\pi \sim -1$)



- Cross section of $\gamma^* N \to N'\pi$ $\frac{d^2\sigma}{d\Omega^*_\pi}$ scales as $\frac{1}{(Q^2)^4}$ (c.f. $\frac{1}{(Q^2)^2}$ for usual HMP)

Twist-3 πN TDA

L.Frankfurt, P.Pobylitsa, M.Polyakov & M.Strikman'99; J.P.Lansberg, B.Pire & L.Szymanowski'07:

$$4(P\cdot n)^3 \int \left[\prod_{i=1}^{3} \frac{dz_i}{2\pi} e^{ix_i z_i (P\cdot n)}\right] \langle \pi(p_\pi)|\, \varepsilon_{c_1 c_2 c_3} \Psi_\rho^{c_1}(z_1 n) \Psi_\tau^{c_2}(z_2 n) \Psi_\chi^{c_3}(z_3 n)\, |N(p_1, s_1)\rangle$$

$$= \delta(2\xi - x_1 - x_2 - x_3) i \frac{f_N}{f_\pi M}$$

$$\times \left[V_1^{\pi N} (\hat{P}C)_{\rho\tau} (\hat{P}U)_\chi + A_1^{\pi N} (\hat{P}\gamma^5 C)_{\rho\tau} (\gamma^5 \hat{P}U)_\chi + T_1^{\pi N} (\sigma_{P\mu} C)_{\rho\tau} (\gamma^\mu \hat{P} U)_\chi \right.$$

$$+ V_2^{\pi N} (\hat{P}C)_{\rho\tau} (\hat{\Delta}U)_\chi + A_2^{\pi N} (\hat{P}\gamma^5 C)_{\rho\tau} (\gamma^5 \hat{\Delta}U)_\chi + T_2^{\pi N} (\sigma_{P\mu} C)_{\rho\tau} (\gamma^\mu \hat{\Delta} U)_\chi$$

$$\left. + \frac{1}{M} T_3^{\pi N} (\sigma_{P\Delta} C)_{\rho\tau} (\hat{P}U)_\chi + \frac{1}{M} T_4^{\pi N} (\sigma_{P\Delta} C)_{\rho\tau} (\hat{\Delta}U)_\chi \right]$$

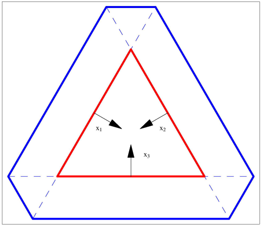
- $P = \frac{1}{2}(p_1 + p_\pi)$; $\Delta = (p_\pi - p_1)$; $n^2 = p^2 = 0$; $2p\cdot n = 1$; $\sigma_{P\mu} \equiv P^\nu \sigma_{\nu\mu}$;
- C: charge conjugation matrix;
- $f_N = 5.2 \cdot 10^{-3}$ GeV^2 (V. Chernyak and A. Zhitnitsky'84);
- 8 TDAs: $H(x_1, x_2, x_3, \xi, \Delta^2, \mu^2) \equiv \{V_i, A_i, T_i\}(x_1, x_2, x_3, \xi, \Delta^2, \mu^2)$

How πN TDAs meet fundamental theoretical requirements:

1. support properties
2. polynomialty in ξ of the Mellin moments in x_1, x_2, x_3
3. QCD evolution
4. symmetry properties: $SU(2)$ isospin + permutation symmetry $\Rightarrow 8$ independent πN TDAs to describe all isospin channels
5. crossing: πN TDA $\leftrightarrow$ πN GDA
6. chiral properties: soft pion theorem

Support properties of πN TDAs

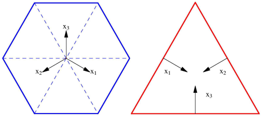
- The natural way to depict physical domains for πN TDAs: Mandelstam plane $x_1 + x_2 + x_3 = 2\xi$ (barycentric coordinates)



- Intersection of three stripes $-1 + \xi \leq x_i \leq 1 + \xi$
- "ERBL-like" region: $x_1,\, x_2,\, x_3 \in [0,\, 2\xi]$
- "DGLAP-like type I" region: one momentum fraction is positive and two are negative
- "DGLAP-like type II" region: one momentum fraction is negative and two are positive

- DGLAP-like domains are bounded by the lines $x_i = \pm 1 + \xi$; $x_i = 0$

Two limiting cases: $\xi = 0$ and $\xi = 1$.



Polynomiality of the Mellin moments of πN TDAs

- $(n_1,\, n_2,\, n_3)$-th Mellin moments in x_1, x_2, x_3 of πN TDA are the form factors of the local twist three operators

$$\widehat{O}^{\mu_1 \dots \mu_{n_1} \nu_1 \dots \nu_{n_2} \lambda_1 \dots \lambda_{n_3}}(0)$$
$$= \left[i\vec{D}^{\mu_1} \dots i\vec{D}^{\mu_{n_1}} \Psi(0) \right] \left[i\vec{D}^{\nu_1} \dots i\vec{D}^{\nu_{n_2}} \Psi(0) \right] \left[i\vec{D}^{\lambda_1} \dots i\vec{D}^{\lambda_{n_3}} \Psi(0) \right]$$

- Polynomiality is the direct consequence of the Lorentz invariance: (n_1, n_2, n_3)-th Mellin moments of πN TDAs are polynomials of ξ order $N+1$ $\quad (N \equiv n_1 + n_2 + n_3)$
- analogue of the D-term for GPDs (C.Weiss and M.Polyakov'99) exists for $V_{1,2}$, $A_{1,2}$, $T_{1,2}$

A note on the evolution

- πN TDAs depend on the renormalization scale μ^2 of operators in their definition (scale at which partons are resolved)
- Generalization of DGLAP equation. Splitting functions are much more complicated: include the pieces different in different kinematical regions (3 types of kinematical regions exist); LO kernels B.Pire &L. Szymanowski'05
- As in the case of PDFs and GPDs evolution of TDAs can be treated in terms of renormalization of the local operators corresponding to their x moments
- Matrix elements of the local operators in question were extensively studied in connection with scale dependence of nucleon DA e.g. N. Stefanis'97, V. Braun et al'98, 99

Spectral representation for πN TDAs:

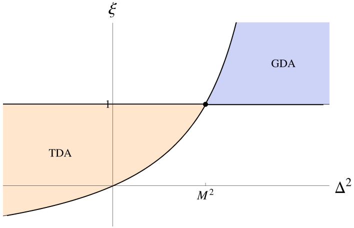
- O. Teryaev'01: the relation between DDs and GPDs is a particular case of the Radon transform
- Polynomiality property $\Leftrightarrow$ the Cavalieri conditions
- Support properties of + polynomiality $\Leftrightarrow$ spectral representation + spectral constraints.
- Spectral representation A. Radyushkin'97 generalized for πN TDAs:

$$
\begin{aligned}
& H(x_1, x_2, x_3 = 2\xi - x_1 - x_2, \xi) \\
& = \left[\prod_{i=1}^{3} \int_{\Omega_i} d\beta_i d\alpha_i \right] \delta(x_1 - \xi - \beta_1 - \alpha_1 \xi)\, \delta(x_2 - \xi - \beta_2 - \alpha_2 \xi) \\
& \times \delta(\beta_1 + \beta_2 + \beta_3) \delta(\alpha_1 + \alpha_2 + \alpha_3 + 1) F(\beta_1, \beta_2, \beta_3, \alpha_1, \alpha_2, \alpha_3);
\end{aligned}
$$

- Ω_i: $\{|\beta_i| \leq 1, \; |\alpha_i| \leq 1 - |\beta_i|\}$ are copies of the usual DD square in spectral parameter space;
- spectral density $F(\ldots)$ is the function of six variables that are subject to two constraints $\Rightarrow$ quadruple distributions
- This form ensures polynomiality and support

Crossing πN TDA $\leftrightarrow$ πN GDA and soft pion theorem

- Crossing relates πN TDAs in $\gamma^* N \to \pi N'$ and πN GDAs (light-cone wave function)
- Physical domain in (Δ^2, ξ)-plane (defined by $\Delta_T^2 \leq 0$) in the chiral limit $(m = 0)$:



- Soft pion theorem Pobylitsa, Polyakov and Strikman'01 $(Q^2 \gg \Lambda^3_{\rm QCD}/m)$ constrains πN GDA at the threshold $\xi = 1$, $\Delta^2 = M^2$.

Soft pion theorem for πN GDA

- Soft pion theorem Pobylitsa, Polyakov and Strikman'01 ($Q^2 \gg \Lambda^3_{\rm QCD}/m$):

$$\langle 0|\widehat{O}^{\alpha\beta\gamma}_{\rho\tau\chi}(z_1,\, z_2,\, z_3)|\pi_a N_\iota\rangle = -\frac{i}{f_\pi}\langle 0|\left[\widehat{Q}^a_5,\, \widehat{O}^{\alpha\beta\gamma}_{\rho\tau\chi}(z_1,\, z_2,\, z_3)\right]|N_\iota\rangle\,,$$

with $\left[\widehat{Q}^a_5,\;\Psi^\alpha_\eta\right] = -\frac{1}{2}(\sigma_a)^\alpha_\delta \gamma^5_{\eta\tau}\Psi^\delta_\tau\,;$

- At the pion threshold ($\xi = 1,\ \Delta^2 = M^2$ in the chiral limit) soft pion theorem fixes πN TDAs/GDAs in terms of nucleon DAs V^p, A^p, T^p (see V. Braun, D. Ivanov, A.Lenz, A.Peters'08).
- E.g. soft pion theorem for uud proton to π^0 TDAs:

$$\{V_1^{p\pi^0},\, A_1^{p\pi^0}\}(x_1,x_2,x_3,\xi=1,\Delta^2=M^2) = -\frac{1}{8}\{V^p,\, A^p\}(\frac{x_1}{2},\frac{x_2}{2},\frac{x_3}{2})\,;$$
$$T_1^{p\pi^0}(x_1,x_2,x_3,\xi=1,\Delta^2=M^2) = \frac{3}{8}T^p(\frac{x_1}{2},\frac{x_2}{2},\frac{x_3}{2})$$
$$\{V_2^{p\pi^0},\, A_2^{p\pi^0},\, T_2^{p\pi^0}\} = -\frac{1}{2}\{V_1^{p\pi^0},\, A_1^{p\pi^0},\, T_1^{p\pi^0}\} \quad T_{3,4}^{p\pi^0} = 0\,;$$

- *C.f.* soft pion theorems for isoscalar and isovector pion GPDs:

$$H^{I=0}(x,\xi=1) = 0; \quad H^{I=1}(x,\xi=1) = \phi_\pi(x)$$

Realistic strategy for modelling πN TDAs

- No enlightening $\xi = 0$ limit as for GPDs
- In the limit $\xi \to 1$ πN TDAs are fixed due to soft pion theorems in terms of nucleon DAs
- $\xi \to 1$ limit allows to fix the overall magnitude
- try to start from $\xi = 1$ limit rather than the forward limit $\xi = 0$

"Skewing" $\xi = 1$ limit for pion isovector GPD (toy exercise)

Let us try to use input at $\xi = 1$ rather than $\xi = 0$ for GPD modeling.

- let us perform the change of variables in the DD representation for GPDs: $\alpha = \frac{\kappa+\theta}{2}$, $\beta = \frac{\kappa-\theta}{2}$ This gives:

$$H(x,\,\xi) = \int_{-1}^{1} d\kappa \int_{-1}^{1} d\theta\, \delta\Big(x + \frac{1-\xi}{2}\,\theta - \frac{1+\xi}{2}\,\kappa \Big)\, \frac{1}{2}\, F(\kappa,\,\theta)\,,$$

  where $F(\kappa,\,\theta) \equiv f\left(\frac{\kappa-\theta}{2},\, \frac{\kappa+\theta}{2} \right)$

- try the following factorized Ansatz:

$$F(\kappa,\,\theta) = \phi_\pi(\kappa) h(\theta)$$

  with the profile $h(\theta)$ normalized according to $\int_{-1}^{1} d\theta h(\theta) = 1\,.$

- Then $H(x, \xi = 1) = \phi_\pi(x)$
- Problem is to implement the so-called "Munich symmetry" $f(\beta, \alpha) = f(\beta,\, -\alpha)$. But *e.g.* $h(\theta) = \phi_\pi(\theta)$: ok.

“Skewing” $\xi = 1$ limit for πN TDAs

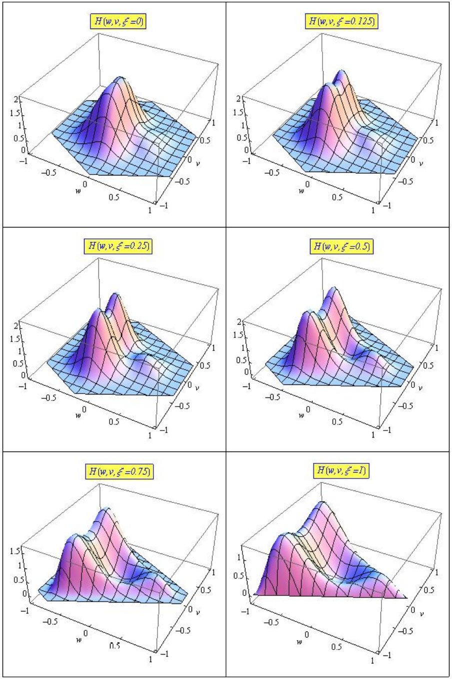
After suitable change of spectral variables ($\kappa = \alpha_3 + \beta_3$, $\theta = \frac{\alpha_1+\beta_1-\alpha_2-\beta_2}{2}$, $\mu = \alpha_3 - \beta_3$, $\lambda = \frac{\alpha_1-\beta_1-\alpha_2+\beta_2}{2}$) and introduction of “quark-diquark” coordinates $w = x_3 - \xi$; $v = \frac{x_1-x_2}{2}$:

$$H(w,\, v,\, \xi) = \int_{-1}^{1} d\kappa \int_{-\frac{1-\kappa}{2}}^{\frac{1-\kappa}{2}} d\theta \int_{-1}^{1} d\mu_i \int_{-\frac{1-\mu}{2}}^{\frac{1-\mu}{2}} d\lambda\, \delta(w - \frac{\kappa-\mu}{2}(1-\xi) - \kappa\xi)$$
$$\times \delta\left(v - \frac{\theta-\lambda}{2}(1-\xi) - \theta\xi\right) F(\kappa,\, \theta,\, \mu,\, \lambda)$$

- A factorized Ansatz for quadruple distribution F_i:

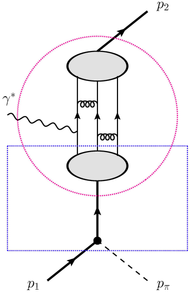
$$F(\kappa,\, \theta,\, \mu,\, \lambda) = V(\kappa,\, \theta)\, h(\mu,\, \lambda)$$

  with the profile $h(\mu,\, \lambda)$ normalized as $\int d\mu \int d\lambda\, h(\mu,\, \lambda) = 1$.
- Since $H(w,\, v,\, \xi = 1) = V(w, v)$ for V one may use input from the soft pion theorem
- A possible choice for the profile: $h(\mu,\, \lambda) = \frac{15}{16}\,(1+\mu)((1-\mu)^2 - 4\lambda^2)$; vanishes at the borders of the definition domain.


H(w,v,ξ=0)
H(w,v,ξ=0.125)
H(w,v,ξ=0.25)
H(w,v,ξ=0.5)
H(w,v,ξ=0.75)
H(w,v,ξ=1)

Nucleon pole contribution

- u-channel nucleon exchange is complementary to the spectral representation (D-term like contributions) non-zero in the ERBL-like region $0 \leq x_i \leq 2\xi$.



- The effective Hamiltonian for $\pi \bar{N} N$:

$$\mathcal{H}_{\text{eff}} = i g_{\pi NN} \bar{N}_\alpha (\sigma_a)^\alpha_{\ \beta} \gamma_5 N^\beta \pi_a$$

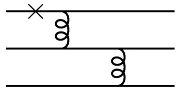
$$\langle \pi_a(p_\pi)| \widehat{O}^{\alpha\,\beta\,\gamma}_{\rho\tau\chi}(\lambda_1 n,\, \lambda_2 n,\, \lambda_3 n)|N_\iota(p_1, s_1)\rangle$$
$$= \sum_{s_p} \langle 0|\widehat{O}^{\alpha\beta\gamma}_{\rho\tau\chi}(\lambda_1 n,\, \lambda_2 n,\, \lambda_3 n)|N_\kappa(-\Delta, s_p)\rangle (\sigma_a)^\kappa_{\ \iota} \frac{i g_{\pi NN}\, \bar{U}_\varrho(-\Delta, s_p)}{\Delta^2 - M^2} \left(\gamma^5 U(p_1, s_1)\right)_\varrho \, .$$

- After decomposition over the Dirac structures:

$$\left\{V_1,\, A_1,\, T_1\right\}^{(\pi N)}(x_1, x_2, x_3)$$
$$= \Theta_{\text{ERBL}}(x_1, x_2, x_3) \times \frac{M f_\pi g_{\pi NN}}{\Delta^2 - M^2} \frac{1}{(2\xi)} \left\{V^p,\, A^p,\, T^p\right\} \left(\frac{x_1}{2\xi}, \frac{x_2}{2\xi}, \frac{x_3}{2\xi}\right) \, ;$$

Calculation of the amplitude

- LO amplitude for $\gamma^* p \to n\,\pi^+$ can be computed as in J.P. Lansberg, B. Pire and L. Szymanowski'07
- 21 diagrams contribute



$$\mathcal{M} \sim \int_{-1+\xi}^{1+\xi} d^3x\,\delta(x_1+x_2+x_3-2\xi) \int_{-1}^{1} d^3y\,\delta(1-y_1-y_2-y_3) \left(\sum_{\alpha=1}^{21} T_\alpha\right)$$

Each T_α, has the structure:

$$T_\alpha \sim K_\alpha(x_1,x_2,x_3) \times Q_\alpha(y_1,y_2,y_3) \times$$
[combination of πN TDAs] $\times$ [combination of nucleon DAs]

$$T_1 = \frac{q^d(2\xi)^2[(V_1^{n\pi^+} - A_1^{n\pi^+})(V^p - A^p) + 4T_1^{n\pi^+}T^p + 2\frac{\Delta_T^2}{M^2}T_4^{n\pi^+}T^p]}{(2\xi - x_1 - i\epsilon)^2(x_3 - i\epsilon)(1-y_1)^2 y_3}$$

Classification of convolution kernels

- Switch to quark-diquark coordinates
- The following types of convolution kernels occur:

$$K_I^{(\pm,\pm)}(x_1,x_2,x_3) = \frac{1}{(w \pm \xi \mp i\epsilon)} \frac{1}{(v \pm \xi' \mp i\epsilon)}$$

$$K_{II}^{(-,\pm)}(x_1,x_2,x_3) = \frac{1}{(w - \xi + i\epsilon)^2} \frac{1}{(v \pm \xi' \mp i\epsilon)}$$

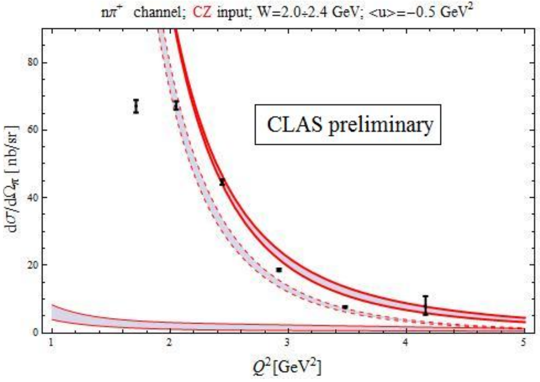
Strategy for the calculation of the P.V. integral

1. Insert the spectral representation for TDA
2. Interchange the order of integration and compute w and v integrals using the two delta functions.
3. One is left with four integrations over spectral parameters. Two of these integrations are to be performed with the principal value prescription.
4. After a suitable change of variables one may perform the two principle value integrations analytically.
5. The double integration over the remaining two spectral parameters has to be performed numerically.

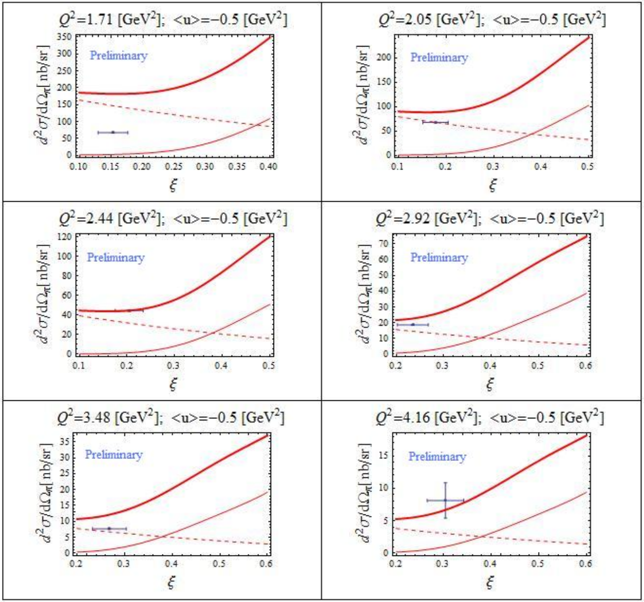
CLAS $\gamma^* p \to \pi^+ n$ very preliminary analysis by Kijun Park I

Table: Determination of kinematic bin

variable	unit	num. bin	range	bin size
W	GeV	1	> 2.0	0.4
Q^2	GeV^2	5	$1.6 \sim 4.5$	various
$\lvert\Delta_T^2\rvert$	GeV^2	1	< 0.5	0.5



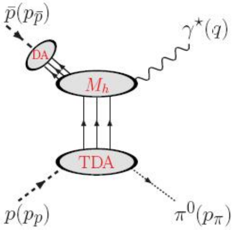
CLAS $\gamma^* p \to \pi^+ n$ very preliminary analysis by Kijun Park II



Ideas for PANDA

- J.P. Lansberg, B. Pire, L. Szymanowski'07: πN TDAs arise in the factorized description of

$$N(p_1) + \bar{N}(p_2) \to \gamma^*(q) + \pi(p_\pi) \to l^+(k_1) + l^-(k_2) + \pi(p_\pi)$$



- $W^2 = (p_1 + p_2)^2$ and $q^2 = Q^2$ - large; $(p_1 - p_\pi)^2$-small ($\theta_\pi \sim 0$ in C.M.S: near forward kinematics)
- Test of universality of TDAs

Conclusions & Outlook

1. hard exclusive electroproduction of baryons off nucleons provide new information about correlation of partons inside hadrons
2. experimental information on πN TDAs can be extracted from $\gamma^* N \to N'\pi$ in the kinematical conditions of Jlab already at 6 GeV; more is expected at 12 GeV.
3. $\bar{p}p \to \pi l^+ l^-$ in PANDA. Check universality of TDAs
4. spectral representation for πN TDA based on quadruple distributions which satisfies the polynomiality condition and respects the support properties is proposed
5. factorized Ansatz for quadruple distributions with input at $\xi = 1$ is proposed
6. a reliable method for the calculation of real and imaginary parts of $\gamma^* N \to N'\pi$ amplitude employing factorized Ansatz for quadruple distributions is proposed.
7. $\gamma^* N \to N'\pi$ cross-section computed to confront the available preliminary CLAS data
8. backward electroproduction of η. Data exists: V.Kubarovsky et al.