About Predictive Power of Nuclear Theories: Strengthening Links with Experiment

Jerzy DUDEK

Department of Subatomic Research, CNRS/IN_2P_3
and
University of Strasbourg, F-67037 Strasbourg, FRANCE

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COLLABORATORS:

Arthur DROMARD, UdS/IPHC-CNRS Strasbourg
Bartłomiej SZPAK, IFJ Kraków
Marie-Geneviève PORQUET, CSNSM Orsay

Bogdan FORNAL, IFJ Kraków
Hervé MOLIQUE, UdS/IPHC-CNRS Strasbourg
Karolina RYBAK, UdS/IPHC-CNRS Strasbourg

In this presentation we use some material from the article:

Nuclear Hamiltonians: The Question of their Spectral Predictive Power and the Associated Inverse Problem

JD, B. Szpak, M-G, Porquet, H. Molique, K. Rybak & B. Fornal
J. Phys. G: Nucl. Part. Phys. **37** *(2010) 064031*

FOCUS Special Issue: Open problems in nuclear structure theory

... as well as some material from the articles:

2. Nuclear Mean Field Hamiltonians and Factors Limiting their Predictive Power: Formalism

JD, K. Rybak, B. Szpak, M-G, Porquet, H. Molique & B. Fornal Int. J. Mod. Phys. E **19** *(2010) 652*

3. Nuclear Mean Field Hamiltonians and Factors Limiting their Predictive Power: Illustrations

B. Szpak, JD, K. Rybak, M-G, Porquet, H. Molique & B. Fornal Int. J. Mod. Phys. E **19** *(2010) 665*

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ANSWER: Yes, we always do when one of our friends winns a bigger amount of money in a poker game...

This presentation is about:

How to help our 100 theorists to arrive at close-lying results

Part I

New Strategies in Constructing Theories: Predictive-Power Perspective

The New Strategies in Constructing Theories

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Conclusion 2: The desired truth remains unknown to us $\longrightarrow$ lack of knowledge $\longrightarrow$ ignorance imposed[#)] by nature

#)... and thus well excused - because not resulting from our lazyness

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Conclusion: We need to introduce probabilities of ignorance!

Comments: Mathematical Sense of Uncertainties

- Theories are incomplete whereas experiments plagued with errors:

$$\text{Theo.} \rightarrow \boxed{e_n = e_n^{true}(p) + \delta e_n^{error}} \;\; \& \;\; \boxed{\varepsilon_n = \varepsilon_n^{true} + \delta\varepsilon_n^{err}} \leftarrow \text{Exp.}$$

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- Conclusion: All the predictions have the probability distributions!

Observe a Paradox:

It is NOT that we need to do something extra:

Our theories already
Contain Information about Uncertainties

It will be sufficient to start NOT ignoring it!!!

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but also by their probability distributions:

$$\boxed{P_1 = P_1(f_1), \;\; P_2 = P_2(f_2), \; \ldots \; P_p = P_1(f_p)}$$

An Example of Theory Predictions

[Uncertainty Distributions]

Inverse Problem and Predictive Power: ^{132}Sn

Neutron levels around N=82					
Level	States in ^{133}Sn				
	e_1^{exc}	shift 1$^{(a)}$	shift 2$^{(b)}$	$\bar{\varepsilon}$	B. E.
$\nu f_{7/2}$	0.0000	0.2	0.6(4)	0.6(4)	-1.9(4)
$\nu p_{3/2}$	(0.8537)	-	-	-	-
$\nu h_{9/2}$	1.5609	0.1	0.6(5)	2.2(5)	-0.3(5)
$\nu p_{1/2}$	(1.6557)	-	-	-	-
Level	States in ^{131}Sn				
	e_1^{exc}	shift 1$^{(a)}$	shift 2$^{(b)}$	$\bar{\varepsilon}$	B. E.
$\nu d_{3/2}$	0.0000	0.25	0.6(4)	0.6(4)	-7.9(4)
$\nu h_{11/2}$	0.0651	0.3	0.6(3)	0.7(3)	-8.0(3)
$\nu s_{1/2}$	0.3317	0.25	0.6(4)	0.9(4)	-8.2(4)
$\nu d_{5/2}$	1.6545	-	0.6(4)	2.3(4)	-9.6(4)
$\nu g_{7/2}$	2.4341	-	0.6(4)	3.0(4)	-10.3(4)

$^{(a)}$*Shifts in energy from level fragmentation measured in neighbouring nuclei.*
$^{(b)}$*The values obtained through analogy by extrapolating from the data on* ^{208}*Pb.*
The numbers in parentheses give errors in the last digit.

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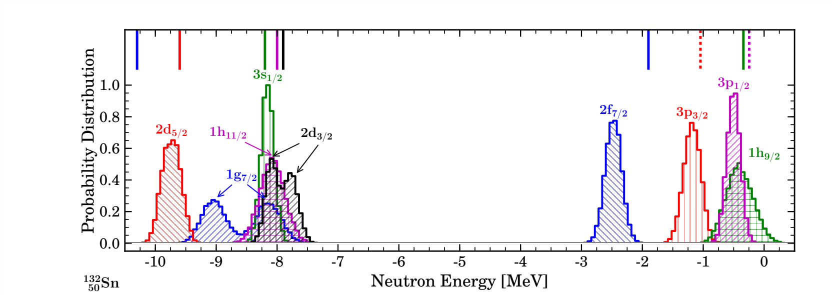

Results of the extrapolation from the ^{208}Pb to the ^{132}Sn nucleus for the neutrons, bars, cf. preceding table. Monte-Carlo simulation with N=20 000 Gaussian-distributed parameter sets, based on ^{208}Pb results; noise width σ=0.1MeV. With each of the so obtained N=20 000 sets of parameters the results for the neutrons in ^{132}Sn nucleus have been obtained. Observe 'pathologies': $1g_{7/2}$ and $2d_{3/2}$ cf. following figures.

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- **...and one may try using similar, a slightly modified wording: What carries certain interest is, possibly, theory's good predictive power!**

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*) *This notion is still to be defined for you here ...*

#) *So is the very notion of probability (12 'official' definitions and 16 interpretations)*

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- An example: the data for $\nu d_{3/2}$ states $(3/2^+)$:

$$
\begin{array}{lcl}
E_1 = 2042 \ keV & \text{with} & S_1 = 0.78 \\
E_2 = 2871 \ keV & \text{with} & S_1 = 0.08 \\
E_3 = 3083 \ keV & \text{with} & S_1 = 0.16 \\
E_4 = 3290 \ keV & \text{with} & S_1 = 0.22 \\
E_2 = 3681 \ keV & \text{with} & \underline{S_1 = 0.16}
\end{array}
$$

$$\boxed{\langle \mathbf{E} \rangle = \left(\sum_\mathbf{i} \mathbf{E_i} \times \mathbf{S_i} \right) / \left(\sum_\mathbf{i} \mathbf{S_i} \right)} \quad \longrightarrow\longrightarrow\longrightarrow\longrightarrow \quad \langle E \rangle = 2592 \ keV$$

$$\boxed{E_{d_{3/2}} = -S_n + \langle E \rangle = (-7194.5 + 2592) \ keV = -4602.5 \ keV}$$

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4. Levels depend, among others, on the spectroscopic factors, defined in the presence of simplifying assumptions in reaction theory; the latter may facilitate the self-control through the sum rule tests

5. Paradoxally, the so-called experimental single-particle levels are highly complicated, model-dependent objects - this leads to errors!

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it follows that their errors are
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Single-Particle Levels - Noise-Simulation Example

• Consider a single particle spectrum $\{e_\nu^o\} \leftrightarrow H\varphi_\nu^o = e_\nu^o\,\varphi_\nu^o$ obtained with the 'optimal' set of parameters $\{p\}_o$ as in the preceding Table;

• Define the "pseudo-experimental" levels $\{e_\nu^{exp}\} \equiv \{e_\nu^o\}$. Applying the minimisation procedure will now reproduce those $\{e_\nu^o\}$ exactly;

• Chose one level, say $e_\kappa^o \in \{e_\nu^o\}$, and arbitrarily modify its position:

$$e_\kappa^o \to e_\kappa \equiv (e_\kappa^o - e) \;\; \text{with, say} \;\; e \in [-2, +2]\ \text{MeV};$$

then refit the χ^2-test $\to$ all other levels will move to new positions

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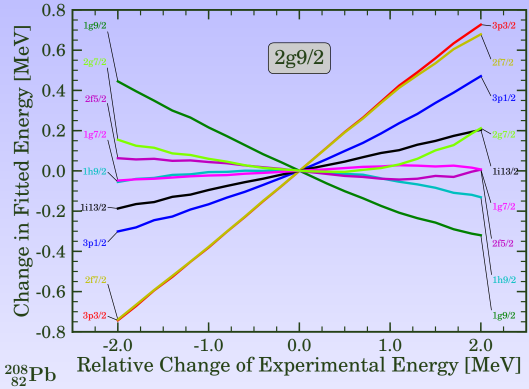
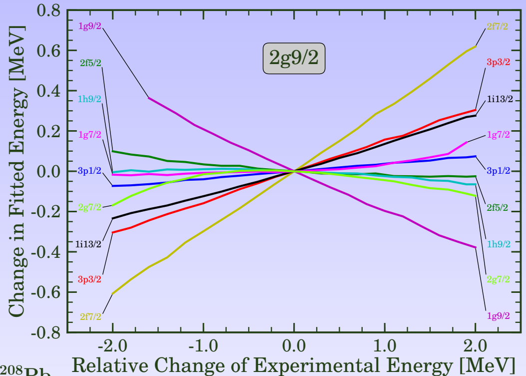
Example: Error Response Functions to $2g_{9/2}$-Orbital



To determine precisely the parameters through fitting the energies of $3p_{3/2}$, $2f_{7/2}$ etc. the right position of $2g_{9/2}$ must be analyzed particularly carefully (associated spectroscopic factors precise, particle vibration subtracted, pairing effect subtracted)

Example: Alternative Representation for $2g_{9/2}$-Orbital



$^{208}_{82}$Pb

Attention: The figure may look similar but it contains a totally opposite information:
All the curves represent the $2g_{9/2}$-level - this is how the fitting will modify $2g_{9/2}$ if we vary the indicated levels

Single-Particle Levels - Monte-Carlo Simulations

• Consider a single particle spectrum $\{e_\nu^o\} \leftrightarrow H\varphi_\nu^o = e_\nu^o\,\varphi_\nu^o$ obtained with the 'optimal' set of parameters $\{p\}_o$ as in the preceding tests;

• For a given level e.g. $i_{13/2}$, define a Gaussian distribution with σ^2-width corresponding to the information we have about this level;

• Vary the energy of the $i_{13/2}$-level and repeat the χ^2 minimisation:

$$e_{13/2}^o \to e_{13/2} \equiv (e_{13/2}^o - e) \text{ with, say } e \in [-3\sigma, +3\sigma] \text{ MeV;}$$

after that $\to$ all other levels will move to the new positions: $e_\kappa \to e'_\kappa$

• Define the new coupling-probability $P_\kappa(e'_\kappa)$ as equal $P_{13/2}(e_{13/2})$; They are functions of $e'_\kappa(e_{13/2})$, and they can be interpreted as the relative-coupling probability distributions

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$$e^o_{13/2} \rightarrow e_{13/2} \equiv (e^o_{13/2} - e) \;\text{ with, say }\; e \in [-3\sigma, +3\sigma] \text{ MeV};$$

after that $\rightarrow$ all other levels will move to the new positions: $e_\kappa \rightarrow e'_\kappa$

• Define the new coupling-probability $P_\kappa(e'_\kappa)$ as equal $P_{13/2}(e_{13/2})$; They are functions of $e'_\kappa(e_{13/2})$, and they can be interpreted as the relative-coupling probability distributions

Single-Particle Levels - Monte-Carlo Simulations

- Consider a single particle spectrum $\{e^o_\nu\} \leftrightarrow H\varphi^o_\nu = e^o_\nu\,\varphi^o_\nu$ obtained with the 'optimal' set of parameters $\{p\}_o$ as in the preceding tests;

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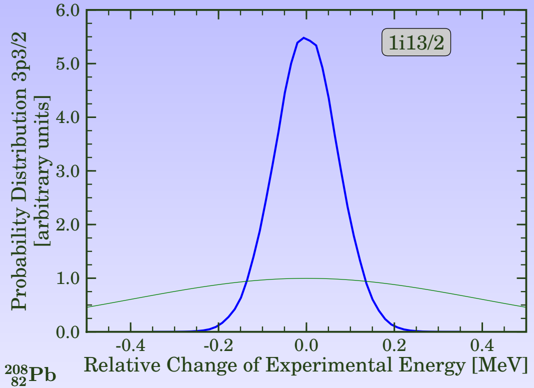
Relative-Coupling Probability Distributions for $1i_{13/2}$



We vary the energy of $e_{13/2}$-level in the spirit of the Monte-Carlo simulations using its test-energy [along the horizontal axis] distributed according to σ^2-normal density.

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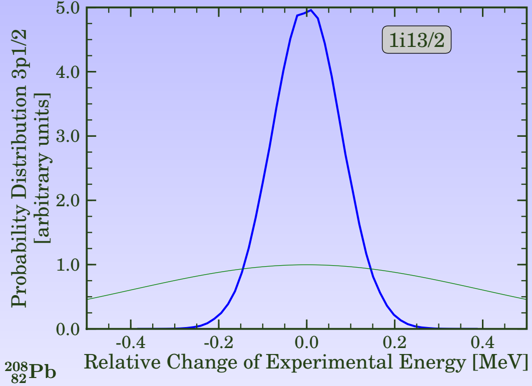

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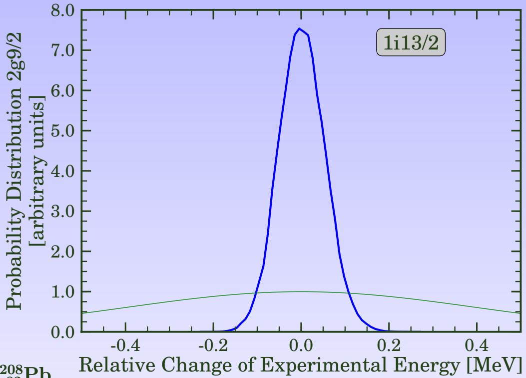

$^{208}_{82}$**Pb**

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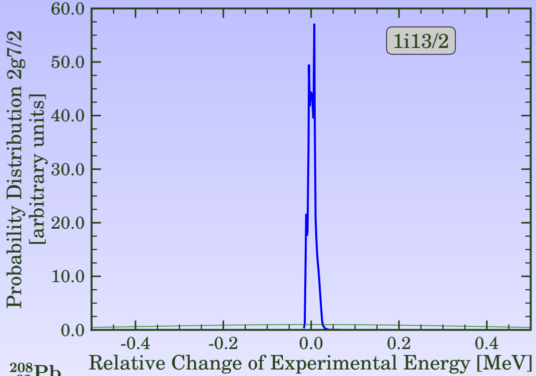

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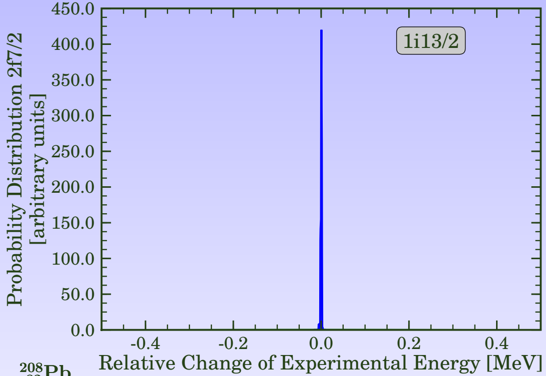

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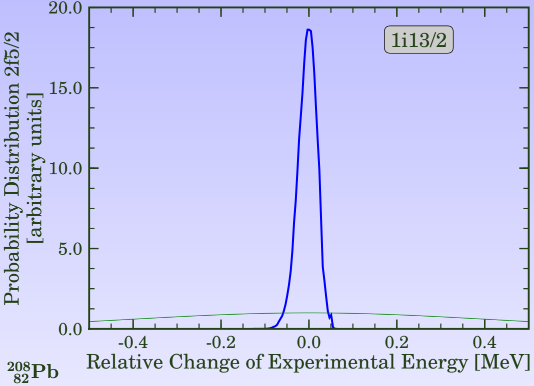

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Conclusions from Discussed Simulations

• Observe rather precise indications as to *'which levels influence which'* what allows to discuss the experimental strategies precisely

• The low-ℓ orbitals (such as $3p_{1/2}$, $3p_{3/2}$) have relatively small impact on the error-response functions ...

• ... while some pairs of orbitals couple very strongly

• The highest-ℓ orbitals do not necessarily couple the strongest way

• ... all that in a particular case presented; analysis of this type may require a case-by-case mode of operating...

Part II

Inverse Problem of Applied Mathematics

Consider a mean-field Hamiltonian: RMF, HF, Phenomenological ...

$$\mathbf{H_{mf} = H_{mf}(\hat{r}, \hat{p}, \hat{s}; \{p\})}; \quad \{\mathbf{p}\} \longrightarrow \text{parameters}$$

After laborious constructions of H_{mf}, we often get terribly exhausted and forget that: *Parameter determination is a noble, mathematically sophisticated procedure based on the statistical theories often more involved than the physical problems under study!*

Theory Predictions - Their Statistical Significance

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• In their introduction to the chapter *'Modeling of Data'*, the authors of *'Numerical Recipes"* (p. 651), observe with sarcasme:

"Unfortunately, many practitioners of parameter estimation never proceed beyond determining the numerical values of the parameter fit. They deem a fit acceptable if a graph of data and model 'l o o k s g o o d'. This approach is known as chi-by-the-eye. Luckily, its practitioners get what they deserve" [i.e. - what is meant is: "they" get a 'statistical nonsense']

- **Starting from a limited experimental data set $\{e_\mu^{exp}\}$ we wish to obtain the information about all of the single particle levels**

Applied Mathematics: Inference & Inverse Problems

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THE PREDICTIVE POWER

- **Statistically sound $\Leftrightarrow$ Instead of saying: $e_{g_{9/2}} = -8.8$ MeV we better provide also the probability function $P = P(e_{g_{9/2}})$, as narrow as possible to allow for the precise energy-indication**

Simple Mathematics of the Nonlinear Inverse Problem

- Consider an example of a spherical mean-field Hamiltonian:

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- In looking for ‘an as adequate approach as possible in a search for coupling constants’ we need some guidelines. Our choice:

The mean-field Hamiltonian should first of all describe optimally the mean field degrees of freedom

$$\boxed{\{\varphi_\nu, \mathbf{e}_\nu\} \text{ in } \mathbf{H}_{\mathbf{mf}}\varphi_\nu(\mathbf{r}, \{\mathbf{p}\}) = \mathbf{e}_\nu(\mathbf{p})\,\varphi_\nu(\mathbf{r}, \{\mathbf{p}\})}$$

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- In an Appendix we explain at length why not the nuclear masses...

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- Introduce linearisation procedure [after simplification $e_j^{th}(p) \to e_j$]

$$e_j(p) \approx e_j(p_0) + \sum_{i=1}^{n}\left(\frac{\partial e_j}{\partial p_i}\right)\Big|_{p=p_0}(p_i - p_{0,i})$$

$$J_{jk} \equiv \sqrt{W_j}\,\left(\frac{\partial e_j}{\partial p_k}\right)\Big|_{p=p_0} \quad \text{and} \quad b_j = \sqrt{W_j}\,[e_j^{exp} - e_j(p_0)]$$

$$\boldsymbol{\chi^2(p) = \frac{1}{m-n}\sum_{j=1}^{m}\left[\sum_{i=1}^{n} J_{ji}\cdot(p_i - p_{0,i}) - b_j\right]^2}$$

- **One may easily show that within a linearized representation**

$$\boxed{\frac{\partial\chi^2}{\partial p_i} = 0 \quad\rightarrow\quad (J^T J)\cdot(p - p_0) = J^T e \quad\leftrightarrow\quad A \stackrel{df}{=} J^T J}$$

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- In *Applied Mathematics* we usually change wording and notation:

$$\{p\} \leftrightarrow \text{`causes'} \text{ and } \{e\} \leftrightarrow \text{`effects'} \leftrightarrow A \cdot p = e$$

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- "An unusual" feature: $\mathbf{J} \leftrightarrow \mathbf{m} \times \mathbf{n}$ rectangular matrix $(\mathbf{m} \neq \mathbf{n})$.

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• The problems with instabilities (i.e. ill-conditioning) can be easily illustrated using the so-called Singular-Value Decomposition of A:

$$\mathbf{A} = \mathbf{U} \cdot \mathbf{D} \cdot \mathbf{V}^{\mathbf{T}} \text{ with } \mathbf{U} \in \mathbb{R}^{\mathbf{m}\times\mathbf{m}}, \quad \mathbf{V} \in \mathbb{R}^{\mathbf{n}\times\mathbf{n}}, \quad \mathbf{D} \in \mathbb{R}^{\mathbf{m}\times\mathbf{n}}$$

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- Formally (but also in practice), the solution 'e' is expressed as

$$\mathbf{e = A^T p; \quad A^T = V \cdot D^T \cdot U^T}$$

where

$$\mathbf{D^T = diag\{\tfrac{1}{d_1}, \tfrac{1}{d_2}, \ldots \tfrac{1}{d_p}; 0, 0, \ldots 0\}}$$

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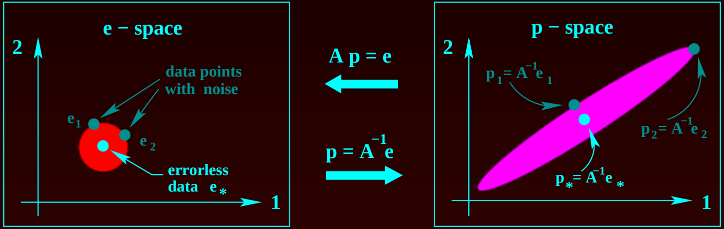

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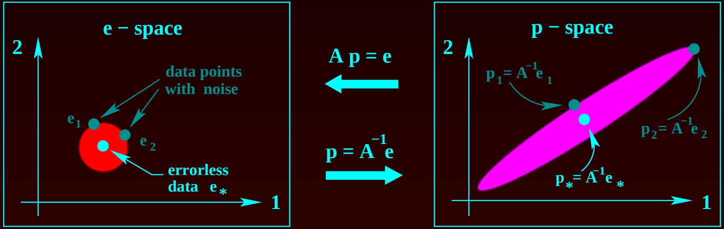

Left: Red circle represents points equally distant from 'noisless' data e_**.*
Right: Purple oval represents the image of the circle through $p = A^{-1} e$*. One may show that instability is directly dependent on the condition number*

$$cond(A) \equiv \frac{d_1}{d_r}$$

- the bigger the condition number the more 'ill-conditioned' the problem

Fitting, Implied Errors and Confidence Intervals

- Let us come back to the underlying χ^2 minimum condition:

$$\frac{\partial \chi^2}{\partial \mathbf{p_j}} \quad \longrightarrow \quad (\mathbf{J^T J})(\mathbf{p} - \mathbf{p_0}) = \mathbf{J^T e}$$

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$\mathbf{J_{ik}} = \sum_{\ell=1}^{\mathbf{r}} \mathbf{U_{i\ell}}\,\mathbf{d}_\ell\,\mathbf{V^T_{\ell k}}$ one shows that $(\mathbf{J^T J})^{-1} = \mathbf{V}\frac{\mathbf{1}}{\mathbf{d}^{\,2}}\mathbf{V^T}$

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- Independently one derives the expression for the correlation matrix

$$\boxed{\langle(\mathbf{p_i}-\langle\mathbf{p_i}\rangle)\cdot(\mathbf{p_j}-\langle\mathbf{p_j}\rangle)\rangle = \chi^2(\mathbf{p})\, \mathbf{t}^2_{\alpha/2,m-n}\, (\mathbf{J}^T\mathbf{J})^{-1}_{ij}}$$

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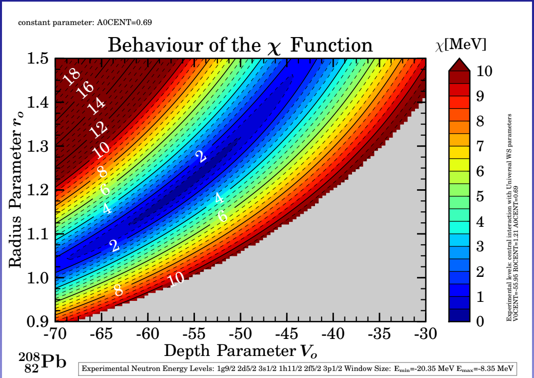
- **If one or more $d_k \to 0$ then $(J^T J)^{-1}$ tends to infinity and generally, the confidence intervals of a l l parameters diverge**

Part III

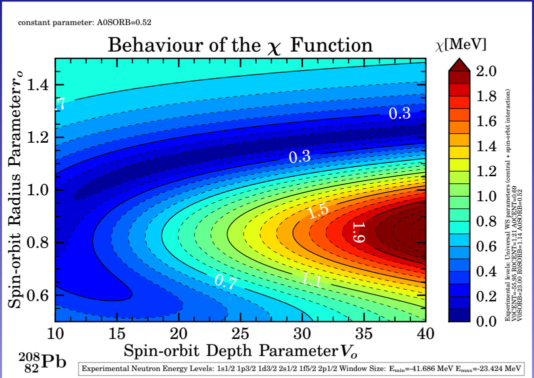
About Undesired Parameteric Correlations

Begin with a Well Known: V_o vs. r_o Are Correlated



A map of χ^2 from the fit based on six levels close to the Fermi level.

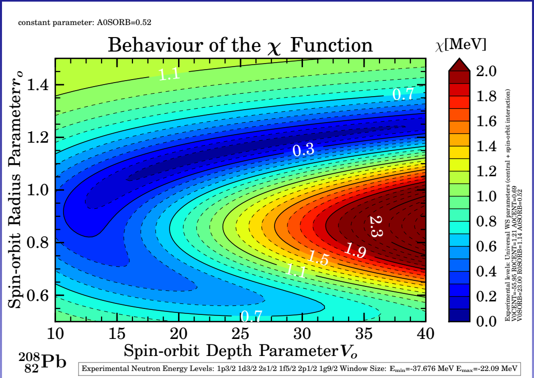
Parametric Correlations and Their Consequences



We plot the χ^2 in function of the S-O strength (horizontal) and the S-O radius (vertical) axis. We start with six very lowest levels. Note: no way to fix reliably the spin-orbit strength in the interval from 15 to 40 units!!

We will gradually increase the energy of the six-level window to approach the nucleon binding region and thus simulate the present-day experimental situation

Limited Experimental Input: How Little is Sufficient?



Increasing the energy of the six levels helps localising the spin-orbit strength only very slowly!

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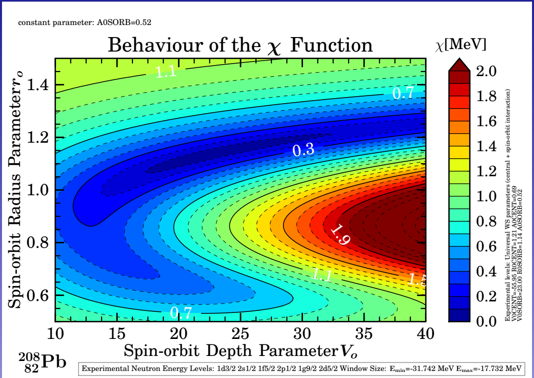

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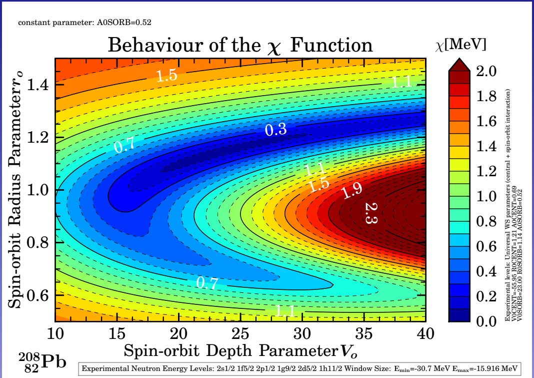

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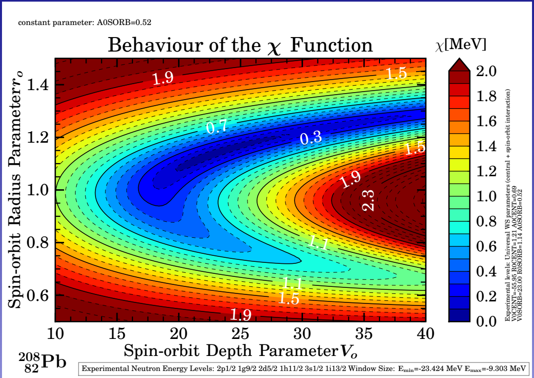

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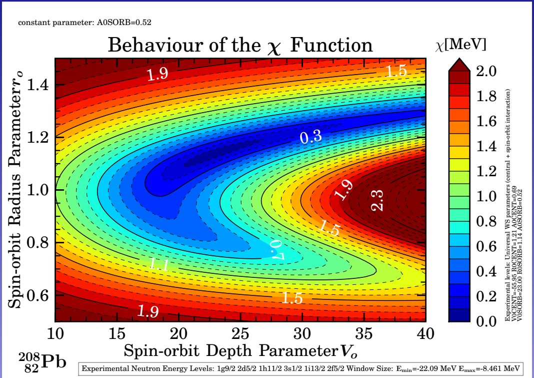

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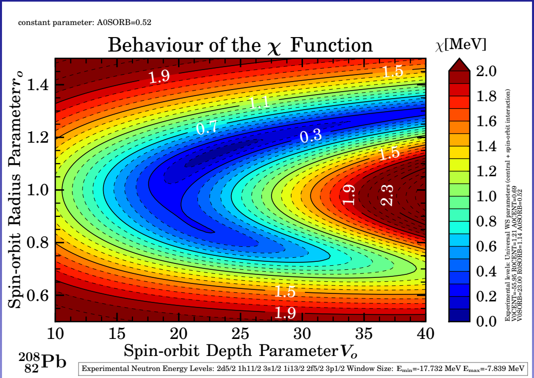

Increasing the energy of the six levels helps localising the spin-orbit strength only very slowly... whereas gradually another solution ...

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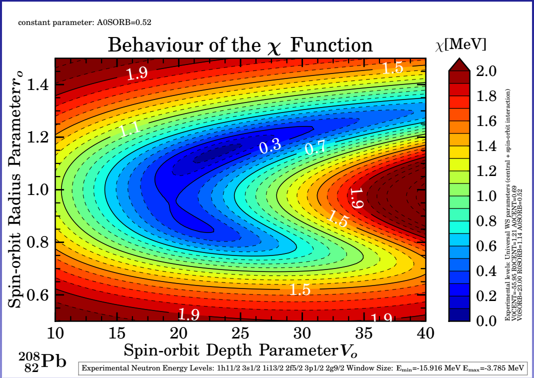

Increasing the energy of the six levels helps localising the spin-orbit strength only very slowly... Attention: Second solution is coming !

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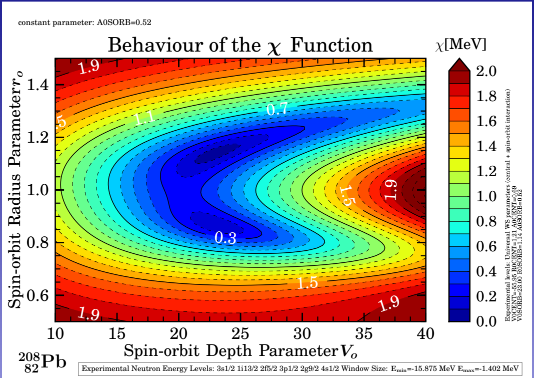

... and here we discover the existence of <u>two solutions</u> - we call them compact and non-compact.

What Can We Conclude from This Set of Tests?

• **First of all, the <u>fitted</u> spin-orbit strength may <u>vary widely</u> from one doubly-magic nucleus to another - there exists a considerable softness in χ^2 dependence on λ_{so}**

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• We confirm the presence of iso-spectral lines also in the space of the spin-orbit potential parameters

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Part IV

Predicitive Power in Terms of Soluble Models

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- **What useful could we learn from such a 'naive' formulation?**

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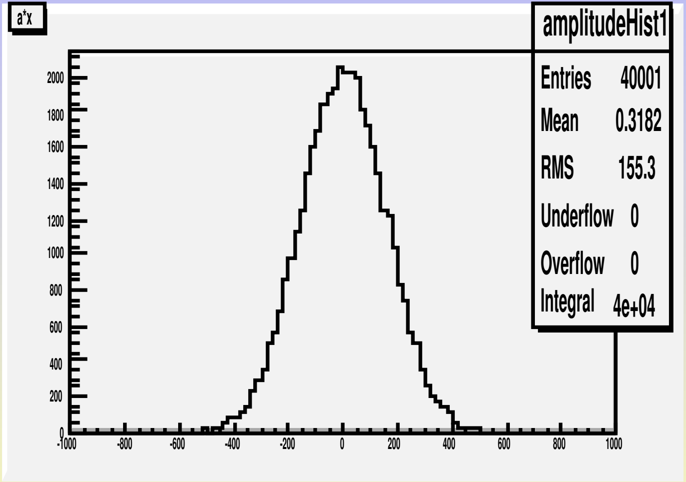
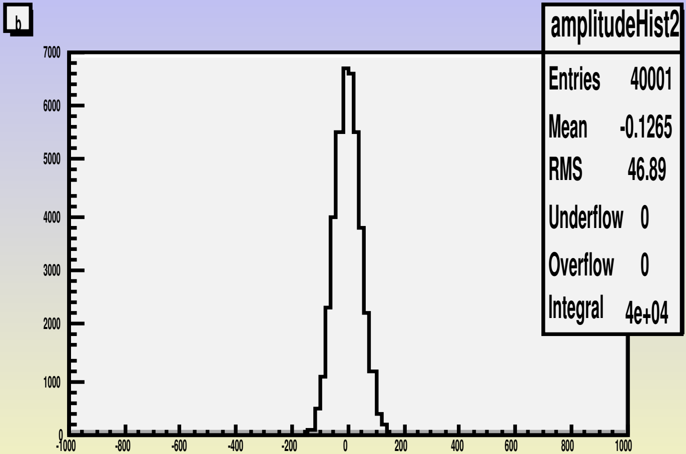
Predictive Power in Terms of Soluble Models



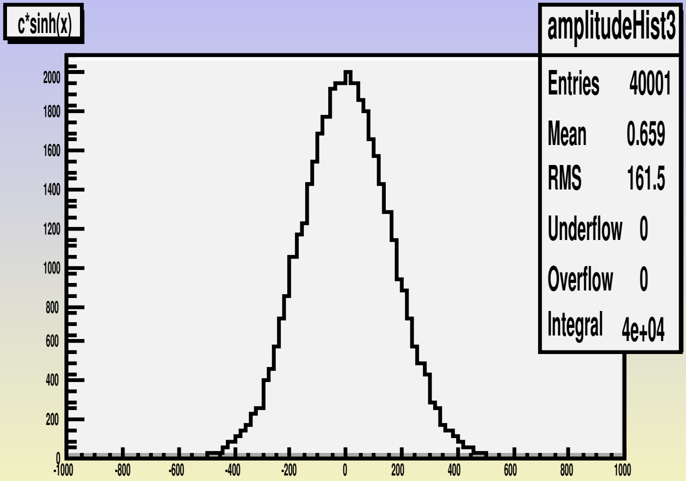
Probability Distribution of the *a*-parameter of the 'theory'

Predictive Power in Terms of Soluble Models

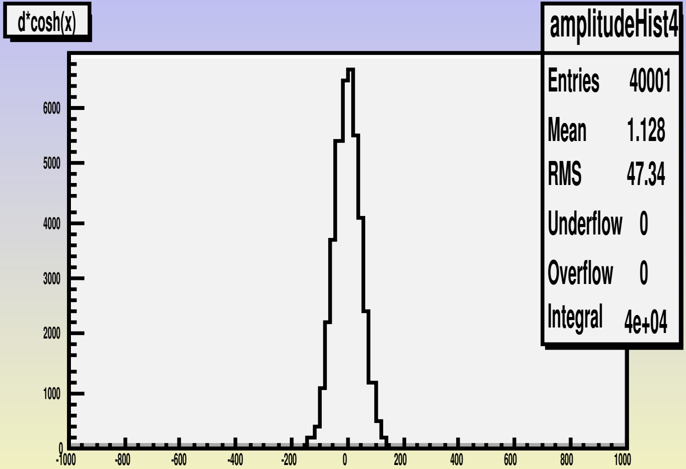


Probability Distribution of the *b*-parameter of the 'theory'

Predictive Power in Terms of Soluble Models

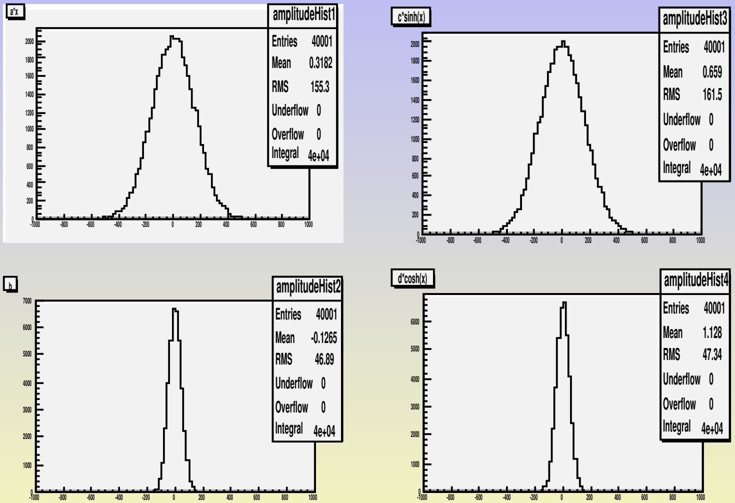


Probability Distribution of the *c*-parameter of the 'theory'



Probability Distribution of the *d*-parameter of the 'theory'

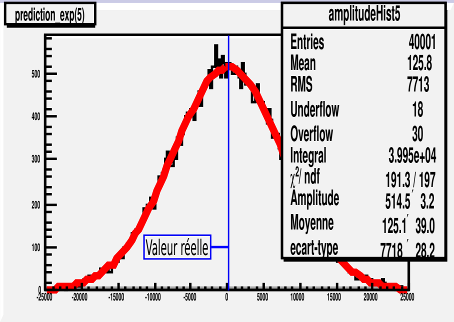
Predictive Power in Terms of Soluble Models



Observe different behaviour of the positive and negative parities

- **Extraneous Predictive Power ↔ Extrapolations by theory:**

$$\mathbf{exp(5) = ?}$$



Probability Distribution of the 'exact theory' prediction for $\exp(5) = 148.4$

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What is the fundamental origin of the 'theory' failure?

The Inverse Problem Related to the Theory

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- When there are correlations $[p_i = f(p_j)]$ at least one $d \rightarrow 0$

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At the end : Predictive Power Disappears

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- **The <u>real</u> problem: <u>How about small but $\neq 0$ singular values?</u>**

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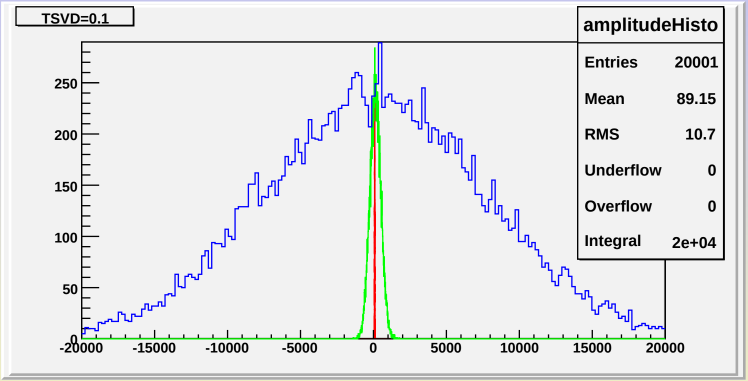
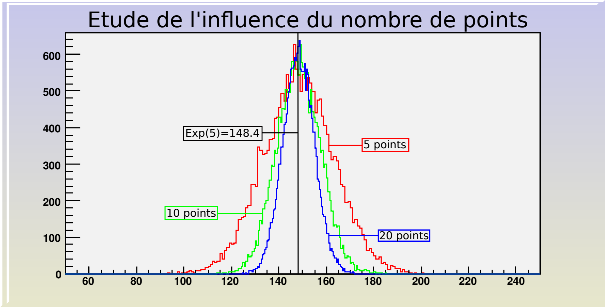
- **Before we start looking for a compromise → an illustration**

- **TSVD: ⇒ efficiently counteracts the problem of instability**



Blue: no TSVD truncation, Green = cut off = 0.01, Red = cut off 0.1

- **Divergence depend on the 'Hamiltonian' and on data points**



Increasing the no. of data points increases the constraint on the model and as a consequence - stabilises the final solution

The Following Messages

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• It is instructive to think about the extentions of the EDF based approaches in terms of increasing number of coupling constants and the preceding illustrations...

About Contemporary Skyrme-Type Functionals

Numbers of terms of different orders in the EDF up to N^3LO. Numbers of terms depending on the time-even and time-odd densities are given separately. The last two columns give numbers of terms when the Galilean or gauge invariance is assumed, respectively. To take into account both isospin channels, the numbers of terms should be multiplied by a factor of two.

Order	T-even	T-odd	Total	Galilean	Gauge
0	1	1	2	2	2
2	8	10	18	12	12
4	53	61	114	45	29
6	250	274	524	129	54
N^3LO	2x312	2x346	2x658	2x188	2x97
	624	692	1316	376	194

For comments about Skyrme HF gauge invariance cf. e.g.
J. Dobaczewski and J. Dudek, PRC 52 (1995) 1827

A Realistic Toy Model - Noise-Simulation Example

- Let us calculate $\{e_\mu\}$-levels for a given W-S parameter set, here:

Woods-Saxon parameters for the neutrons in ^{208}Pb reproduce the experimental levels with the r.m.s. deviation of 0.164 MeV and maximum error of 0.353 MeV.

V_o^c	r_o^c	a_o^c	λ	r_o^{so}	a^{so}
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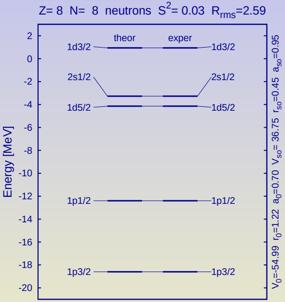
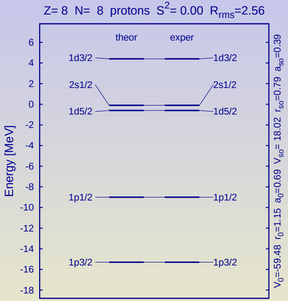
- We will obtain the response of all the levels to a 'linear noise' - vary a level position within a window and refit the H-parameters $\{p\}$

- We fit the single-particle experimental levels in ^{16}O using Woods-Saxon potential (six parameters for protons and neutrons each)





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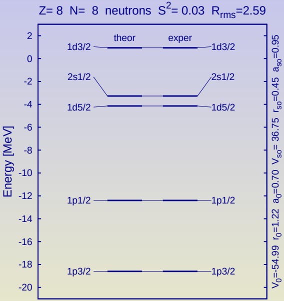

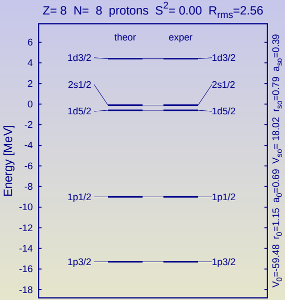

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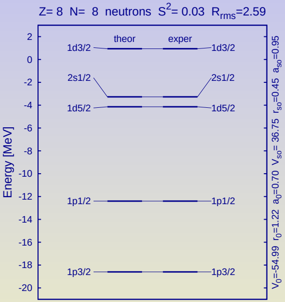

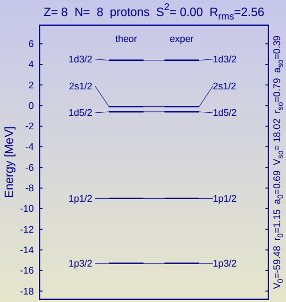

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• On the other hand: If we trust the model - we may hope that also the remaining levels are close to the experimental results to come

Unprecedented Precision of the Fits: 10^{-1} keV!

→ The standard Woods-Saxon Hamiltonian has been used:

No.	E_{calc}	E_{exp}	Level	Err.(th-exp)
1.	-15.300	-15.300	$1p_{3/2}$	-0.0001
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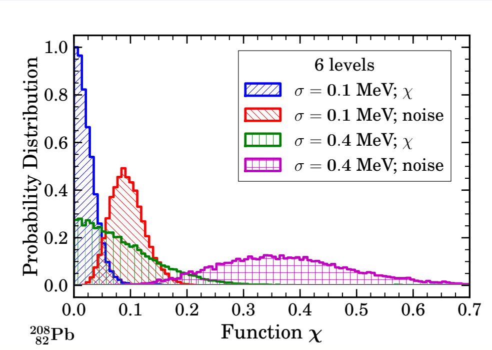
- *What is the mathematical/physical significance of the result?*

Unprecedented Precision ... Known as 'Overfitting'



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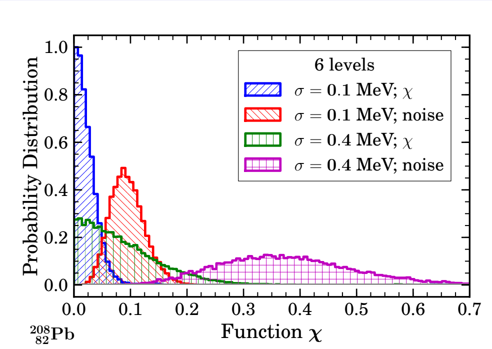

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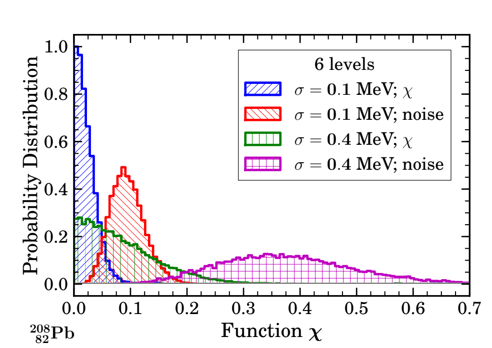

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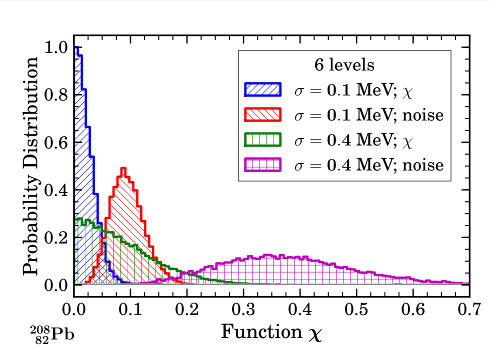

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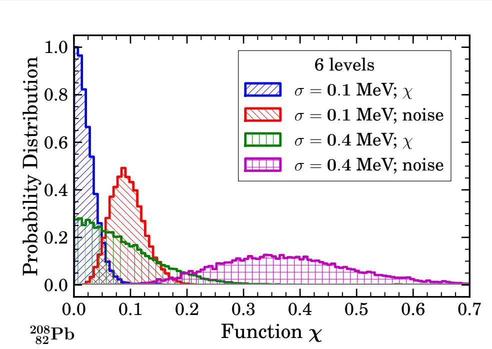

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- Under the mathematical conditions discussed there are $N = \infty^6$ exact fits possible. Is it totally trivial?

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• We improve the model by reducing the number of parameters

Part V

Nuclear Hamiltonians: Sampling vs. Microscopic Structure

Decrease Parametric Freedom: ‘Be More Microscopic’

• We suggest to replace a too neat W-S spin-orbit parametrisation by using the density gradient

$$V_{ws}^{so} \sim \frac{1}{r}\frac{d}{dr}\left\{\frac{\lambda}{1+\exp[(r-R_o)/a_o]}\right\} \quad \leftrightarrow \quad V_{ws}^{so} \sim \frac{\lambda'}{r}\frac{d\rho}{dr}$$

• The nucleonic density can be seen as describing the interaction source: in systems with short range interactions, on the average, the higher the density (gradient) - the more chance to S-O interact.

• Similarly, in the relativistic approach

$$V_{rel}^{so} \sim \frac{1}{r}\frac{d}{dr}[S(r) - V(r)]$$

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A New Toy-Model, Half-Microscopic Hamiltonian

- Consider a given nucleus with the density ρ and a series of neighbouring nuclei with extra occupied orbitals $j_1 \leftrightarrow \rho_{j_1}$, $j_2 \leftrightarrow \rho_{j_2}$, etc. We expect that the density-dependent spin-orbit potentials

$$\mathbf{V^{so}} \sim \frac{\mathbf{d\rho}}{\mathbf{dr}}, \quad \frac{\mathbf{d\rho_{j_1}}}{\mathbf{dr}}, \quad \frac{\mathbf{d\rho_{j_2}}}{\mathbf{dr}} \quad \dots$$

account much better for these extra orbitals than just a flat WS potential introduced long ago for numerical simplicity

- Therefore we will test the following Hartree-Fock like hypothesis

$$V_\pi^{so} = \frac{\lambda_{\pi\pi}}{r}\frac{d\rho_\pi}{dr} + \frac{\lambda_{\pi\nu}}{r}\frac{d\rho_\nu}{dr}$$

and

$$V_\nu^{so} = \frac{\lambda_{\nu\pi}}{r}\frac{d\rho_\pi}{dr} + \frac{\lambda_{\nu\nu}}{r}\frac{d\rho_\nu}{dr}$$

with the central WS potentials

$$V_\pi^c = \frac{V_o^\pi}{1+\exp[(r-R_o^\pi)/a_o^\pi]}$$

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• On the other hand we will modify the HF idea of self-consistency from the usual variational context to the spectroscopic context:

$$\mathbf{H}(\rho)\psi_{\mathbf{n}} = \mathbf{e}_{\mathbf{n}}\,\psi_{\mathbf{n}} \;\rightarrow\; \rho = \sum \psi^*\psi \;\rightarrow\; \mathbf{H}(\rho)\psi_{\mathbf{n}} = \mathbf{e}_{\mathbf{n}}\,\psi_{\mathbf{n}} \ldots$$

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• **In other words: We will reduce the number of spin-orbit parameters from 6 (3 per particle kind: λ, r_o^{so} and a^{so}) to one!**

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• In other words: We will reduce the number of spin-orbit parameters from 6 (3 per particle kind: λ, r_o^{so} and a^{so}) to one!

Self-consistent Formulation: Minimum Coupling

• For the first tests we apply what we call a minimum coupling hypothesis

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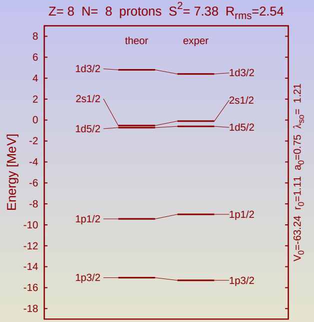
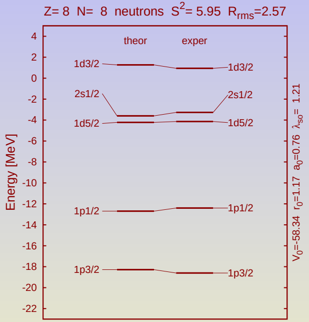
Comparing with Experimental Results: Example ^{16}O

Single particle proton and neutron states in ^{16}O





These results corresponds to just one λ-parameter fit instead of 6.

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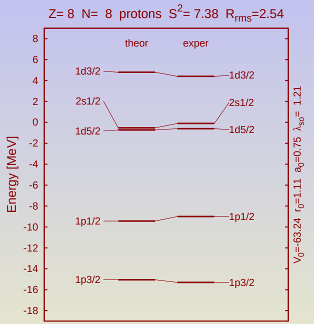

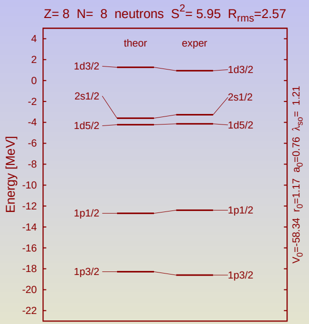

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Recall: spin-orbit strength parameter $\lambda = \lambda_{\nu\nu} = \lambda_{\nu\pi} = \lambda_{\pi\nu} = \lambda_{\pi\pi}$.

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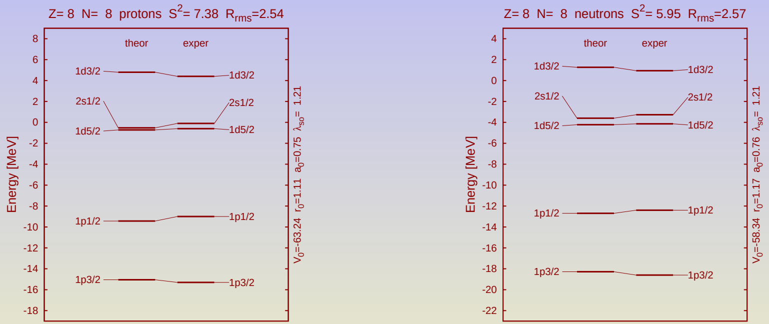

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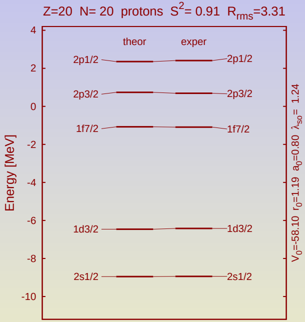
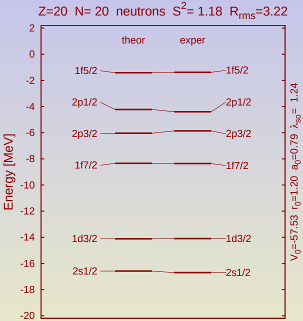
The maximum error $\sim$ 200 keV

Comparing with Experimental Results: Example ^{40}Ca

Similarly the single particle proton and neutron states in ^{40}Ca





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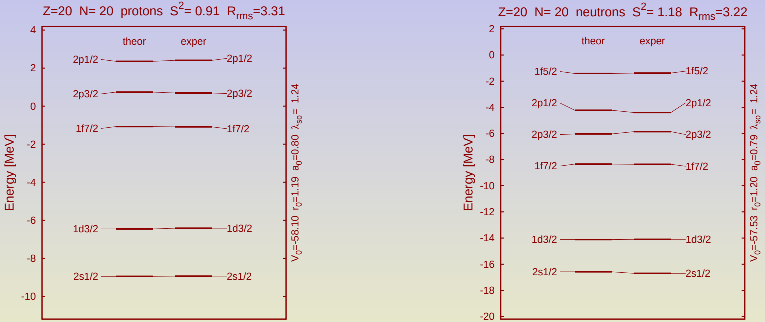

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Comparing with Experimental Results: Example ^{40}Ca

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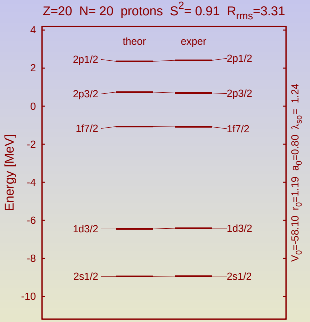

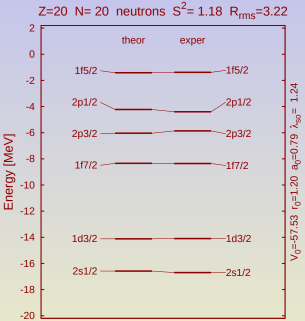

This result corresponds to just one (λ) parameter fit instead of 6; similar results hold for nuclei up to ^{208}Pb.

We have V_o^π, V_o^ν, r_o^π, r_o^ν and $a_o^\pi = a_o^\nu$ parameters. In the case of ^{208}Pb we have $13_\nu + 11_\pi$ data points.

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   - Most importantly, Fits show that the density fluctuations are needed for the gradients in the realistic spin-orbit terms!

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• Needless to say - we aim at the microscopic level (theories), in particular HF - but today we have presented some simple semi-quantitative illustrations