Dalitz analysis of $B^{\pm} \rightarrow K^+ K^- K^{\pm}$ decays and $K^+ K^-$ interaction amplitudes

In collaboration with :

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Recent paper:

1. *Final state interactions in* $\boldsymbol{B^{\pm} \rightarrow K^+ K^- K^{\pm}}$ *decays* ,
   Phys. Lett. B 699 (2011) 102, arXiv:1101.4126 [hep-ph].

Motivation

1. search for CP violation,
2. studies of weak and strong interaction amplitudes,
3. tests of standard model and search for ``new physics'' effects,
4. determination of short and long-distance mechanisms in B-decays,
5. analysis of final state interactions,
6. studies of Dalitz diagrams.

Unitarity is important !

Unitary model allows for:

1. proper construction of B-decay amplitudes,
2. partial wave analyses of final states,
3. explanation of structures seen in Dalitz plots,
4. adequate determination of branching fractions and CP asymmetries for different quasi-two-body decays,
5. extraction of standard model parameters (weak amplitudes) or estimation of ''new physics'' amplitudes,
6. application not only in analyses of B decays but also in studies of other reactions.

Problems in the isobar model

1. Amplitudes used in the isobar model are **not unitary** neither in three-body decay channels nor in two-body subchannels.
2. **Difficulty** to distinguish the **S**-wave amplitude from the **background** terms. Their interference is often very strong.
3. The sum of quasi-two-body branching fractions frequently exceeds 100 % (e.g. larger than 300 % in the BaBar analysis of the $B^{\pm} \to K^{+} K^{-} K^{\pm}$ decays).
4. The **branching fractions** extracted in such cases could be unreliable (substantially **overestimated**).
5. Too many free parameters (at least two fitted parameters for each amplitude component).

Theoretical constraints

1. Derivation of unitary three-body strong interaction amplitudes is difficult. It is easier to satisfy unitarity in quasi two-body subchannels of the decay process.

2. Case discussed in this talk: final state interactions in $B \rightarrow (K\ K)\ K$ processes. Unitary two-body strong interaction amplitudes are used for the S-wave in the limited effective mass range up to about 1.8 GeV.

3. Other theoretical tools: QCD factorization approach, analyticity and chiral symmetry at low energies.

Decay amplitudes

Rare weak decays:

$b \to \bar{u}us$, $b \to \bar{d}ds$ and $b \to \bar{s}ss$ in **$B^{\pm} \to 3K^{\pm}$**.

The theoretical model is based on:

1. application of the **QCD factorization** in quasi-two-body approach for a limited range of the K^+K^- effective masses smaller than 1.8 GeV,
2. description of the final state K^+K^- interactions in terms of the kaon scalar and vector **form factors** ,
3. introduction of unitary constraints in the S-wave coupled channel approach with coupling of the $\pi^+\pi^-$, $\pi^0\pi^0$, K^+K^-, $K^0\bar{K}^0$ and 4π (effective $\sigma\sigma$ or $\rho\rho$) states.

Non-strange scalar form factor:

$$\langle 0 | \bar{n}n | \bar{K}K \rangle = \sqrt{2} B_0 \Gamma_2^n(E)$$

$$\bar{n}n = \tfrac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$$

$$E = m_{KK}$$

Strange scalar form factor:

$$\langle 0 | \bar{s}s | \bar{K}K \rangle = \sqrt{2} B_0 \Gamma_2^s(E)$$

kaon form factor

$$B_0 = \frac{m_\pi^{\ 2}}{m_u + m_d}$$

Coupled channel expressions for the scalar form factors

3 coupled channels: $\pi\pi, \overline{K}K$, effective 4π ($\sigma\sigma$ or $\rho\rho$)

$$\boxed{\Gamma^{n,s*} = R^{n,s} + TGR^{n,s}}$$

G – Green's functions

$\boldsymbol{R_j}$ – three **production** functions, T – 3x3 matrix of **amplitudes**

$$\boxed{R_j(E) = \frac{\alpha_j + \tau_j E + \omega_j E^2}{1 + cE^4} \qquad j = 1,2,3}$$

The parameters $\alpha_j, \tau_j, \omega_j$ are calculated using the chiral perturbation model.
The fitted parameter c controls the high energy behaviour of R.

Multichannel model of the coupled amplitudes $\boldsymbol{T}$ is constrained by the data on the pion-pion, kaon-antikaon and four pion production (R. Kamiński, L. Leśniak, B. Loiseau, Phys. Lett. B 413 (1997) 130).

Chiral symmetry constraints

Low energy constraints on the scalar form factors:

$$\Gamma_2^{n,s} \approx a_2^{n,s} + b_2^{n,s} E^2, \qquad \Gamma_3^{n,s} \approx 0, \qquad E \to 0.$$

Parameters a_2 and b_2 are calculated using the **chiral** model of **Meissner** and **Oller** (Phys. Rev. D 65 (2002) 094004), and the **lattice QCD** results (from RBC and UKQCD Collaborations, Phys. Rev. D 78 (2008) 114509).

Then $\alpha_j, \tau_j, \omega_j$ coefficients are expressed in terms of a_j and b_j.

Unitarity conditions

$$\operatorname{Im}\Gamma_i^*(E) = \sum_{j=1}^{3} T_{ji}^*(E) r_j \Gamma_j^*(E)\theta(E-2m_j), \qquad i = 1,2,3$$

$$r_j = -\frac{k_j E}{8\pi}, \quad k_j - \text{channel momenta}$$

Watson's theorem: for a single channel (i=1) the phases of T_{11} and Γ_1* are equal.

Unitarity relations for the meson - meson amplitudes :

$$\operatorname{Im} T_{ik}(E) = \sum_{j=1}^{3} T_{kj}^*(E) r_j T_{ij}(E)\theta(E-2m_j), \qquad i,k = 1,2,3$$

$B \to R_S$ transition matrix elements

$B \to R_S$ **R_S = scalar** state

$R_S \to ($ K_2 $K_3)$

Assumption:

$$< [K_2 K_3]_S \,|\, (\bar{u} b)_{V-A} \,|\, B > = G_{R_S K_2 K_3}(s_{23}) < R_S \,|\, (\bar{u} b)_{V-A} \,|\, B >$$

$$G_{R_S K_2 K_3}(s_{23}) \propto \chi\, \Gamma_2^{n^*}(s_{23}), \qquad s_{23} = E^2$$

χ = real parameter

Physical observables in $B^- \to K_1^- K_2^+ K_3^-$

1. Double effective mass and helicity angle branching fraction:

$$\boxed{\frac{dBr}{dm_{23}d\cos\theta_{12}} = PH\,|A|^2} \quad PH = \frac{m_{23}\,|\vec{p}_1||\vec{p}_2|}{8(2\pi)^3 M_B{}^3\Gamma_{B^-}}$$

$$A = \frac{1}{\sqrt{2}}[A_S(m_{23}) + A_P(m_{23})\cos\theta_{12} + \{1 \leftrightarrow 3\}]$$

$$\cos\theta_{12} = \frac{\vec{p}_1 \cdot \vec{p}_2}{|\vec{p}_1||\vec{p}_2|}$$ momenta in the $(K_2 K_3)$ c.m. system

2. Effective mass m_{23} distributions
3. **Helicity angle** θ_{12} distributions

Amplitudes for $B^- \to K_1^- K_2^+ K_3^-$ decay

$$A_S = -\tfrac{1}{2} G_F f_K (M_B^2 - m_{23}^2) F_0^{B^- \to (K^+K^-)_S}(m_K^2)\, y \chi \Gamma_2^{n*}(m_{23})$$
$$+ \tfrac{1}{2} G_F \frac{2\sqrt{2}B_0}{m_b - m_s} (M_B^2 - m_K^2) F_0^{B^- \to K^-}(m_{23})\, v \Gamma_2^{s*}(m_{23})$$

Γ_2^n and Γ_2^s are the kaon scalar non-strange and strange form factors.

$$A_P = 2\sqrt{2}\,\vec{p}_1 \cdot \vec{p}_2 G_F \{ \frac{f_K}{f_\rho} A_0^{B\to\rho}(m_K^2) F_u^{K^+K^-}(m_{23})\, y$$
$$- F_1^{BK}(m_{23})[F_u^{K^+K^-}(m_{23}) w_u + F_d^{K^+K^-}(m_{23}) w_d + F_s^{K^+K^-}(m_{23}) w_s]\}$$

$F_u^{K^+K^-}, F_d^{K^+K^-}, F_s^{K^+K^-}$ are the kaon vector form factors.

$\vec{p}_1, \vec{p}_2$ are the K_1, K_2 momenta in the K_2K_3 c.m. frame.

$y, v, w_u, w_d, w_s =$ weak amplitudes, functions of $\lambda_u = V_{ub}V_{us}^*, \lambda_c = V_{cb}V_{cs}^*$

Vector kaon form factors

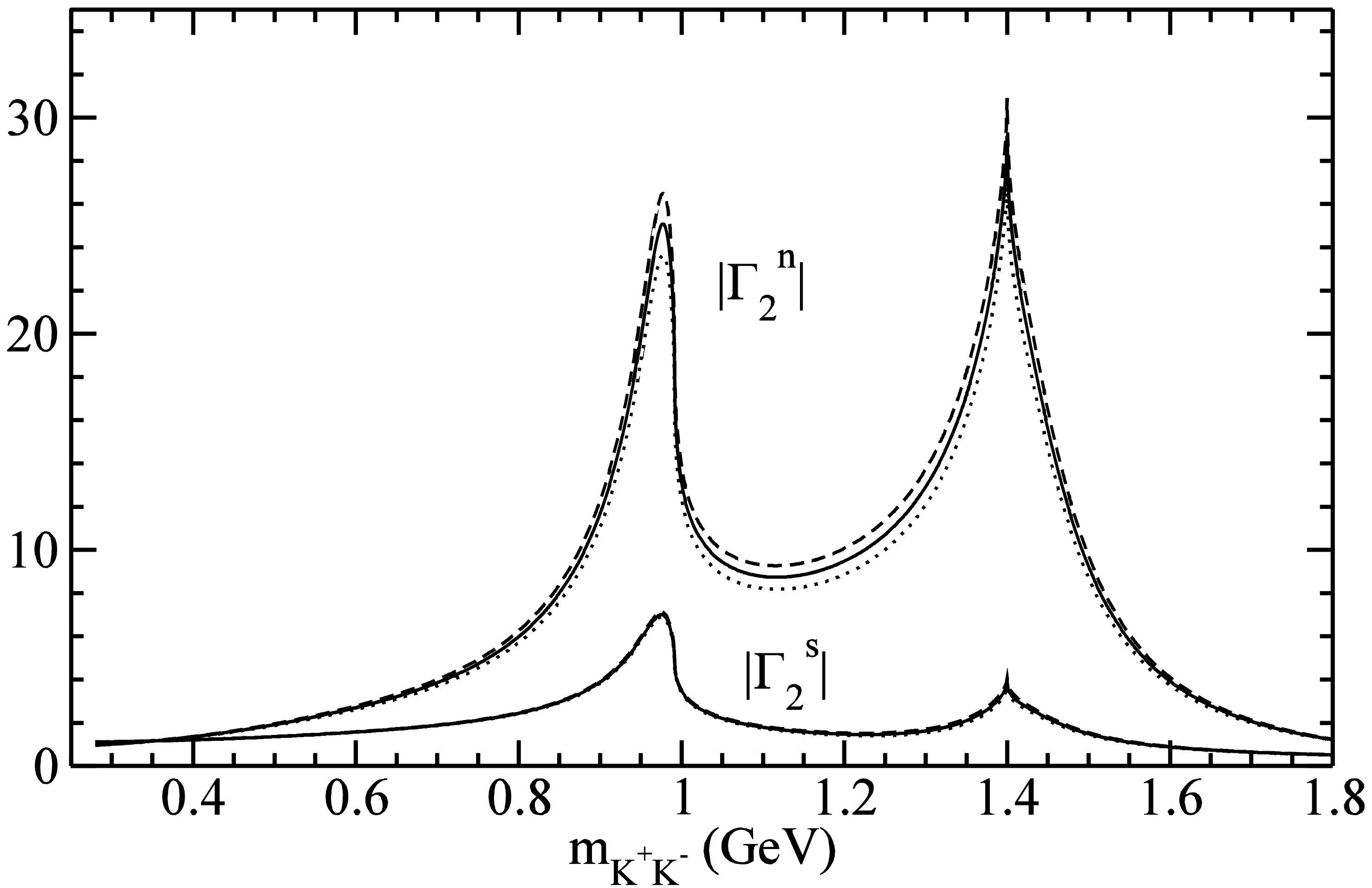
Contributions to vector form factors from 8 vector mesons V:

[ρ, ρ’, ρ’’] = [ρ(770), ρ(1450), ρ(1700)],
[ω, ω’, ω’’]= [ω(782), ω(1420), ω(1650)],
[ϕ, ϕ’] = [ϕ(1020), ϕ(1680)].

$$F_u^{K^+K^-} = F_\rho + F_{\rho'} + F_{\rho''} + 3(F_\omega + F_{\omega'} + F_{\omega''})$$
$$F_d^{K^+K^-} = -(F_\rho + F_{\rho'} + F_{\rho''}) + 3(F_\omega + F_{\omega'} + F_{\omega''})$$
$$F_s^{K^+K^-} = -3(F_\phi + F_{\phi'})$$

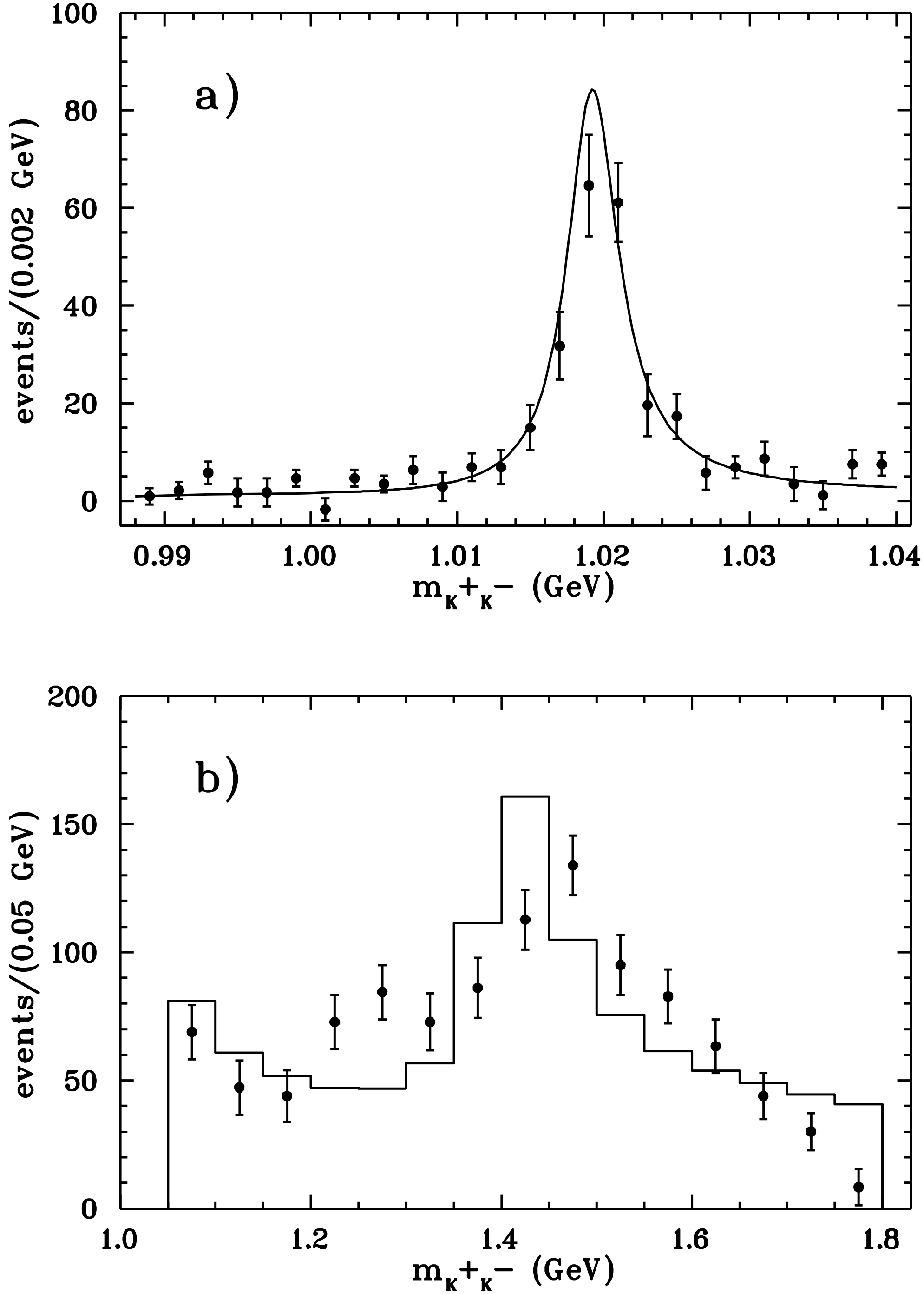
Components F_V are taken from the model of Bruch, Khodjamirian and Kuhn, Eur. Phys. J. C 39 (2005) 41.

Scalar kaon form factors



Comparison with the BaBar data: effective K^+K^- mass distributions

Data from BaBar Collaboration: Phys. Rev. D 74 (2006) 032003



Comparison with the Belle data

a) $m_{12}^2 \leq 5\,\mathrm{GeV}^2$

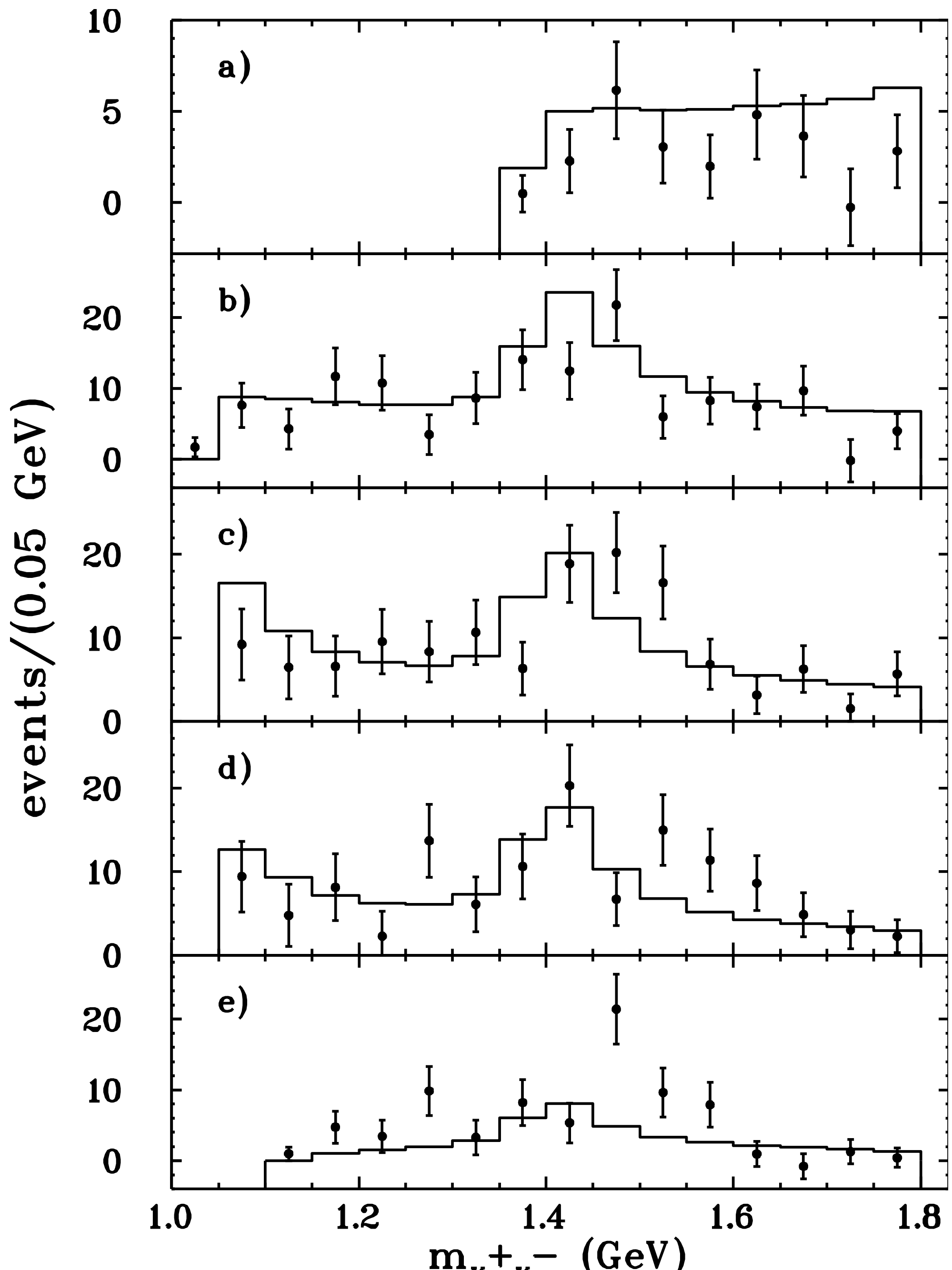
b) $5\,\mathrm{GeV}^2 \leq m_{12}^2 \leq 10\,\mathrm{GeV}^2$

c) $10\,\mathrm{GeV}^2 \leq m_{12}^2 \leq 15\,\mathrm{GeV}^2$

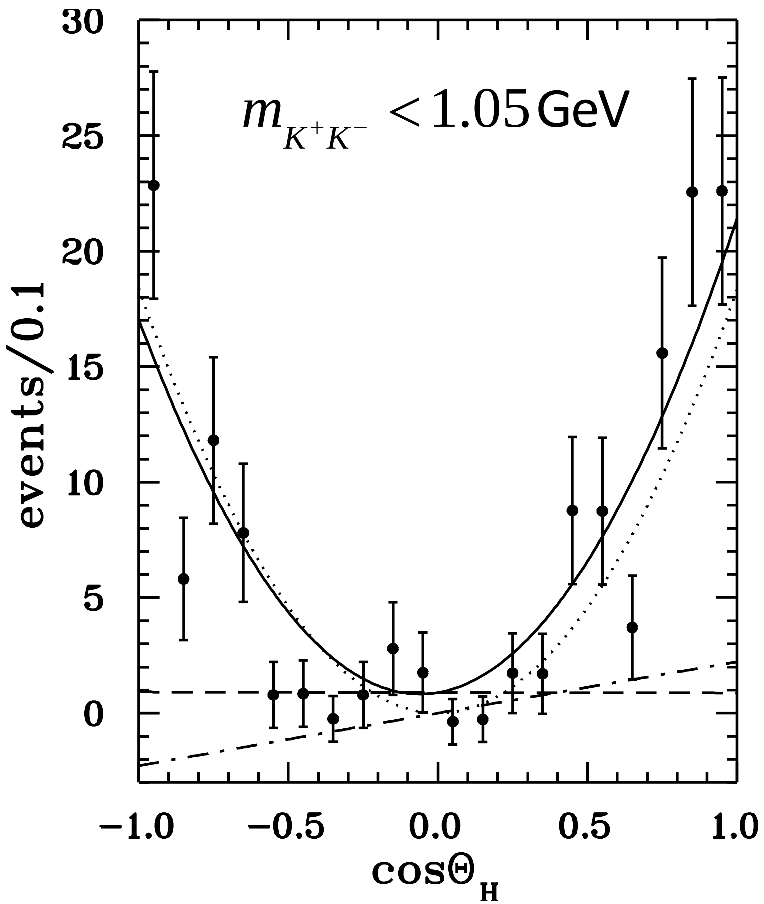
d) $15\,\mathrm{GeV}^2 \leq m_{12}^2 \leq 20\,\mathrm{GeV}^2$

e) $20\,\mathrm{GeV}^2 \leq m_{12}^2$

Belle Collaboration:
Phys. Rev. D 71 (2005) 092003



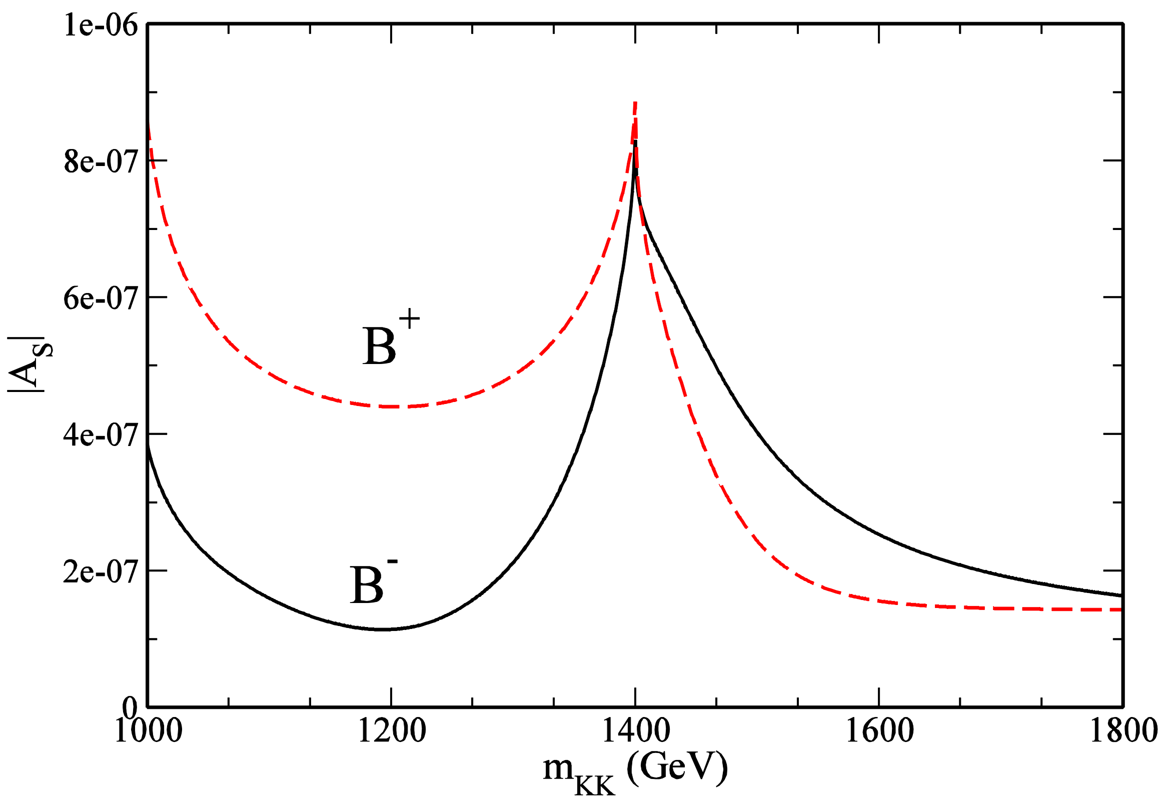
Helicity angle distribution



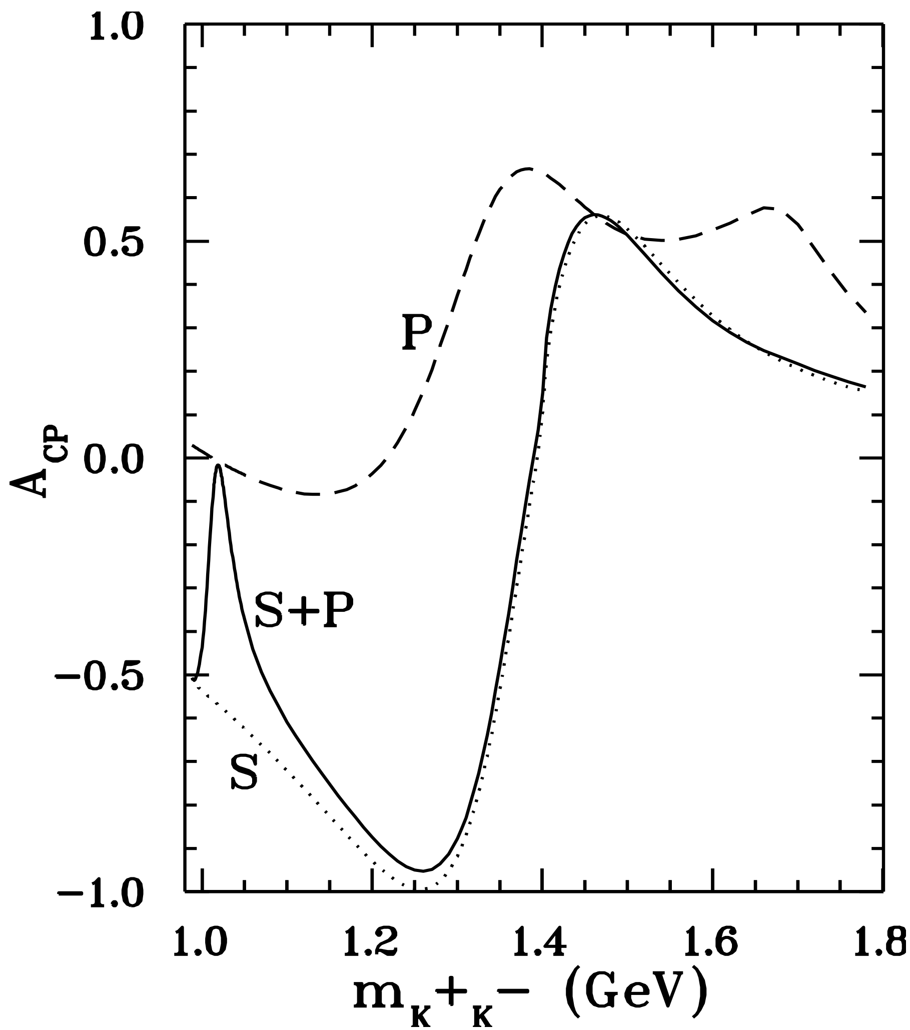
······· P-wave, – – – – – S-wave, – · – · – · · interference

Data from Belle Colaboration: Phys. Rev. D 71 (2005) 092003, here $\cos\Theta_H = -\cos\Theta_{12}$.

Moduli of scalar amplitudes



CP asymmetry



Summary

1. Final state $K^+ K^-$ interactions are described using the **scalar strange and non-strange form factors** for the S-wave and the vector kaon form factor for the P-wave.
2. The scalar resonance **f_0 (980)** leads to the **threshold enhancement** of the S-wave amplitude. The $K^+ K^-$ structure around 1.5 GeV can be attributed to another scalar resonance, coupled to the $K^+ K^-$, to $\pi\pi$ and to the 4π system.
3. Away from $\Phi(1020)$, a possibility of a large CP asymmetry is identified in the part of the mass spectrum dominated by the **S**-wave.
4. Using this model Dalitz plot analyses of the $B^\pm \to K^+ K^- K^\pm$ decays can be improved using smaller number of fitted parameters.
6. The model can be extended to study other charmless B decays and to analyse new high-statistics data coming from Belle, BaBar, LHCb and from future super-B factories.

Conclusions

1. CP violation and final state interactions are studied in the charged B decays into three pions and three kaons.
2. The weak decay amplitudes are derived in the QCD factorization approach.
3. The long-distance S-wave final state interactions are described by the **meson scalar form factors** which satisfy the unitarity conditions and the constraints coming from the chiral perturbation theory.
4. A **unitary model** is constructed for the scalar non-strange and strange form factors in which **three scalar resonances** $\mathbf{f_0}$ **(600),** $\mathbf{f_0}$ **(980)** and $\mathbf{f_0}$ **(1400)** are naturally incorporated.
5. Using this model Dalitz plot analyses of the $B^{\pm} \to \pi^{+}\pi^{-}\pi^{\pm}$ and $B^{\pm} \to K^{+}K^{-}K^{\pm}$ decays can be improved using smaller number of fitted parameters.
6. The model can be extended to study other charmless B decays and to analyse new high-statistics data coming from Belle, BaBar, LHCb and from future super-B factories.

$B^{\pm} \to \pi^{+} \pi^{-} \pi^{\pm}$ decays

Recent papers:

1. *Final state interactions and CP violation in* $\boldsymbol{B^{\pm} \to \pi^{+} \pi^{-} \pi^{\pm}}$ *decays* , arXiv:1011.0960 [hep-ph].
2. *CP violation and kaon-pion interactions in* $\boldsymbol{B \to \pi^{+} \pi^{-} K}$ *decays,* Phys. Rev. D79 (2009) 094005.

S wave amplitude in $B^- \to \pi_1^- \pi_2^+ \pi_3^-$

$$A_S = -\frac{1}{\sqrt{3}} G_F \chi\, f_\pi (M_B^2 - m_{23}^2) F_0^{B^- \to (\pi^+\pi^-)_S}(m_\pi^2) U \Gamma_1^{n*}(m_{23}) + \frac{1}{\sqrt{3}} G_F \frac{B_0}{m_b - m_d} (M_B^2 - m_\pi^2) F_0^{B^- \to \pi^-}(m_{23}) v \Gamma_1^{n*}(m_{23})$$

$$U = \Lambda_u [a_1 + a_4^u + a_{10}^u - (a_6^u + a_8^u) r)] + \Lambda_c [a_4^c + a_{10}^c - (a_6^c + a_8^c) r)]$$

$$v = \Lambda_u (-2a_{6v}^u + a_{8v}^u) + \Lambda_c (-2a_{6v}^c + a_{8v}^c), \quad B_0 = m_\pi^2 / (m_u + m_d)$$

$$r = \frac{2B_0}{m_b + m_u}; \quad \Lambda_u = V_{ub} V_{ud}^*, \quad \Lambda_c = V_{cb} V_{cd}^*,$$ χ - fitted parameter

$\Gamma_1^n(m_{23})$ is the **pion non-strange scalar form factor**.

P wave amplitude in $B^- \to \pi_1^- \pi_2^+ \pi_3^-$

$$\boxed{A_P = 2\sqrt{2}\vec{p}_1 \cdot \vec{p}_2 G_F [\frac{f_\pi}{f_\rho} A_0^{B\rho}(m_\pi^2) U + F_1^{B\pi}(m_{23}) W] F_1^{\pi^+\pi^-}(m_{23})}$$

$\vec{p}_1, \vec{p}_2$ are the π_1, π_2 momenta in the $(\pi_2\pi_3)$ c.m. frame.

$$W = \Lambda_u [a_2 - a_{4w}^u + \frac{3}{2}(a_7 + a_9) + \frac{1}{2} a_{10w}^u] + \Lambda_c [-a_{4w}^c + \frac{3}{2}(a_7 + a_9) + \frac{1}{2} a_{10w}^c]$$

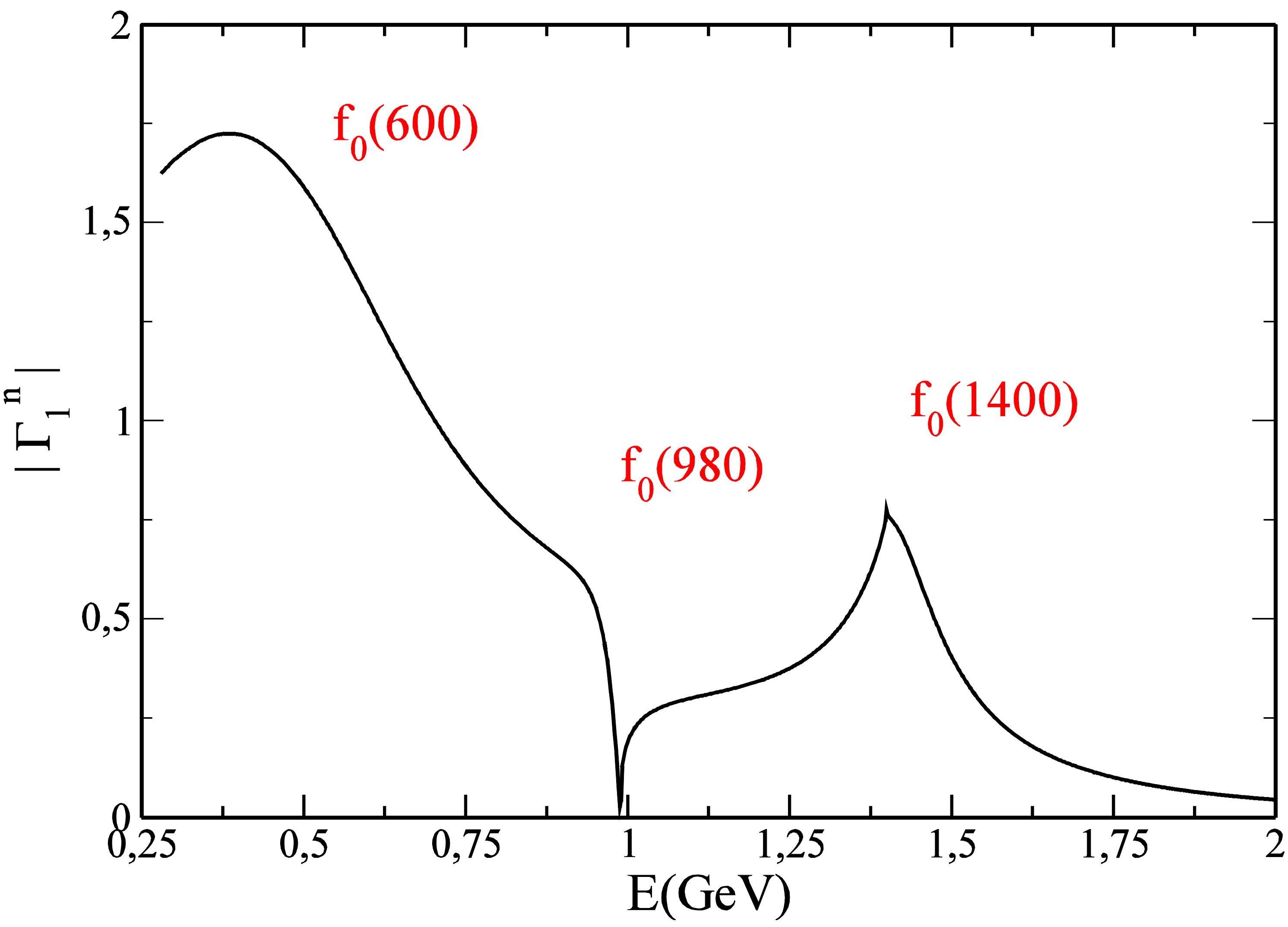
$A^{B\rho}$ and $F_1^{B\pi}$ are the transition form factors.

$\boxed{F_1^{\pi^+\pi^-} = F_\rho + F_{\rho'} + F_{\rho''} \text{ is the pion vector form factor.}}$

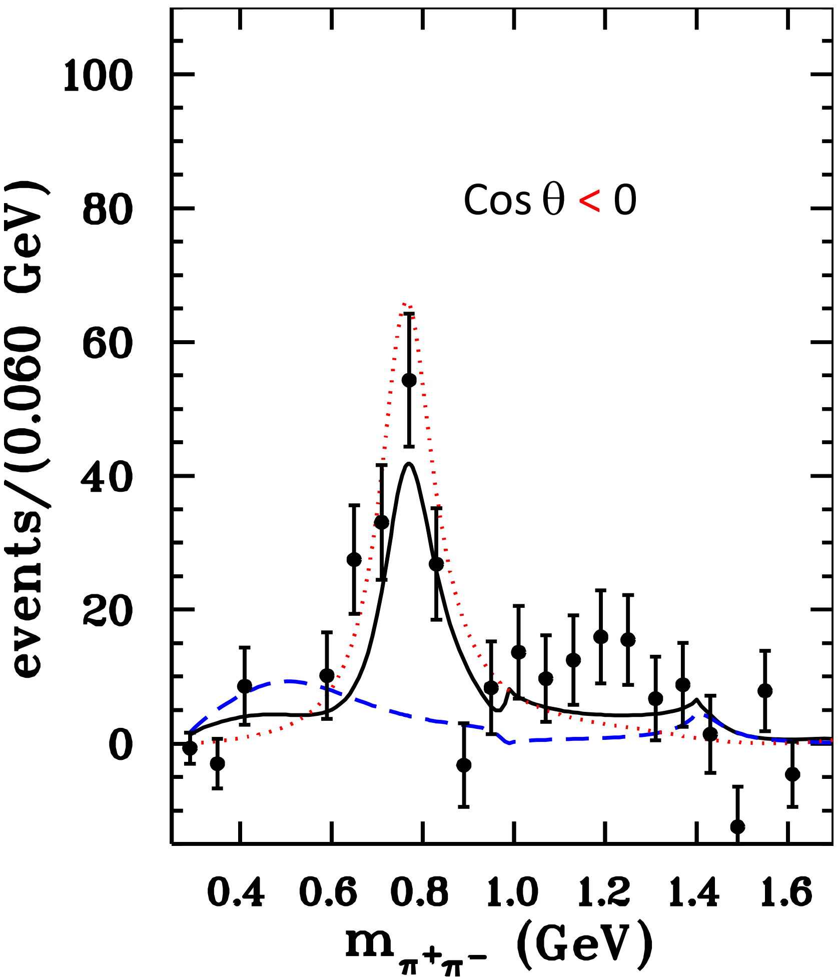
[ρ, ρ', ρ''] = [ρ(770), ρ(1450), ρ(1700)].

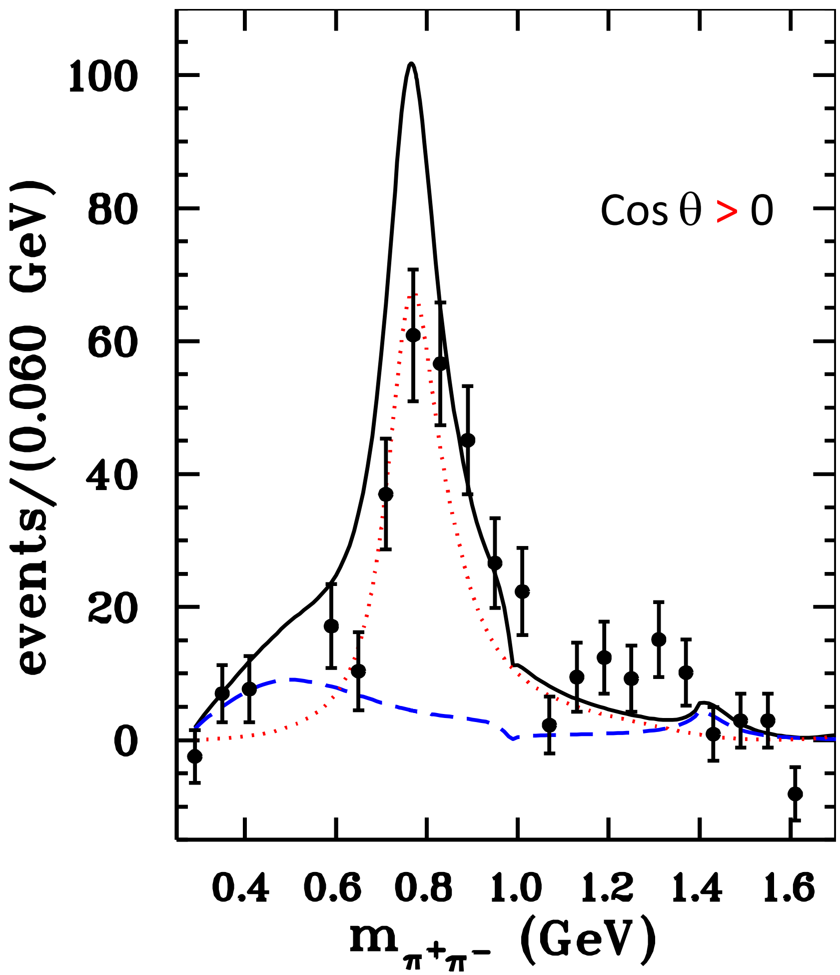
The components F_V are taken from the phenomenological model of Fujikawa et al. (Belle Collaboration, Phys. Rev. D78 (2008) 072006), used to describe the high-statistics data of the $\tau^- \to \pi^- \pi^0 \nu_\tau$.

Scalar pion non-strange form factor

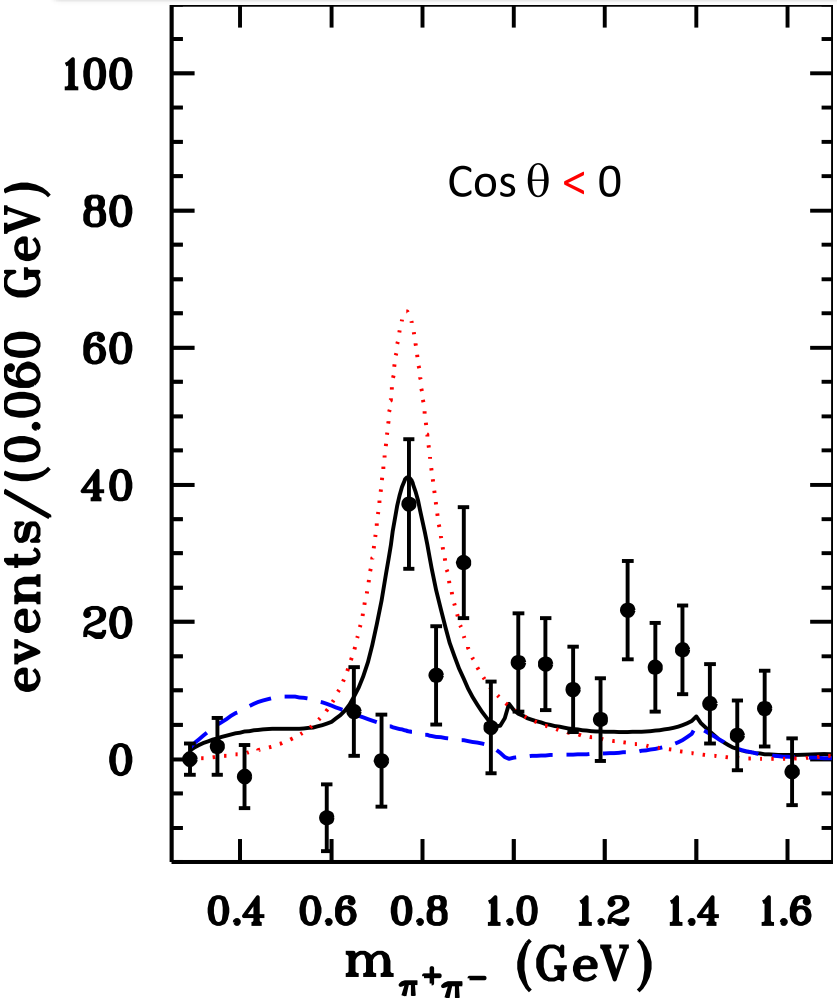


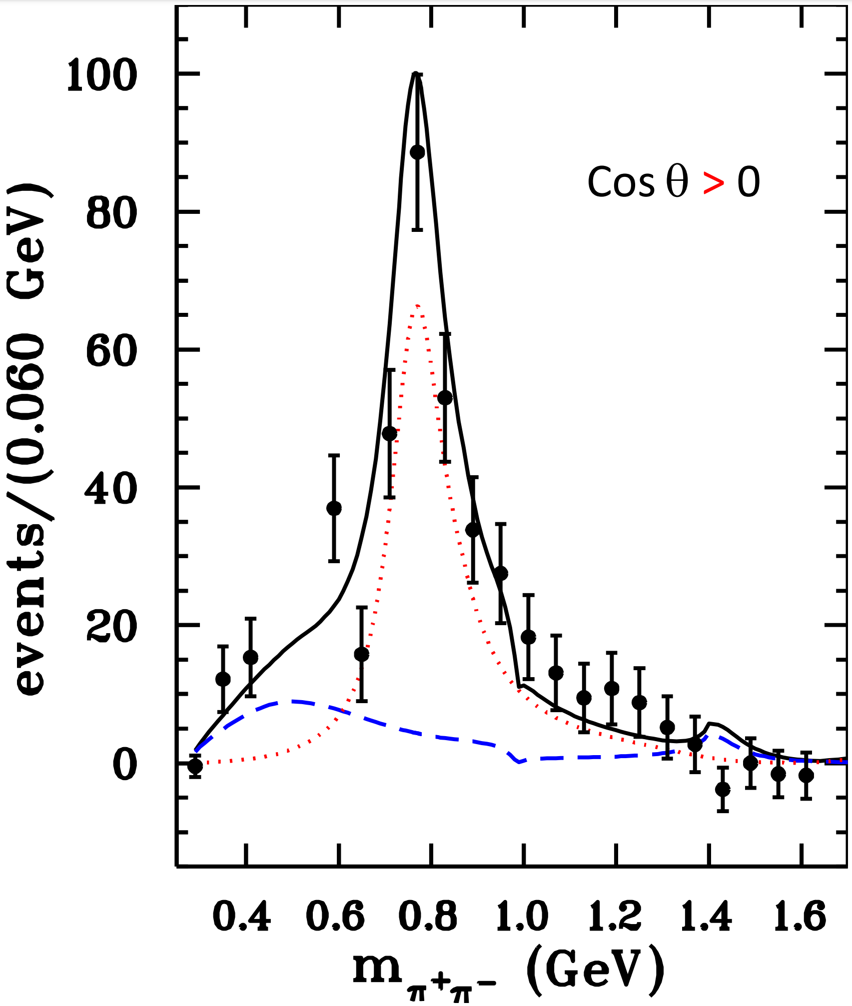
$\pi^+\pi^-$ effective mass distributions for B^- decays





$\pi^+\pi^-$ effective mass distributions for B^+ decays





Summary for $B^{\pm} \rightarrow \pi^{+} \pi^{-} \pi^{\pm}$ decays

1. The $\pi^{+}\pi^{-}$ spectrum is dominated by the $\rho(770)$ but the **scalar resonance f_0 (600)** **(**or σ**)** is **visible.** It leads to the **threshold enhancement** and to the **strong interference with the ρ resonance.**

2. The **f_0 (980)** is not observed as a peak in the effective mass distribution since the pion scalar form factor has a **dip** near 1 GeV.

$B^{\pm} \rightarrow K^{+} K^{-} K^{\pm}$ decays

S wave amplitude for $B^- \to K_1^- K_2^+ K_3^-$

$$A_S = -\tfrac{1}{2} G_F f_K (M_B^2 - m_{23}^2) F_0^{B^- \to (K^+K^-)_S}(m_K^2)\, y \chi \Gamma_2^{n*}(m_{23})$$

$$+ \tfrac{1}{2} G_F \frac{2\sqrt{2}B_0}{m_b - m_s} (M_B^2 - m_K^2) F_0^{B^- \to K^-}(m_{23})\, v \Gamma_2^{s*}(m_{23})$$

$$y = \lambda_u [a_{1y} + a_{4y}^u + a_{10y}^u - (a_{6y}^u + a_{8y}^u r)] + \lambda_c [a_{4y}^c + a_{10y}^c - (a_{6y}^c + a_{8y}^c r)]$$

$$v = \lambda_u (-a_{6v}^u + \tfrac{1}{2} a_{8v}^u) + \lambda_c (-a_{6v}^c + \tfrac{1}{2} a_{8v}^c), \quad B_0 = m_\pi^2 / 2m_u$$

$$r = \frac{2m_K^2}{(m_b + m_u)(m_s + m_u)}; \quad \lambda_u = V_{ub} V_{us}^*, \quad \lambda_c = V_{cb} V_{cs}^*$$

Γ_2^n and Γ_2^s are the kaon scalar non-strange and strange form factors.

P wave amplitude in $B^- \to K_1^- K_2^+ K_3^-$

$$\boxed{\begin{aligned} A_P = {} & 2\sqrt{2}\,\vec{p}_1 \cdot \vec{p}_2 G_F \{ \frac{f_K}{f_\rho} A_0^{B\to\rho}(m_K^2) F_u^{K^+K^-}(m_{23}) y \\ & - F_1^{BK}(m_{23})[F_u^{K^+K^-}(m_{23}) w_u + F_d^{K^+K^-}(m_{23}) w_d + F_s^{K^+K^-}(m_{23}) w_s]\} \end{aligned}}$$

$F_u^{K^+K^-}, F_d^{K^+K^-}, F_s^{K^+K^-}$ are the vector kaon form factors.

$\vec{p}_1, \vec{p}_2$ are the K_1, K_2 momenta in the K_2, K_3 c.m. frame.

$$w_u = \lambda_u (a_{2w} + a_{3w} + a_{5w} + a_{7w} + a_{9w}) + \lambda_c (a_{3w} + a_{5w} + a_{7w} + a_{9w})$$

$$w_d = \lambda_u [a_{3w} + a_{5w} - \tfrac{1}{2}(a_{7w} + a_{9w})] + \lambda_c [a_{3w} + a_{5w} - \tfrac{1}{2}(a_{7w} + a_{9w})]$$

$$\begin{aligned} w_s = {} & \lambda_u [a_{3w} + a_{4w}^u + a_{5w} - \tfrac{1}{2}(a_{7w} + a_{9w} + a_{10w}^u)] + \lambda_c [a_{3w} + a_{4w}^c + a_{5w} \\ & - \tfrac{1}{2}(a_{7w} + a_{9w} + a_{10w}^c)] \end{aligned}$$

Model parameters $B^{\pm} \rightarrow K^{+} K^{-} K^{\pm}$

κ =3.51 GeV

χ = 6.44 GeV^{-1}

c= 0.084 GeV^{-4}

N_P = 1.037

Some numerical results

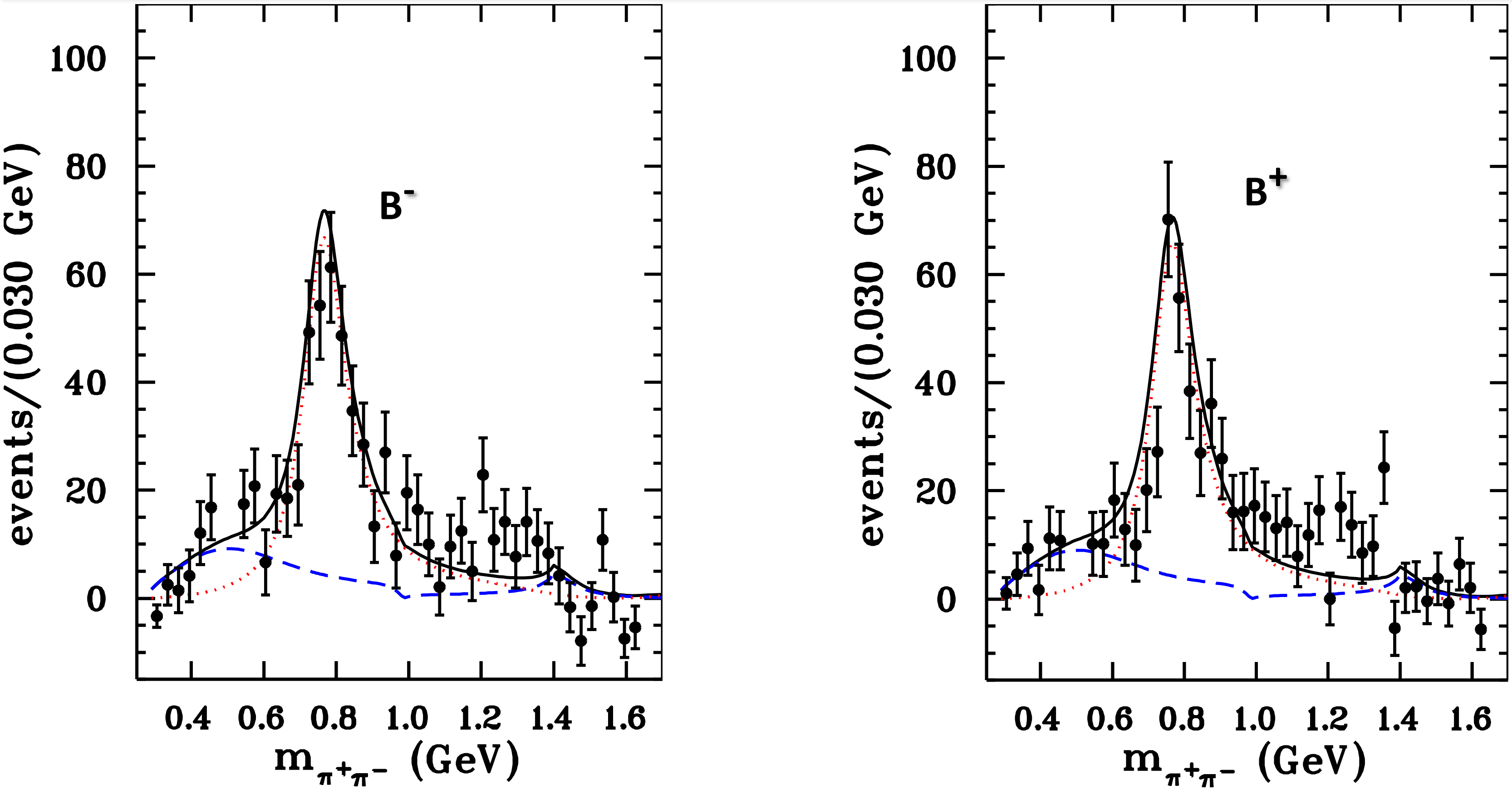
Branching fractions in units of 10^{-6}

decay		BaBar	our model
$B^{\pm} \to (\rho^0 \to \pi^+ \pi^-)\pi^{\pm}$		8.1±1.6	8.2±0.5
$B^{\pm} \to \pi^+ \pi^- \pi^{\pm}$	total	15.2±1.4	13.2±1.4

Model parameters: κ =5 GeV χ = -18.3 GeV^{-1}

c= 31.5 GeV^{-4} N_P = 1.132

Comparison with the BaBar data: effective $\pi^+\pi^-$ mass distributions



P- wave, S-wave

Data from BaBar Collaboration: Phys. Rev. D 79 (2009) 072006