

# A phenomenological study of helicity amplitudes of high energy exclusive lepton production of the $\rho$ meson

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# Outline

- A phenomenological model of the helicity amplitudes of high energy exclusive lepto-production of the  $\rho$  meson

PhysRevD.84.054004

in collaboration with

I. V. Anikin (JINR, Dubna), D .Yu. Ivanov (SIM, Novosibirsk), B. Pire (CPhT, Palaiseau), L. Szymanowski (SINS, Warsaw) and S. Wallon (LPT, Orsay)

- Impact factor  $\gamma^* \rightarrow \rho$  up to twist 3 - link to colour dipole model

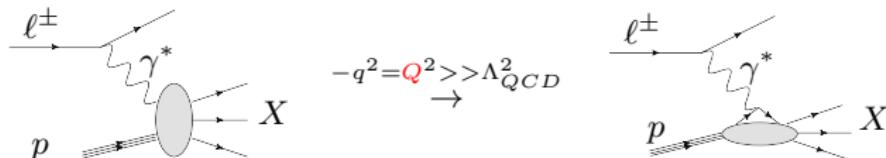
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# Introduction

## Deep Inelastic Scattering

- Factorisation of inclusive process: Deep Inelastic Scattering



- Parton distribution functions (PDFs)

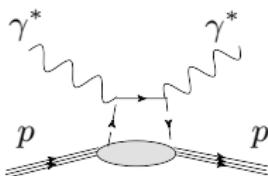
$$\left| \frac{\ell^\pm}{p} \frac{p}{X} \right|^2 = 2 \operatorname{Im}$$

$\ell^\pm$

$p$

$\gamma^*$

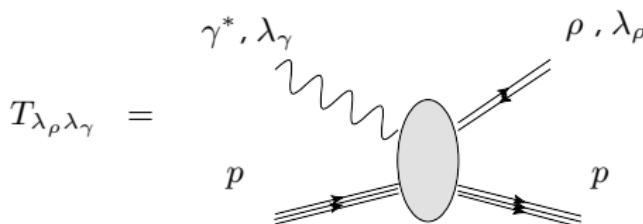
$X$



# Introduction

Helicity amplitudes of the diffractive lepto-production of the  $\rho$  meson

- **Helicity Amplitudes**  $T_{\lambda_\rho \lambda_\gamma}$



Examples :

$$T_{00} \iff \gamma_L^* p \rightarrow \rho_L p$$

$$T_{11} \iff \gamma_T^* p \rightarrow \rho_T p$$

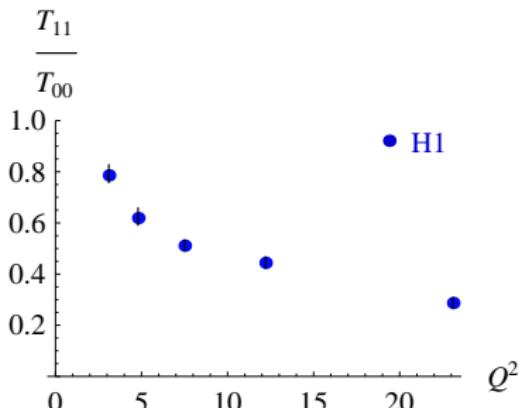
- **Perturbative Regge Limit :**

- **Regge Limit** :  $s = W^2 \gg Q^2, |t|, M_{\text{hadron}}^2$
- **Hard scale** :  $Q \gg \Lambda_{QCD}$

# Introduction

Experimental data of helicity amplitudes at high energy

- Helicity amplitudes  $T_{\lambda_\rho \lambda_\gamma}$  :  $\gamma_{\lambda_\gamma}^* + p \rightarrow \rho_{\lambda_\rho} + p$
- H1 and ZEUS data for Helicity Amplitudes at HERA:



S. Chekanov et al. (2007), F.D Aaron et al. (2010)

- Kinematics

- High energy in the center of mass  $30 \text{ GeV} < W < 180 \text{ GeV}$
- Photon Virtuality  $2.5 \text{ GeV}^2 < Q^2 < 60 \text{ GeV}^2$
- $|t| < 1 \text{ GeV}^2$

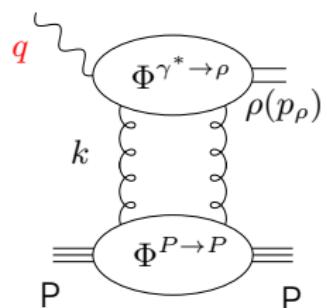
$$\Rightarrow s_{\gamma^* p} = W^2 \gg Q^2 \gg \Lambda_{QCD}^2$$

# Introduction

A Theoretical approach within  $k_T$  factorisation

## $k_T$ factorisation

Amplitudes with gluons exchange in t-channel dominate at large  $s$  ( $s = W^2$ )



Born order: 2 t-channel gluons

$$T_{\lambda_\rho \lambda_\gamma} = is \int \frac{d^2 \underline{k}}{(2\pi)^2 (\underline{k}^2)^2} \Phi^{\gamma^*(\lambda_\gamma) \rightarrow \rho(\lambda_\rho)}(\underline{k}) \Phi^{P \rightarrow P}(-\underline{k})$$

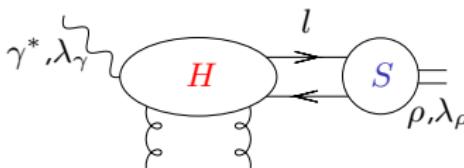
# Introduction

## A Theoretical approach

Impact factors  $\Phi^{\gamma^* \rightarrow \rho}$  and  $\Phi^{P \rightarrow P}$

- $\Phi^{\gamma^* \rightarrow \rho}$  Light-Cone Collinear factorisation

$$Q^2 \gg \Lambda_{QCD}^2$$



- **Twist**  $t$ : Impact factor behaves as  $1/Q^{t-1}$
- $T_{00} \equiv \gamma_L^* \rightarrow \rho_L$  impact factor : Dominant term at **twist 2**  $\equiv 1/Q$
- $T_{11} \equiv \gamma_T^* \rightarrow \rho_T$  impact factor : Dominant term at **twist 3**  $\equiv 1/Q^2$

Recently computed at  $t = t_{min} \approx 0$

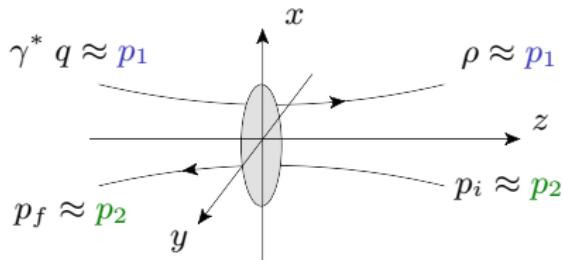
Nucl. Phys. B **828** (2010) 1-68. by Anikin et al.

- Phenomenological model for  $\Phi^{P \rightarrow P}$

# Impact factor for exclusive processes

$k_T$  factorisation

- Light-cone vectors  $p_1$  and  $p_2$ :  $p_1^2 = p_2^2 = 0$ ,  $2p_1 \cdot p_2 = s$
- We can choose:



# Impact factor for exclusive processes

$k_T$  factorisation

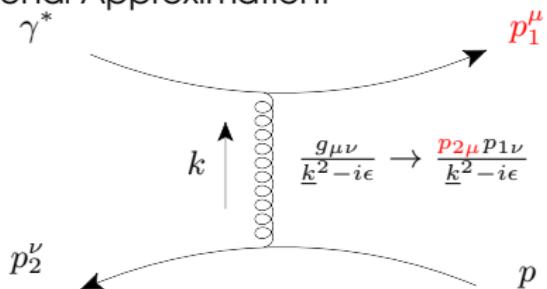
- Particles in  $t$ -channel:

- $\mathcal{M} \propto s^{\sum \sigma_i - N + 1} \Rightarrow$  gluons in  $t$ -channel :  $\mathcal{M} \propto s$

- Gluon in  $t$ -channel :  $k = \alpha p_1 + \beta p_2 + k_\perp$

$$\Rightarrow \frac{g^{\mu\nu}}{k^2 + i\epsilon} \approx \frac{g^{\mu\nu}}{-\underline{k}^2 + i\epsilon}$$

- Eikonal Approximation:



Non-sens polarisations:

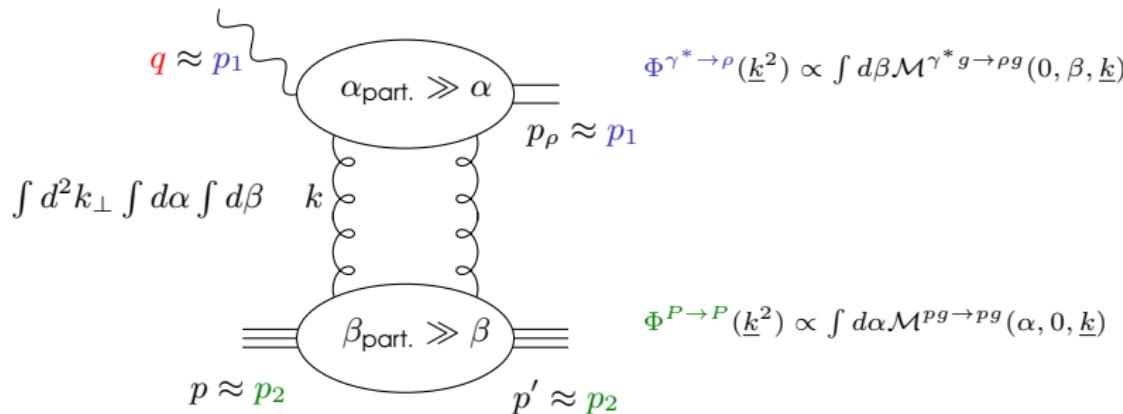
$$\epsilon_{NS}^{up} = \sqrt{\frac{2}{s}} \, p_2$$

$$\epsilon_{NS}^{down} = \sqrt{\frac{2}{s}} \, p_1$$

# Impact factor for exclusive processes

$k_T$  factorisation

- Impact Factors  $\Phi^{\gamma^* \rightarrow \rho}$ ,  $\Phi^{P \rightarrow P}$ :



- Impact factor representation of the helicity amplitudes

$$T_{\lambda_\rho \lambda_\gamma} = is \int \frac{d^2 \underline{k}}{(2\pi)^2 (\underline{k}^2)^2} \Phi^{\gamma^*(\lambda_\gamma) \rightarrow \rho(\lambda_\rho)}(\underline{k}) \Phi^{P \rightarrow P}(-\underline{k})$$

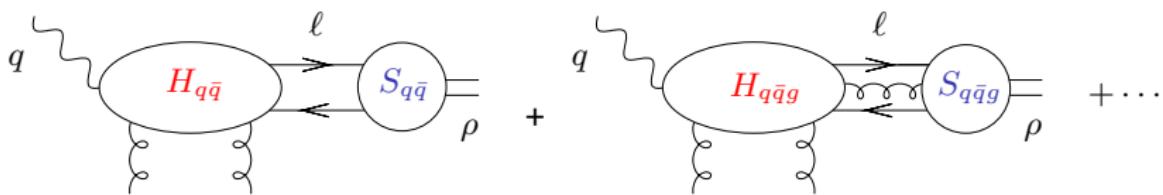
# Collinear factorization

Light-Cone Collinear approach

- The impact factor  $\Phi^{\gamma^*(\lambda_\gamma) \rightarrow \rho(\lambda_\rho)}$  can be written as

$$\Phi^{\gamma^*(\lambda_\gamma) \rightarrow \rho(\lambda_\rho)} = \int d^4 \ell \dots \text{tr}[H(\ell \dots) \quad S(\ell \dots)]$$

hard part      soft part



- At the 2-body level:

$$S_{q\bar{q}}(\ell) = \int d^4 z e^{-i\ell \cdot z} \langle \rho(p) | \psi(0) \bar{\psi}(z) | 0 \rangle,$$

# Collinear factorization

Light-Cone Collinear approach: 2 steps of factorization (2-body case)

## 1 - Momentum factorization

- Decomposition of partons momenta  $\ell$ :

$$\ell = \textcolor{red}{y} p_\rho + \textcolor{violet}{\ell^\perp} + (\ell \cdot p_\rho) n$$

- Taylor Expand  $H(\ell)$  around the  $\textcolor{red}{p}$  direction:

$$H(\ell) = H(yp) + \left. \frac{\partial H(\ell)}{\partial \ell_\alpha} \right|_{\ell=yp} \ell_\alpha^\perp + \dots$$

- $\int \textcolor{red}{dy}$  performs the longitudinal momentum factorization

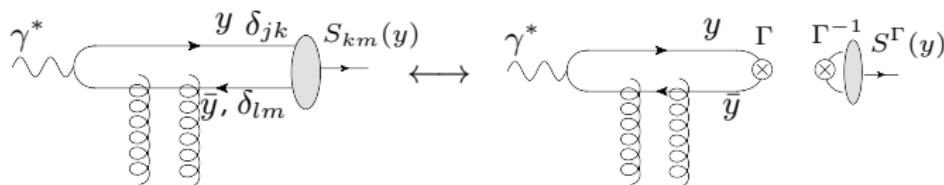
$$\Phi = \int \textcolor{red}{dy} \left\{ \text{tr} [\textcolor{green}{H}_{q\bar{q}}(\textcolor{red}{y} p) S_{q\bar{q}}(\textcolor{red}{y})] + \text{tr} [\textcolor{violet}{\partial}_\perp H_{q\bar{q}}(\textcolor{red}{y} p) \partial_\perp S_{q\bar{q}}(\textcolor{red}{y})] \right\}$$

# Collinear factorization

Light-Cone Collinear approach: 2 steps of factorization (2-body case)

## Momentum and spinorial factorization

- Fierz Identity:  $\delta_{jk}\delta_{lm} = -\frac{1}{4}(\Gamma)_{jl}(\Gamma^{-1})_{km}$



- Spinor (and color) factorisation:

$$\Phi = \int dy \left\{ \text{tr}[H_{q\bar{q}}(y p) \Gamma] S_{q\bar{q}}^\Gamma(y) + \text{tr}[\partial_\perp H_{q\bar{q}}(y p) \Gamma] \partial_\perp S_{q\bar{q}}^\Gamma(y) \right\}$$

$$S_{q\bar{q}}^\Gamma(y) = \int \frac{d\lambda}{2\pi} e^{-i\lambda y} \langle \rho(p) | \bar{\psi}(\lambda n) \Gamma \psi(0) | 0 \rangle$$

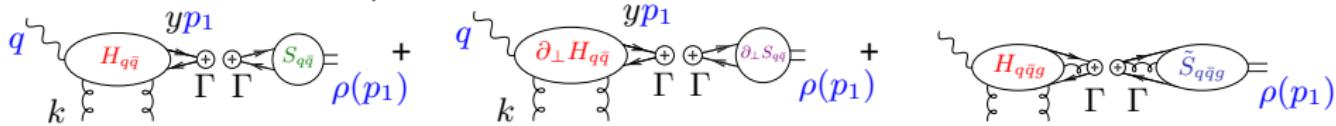
$$\partial_\perp S_{q\bar{q}}^\Gamma(y) = \int \frac{d\lambda}{2\pi} e^{-i\lambda y} \langle \rho(p) | \bar{\psi}(\lambda n) \Gamma i \overleftrightarrow{\partial}_\perp \psi(0) | 0 \rangle$$

# Collinear factorization

Light-Cone Collinear approach

## Collinear factorization of 2-body and 3-body contributions

- Momentum, spinorial and color factorizations



- vector correlator  $\Gamma = \gamma_\mu$

$$\langle \rho(p) | \bar{\psi}(z) \gamma_\mu \psi(0) | 0 \rangle = \int dy e^{-i(\bar{y}p) \cdot z} m_\rho f_\rho \left[ \varphi_1(y) (e^* \cdot n) p_\mu + \varphi_3(y) e_\mu^{*T} \right]$$

- Parametrization of Soft parts  $S_{q\bar{q}}$ ,  $\partial_\perp S_{q\bar{q}}$ ,  $S_{q\bar{q}g}$

- $\Rightarrow$  5 2-body Distribution Amplitudes "DAs"  $\{\varphi_1, \varphi_A, \varphi_3, \varphi_{1T}, \varphi_{AT}\}$

- $\Rightarrow$  2 3-body DAs  $\{B(y_1, y_2), D(y_1, y_2)\}$

# Collinear factorization

Equations of motion and n-independence

Relation between DAs:

- Dirac equation leads to

$$\langle i(\vec{\not{D}}(0)\psi(0))_\alpha \bar{\psi}_\beta(z) \rangle = 0 \quad (i \vec{D}_\mu = i \vec{\partial}_\mu + g A_\mu)$$

⇒ 2 Equations of motion:

$$\begin{aligned} \bar{y}_1 \varphi_3(y_1) + \bar{y}_1 \varphi_A(y_1) + \varphi_1^T(y_1) + \varphi_A^T(y_1) \\ + \int dy_2 \left[ \zeta_3^V B(y_1, y_2) + \zeta_3^A D(y_1, y_2) \right] = 0 \quad \text{and} \quad (\bar{y}_1 \leftrightarrow y_1) \end{aligned}$$

- **n-independence** of the amplitude:  $\frac{d}{dn} \mathcal{M} = 0$
- ⇒ 3 independent DAs :  $\{\varphi_1(y), B(y_1, y_2), D(y_1, y_2)\}$

# Collinear factorization

Wandzura-Wilczek and Genuine contributions

- Solution in the Wandzura-Wilczek Approximation (WW)  $\equiv$  Only 2-body contributions

$$\varphi_1 \Rightarrow \{\varphi_3^{WW}(y), \varphi_A^{WW}(y), \varphi_{1T}^{WW}(y), \varphi_{AT}^{WW}(y)\}$$

- Genuine solutions

$$\{B(y_1, y_2), D(y_1, y_2)\} \Rightarrow \{\varphi_3^{gen}(y), \varphi_A^{gen}(y), \varphi_{1T}^{gen}(y), \varphi_{AT}^{gen}(y)\}$$

- Evolution of the DAs P. Ball, V.M Braun, Y. Koike, K. Tanaka

$$\varphi_1(y, \mu^2) = 6y\bar{y}(1 + a_2(\mu^2) \frac{3}{2}(5(y - \bar{y})^2 - 1)) \xrightarrow{\mu^2 \rightarrow \infty} 6y\bar{y}$$

$$B(y_1, y_2; \mu^2) = -5040y_1\bar{y}_2(y_1 - \bar{y}_2)(y_2 - y_1)$$

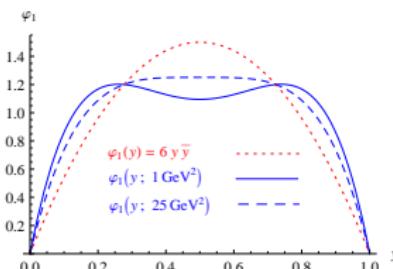
$$D(y_1, y_2; \mu^2) = -360y_1\bar{y}_2(y_2 - y_1)\left(1 + \frac{\omega_{\{1,0\}}^A(\mu^2)}{2}(7(y_2 - y_1) - 3)\right)$$

with  $\mu^2 \approx Q^2$  the collinear factorisation scale

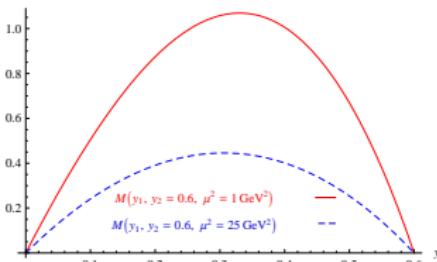
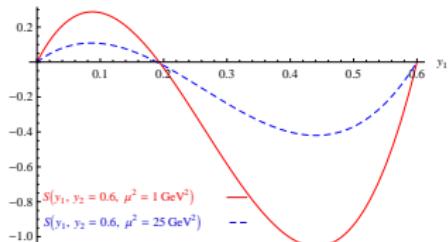
# Collinear factorisation

DAs dependence on  $\mu^2$

- $\varphi_1(y, \mu^2)$



- $M(y_1, y_2) = \zeta_\rho^V(\mu^2)B(y_1, y_2; \mu^2) - \zeta_\rho^A(\mu^2)D(y_1, y_2; \mu^2) \xrightarrow{\mu^2 \rightarrow \infty} 0$   
 $S(y_1, y_2) = \zeta_\rho^V(\mu^2)B(y_1, y_2; \mu^2) + \zeta_\rho^A(\mu^2)D(y_1, y_2; \mu^2) \xrightarrow{\mu^2 \rightarrow \infty} 0$

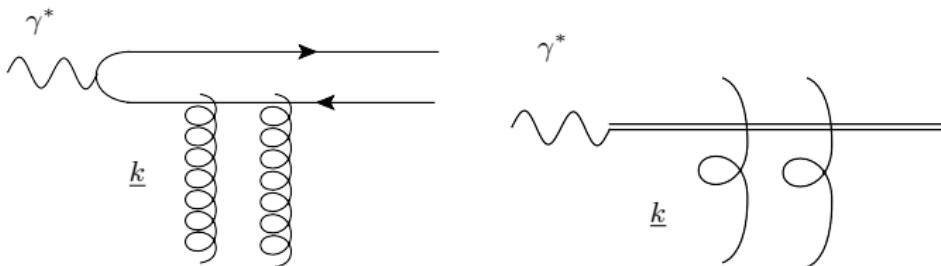


# Ratios of Helicity Amplitudes

A model for the proton impact factor

- $T_{\lambda_\rho \lambda_\gamma}(Q, M) = is \int \frac{d^2 \underline{k}}{(2\pi)^2} \frac{1}{(\underline{k}^2)^2} \Phi^{P \rightarrow P}(\underline{k}; M^2) \Phi^{\gamma^*(\lambda_\gamma) \rightarrow \rho(\lambda_\rho)}(\underline{k}; Q^2)$

- Universal property of impact factor:



- Phenomenological Model for  $\Phi^{P \rightarrow P}$

$$\Phi^{P \rightarrow P}(\underline{k}; M^2) \propto \left[ \frac{1}{M^2} - \frac{1}{M^2 + \underline{k}^2} \right] \quad \text{J.F Gunion, D.E Soper}$$

# Ratios of Helicity Amplitudes

A model for the proton impact factor

- $T_{\lambda_\rho \lambda_\gamma}(Q, M) = is \int \frac{d^2 \underline{k}}{(2\pi)^2} \frac{1}{(\underline{k}^2)^2} \Phi^{P \rightarrow P}(\underline{k}; \textcolor{red}{M}^2) \Phi^{\gamma^* (\lambda_\gamma) \rightarrow \rho(\lambda_\rho)}(\underline{k}; Q^2)$
- $\gamma_L^* \rightarrow \rho_L$  helicity amplitude:

$$\begin{aligned} T_{00} &\propto \frac{is}{(2\pi)} \int_{\textcolor{red}{x}^2}^{\infty} d\underline{k}^2 \frac{1}{(\underline{k}^2)^2} \left( \frac{1}{M^2} - \frac{1}{\underline{k}^2 + M^2} \right) \\ &\times \frac{1}{Q} \int_0^1 dy \varphi_1(y, \mu^2) \frac{\underline{k}^2}{\underline{k}^2 + y\bar{y}Q^2} \end{aligned}$$

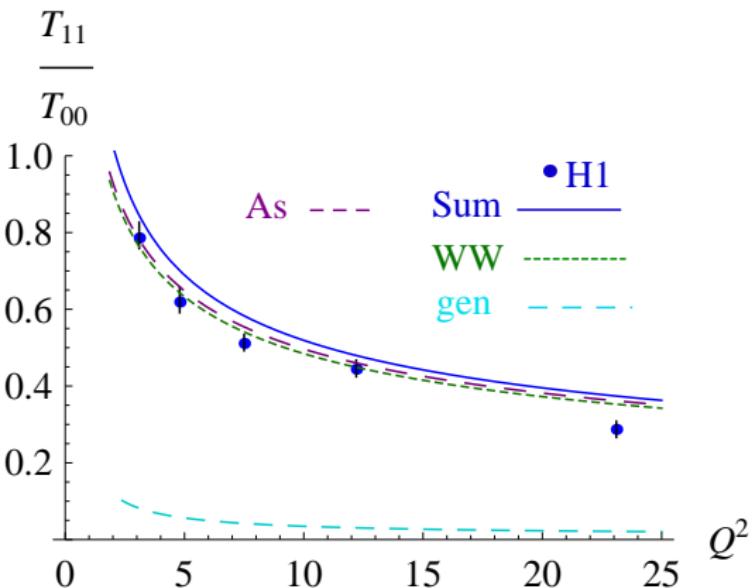
- The  $\gamma_T^* \rightarrow \rho_T$ , WW contribution:

$$\begin{aligned} T_{11}^{WW} &\propto \frac{is}{2\pi} \int_{\textcolor{red}{x}^2}^{\infty} d(\underline{k}^2) \frac{1}{(\underline{k}^2)^2} \left( \frac{1}{M^2} - \frac{1}{\underline{k}^2 + M^2} \right) \\ &\times \left( \frac{(\epsilon_\gamma \cdot \epsilon_\rho^*) m_\rho}{Q^2} \int_0^1 du \frac{\varphi_1(u; \mu^2)}{u} \int_0^u dy \frac{\underline{k}^2(\underline{k}^2 + 2y\bar{y}Q^2)}{(\underline{k}^2 + y\bar{y}Q^2)^2} \right) \end{aligned}$$

# Ratios of Helicity Amplitudes

Comparison with H1 data :  $T_{11}/T_{00}$

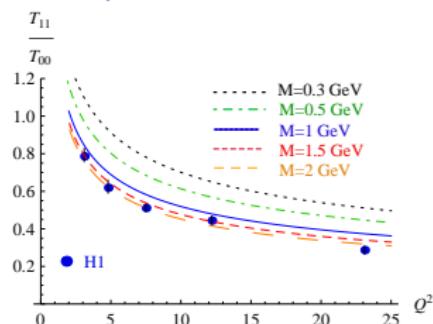
- Genuine and Wandzura-Wilczek Contributions at  $M = 1 \text{ GeV}$



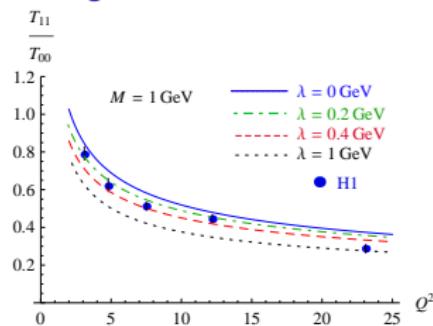
# Ratios of Helicity Amplitudes

Dependence on parameters  $M$  and  $\lambda$

- $M$  dependence of the ratio  $T_{11}/T_{00}$



- Soft gluon effect :  $\lambda$  IR cut-off on  $k_T$  integrals



# Conclusion I : Perspectives for $\Phi \gamma_T^* \rightarrow \rho_T$

- Good agreement with Experimental data

- reasonable values of  $M \approx M_p$  and  $\lambda \approx 0 \text{ GeV}$
- weak sensitivity in the parameters  $M$  and  $\lambda$

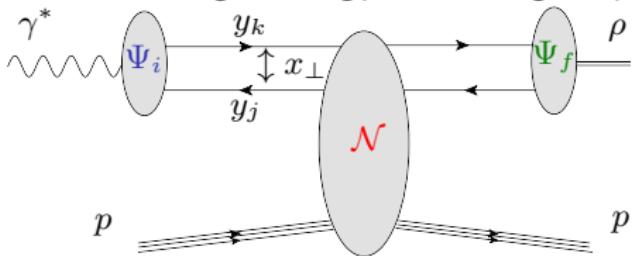
- Perspectives :

- Extend the kinematic to  $t \neq t_{min} \Rightarrow$  access to all spin density matrix elements.
- Link with the Dipole model and implementation of saturation effects

# Dipole Models

## Dipole model picture

- Factorization of a high energy scattering amplitude into:

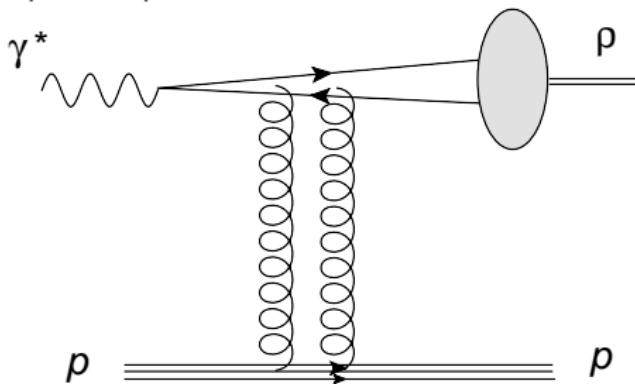


- Initial  $\Psi_i$  and final  $\Psi_f$  states wave functions of projectiles
- Universal scattering amplitude  $\mathcal{N} \equiv \mathcal{N}_{\text{dipole-target}}$
- Dipole models are consistent with LO Collinear approximation but are they still consistent with collinear factorization at higher twist order?

# Dipole Models

The  $\gamma^* \rightarrow \rho$  impact factor in a dipole model

- Dipole representation at lowest Fock state ( $q\bar{q}$  pair)



- In the dipole model representation:

$$\mathcal{A} = is \int d^2 \underline{x} \int dy \bar{\Psi}^{\rho \lambda_{\rho}}(y, \underline{x}) \mathcal{N}(x_{Bj}, \underline{x}) \Psi^{\gamma^* \lambda_{\gamma}}(y, \underline{x})$$

(from Bartels, Golec-Biernat, Peters )

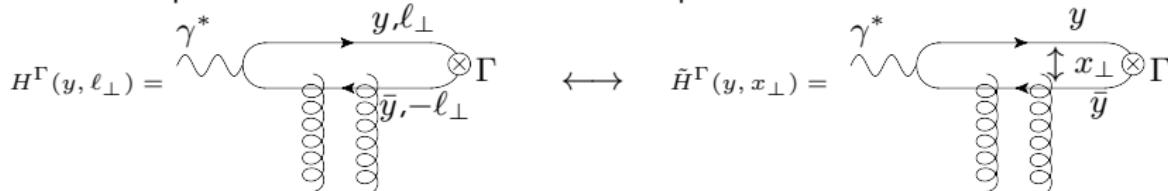
with

$$\mathcal{N}(x_{Bj}, \underline{x}) \propto \alpha_s \frac{\delta^{ab}}{N_c} \int \frac{d^2 \underline{k}}{(\underline{k}^2)^2} (1 - e^{i\underline{k} \cdot \underline{x}})(1 - e^{-i\underline{k} \cdot \underline{x}}) f(x_{Bj}, \underline{x})$$

# Dipole Models

Fourrier transform of the  $\gamma^* \rightarrow \rho$  impact factor

- Hard parts in transverse coordinate space :



- 

$$\begin{aligned} \tilde{H}^{\gamma^\mu}(y, \underline{x}) = & -4 \frac{2\pi e g^2}{\sqrt{2}s} \frac{\delta^{ab}}{2N_c} \{ y \bar{y} s K_0(\mu |\underline{x}|) \textcolor{red}{e_{\gamma T}^\mu} \\ & - (y - \bar{y}) i \mu \frac{e_{\gamma T} \cdot \underline{x}}{|\underline{x}|} K_1(\mu |\underline{x}|) ((1 - e^{i\underline{k} \cdot \underline{x}})(1 - e^{-i\underline{k} \cdot \underline{x}}) - 1) p_2^\mu \} \end{aligned}$$

- Equations of motion:

$$\text{Termes} \times (1 - e^{i\underline{k} \cdot \underline{x}})(1 - e^{-i\underline{k} \cdot \underline{x}}) + \text{Termes} \times \underbrace{2y\bar{y}\varphi_3(y) + (y - \bar{y})\varphi_{1T}(y) + \varphi_{AT}(y)}_{=0} = 0$$

# Dipole Models

Fourrier transform of the  $\gamma^* \rightarrow \rho$  impact factor

Twist 3, 2-body impact factors:

- Non-flip part:

$$\Phi_{nf} = \frac{1}{4} \int dy \int \frac{d^2 \underline{x}}{2\pi} \frac{e}{\sqrt{2}} \mu |\underline{x}| K_1(\mu |\underline{x}|) g^2 \delta^{ab} (1 - e^{i\underline{k} \cdot \underline{x}}) (1 - e^{-i\underline{k} \cdot \underline{x}}) \frac{m_\rho f_\rho}{2N_c} \{(y - \bar{y}) \varphi_{1T}(y) + \varphi_{AT}(y)\}$$

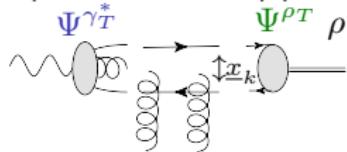
- Flip part:

$$\Phi_f = \frac{1}{2} \int dy \int \frac{d^2 \underline{x}}{2\pi} \frac{e}{\sqrt{2}} \mu |\underline{x}| K_1(\mu |\underline{x}|) g^2 \delta^{ab} (1 - e^{i\underline{k} \cdot \underline{x}}) (1 - e^{-i\underline{k} \cdot \underline{x}}) \frac{m_\rho f_\rho}{2N_c} \{(y - \bar{y}) \varphi_{1T}(y) - \varphi_{AT}(y)\}$$



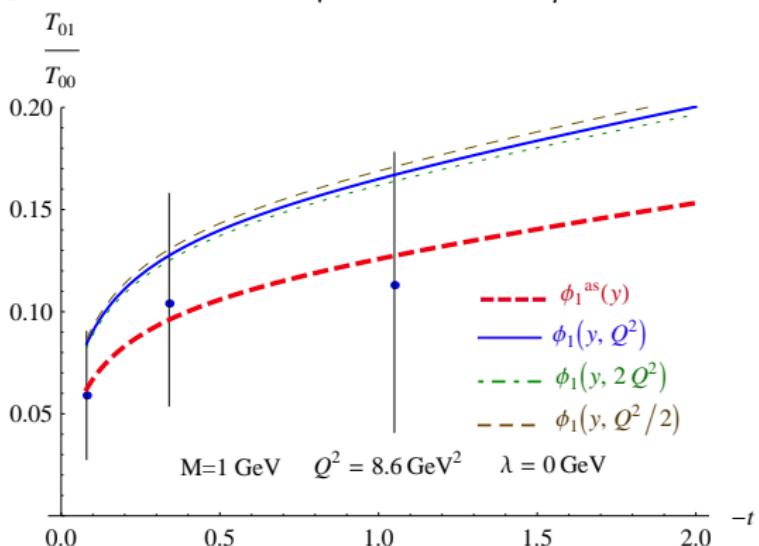
## Conclusion II : Perspectives for $\Phi \gamma_T^* \rightarrow \rho T$

- Agreement between the higher twist computation and the dipole representation:
- Dipole factors appear in the 3-body impact factor:



- Improvement of the phenomenological model by taking into account **Saturation effects** in the previous phenomenological model.

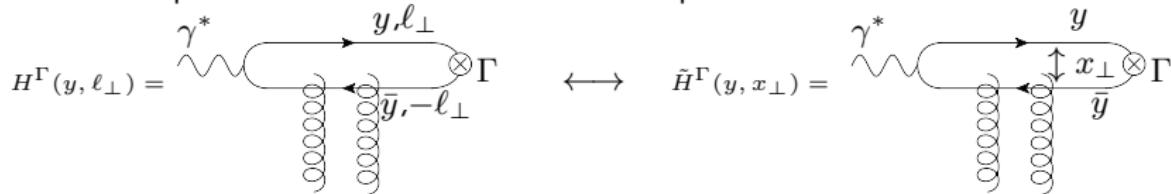
- $T_{01}/T_{00}$  at  $M = 1 \text{ GeV}$ , Dependence on  $\mu^2$  at  $M = 1 \text{ GeV}$ :



- Impact factor in transverse coordinate space:

$$\begin{aligned}\Phi_{\text{body}}^{\gamma^* \rightarrow \rho} &= \int d^4 \ell H(\ell) S(\ell) = -\frac{1}{4} \int d^4 \ell H^\Gamma(\ell) S_\Gamma(\ell) \\ &= -\frac{1}{4} \int dy \int d^2 \ell_\perp \int \frac{d^2 x_\perp}{2\pi} e^{-i\ell_\perp \cdot x_\perp} \tilde{H}^\Gamma(y, x_\perp) \\ &\quad \int \frac{d^2 z_\perp}{2\pi} e^{-i\ell_\perp \cdot z_\perp} \int \frac{d\lambda}{2\pi} e^{-i\lambda y} \langle \rho | \bar{\psi}(\lambda n + z_\perp) \Gamma \psi(0) | 0 \rangle\end{aligned}$$

- Hard parts in transverse coordinate space :



## Hard parts in coordinate space:

- $\Gamma \equiv \gamma^\mu$

$$\begin{aligned}\tilde{H}^{\gamma^\mu}(y, \underline{x}) = & -4 \frac{2\pi e g^2}{\sqrt{2}s} \frac{\delta^{ab}}{2N_c} \{ y \bar{y} s K_0(\mu |\underline{x}|) \textcolor{green}{e_{\gamma T}^\mu} \\ & - (y - \bar{y}) i \mu \frac{\textcolor{red}{e_{\gamma T} \cdot \underline{x}}}{|\underline{x}|} K_1(\mu |\underline{x}|) ((1 - e^{i\underline{k} \cdot \underline{x}})(1 - e^{-i\underline{k} \cdot \underline{x}}) - 1) \textcolor{blue}{p_2^\mu} \}\end{aligned}$$

- Hard part  $\Gamma \equiv \gamma_5 \gamma^\mu$

$$\begin{aligned}\tilde{H}^{\gamma_5 \gamma^\mu}(y, \underline{x}) = & 4i \frac{2\pi e g^2}{\sqrt{2}s} \frac{\delta^{ab}}{2N_c} \varepsilon^{\mu\nu\rho\sigma} \{ -y \bar{y} K_0(\mu |\underline{x}|) (e_{\gamma T \nu} p_{1\rho} p_{2\sigma} + p_{2\nu} p_{1\rho} e_{\gamma T \sigma}) \\ & - i \mu K_1(\mu |\underline{x}|) ((1 - e^{i\underline{k} \cdot \underline{x}})(1 - e^{-i\underline{k} \cdot \underline{x}}) - 1) (y \textcolor{blue}{e_{\gamma T \nu}} \frac{x_{\perp\rho}}{|\underline{x}|} p_{2\sigma} - \bar{y} p_{2\nu} \frac{x_{\perp\rho}}{|\underline{x}|} p_{1\sigma}) \}\end{aligned}$$

- Equations of motion:

$$\text{Termes} \times (1 - e^{i\underline{k} \cdot \underline{x}})(1 - e^{-i\underline{k} \cdot \underline{x}}) + \text{Termes} \times \underbrace{2y \bar{y} \varphi_3(y) + (y - \bar{y}) \varphi_{1T}(y) + \varphi_{AT}(y)}_{=0}$$