Bureau International des Poids et Mesures

Séminaire, Laboratoire de Physique Nucléaire et de Hautes Energies
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On the redefinition of the kilogram: Recent advances

Richard Davis

with thanks to BIPM colleagues, especially Alain Picard, Hao Fang, Michael Stock

Topics to be presented

- Reminder of the present definition.
- Who must approve a change to the SI base units?
- Why do electrical metrologists and chemists care about the definition of the kilogram?
- Which group of physicists cares about the kilogram definition?
- What do the electrical and chemical communities recommend?
- When could/should the kilogram be redefined?
- How will a redefinition affect mass metrology?
- What is the mass community doing to prepare?

Definition of the kilogram

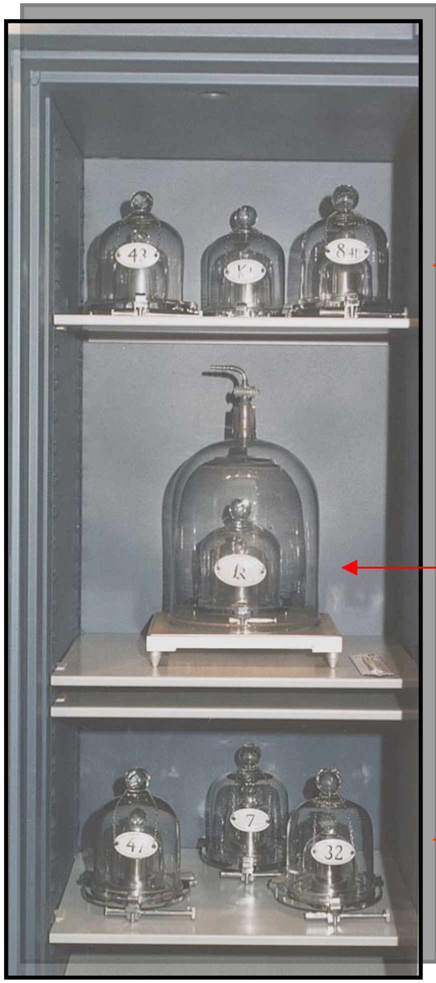
3rd CGPM, 1901 :

"Le kilogramme est l'unité de masse;
il est égal à la masse du prototype international du kilogramme."

"The kilogram is the unit of mass;
it is equal to the mass of the international prototype of the kilogram."

(international prototype manufactured in 1880s,
put into service in 1889)

Conservation of the international prototype



official copies (3 of 6)

international prototype

official copies (3 of 6)

1927

from ***La Création du BIPM et son oevre***

«Il semble donc que l'unité de masse soit guarantie au cent-millionième près pour plus de 10 000 ans, et cette durée est à peine commencée.

Sans doute, bien avant qu'elle soit écoulée, les travaux exécutés par les métrologistes des siècles futurs auront conduit à des solutions encore plus parfaites.»

Ch.-Ed. Guillaume

BIPM Director
Nobel laureate

What the definition means in practice

The mass in kilograms of any
object X is given by :

$$\{m_X\}[\text{kg}] = \left\{\frac{m_X}{m_n}\right\} \cdot \left\{\frac{m_n}{m_{n-1}}\right\} \cdot \ldots \cdot \left\{\frac{m_2}{m_1}\right\} \cdot \left\{\frac{m_1}{m_{\mathcal{K}}}\right\} \cdot [\text{kg}]$$

$$\{m_X\} \cdot [\text{kg}] = \left\{\frac{m_X}{m_{\mathcal{K}}}\right\} \cdot [\text{kg}]$$

For highest accuracy, $\{m_X/m_{\mathcal{K}}\}$ is a measurement carried out on a precision balance (mass comparator):

$$m_X - m_{\mathcal{K}} = \varepsilon \quad ; \quad \frac{m_X}{m_{\mathcal{K}}} = 1 + \frac{\varepsilon}{m_{\mathcal{K}}}$$

$$\varepsilon << m_{\mathcal{K}}$$

What's new in physics since 1889

- atomic view of matter
- special and general relativity; weak equivalence principle
- quantum electrodynamics
- quantum chromodynamics

Leading to kilogram involvement in:

- fundamental physical constants whose SI values are manifestly measured in terms of the kilogram: m_e, h, $m(^{12}\mathrm{C})$ etc.
- other fundamental constants whose SI values are measured less obviously in terms of the kilogram : e, K_J, R_K etc.
- Conversion factors between the microscopic and macroscopic worlds: N_A, F, etc.
- Conversion factors to other unit systems: eV in J, u in kg, etc.

Consequences of 1901 definition and 2007 physics

Even though m_X may represent something fundamental--
m_e, m_p, $m(^{12}C)$, $(\hbar c/G)^{1/2}$ etc., nevertheless

$$\{m_X\}[\mathrm{kg}] = \left\{\frac{m_X}{m_{\mathcal{K}}}\right\}[\mathrm{kg}]$$

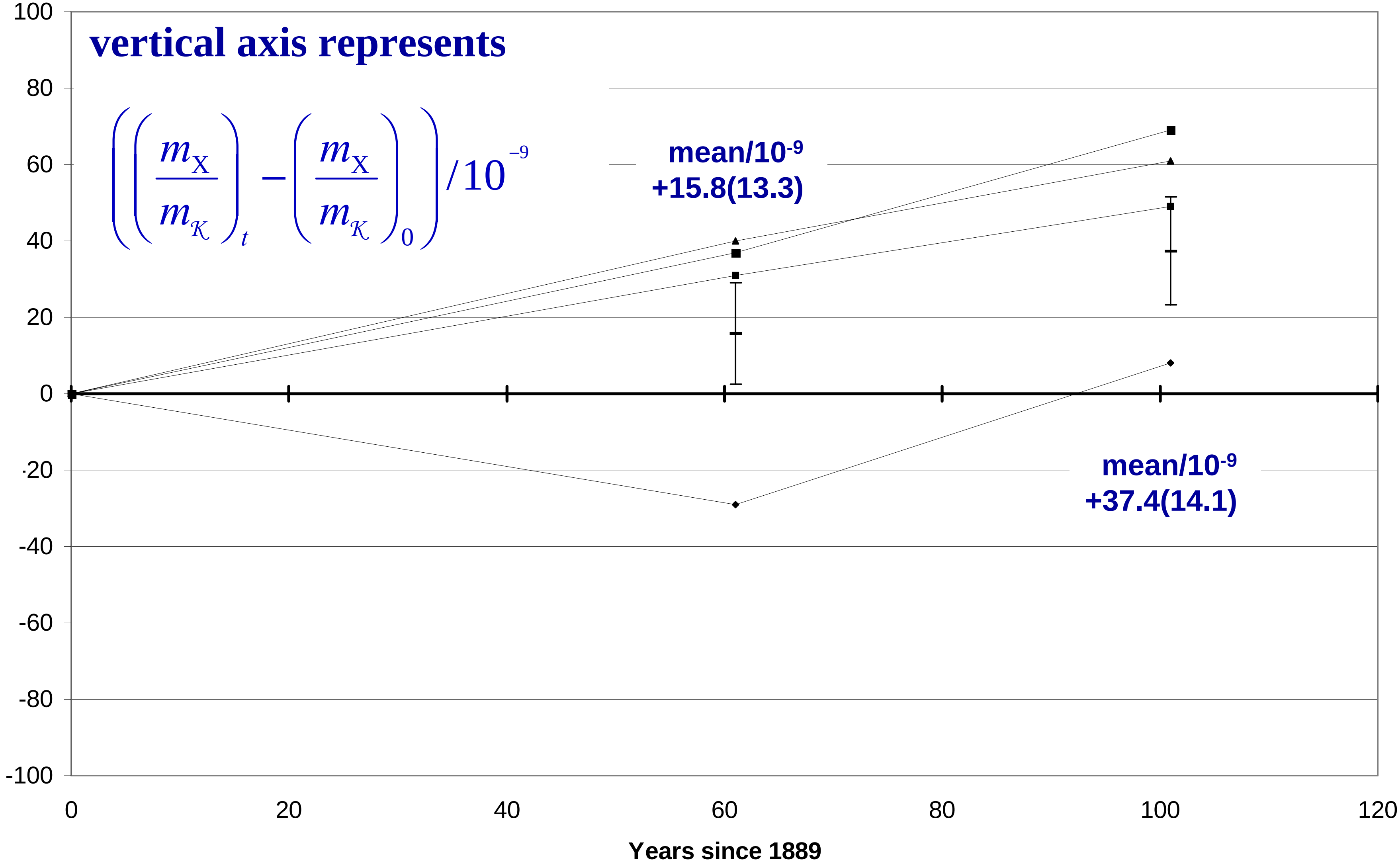
This curious situation persists largely because:

Experimental uncertainties of $\{m_X/m_{\mathcal{K}}\}$ are still much larger than the precision of the best commercial balances;

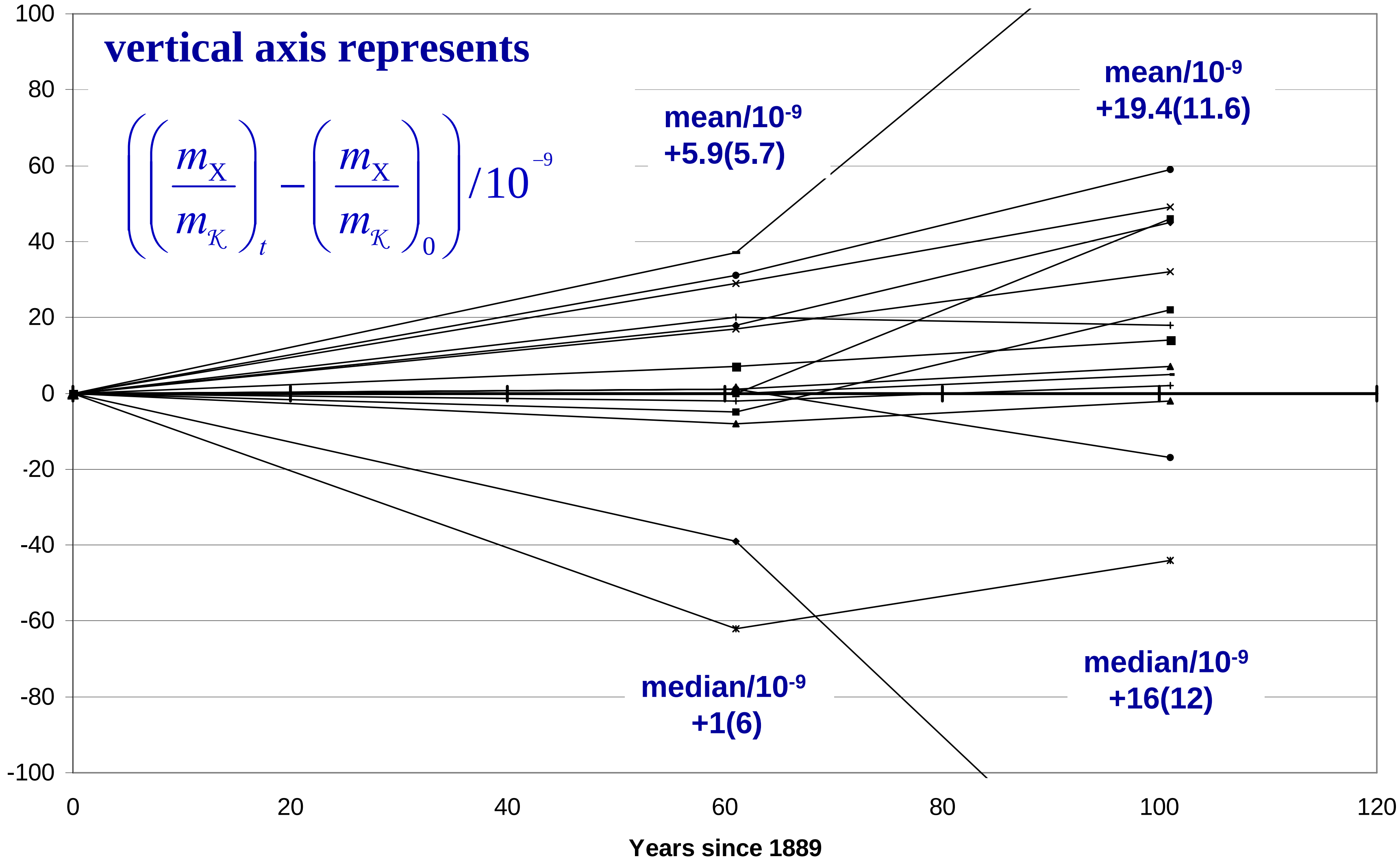
Inconclusive evidence that $\{m_X/m_{\mathcal{K}}\}$ is changing, where m_X is "something" (anything!) that can reasonably be considered more stable than the mass of $\mathcal{K}$;

When comparing molecular, atomic, and subatomic masses amongst themselves, it is traditional to use the dalton, Da, a non-SI unit which avoids correlations to $m_{\mathcal{K}}$.

Ensemble average of $\mathcal{K}$ and four oldest official copies

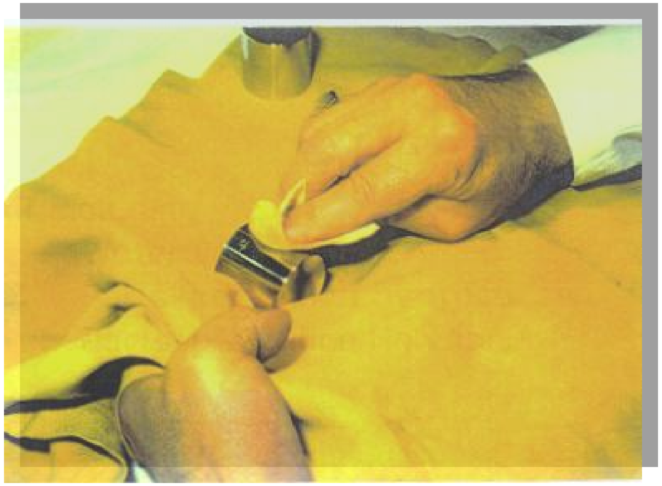


Ensemble average of $\mathcal{K}$ and the oldest national prototypes



Summary: real problems with the artefact definition

- mass ratios of "identical" prototypes have clearly changed during a century;
- as experiments improve, we will find evidence of $\{m_{\mathrm{FC}}/m_{\mathcal{K}}\}$ change with time;
- mass artefacts suffer surface contamination over long periods of time;



in 1989, the International Committee for Weights and Measures (CIPM) decided to interpret the 1901 definition of the kilogram as referring to the mass of $\mathcal{K}$ just after cleaning and washing using “the BIPM method”

Mass metrologists are not alone

The definition of the kilogram has an impact on other areas of science and metrology:

- Electrical metrology
- Chemistry
- Physics (especially the fundamental constants)

Why electrical metrologists care about the kilogram definition-1

"The **ampere** is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2 x 10^{-7} **newton** per metre of length."

The definition ensures the consistency of SI units.
For instance, thanks to this definition, the SI unit "watt" is the same for

$I^2 \cdot R$ (electrical power) and $m \cdot a \cdot v$ (mechanical power)

The ampere definition implicitly fixes the permeability of vacuum, μ_0, (sometimes called the magnetic constant)

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{N/A^2} \quad (1\ \mathrm{N/A^2} = 1\ \mathrm{H/m})$$

Why electrical metrologists care about the kilogram definition-2

Present voltage and resistance metrology rely on quantum-mechanical phenomena and two 'fundamental' constants of physics.

Josephson constant, K_J ,
and
von Klitzing constant, R_K .

According to current knowledge, $K_J = 2e/h$ and $R_K = h/e^2$.

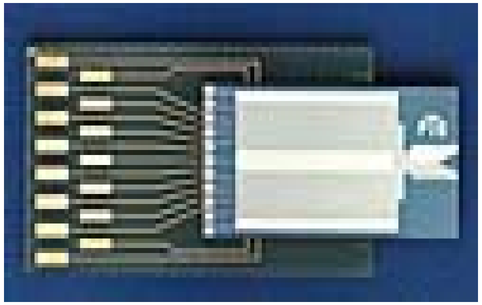
Therefore, fixing the values of *h* and of *e*

- **fixes the value of the Josephson constant,**
- **fixes the value of the von Klitzing constant,**

thereby eliminating the need for conventional (non SI) values, $K_{J\text{-}90}$ and $R_{K\text{-}90}$ that are used today.

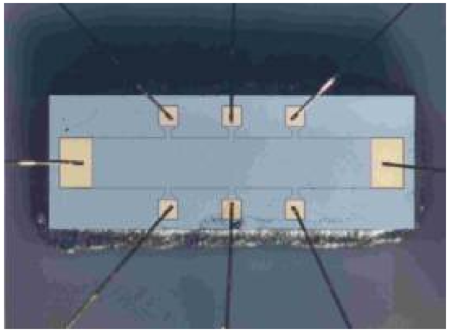
More on the importance of *h* and *e* to electrical metrology

Josephson effect



$$U_{\mathrm{J}}(n) = \frac{n\, f}{K_{\mathrm{J}}}, \quad K_{\mathrm{J}} = \frac{2e}{h}$$

quantized-Hall effect



$$R_{\mathrm{H}}(i) = \frac{R_{\mathrm{K}}}{i}, \quad R_{\mathrm{K}} = \frac{h}{e^2}$$

Conventional (non SI values)

$$\frac{h}{e^2} = \frac{\mu_0 c}{2\alpha}$$

$K_{\text{J-90}} \equiv 483\ 597.9$ GHz/V

$R_{\text{K-90}} \equiv 25\ 812.807\ \Omega$

But…

c : speed of light in vacuum. This already has a fixed value in the SI.

μ_0 already has a fixed value in the SI due to the definition of the Ampere

Therefore, it is impossible for *e* , *h* to be fixed as well:

$$\alpha = \frac{\mu_0 c e^2}{2h}$$

A choice must be made.

Many interesting proposals, including those of the *Académie des Sciences*

How should electrical units be derived in the new SI?

1. Keep μ_0 as a defined constant?

Consequence: the fundamental electromagnetic unit is then related to the properties of vacuum:

e.g. $Z_0 = \mu_0 c \approx 377\ \Omega$

but $e = \sqrt{\dfrac{2h\alpha}{\mu_0 c}}$

2. Make e a defined constant?

Consequence: $\mu_0 = \dfrac{2h\alpha}{e^2 c}$

but K_J and R_K are now exact.

Why chemists care about the kilogram definition

“The **mole** is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 **kilogram** of carbon 12; its symbol is ‘mol’.

“When the mole is used, the elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles.”

This definition is closely linked to the Avogadro Constant

$$N_{\mathrm{A}} = \frac{0.012\left[\mathrm{kg} \cdot \mathrm{mol}^{-1}\right]}{m\left({}^{12}\mathrm{C}\right)[\mathrm{kg}]} = \frac{\textit{molar mass of } x}{\textit{mass of } x}$$

The dalton (example)

$$\{m_{^{12}\mathrm{C}}\}[\mathrm{kg}] = \left\{\frac{m_{^{12}\mathrm{C}}}{m_{\mathcal{K}}}\right\}[\mathrm{kg}] \qquad = 12[\mathrm{u}]$$

$$\{m_{\mathrm{e}}\}[\mathrm{kg}] = \left\{\frac{m_{\mathrm{e}}}{m_{^{12}\mathrm{C}}}\right\}\left\{\frac{m_{^{12}\mathrm{C}}}{m_{\mathcal{K}}}\right\}[\mathrm{kg}] \qquad = 12\left\{\frac{m_{\mathrm{e}}}{m_{^{12}\mathrm{C}}}\right\}[\mathrm{u}]$$

From 2006 CODATA adjustment

quantity	relative uncertainty (ppm)
$m_{^{12}\mathrm{C}}$	0.05
m_{e}	0.05
$m_{\mathrm{e}} / m_{^{12}\mathrm{C}}$	0.000 42

(1 u = 1 Da)

$$[\mathrm{u}] = \frac{1}{12}\left\{\frac{m_{^{12}\mathrm{C}}}{m_{\mathcal{K}}}\right\}[\mathrm{kg}]$$

unified atomic mass unit, or dalton

Summary of why physicists care about the kg definition

- Conceptually peculiar to measure fundamental physical constants in terms of an artefact manufactured in the 19th century.

- Comparison to the international prototype is the dominant uncertainty in the SI values for many fundamental constants of physics; therefore: either large covariances must be taken into account or non-SI units like the dalton must be used.

Corollary: SI values of many fundamental constants change whenever there is a newly-measured link to $\mathcal{K}$, and yet the link to $\mathcal{K}$ is not "fundamental".

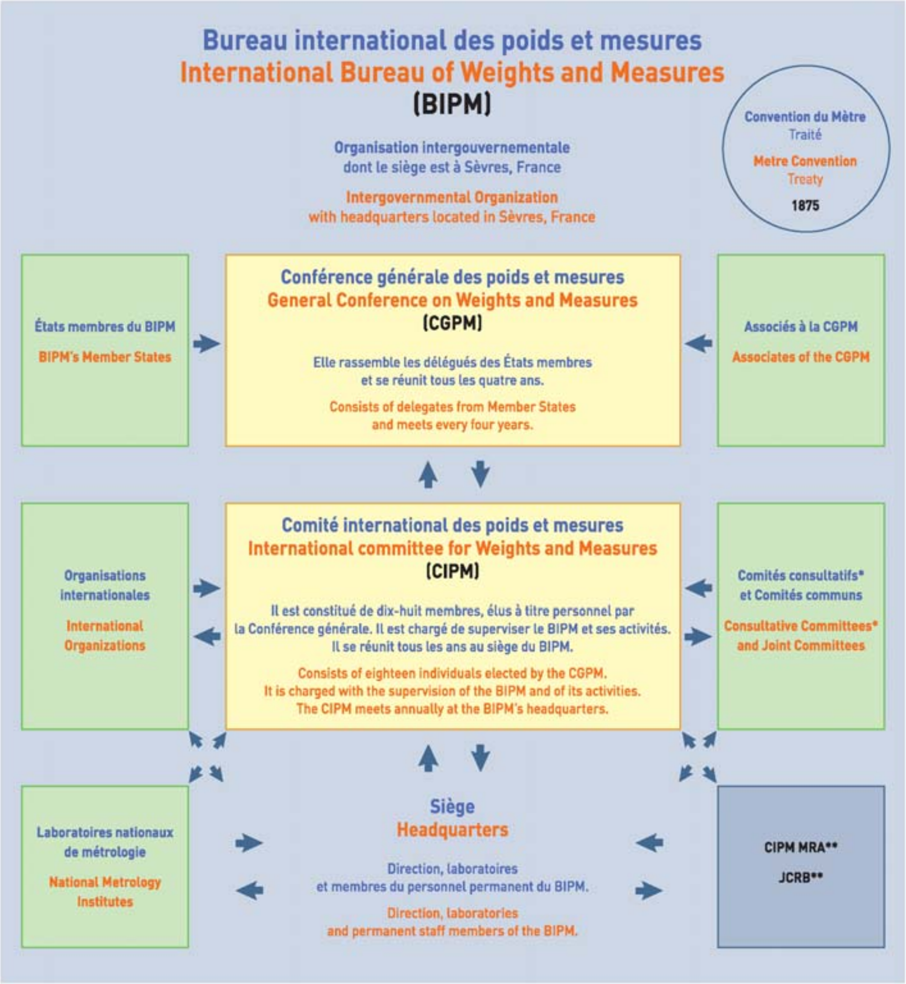
Administrative and legal structure

The **Metre Convention**. A diplomatic treaty signed in 1875. There are now 51 member states.

The treaty established the International Bureau of Weights and Measures (**BIPM**)

as well as a scientific oversight committee (**CIPM**) that meets yearly.

Every four years, the General Conference (**CGPM**) of member states meets to take of business at the diplomatic level. The CGPM must approve the budget of the BIPM for the following 4 years. It also must approve any changes to the units of the International System of units (SI).



* ▶ Actuellement au nombre de dix, ils sont chargés de conseiller le CIPM et le Siège, notamment sur des questions techniques, et d'aider à l'administration du CIPM MRA.
▶ There are currently ten CCs which advise the CIPM and the Headquarters, e.g. on technical matters, and the administration of CIPM MRA.

** ▶ Le JCRB est le sigle utilisé pour désigner le Comité mixte des organisations régionales de métrologie et du BIPM. Le CIPM MRA fait référence à l'Arrangement de reconnaissance mutuelle du BIPM rédigé par le CIPM.
▶ The JCRB refers to the Joint Committee of the Regional Metrology Organizations and the BIPM. The CIPM MRA refers to the Mutual Recognition Arrangement of the BIPM drawn up by the CIPM.

CCU
CCM
CCEM
CCQM
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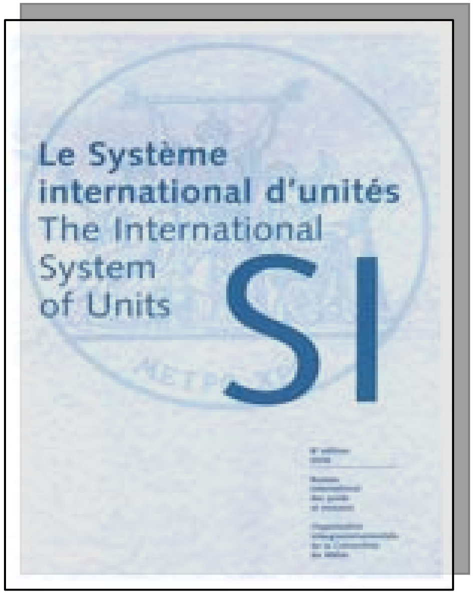
Among the Consultative Committees

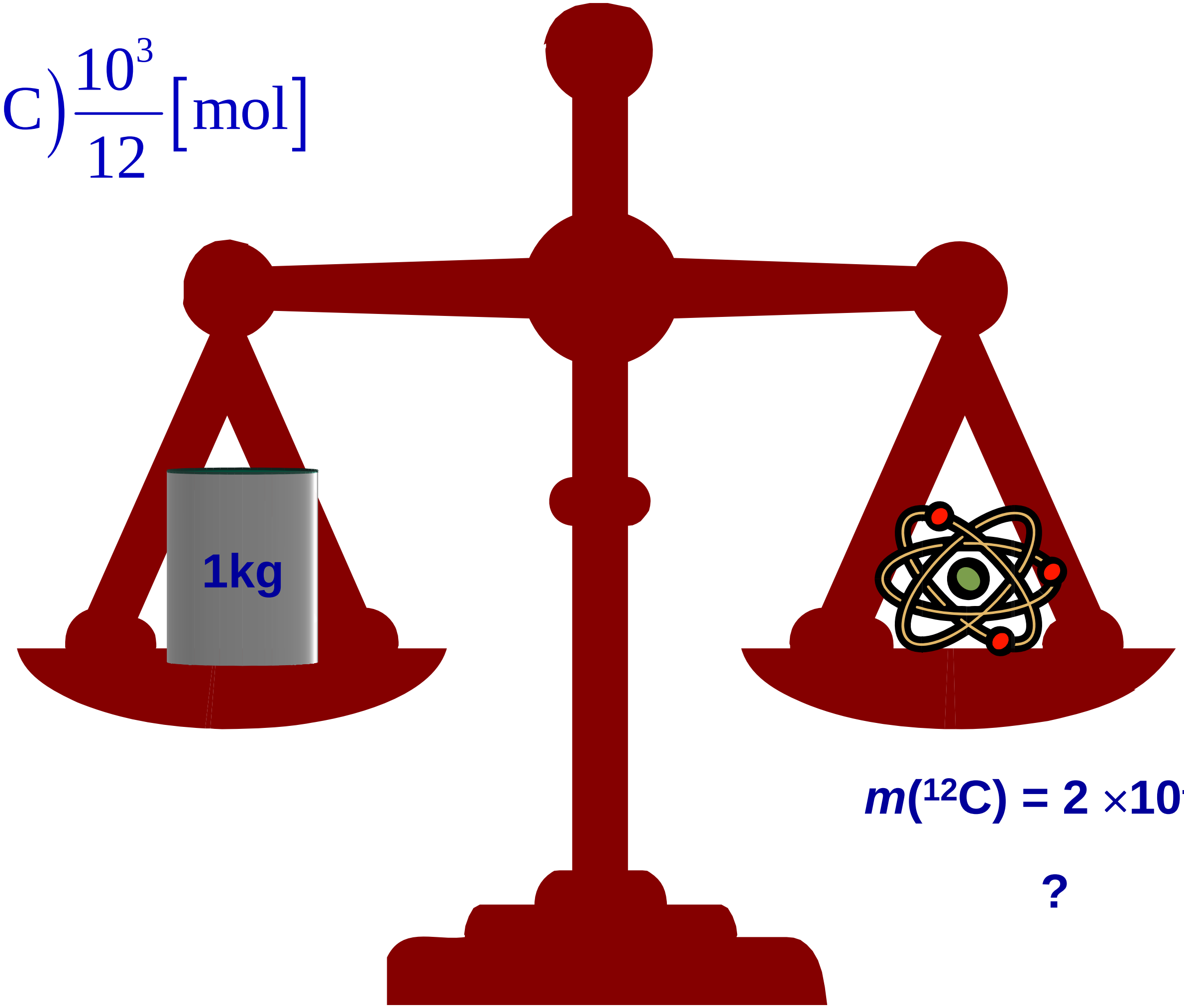
CCM : Consultative Committee for Mass and Related Quantities
CCEM: Consultative Committee for Electricity and Magnetism
CCQM: Consultative Committee for Metrology in Chemistry
CCU : Consultative Committee for Units



How can we link the kg to a fundamental constant?

A large mismatch between 1 kg and the mass of a carbon-12 atom

$$\frac{1}{N_A} = m\left({}^{12}\mathrm{C}\right)\frac{10^3}{12}[\mathrm{mol}]$$



$m(^{12}\mathrm{C}) = 2 \times 10^{-26}$ **kg**

?

N_A by the X-ray Crystal Density (XRCD) method

n = number of ^{28}Si atoms in the sphere:

$$n = \left\{\frac{m_{\mathrm{sph}}}{m_{^{28}\mathrm{Si}}}\right\} = \left\{\frac{m_{\mathrm{sph}}}{m_{\mathcal{K}}}\right\}\left\{\frac{m_{\mathcal{K}}}{m_{^{12}\mathrm{C}}}\right\}\left\{\frac{m_{^{12}\mathrm{C}}}{m_{^{28}\mathrm{Si}}}\right\}$$

$$n = 8\frac{V}{a^3}$$

V = volume of sphere

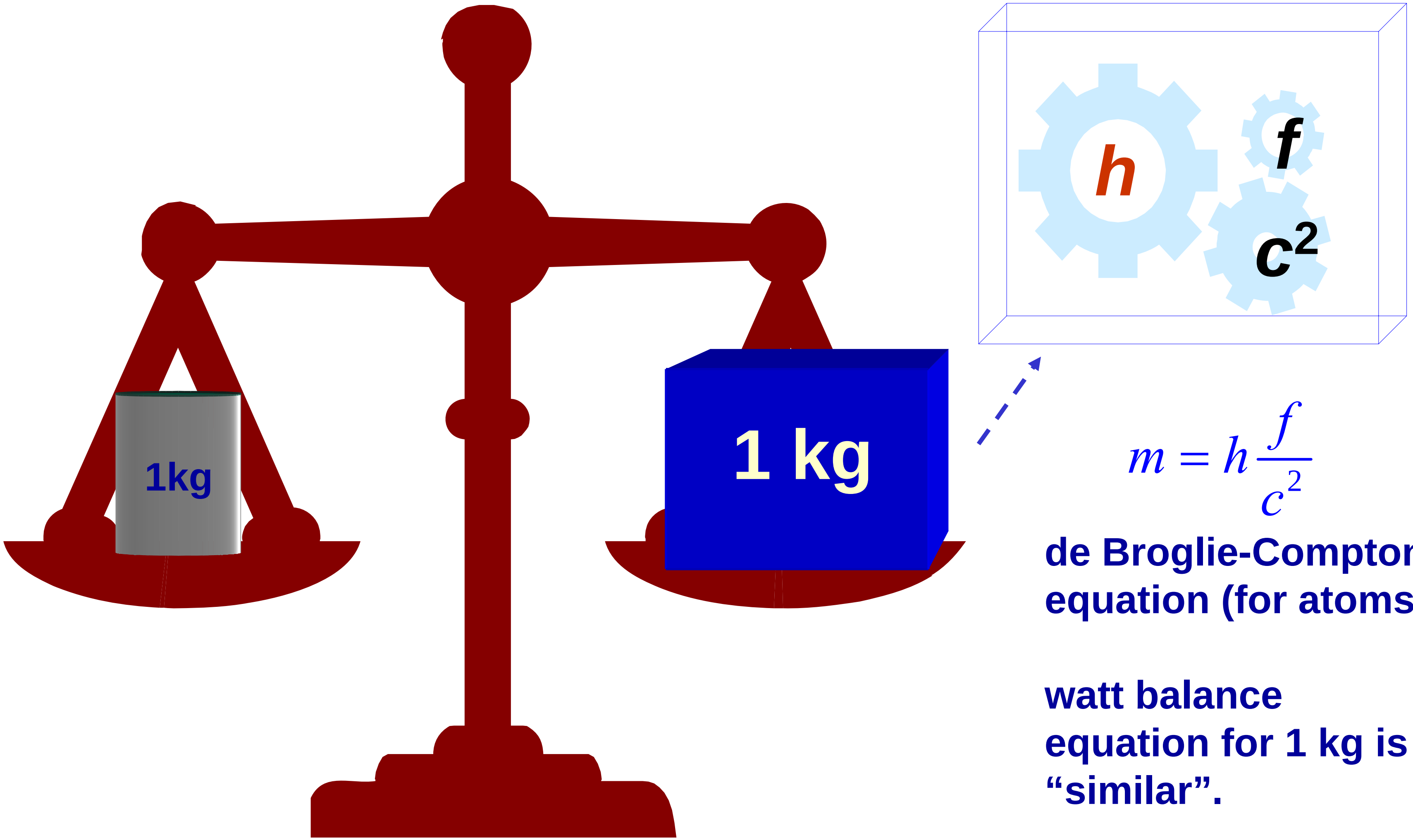
a^3 = volume of unit cell

$$\left\{\frac{m_{^{12}\mathrm{C}}}{m_{\mathcal{K}}}\right\} = \left\{\frac{m_{\mathrm{sph}}}{m_{\mathcal{K}}}\right\}\left\{\frac{m_{^{12}\mathrm{C}}}{m_{^{28}\mathrm{Si}}}\right\}\frac{a^3}{8V}$$

mass comparator ; mass spectrometer
X-ray interferometer; optical interferometer
purity: chemical, atomic, crystallographic

$$\frac{1}{N_{\mathrm{A}}} = \left\{\frac{m_{^{12}\mathrm{C}}}{m_{\mathcal{K}}}\right\}\frac{10^3}{12}[\mathrm{mol}] = \left\{\frac{m_{^{12}\mathrm{C}}}{m_{\mathcal{K}}}\right\}[\mathrm{kg}]\cdot\frac{1}{12}\left[\frac{\mathrm{mol}}{\mathrm{kg}}\right]\cdot 10^3$$

How the Planck constant can be linked to the kilogram



$$m = h\frac{f}{c^2}$$

de Broglie-Compton equation (for atoms);

watt balance equation for 1 kg is "similar".

Watt Balance

Part 1

$$m_{\mathcal{K}}\, g = I \cdot f\left(\vec{B},\vec{r}\right)$$

g is local grav. accel.; I is a current

Part 2

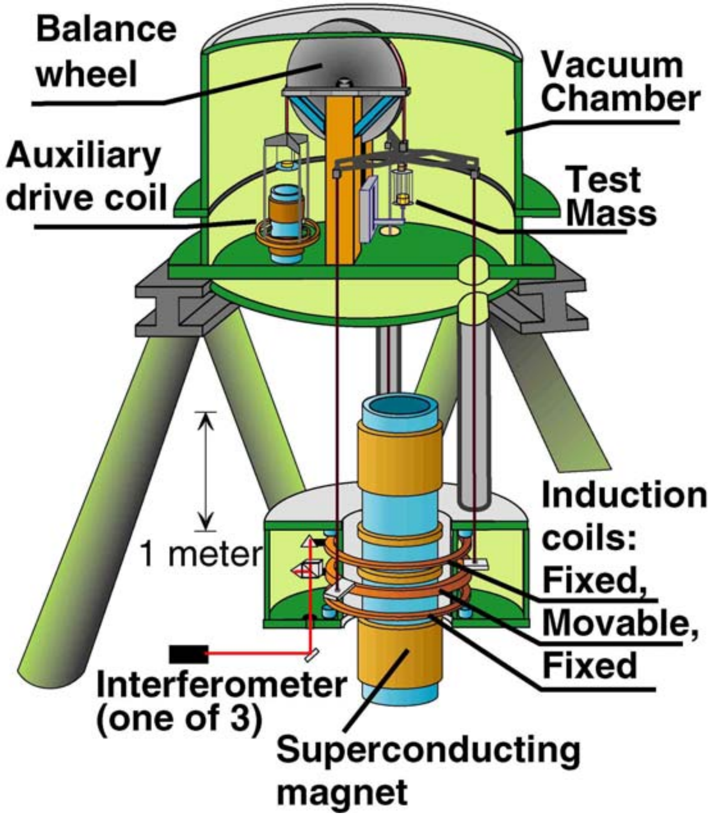
$$U = v \cdot f\left(\vec{B},\vec{r}\right)$$

U is a voltage; v is a velocity

$$m_{\mathcal{K}}\, g v = I U \qquad \text{[Watt]} = \text{[Watt]} \qquad I = \frac{U'}{R}$$

$$m_{\mathcal{K}} = h\left(\frac{U'_{90}\, U_{90}}{R_{90}\, g v} \cdot (\text{cwnu})\right)$$

(cwnu) means “constants with no uncertainty”



Schematic of NIST apparatus
(courtesy of NIST)

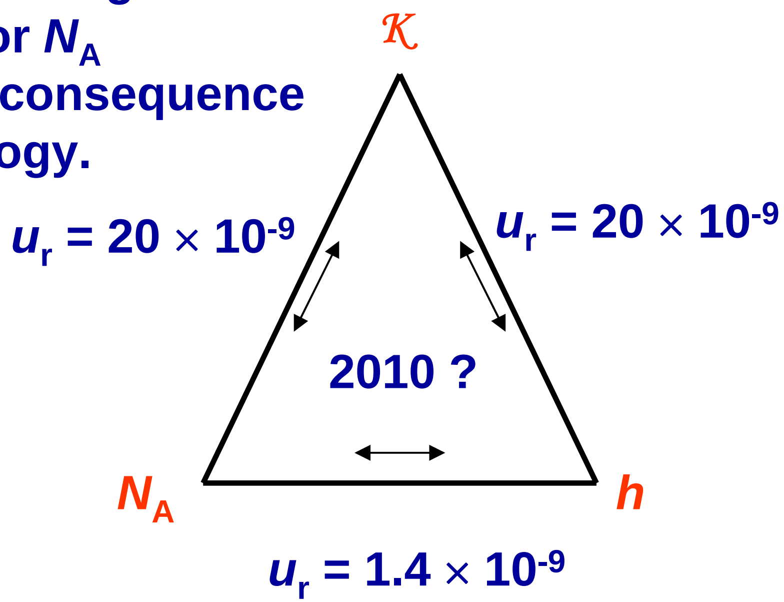
The international prototype, N_A, and h form a metrological triangle

$$N_A\, h = 0.012 \frac{m_e}{m\left({}^{12}\mathrm{C}\right)} \frac{c\alpha^2}{2R_\infty}$$

$$u_r\left(N_A\, h\right) = 1.4 \times 10^{-9}$$

From CODATA 2006

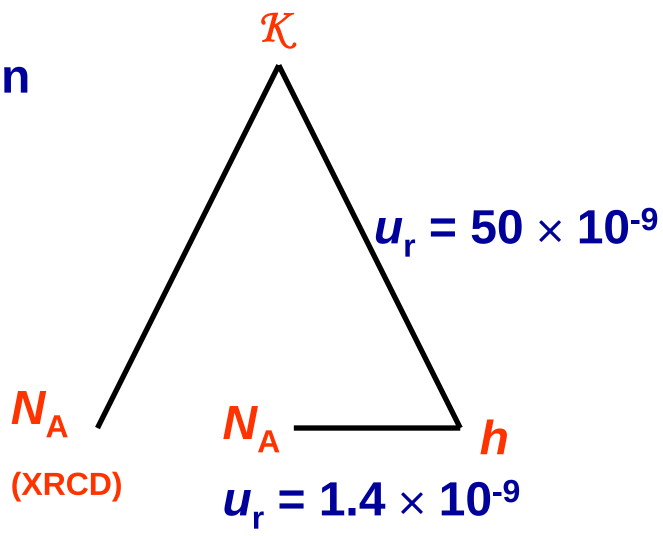
Whether the "new" kg is defined by h or N_A should have no consequence for mass metrology.



what we are working toward

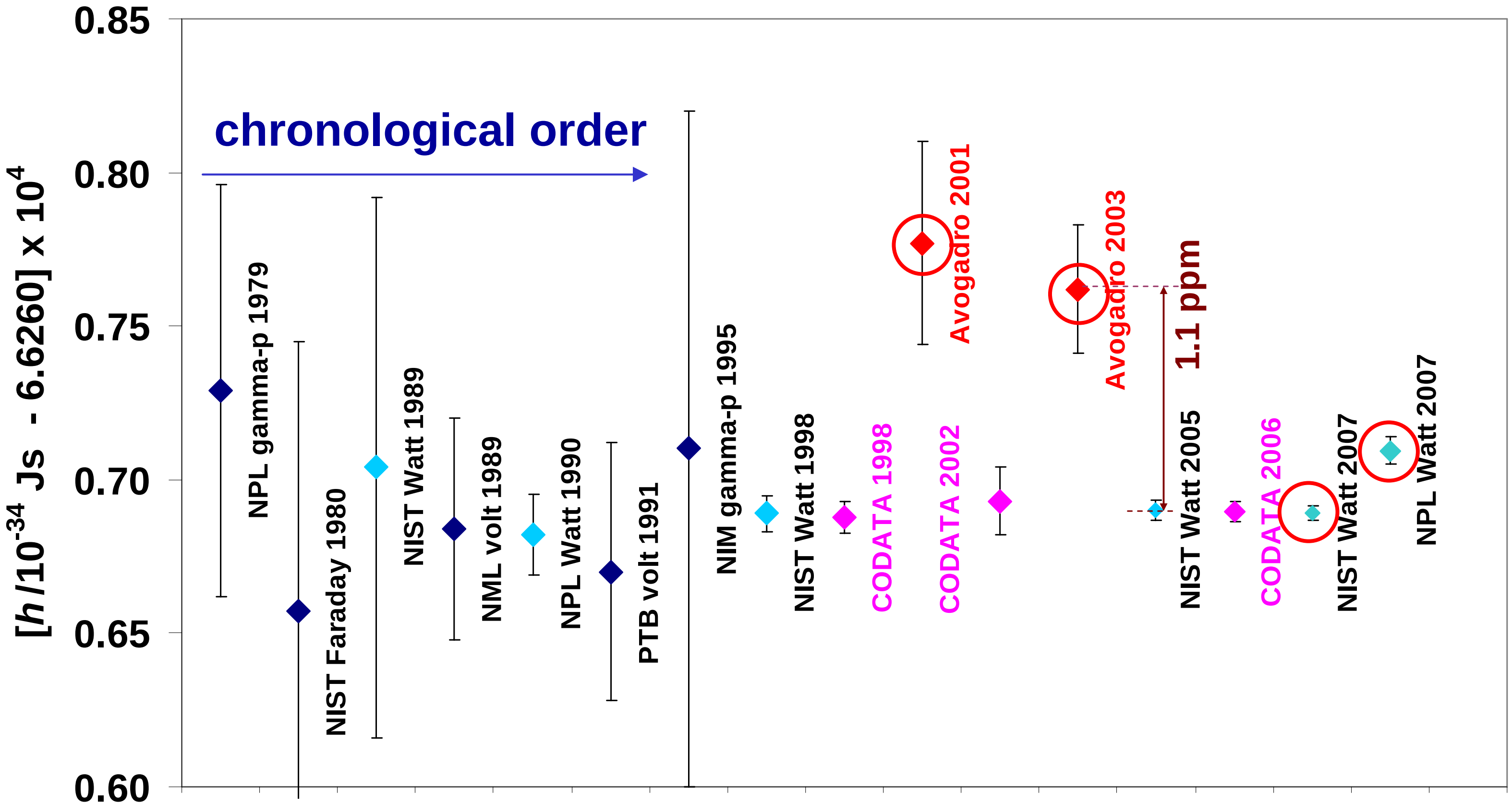
At present, the triangle does not close

CODATA 2006 recommendation



CIPM, CCU, CCM, CCEM agree that this situation must be clarified before a redefinition of the kilogram

History of measurements of the Planck constant



***N_A* and *h* are related by**

$$N_A\ h = 0.012 \frac{m_e}{m\left({}^{12}\mathrm{C}\right)} \frac{c\alpha^2}{2R_\infty} \qquad u_r\left(N_A\ h\right) = 1.4\times10^{-9}$$

Steps to redefine the kilogram

CCU, CCM, etc. send proposals/counter proposals to the CIPM.

The CIPM takes action, or it does not.

In 2005, the CIPM approved a Recommendation whose major points for mass metrologists are:

- approve in principle the preparation of new definitions and *mises en pratique* of the kilogram, the ampere and the kelvin so that if the results of experimental measurements over the next few years are indeed acceptable, all having been agreed with the various Consultative Committees and other relevant bodies, the CIPM can prepare proposals to be put to Member States of the Metre Convention in time for possible adoption by the 24th CGPM in 2011;

invites all Consultative Committees

- particularly the CCM, CCEM, CCQM and CCT, to consider the implications of changing the definitions of the above-mentioned base units of the SI, and to submit a report to the CIPM not later than June 2007;

The CCM reply -1

Following a general meeting in March 2007, the CCM position is:

- A redefinition should not take place until there is a practical way to apply it to a mass of 1 kg. (*mise en pratique* : MeP)
- There should be a sufficient number of laboratories with the capability of realizing the definition at or near 1 kg to assure robustness in the system. It will be necessary to compare these realizations to arrive at the best representation of the definition.
- Our concern is stability of the realized unit at 1 kg. Therefore, the choice among h, N_A, etc. is less important to the CCM than is the practical realization of the kilogram to about $u_r = 20 \times 10^{-9}$.
- The current discrepancies between XRCD measurements (of N_A) and watt balance measurements (of h) must be resolved.

Some consequences for mass metrologists

With m_{X} defined, the present uncertainty of $\{\boldsymbol{m_{\mathrm{X}}/m_{\mathcal{K}}}\}$ will be attributed to the mass of the international prototype and will propagate to all other, macroscopic, masses derived from the international prototype.

Here X might be the mass, now fixed, of a fundamental constant such as $m(^{12}\mathrm{C})$; or the product of a fundamental constant, such as h, multiplied by an appropriate factor; or…

Since the international prototype becomes a derived mass standard, the possibility is open to replace it by a better artefact or by the average of a group of artefacts.

Some consequences for everybody

- There will be no discontinuity in the kilogram, therefore no immediate consequences to measurements.
- The relative uncertainties of all mass standards, including the international prototype, will have an additional (but identical) component.
- Because the relative uncertainty component is exactly the same for all mass standards and all masses derived from mass standards, this component does not increase the uncertainty of comparisons between mass standards.
- Therefore, the consequence for end users will be negligible.
- Nevertheless, we must forge the strongest possible experimental links to a new definition to ensure that values of macroscopic masses remain traceable to the SI to sufficient accuracy.
- It seems unlikely that the public will understand the new definition of the kilogram. (This is a challenging problem in communications.)

Resolution of the CGPM, November 2007 - 1

…

The CGPM notes

- That any changes to the definitions of the SI units must be self-consistent.
- That it is desirable that definitions of the base units should be easily understood.

…

- **The importance of soliciting comments and contributions from the wider scientific and user communities.**

…

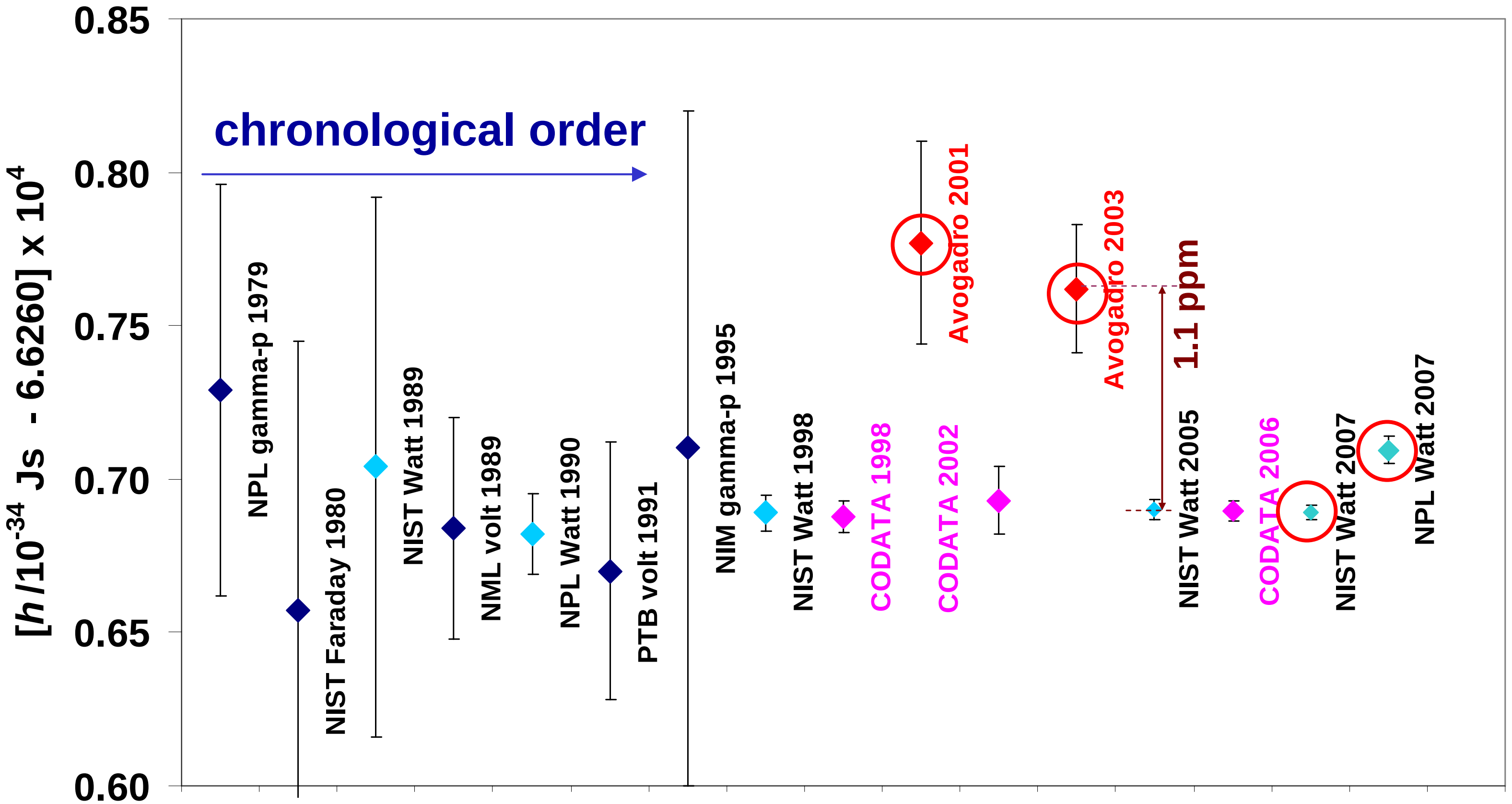
Resolution of the CGPM, November 2007 - 2

The CGPM Recommends that NMIs and the BIPM

- pursue the relevant experiments so that the CIPM can decide whether it may be possible to redefine the kg, A, K and mol using fixed values of the fundamental constants at the time of the 24th CGPM (2011).
- together with CIPM and appropriate Working Groups, work on practical ways of realizing the new definitions (MeP) and consider the most appropriate way of explaining the new definitions to users.
- initiate awareness campaigns to alert user communities to the possibility of redefinitions (technical and legislative implications, MeP)…

Requests the CIPM to report to the 24th CGPM in 2011 and to undertake whatever preparations are considered necessary so that, if the results of experiments are found to be satisfactory and the needs of users met, formal proposals for changes to the definitions of the kg, A, K, and mol can be put to the 24th CGPM.

A reminder of the present situation



***N_A* and *h* are related by**

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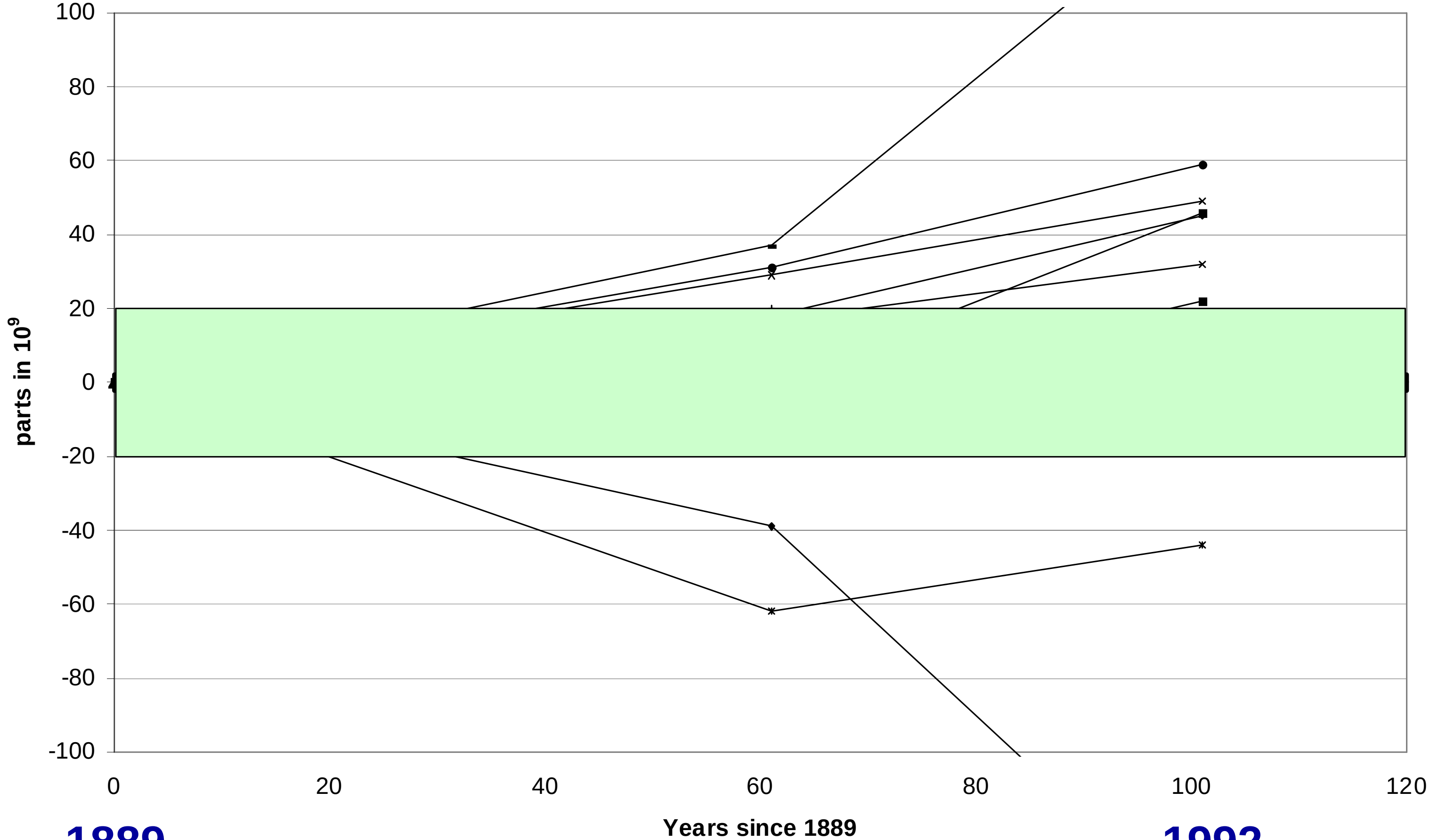
Changes in mass of national prototypes manufactured in 1889

CODATA/NPL_WB: 280 x 10^{-9} ; CODATA/XRCD: 1000 x 10^{-9}



1889 **1992**

Summary

- Artefacts are inherently unstable with respect to fundamental physical constants.
- The masses of "identical" 1 kg Pt-Ir artefacts are not stable with respect to each other.
- There is no experimental evidence (so far) that $m_{\mathcal{K}}$ is changing with respect to the fundamental physical constants.
- Present results between Watt Balance and XRCD values of h (or N_{A}) disagree at the 10^{-6} level.
- Two most recent Watt Balance results disagree by 3×10^{-7}.
- An immediate redefinition of the kilogram would benefit electrical metrology and some important areas of physics.
- Many groups are active in this research and progress is being made.

Conclusion

There is much to do in the next few years if we are to redefine the kilogram in a useful way for everybody.