# Low-lying dipole response within the Second RPA in $^{40,48}$Ca

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# Outline

## Motivation

- Experimental low-lying dipole (from 5 to 10 MeV) response in $^{48}$Ca
- Not described in relativistic and non-relativistic RPA models
  - not good excitation energies (too high energies)
  - and/or do not predict the experimental fragmentation of the peaks
- What happens in Second RPA?

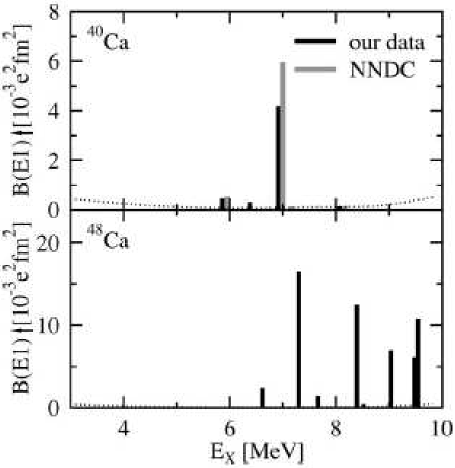
## RPA and Second RPA

- Excitation Operators and Equations

## Second RPA Calculations for $^{40,48}$Ca

- Low-Lying Strength Distributions
- Collectivity and Transition densities

# Experimental low-lying dipole strength in $^{40,48}$Ca. (Photon Scattering)




$\sum$ B(E1)=5.1 $\pm$ 0.8 ($10^{-3}e^2fm^2$),

$\sum$ B(E1)=68.7 $\pm$ 7.5 ($10^{-3}e^2fm^2$),

From T. Hartmann *et al.*, PRC 65, 034301, (2002)

# Relativistic RPA results

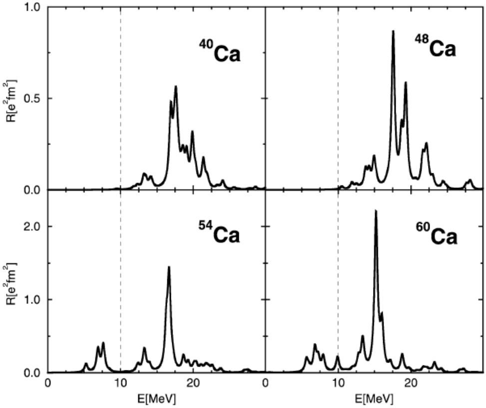
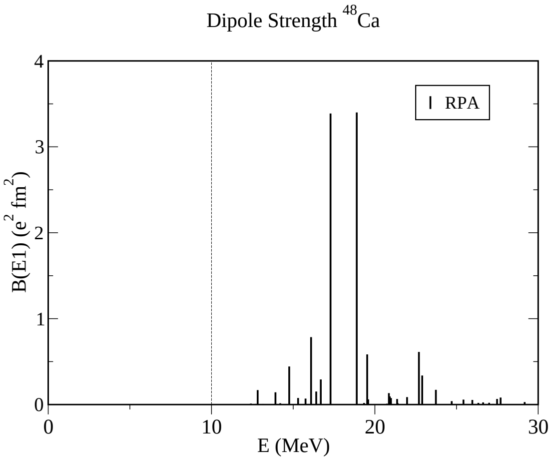


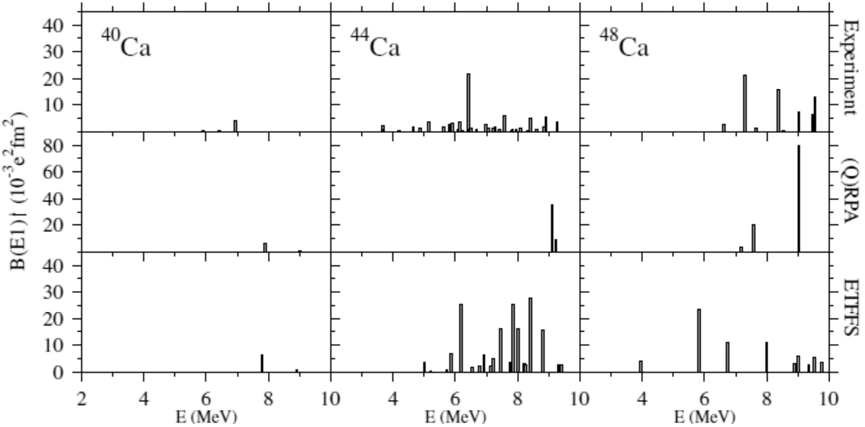

Fig. 5. RRPA isovector dipole strength distributions in Ca isotopes. The thin dashed line tentatively separates the region of giant resonances from the low-energy region below 10 MeV.

From D. Vretenar *et al.*, Nucl. Phys. A 692, 496 (2001)

# Skyrme-RPA results (SGII)



# Theory vs Experimental




From T. Hartmann, *et al.* PRL **79**, 044305 (2004)

# RPA and SRPA

## RPA and SRPA

Excitation Operators and Equations

# RPA Excitation Operators and Equations

## Phonon Operators

$$Q_\nu^\dagger = \sum_{ph} X_{ph}^{(\nu)} a_p^\dagger a_h - \sum_{ph} Y_{ph}^{(\nu)} a_h^\dagger a_p$$

## RPA Equations of Motion (1$\mapsto$ 1p1h)

$$\begin{pmatrix} \mathcal{A}_{11} & \mathcal{B}_{11} \\ -\mathcal{B}_{11}^* & -\mathcal{A}_{11}^* \end{pmatrix} \begin{pmatrix} \mathcal{X}_1^\nu \\ \mathcal{Y}_1^\nu \end{pmatrix} = \omega_\nu \begin{pmatrix} \mathcal{X}_1^\nu \\ \mathcal{Y}_1^\nu \end{pmatrix}$$

# SRPA Excitation Operators and Equations

## Phonon Operators

$$Q_\nu^\dagger = \sum_{ph} (X_{ph}^{(\nu)} a_p^\dagger a_h - Y_{ph}^{(\nu)} a_h^\dagger a_p)$$

$$+ \sum_{p_1 < p_2, h_1 < h_2} (X_{p_1 h_1 p_2 h_2}^{(\nu)} a_{p_1}^\dagger a_{h_1} a_{p_2}^\dagger a_{h_2} - Y_{p_1 h_1 p_2 h_2}^{(\nu)} a_{h_1}^\dagger a_{p_1} a_{h_2}^\dagger a_{p_2})$$

## SRPA Equations of Motion (1$\mapsto$ 1p1h, 2$\mapsto$ 2p2h)

$$\begin{pmatrix} \mathcal{A}_{11} & \mathcal{A}_{12} & \mathcal{B}_{11} & \mathcal{B}_{12} \\ \mathcal{A}_{21} & \mathcal{A}_{22} & \mathcal{B}_{21} & \mathcal{B}_{22} \\ -\mathcal{B}_{11}^* & -\mathcal{B}_{12}^* & -\mathcal{A}_{11}^* & -\mathcal{A}_{12}^* \\ -\mathcal{B}_{21}^* & -\mathcal{B}_{22}^* & -\mathcal{A}_{21}^* & -\mathcal{A}_{22}^* \end{pmatrix} \begin{pmatrix} \mathcal{X}_1^\nu \\ \mathcal{X}_2^\nu \\ \mathcal{Y}_1^\nu \\ \mathcal{Y}_2^\nu \end{pmatrix} = \omega_\nu \begin{pmatrix} \mathcal{X}_1^\nu \\ \mathcal{X}_2^\nu \\ \mathcal{Y}_1^\nu \\ \mathcal{Y}_2^\nu \end{pmatrix}$$

# SRPA Matrices

## RPA Matrices (1p-1h configurations)

$$A_{1,1'} = \langle HF \mid \left[a_h^\dagger a_p, [H, a_{p'}^\dagger a_{h'}]\right] \mid HF\rangle$$

$$B_{1,1'} = -\langle HF \mid \left[a_h^\dagger a_p, [H, a_{h'}^\dagger a_{p'}]\right] \mid HF\rangle.$$

## Beyond RPA Matrices (1p-1h and 2p-2h configurations)

$$A_{1,2} = A^*_{2,1} = \left\langle HF \middle| \left[a_h^\dagger a_p, [H, a_{p_1}^\dagger a_{p_2}^\dagger a_{h_1} a_{h_2}]\right] \middle| HF \right\rangle$$

$$B_{1,2} = B^*_{2,1} = -\left\langle HF \middle| \left[a_p^\dagger a_h, [H, a_{p_2}^\dagger a_{p_2}^\dagger a_{h_1} a_{h_2}]\right] \middle| HF \right\rangle$$

$$A_{2',2} = \left\langle HF \middle| \left[a_{h_2'}^\dagger a_{h_1'}^\dagger a_{p_2'} a_{p_1'}, [H, a_{p_1}^\dagger a_{p_2}^\dagger a_{h_1} a_{h_2}]\right] \middle| HF \right\rangle$$

$$B_{2',2} = -\left\langle HF \middle| \left[a_{p_1'}^\dagger a_{p_2'}^\dagger a_{h_1'} a_{h_2'}, [H, a_{p_1}^\dagger a_{p_2}^\dagger a_{h_1} a_{h_2}]\right] \middle| HF \right\rangle$$

# Calculations for $^{40,48}Ca$

## Calculation Details

- The Skyrme (SGII) interaction is used
- In SRPA all kinds of couplings among all 1p-1h and 2p-2h
- No use of the diagonal approximation is made
- Rearrangement Terms also in beyond RPA matrices (see JPG 38 035103 (2011))
- No Coulomb and Spin-orbit terms in the residual interaction. Deviations of EWSR are at worse of $3 - 5\%$
- For more details see PRC 84, 034301 (2011)

# $^{48}$Ca Stability of SRPA Results vs Energy Cutoff on 2p2h (ECUT)

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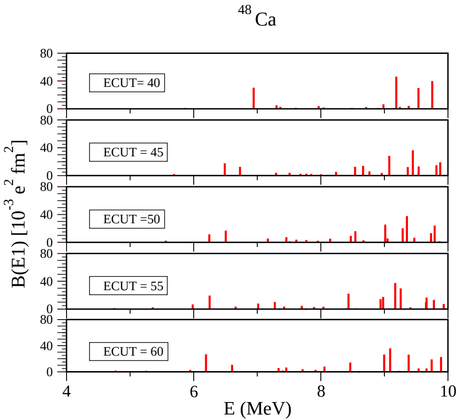



$\sum$ B(E1)=0.184 , EWSR=1.623

$\sum$ B(E1)=0.218, EWSR=1.895

$\sum$ B(E1)=0.226, EWSR=1.944

$\sum$ B(E1)=0.240, EWSR=2.049

$\sum$ B(E1)=0.230, EWSR=1.964

[$e^2$ $fm^2$], [$e^2$ $fm^2$ MeV]

# Total $B(E1)$ and EWSRs (up to 10 MeV)

| | $^{48}$Ca | | $^{40}$Ca | |
|---|---|---|---|---|
| *ECUT* | $\sum B(E1)$ | EWSRs | $\sum B(E1)$ | EWSRs |
| 40 | 0.184 | 1.623 | 0.009 | 0.091 |
| 45 | 0.218 | 1.895 | 0.002 | 0.022 |
| 50 | 0.226 | 1.944 | 0.015 | 0.139 |
| 55 | 0.240 | 2.049 | 0.025 | 0.237 |
| 60 | 0.230 | 1.964 | 0.023 | 0.211 |

Table: Total $B(E1)$ $(e^2 fm^2)$ and EWSRs $(e^2 fm^2 MeV)$ integrated up to 10 MeV, for $^{48}$Ca and $^{40}$Ca, as a function of the energy cutoff $ECUT$ $(MeV)$

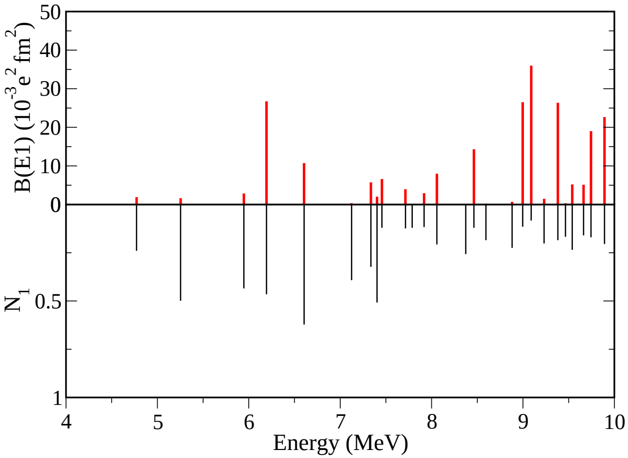
# $1p1h$ composition of the states

**Composition of the states excitation in terms of $1p1h$ and $2p2h$ configurations**

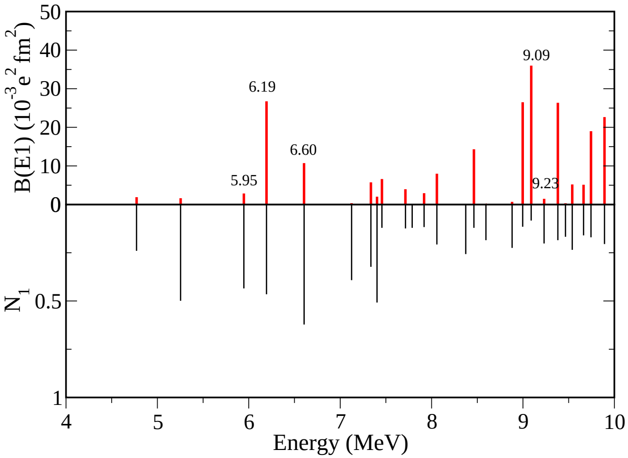
$$\langle\nu|\nu\rangle = \sum_{ph}(|\, X^{\nu}_{ph}\,|^2 - |\, Y^{\nu}_{ph}\,|^2) + \sum_{p_1<p_2,h_1<h_2}(|\, X^{\nu}_{p_1 h_1 p_2 h_2}\,|^2 - |\, Y^{\nu}_{p_1 h_1 p_2 h_2}\,|^2)$$

$$= \quad N_1 \quad + \quad N_2 \quad = 1$$

# $^{48}$Ca 1p1h components of the states



# $^{48}$Ca 1p1h components of the states



# Coherence and Collectivity

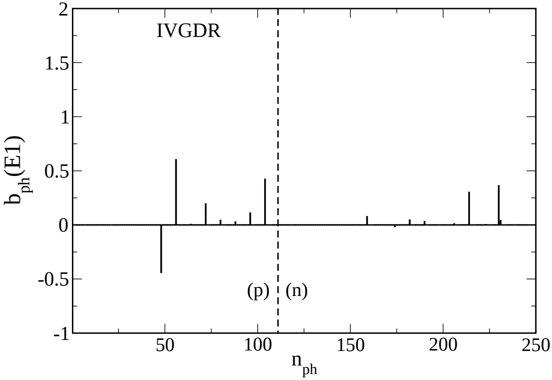
## Coherence of the different $1p1h$ configurations in building the B(E1)

$$B^{\nu}(E\lambda) = |\sum_{ph}(X^{\nu}_{ph} - Y^{\nu}_{ph})F^{\lambda}_{ph}|^2 = |\sum_{ph} b^{\nu}_{ph}(E\lambda)|^2$$

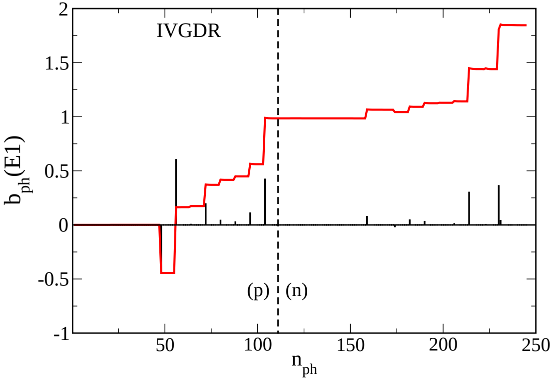
where $F^{\lambda}_{ph}$ are the multipole transition amplitudes of the operator

$$F = e\frac{N}{A}\sum_{i=1}^{Z} r_i Y_{10}(\Omega_i) - e\frac{Z}{A}\sum_{i=1}^{N} r_i Y_{10}(\Omega_i)$$

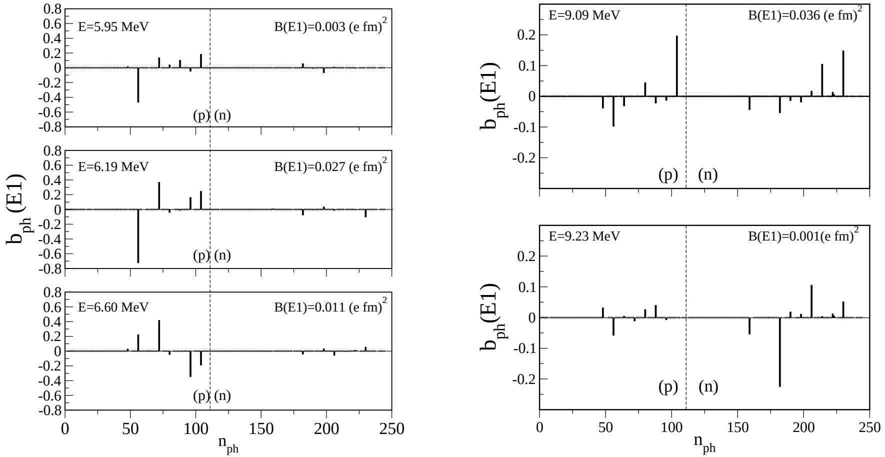
# The IVGDR case



# The IVGDR case

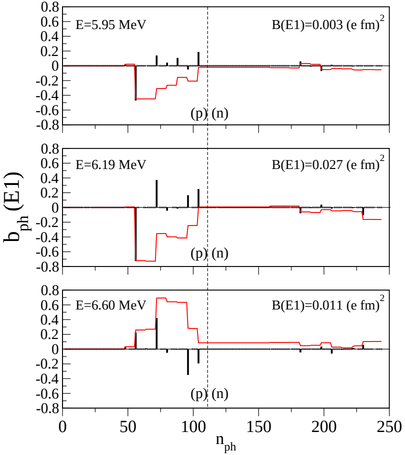


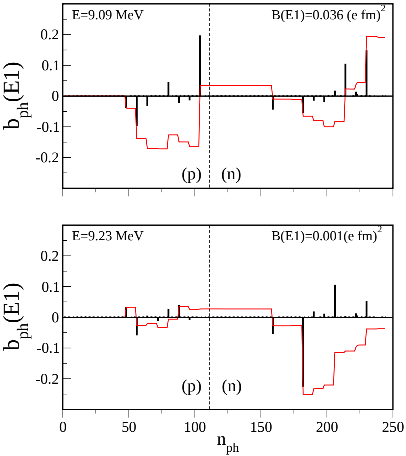
# Collectivity: coherence of the different contributions




Bars are the single $b_{ph}$, the continuous line is the cumulative sum of the $b_{ph}$

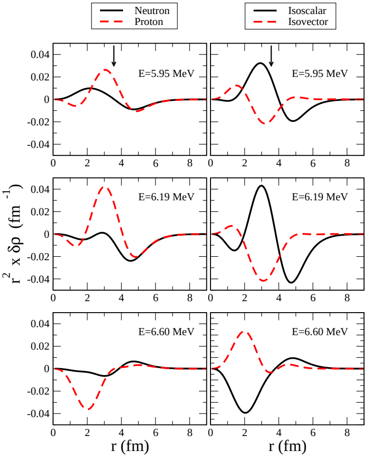
# Collectivity: coherence of the different contributions

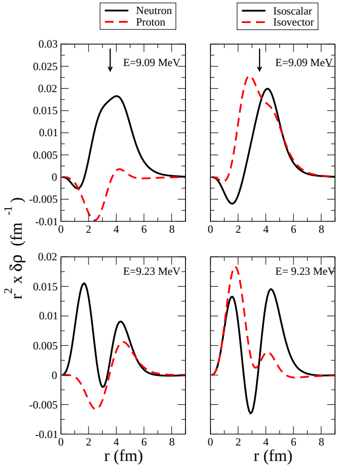


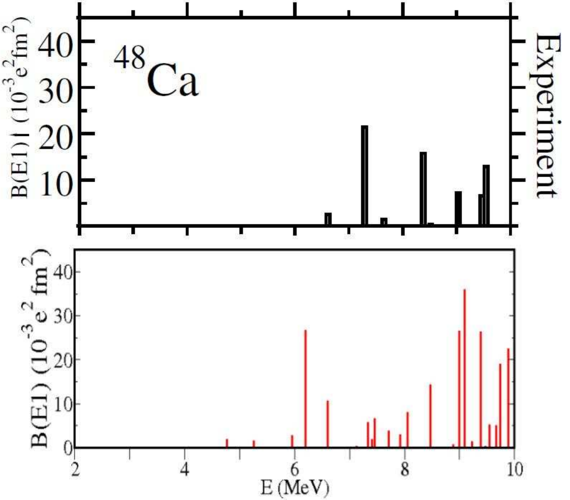

Bars are the single $b_{ph}$, the continuous line is the cumulative sum of the $b_{ph}$

# Transition densities



# Strength distributions SRPA vs Exp



# Total $B(E1)$ and EWSRs (up to 10 MeV)

## Total $B(E1)$ and EWSRs (up to 10 MeV)

| | | $^{48}$Ca |
|---|---|---|
| $\sum B(E1)$ ($10^{-3}$ e$^2$ fm$^2$) | SRPA | 230 |
| | Exp | $68.7 \pm 7.5$ |
| $\sum_i E_i B_i(E1)$ ($10^{-3}$ e$^2$ fm$^2$ MeV) | SRPA | 1964 |
| | Exp | $570 \pm 62$ |
| $E_{centroid}$ MeV | SRPA | 8.54 |
| | Exp | 8.40 |

## Some comments on that

- The double commutator sum rule is enhanced with respect to the classical sum rule by a factor 1.35 for SGII
- The same situation occurs in ETFFS for $^{44}$Ca, it was solved by including pairing effects and continuum
- Use of different Skyrme forces......

# Conclusions and Outlook

## Conclusions

- Low-lying energy dipole spectrum (from 5 to 10 MeV) for $^{40}$Ca and $^{48}$Ca
- No strength is generally provided by RPA models
- SRPA: reasonable agreement for the distribution and the fragmentation of the peaks
- Collectivity and Coherence
- Transition densities: in general no typical profiles of pygmy-like resonance
- Theoretical total B(E1) larger than the experimental one
- Use of different Skyrme forces ($E_s$, $L$, ...)?

# Width Estimation

## Width Γ

$$\Gamma = \sqrt{\frac{m_2}{m_0} - \left(\frac{m_1}{m_0}\right)^2}$$

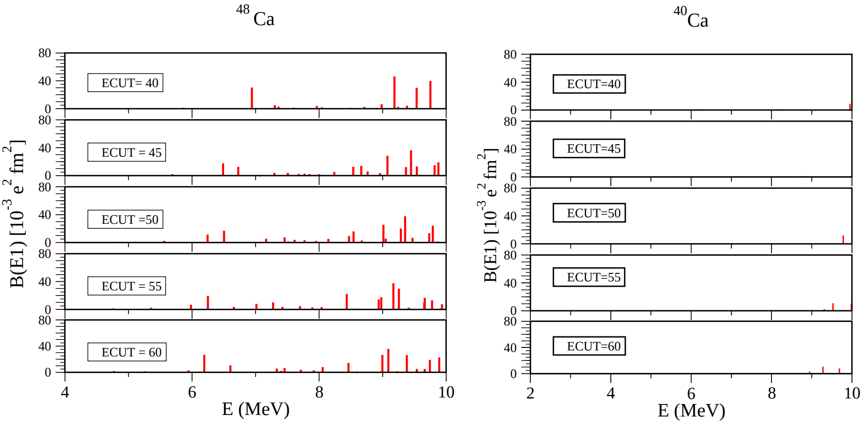
where

$$m_k = \sum_\nu (\omega_\nu)^k |\langle \nu \mid F \mid 0 \rangle|^2$$

## Width in $^{48}$Ca

| ΔE (MeV) | B(E1)($e^2$ $fm^2$) | EWSR ($e^2$ $fm^2$ MeV) | Γ (MeV) |
|---|---|---|---|
| 4-10 | 0.23 | 1.96 | 1.26 |
| 4-7 | 0.05 | 0.27 | 0.41 |
| 7-10 | 0.18 | 1.69 | 0.71 |

# $^{40,48}$Ca Stability of Results vs Energy Cutoff on 2p2h (ECUT)



# Spurious Components

## Spurious State

- Both in RPA and SRPA it should lie at zero energy (**Thouless Theorem**)
- But we are not fully self-consistent
- No Coulomb and Spin Orbit terms in RPA and SRPA

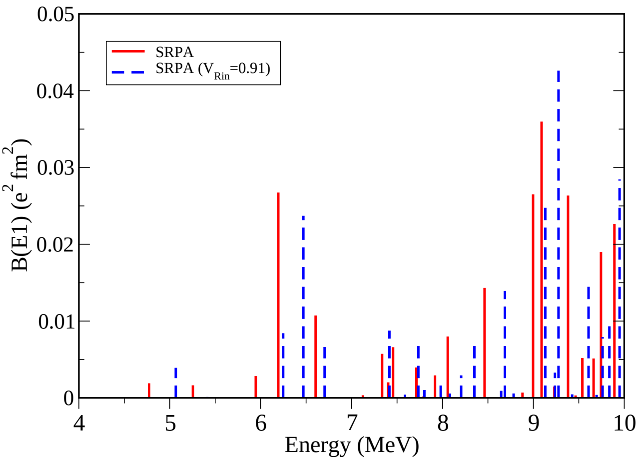
## Spurious State in RPA

- $E_{ss} \sim 3MeV$ (more then 98% of the EWSR)
- The first excited (physical) state is at $11MeV$
- By renormalizing the residual interaction by a factor 1.09 $E_{ss} \sim 0.2MeV$
- The rest of the strength distribution is practically unaffected

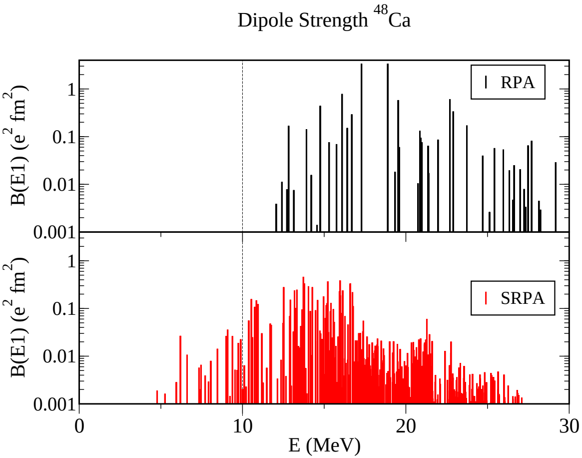
## Spurious State in SRPA

- The coupling with $2p - 2h$ configurations pushes it down (negative or imaginary solution)
- By renormalizing the residual interaction by a factor 0.91 $E_{ss} \sim 0.12MeV$
- The total $B(E1)$ changes from 0.230 to 0.221 $e^2 fm^2$
- The EWSR changes from 1.964 to 1.905 $e^2 fm^2 MeV$

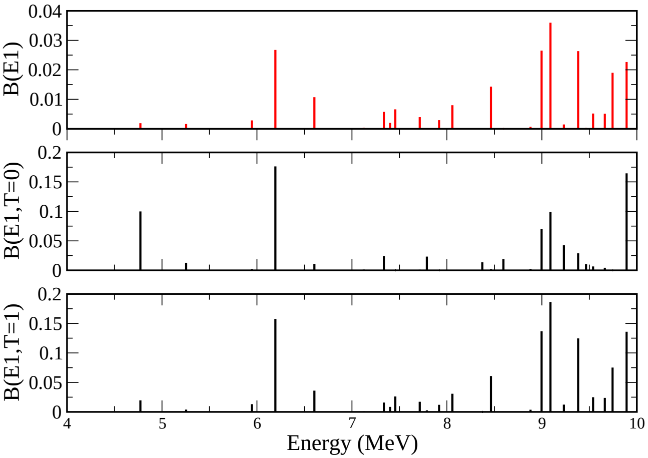
# SRPA Dipole Strength



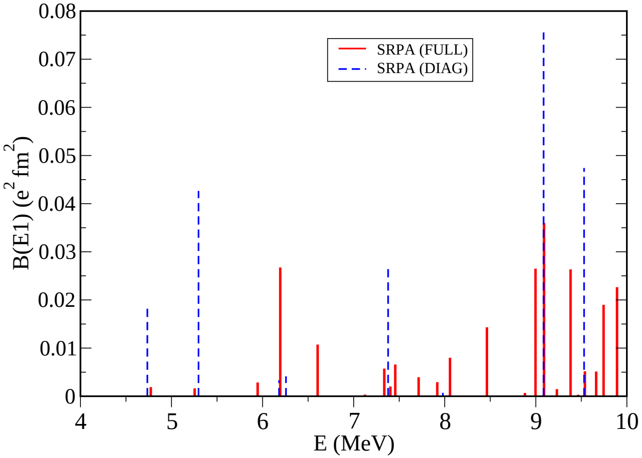
# RPA and SRPA Dipole Strength



# Isoscalar and Isovector B(E1)



# SRPA Full vs Diagonal



# Matrices

## RPA Matrices (1p-1h configurations)

$$A_{1,1'} \sim \epsilon_{HF} + \langle ph|V|ph\rangle$$
$$B_{1,1'} \sim +\langle pp|V|hh\rangle$$

## SRPA Matrices (1p-1h and 2p-2h configurations)

$$A_{1,2} \sim \langle ph|V|pp\rangle + \langle hh|V|hp\rangle$$
$$A_{2',2} \sim \epsilon_{HF} + \langle ph|V|ph\rangle + \langle pp|V|pp\rangle + \langle hh|V|hh\rangle$$

# Variational Approach

## The RPA case

- Thouless theorem:

$$| \Psi(t)\rangle = e^{\hat{S(t)}} \mid HF\rangle$$

- RPA

$$\hat{S} = \sum_{ph} C_{ph}(t) a_p^{\dagger} a_h$$

- $\langle H \rangle = \langle \Psi \mid H \mid \Psi \rangle / \langle \Psi \mid \Psi \rangle$
- Minimizing $\langle H \rangle$ with respect to $C$ and $C^*$ the RPA is obtained

$$A_{ph,p'h'} = \left[\frac{\partial^2 \langle H \rangle}{\partial C_{ph} \partial C^*_{p'h'}}\right]_{C=C^*=0} \qquad B_{ph,p'h'} = \left[\frac{\partial^2 \langle H \rangle}{\partial C^*_{ph} \partial C^*_{p'h'}}\right]_{C^*=0}$$

## Residual Interaction

- If the interaction is density dependent
- $\rho(t) = \rho(C(t), C^*(t)) \Rightarrow$ Rearrangement terms appear

# Variational Approach

## SRPA

- $|\Psi(t)\rangle = e^{\hat{S(t)}} | HF\rangle$
- $$\hat{S} = \sum_{ph} C_{ph}(t) a_p^\dagger a_h + \frac{1}{2} \sum_{php'h'} \hat{C}_{pp'hh'}(t) a_p^\dagger a_{p'}^\dagger a_h a_{h'}$$
- $\langle H\rangle = \langle\Psi | H | \Psi\rangle / \langle\Psi | \Psi\rangle$
- $$\rho(t) = \rho(C(t), C^*(t), \hat{C}(t), \hat{C}^*(t))$$
- The $C$ and $\hat{C}$ coefficients are used as variational parameters

# Variational Approach

## SRPA

- $| \Psi(t)\rangle = e^{\hat{S(t)}} | HF\rangle$
- $$\hat{S} = \sum_{ph} C_{ph}(t) a_p^\dagger a_h + \frac{1}{2} \sum_{php'h'} \hat{C}_{pp'hh'}(t) a_p^\dagger a_{p'}^\dagger a_h a_{h'}$$
- $\langle H\rangle = \langle \Psi | H | \Psi\rangle / \langle \Psi | \Psi\rangle$
- $$\rho(t) = \rho(C(t), C^*(t), \hat{C}(t), \hat{C}^*(t))$$
- The $C$ and $\hat{C}$ coefficients are used as variational parameters

## $\hat{V}$ expanded around the HF density up to quadratic terms

$$\hat{V}(\rho) \sim \hat{V}(\rho^{(0)}) + \left[\frac{\delta \hat{V}}{\delta \rho}\right]_{\rho=\rho^{(0)}} \delta\rho + \frac{1}{2} \left[\frac{\delta^2 \hat{V}}{\delta^2 \rho}\right]_{\rho=\rho^{(0)}} \delta\rho^2$$

# About the residual interaction

## RPA Rearrangement Terms

$$\mathcal{V}^{(rearr)}_{mkip} = \sum_h \left[\frac{\delta \hat{V}_{mhih}}{\delta \rho_{kp}}\right]_{\rho=\rho^{(0)}} \rho_{kp} + \sum_h \left[\frac{\delta \hat{V}_{hkhp}}{\delta \rho_{mi}}\right]_{\rho=\rho^{(0)}} \rho_{mi} + \frac{1}{2}\sum_{hh'} \left[\frac{\delta^2 \hat{V}_{hh'hh'}}{\delta \rho_{mi}\delta \rho_{kp}}\right]_{\rho=\rho^{(0)}} \rho_{mi}\rho_{kp},$$

## SRPA Rearrangement Terms

$$A_{12} = A^{(rearr)}_{mi,pqkl} = \left[\frac{\delta \hat{V}_{klpq}}{\delta \rho_{im}}\right]_{\rho=\rho^{(0)}} \rho_{im}.$$

# SRPA with Density Dependent Force

## We derived RT within a Variational Approach

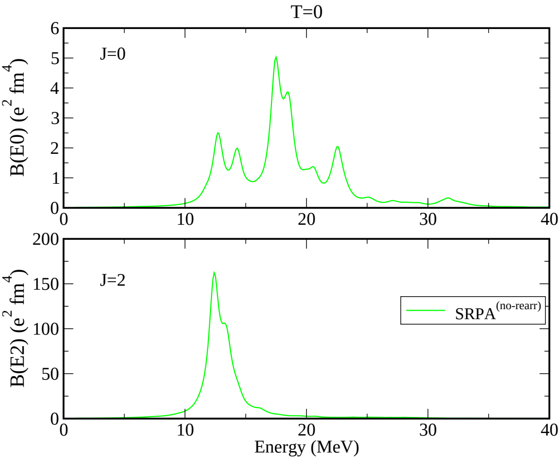
Main Results

- **RT** are present also in matrix elements beyond RPA [a]
- In the $A_{22}$ matrix we have only **RT** giving the correct HF s.p. energies
- In in $A_{12}$, $B_{12}$ **RT** appear in residual interaction and they have a **different expression** from the ones of the RPA matrices

**Results (see next)**

[a] arXiv:1003.2021v1 [nucl-th]

# Correct Rearrangement Terms (Journal of Physics G 38 035103 (2011))




**Monopole and Quadrupole Isoscalar Strength Distributions in $^{16}$o**

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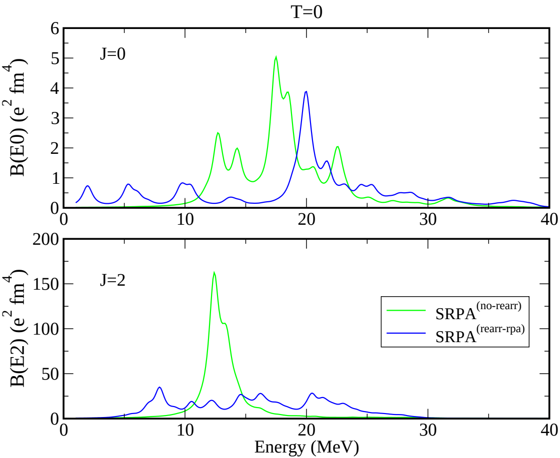


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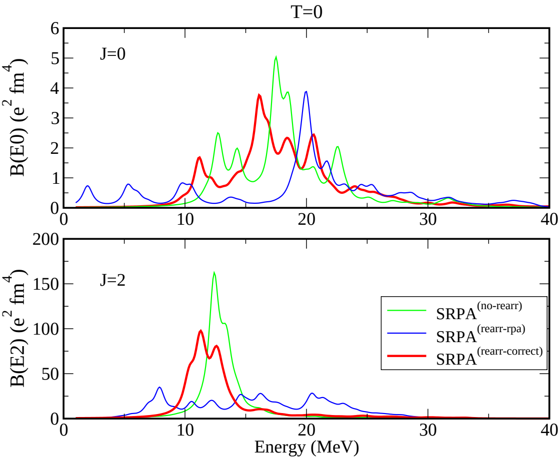


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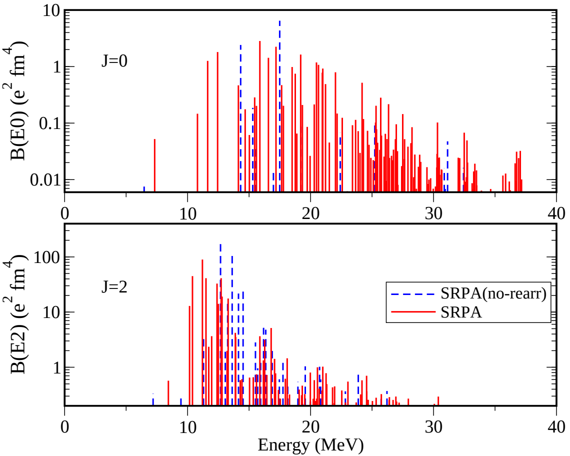

# Self-Energy

- As above mentioned, the SRPA problem can be reduced to an **energy-dependentt** RPA problem

$$\tilde{A}_{1,1'} = A_{1,1'} + \Sigma_{1,1'}(\omega)$$

- $\Sigma_{1,1'}(\omega)$ is the p-h **Self-Energy**
- Its **real part** gives a **shift** of the (RPA) resonance energies [a]
- While its **imaginary part** takes into account **spreading width** effects
- Self Energy Correction

$$\Sigma_{\nu}^{RPA}(\omega) = \sum_{2p2h} \frac{|< \nu|V|2p2h >|^2}{\omega - \epsilon_{2p2h}}$$

[a] S. Adachi and S. Yoshida, Nucl. Phys. A306 (1978) 53, J. Wambach Rep. Prog. Phys. 51 (1988) 980