# NNLL/NLO resummation for photon pair production in ResBos

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## References

*Balazs, Berger, Mrenna, Yuan, PRD 57 (1998);*
*Balazs, Nadolsky, Schmidt, Yuan, PLB 489, 157 (2000);*
*Nadolsky, Schmidt, PLB 558, 63 (2003);*
*C. Balazs, E. Berger, Nadolsky, C.-P. Yuan, Phys. Lett., B637, 235 (2006); PRD 76, 013008 (2007) and 013009 (2007)*
*Zhao Li, C.-P. Yuan, et al., in preparation*

March 30, 2012

# What is ResBos?

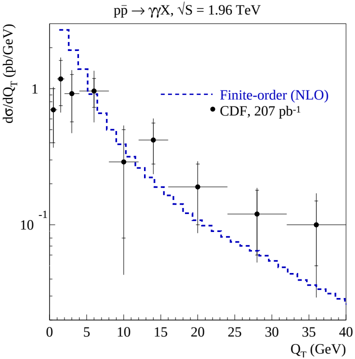
ResBos is a program for NNLL/NLO transverse momentum ($Q_T$) resummation in Drell-Yan-like processes, including

- Drell-Yan pair production ($e^+e^-$)
- Production and decay of $W$, $Z$, Higgs bosons in SM and MSSM
- Photon pair production

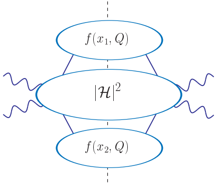
To speed up MC integration and perform matching, resummation is performed in two stages.

1. Tables of structure functions for resummed cross sections (not including final-state decays) are produced by a program Legacy.
2. ResBos interpolates the resummed tables, includes final-state decays, and performs Monte-Carlo integration with user-specified cuts. The output is in the form of fully differential cross section for the final EW particles. (All hadronic final states are integrated out.)

# $Q_T$ resummation in impact parameter space




$$Q_T^2 \sim Q^2$$

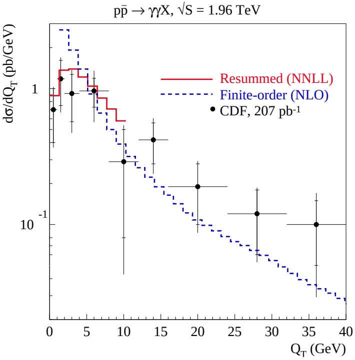



Collinear factorization at NLO:
reliable at $Q_T^2 \approx Q^2$;
unstable at $Q_T^2 \ll Q^2$

$$\frac{d\sigma}{dQ^2 dy dQ_T^2} = \sum_{partons} \int dx_1 dx_2 f_a(x_1, Q) f_b(x_2, Q) \left|\mathcal{H}(ab \to V)\right|^2$$

$$\left(\frac{d\sigma}{dQ_T^2}\right)_{Q_T^2 \ll Q^2} \approx \sum_{k=0}^{\infty} \alpha_s^k \left[ c_k \delta(\vec{Q}_T) + Q_T^{-2} \cdot \sum_{n=0}^{2k-1} d_{nk} \ln^n(Q^2/Q_T^2) \right]$$

# $Q_T$ resummation in impact parameter space

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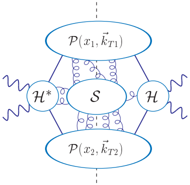
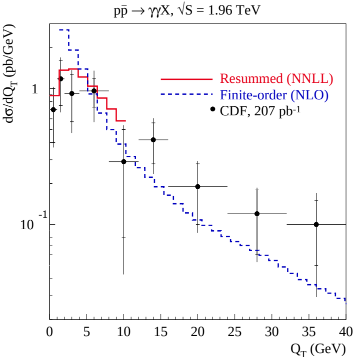



Factorization in terms of a soft factor $S$, $\vec{k}_T$-dependent PDF's $\mathcal{P}_a$, and LO hard vertex $\mathcal{H}_{ab}^{LO}$

$$\frac{d\sigma}{dQ^2 dy dQ_T^2} = \sum_{partons} \int \frac{d^2 b}{(2\pi)^2} e^{-i\vec{Q}_T \cdot \vec{b}} \widetilde{W}_{ab}(b, Q, x_1, x_2)$$

$$\widetilde{W}_{ab}(b, Q, x_1, x_2) = \left|\mathcal{H}_{ab}^{LO}\right|^2 e^{-\mathcal{S}(b,Q)} \overline{\mathcal{P}}_a(x_1, b) \overline{\mathcal{P}}_b(x_2, b)$$

# $Q_T$ resummation in impact parameter space

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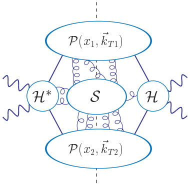


The region $b \lesssim 1$ GeV$^{-1}$ , where $\mathcal{S}$ and $\overline{\mathcal{P}} \approx [\mathcal{C} \otimes f]$ are computed in PQCD, dominates at $Q \gtrsim 10$ GeV

$$\frac{d\sigma}{dQ^2 dy dQ_T^2} = \sum_{partons} \int \frac{d^2 b}{(2\pi)^2} e^{-i\vec{Q}_T \cdot \vec{b}} \widetilde{W}_{ab}(b, Q, x_1, x_2)$$

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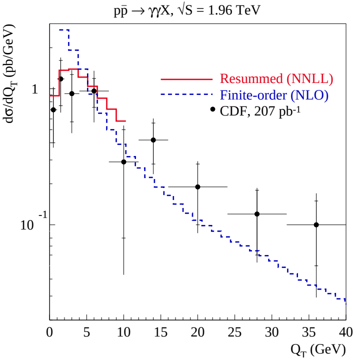
# $Q_T$ resummation in impact parameter space

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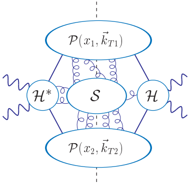


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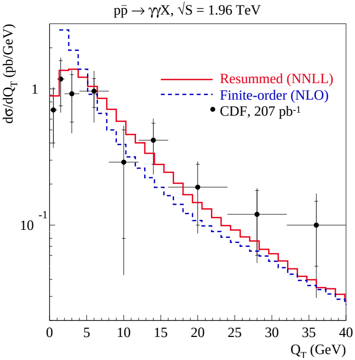
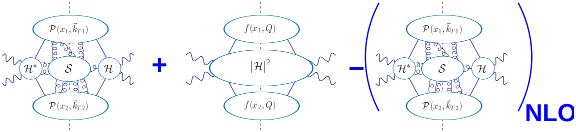
At NNLL accuracy, we include perturbative coefficients up to orders $\mathcal{A}^{(3)}$ *(from Moch, Vermaseren, Vogt, 2004)*; $\mathcal{B}^{(2)}$ ; and $\mathcal{C}^{(1)}$

# $Q_T$ resummation in impact parameter space



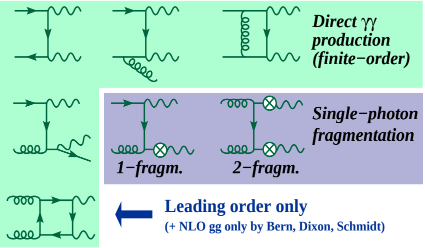

$$Q_T^2 \lesssim Q^2$$

Add small-$Q_T$ and large-$Q_T$ X-sections, subtract the overlap



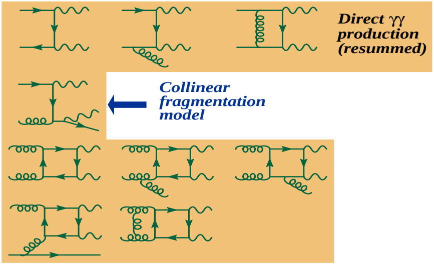
# Compare ResBos and DIPHOX *(Binoth, Guillet, Pilon, Werlen)*

Binoth, Guillet, Pilon, Werlen, 2001



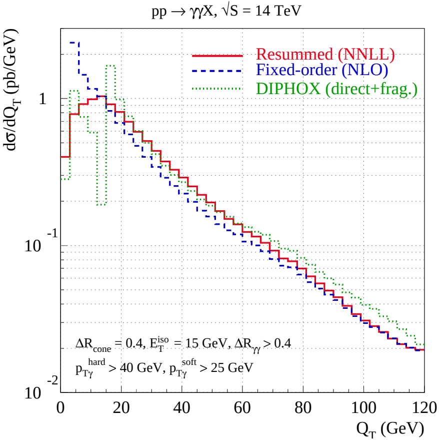

- fixed order in $\alpha_s$, no $gq_S$
- predicts inclusive & large-$Q_T$ rates including fragmentation @ NLO
- discontinuities in $d\sigma/dQ_T$ at $Q_T \to 0$

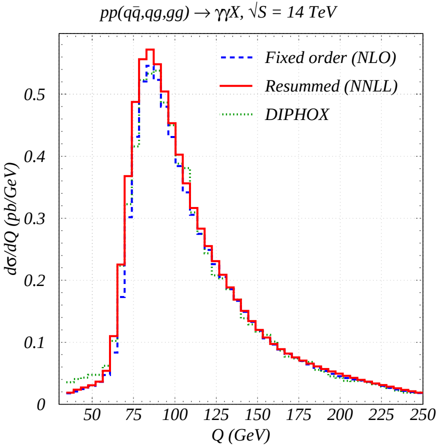
Balazs, Berger, Nadolsky, Yuan, 2006




- resummed ISR
- a model for fragmentation
- agrees with the inclusive & large-$Q_T$ from DIPHOX
- $d\sigma/dQ_T$ is finite everywhere
- NNLL/NLO resummation of the $gg$ spin-flip term

# $\gamma\gamma$ production at the LHC (ATLAS cuts, $E_T^{iso} = 15$ GeV, $\Delta R = 0.4$)



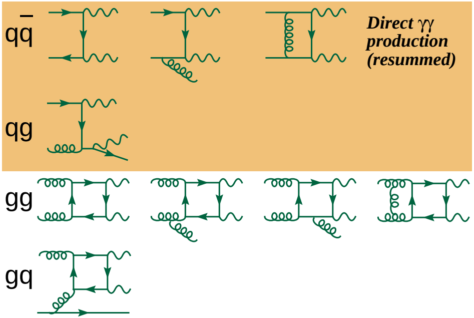




- DIPHOX agrees with the resummation at large $Q_T$; exhibits integrable logarithmic singularities at $Q_T < E_T^{iso}$
- RESBOS shows a mild discontinuity at $Q_T = E_T^{iso}$

# Direct diphotons

The dominant production mode; evaluated up to NLO in $\alpha_s$

Balazs, Berger, Nadolsky, Yuan, 2006




## $q\bar{q} + qg$ channel

- $qg$ scattering is strongly enhanced at the LHC by photon radiation off final-state quarks
- a fragmentation model regularizes the final-state (FS) singularity in $q_1 g_2 \to \gamma_3 \gamma_4 q_5$ when $3||5$ or $4||5$
- the inclusive rate agrees with that in DIPHOX

# Direct diphotons

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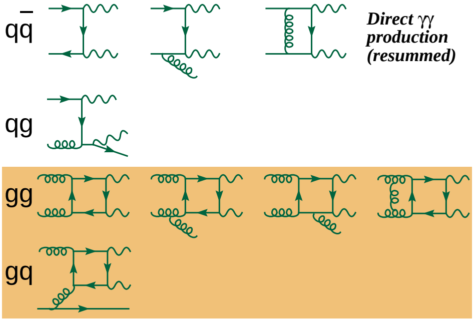
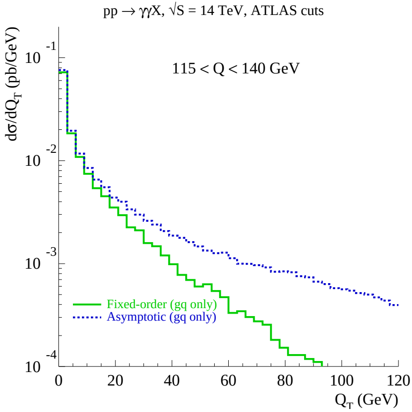
Balazs, Berger, Nadolsky, Yuan, 2006




## $gg + gq$ channel (via the quark box)

- contributes $\sim 20\%$ at the LHC
- NLO $gg$ matrix elements:
  *Balazs, PN, Schmidt, Yuan; de Florian, Kunzst; Bern, De Freitas, Dixon; Bern, Dixon, Schmidt*
- $gq$ matrix element: derived from the $q\bar{q}ggg$ matrix element

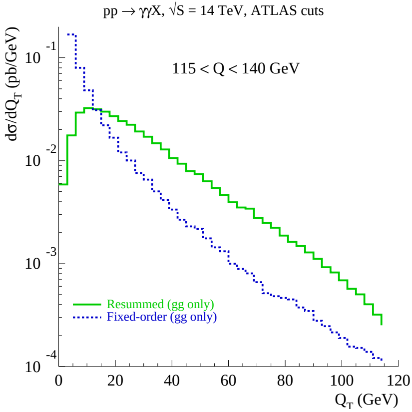
# Resummation for $gg + gq \to \gamma\gamma X$ at $\mathcal{O}(\alpha_s^3)$




## $gq$ 5-leg matrix element

- $\mathcal{M}_5(gq \to \gamma\gamma q)$ correctly reproduces its known behavior in the collinear asymptotic limit ($Q_T \to 0$)

# Resummation for $gg + gq \to \gamma\gamma X$ at $\mathcal{O}(\alpha_s^3)$




## $gq$ 5-leg matrix element

- The $gq$ contribution is subleading in the inclusive rate, but has more pronounced effect at high $Q_T$ (improves matching)

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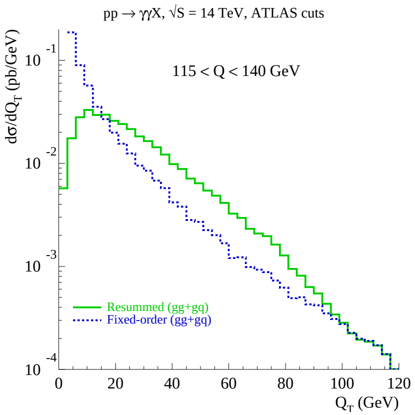


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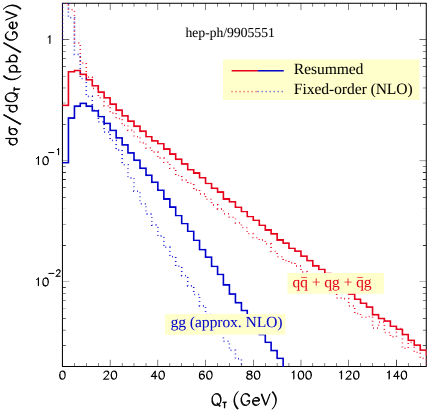
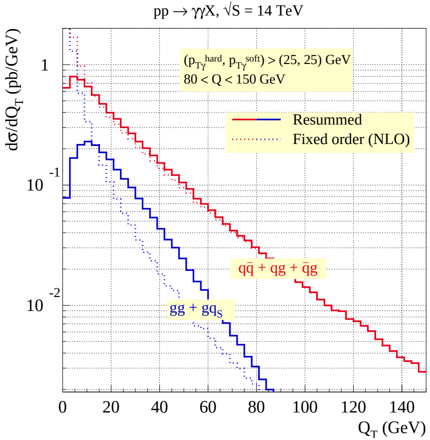
# Matching at large $Q_T$

ResBos 1999




ResBos 2007




At large $Q_T$, the new resummed cross sections reduce to fixed-order cross sections (more reliable theory predictions)

# Spin-flip collinear term in the $g_1 g_2 \to \gamma_3 \gamma_4 g_5$ amplitude

**(PRD 76, 013008 (2007); see also Bern, Dixon, Schmidt, hep-ph/0206194)**

- includes a novel $1/Q_T^2$ term proportional to interference between $2 \to 2$ matrix elements with opposite spins of gluon 1 and a universal spin-flip splitting function $P'_{g/g}(x)$
- is also present in the Catani-Seymour dipole formalism
- arises because of incomplete factorization of helicity dependence in TMD distributions of linearly polarized gluons
- affects dependence on the azimuthal ($\varphi_*$) and polar ($\theta_*$) angles of photons in the Collins-Soper $\gamma\gamma$ rest frame
- The full formalism for resummation of the spin-flip term is developed by *Catani & Grazzini, arXiv:1011.3918*
- Resummation of the spin-flip term is now implemented in ResBos (Z. Li, C.-P. Yuan, in preparation)

# Spin-flip collinear term in the $g_1 g_2 \to \gamma_3 \gamma_4 g_5$ amplitude

**(PRD 76, 013008 (2007); see also Bern, Dixon, Schmidt, hep-ph/0206194)**

$$|\mathcal{M}_5(1,2,3,4,5)|^2 \xrightarrow{5\|1} \frac{\sigma_g^{(1)}}{2\widehat{x}_1 p_1 \cdot p_5} \left\{ P_{g/g}(\widehat{x}_1) L_g(\theta_\star) + P'_{g/g}(\widehat{x}_1)\, L'_g(\theta_\star)\, \cos 2\varphi_\star \right\}$$

$P_{g/g} L_g$ is the usual collinear term, with

$$, \qquad P_{g/g} = 2C_A \left[ \frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) \right] + \beta_0 \delta(1-x),$$

$$L_g(\theta_\star) \equiv \sum_{\lambda_1,\lambda_2,\lambda_3,\lambda_4=\pm 1} |\mathcal{M}_4(\lambda_1,\lambda_2,\lambda_3,\lambda_4)|^2$$

$P'_{g/g} L'_g \cos 2\varphi_\star$ is the interference (spin-flip) term, with

$$P'_{g/g}(x) = 2C_A(1-x)/x$$

$$L'_g(\theta_\star)\cos 2\varphi_\star = \sum_{\lambda_1,\lambda_2,\lambda_3,\lambda_4=\pm 1} \mathcal{M}_4^*(\lambda_1,\lambda_2,\lambda_3,\lambda_4)\mathcal{M}_4(-\lambda_1,\lambda_2,\lambda_3,\lambda_4)$$

# Main features of the current ResBos version

*C. Balazs, E. Berger, P. N., C.-P. Yuan, Phys. Lett., B637, 235 (2006), PRD 76, 013008 (2007) and 013009 (2007)*

- NNLL/NLO $Q_T$ resummation for $gg + gq \to \gamma\gamma$ at two loops
- combined with the NNLL/NLO **direct** $q\bar{q} + qg$ cross section
- sharp and smooth cone **isolation;** a tunable **fragmentation model**
- a recent model for nonperturbative resummed contributions

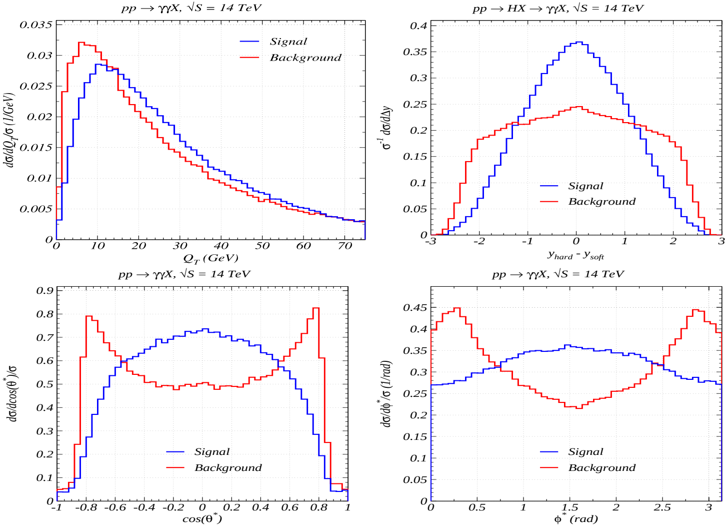
  *(A. Konychev, **P. N.**, PLB 633, 710 (2006))*
- resummation of the spin-flip contribution to azimuthal cross sections
- no singularities or negative cross sections
- streamlined and debugged MC integrator ResBos; automated matching of PAW/ROOT ntuples for the resummed and fixed-order cross sections

# NNLL/NLO distributions for Higgs $\rightarrow \gamma\gamma$ signal and background (ResBos, normalized; $M_H = 130$ GeV, $128 < Q < 132$ GeV)

$Q_T$ and $y_{\gamma_1} - y_{\gamma_2}$ in the lab frame

Decay angles $\theta_*, \varphi_*$ in the $\gamma\gamma$ rest frame

no singularities, in contrast to the fixed-order rate



# Backup slides

# Main differences with our previous calculation

An NLL/NLO calculation for the LHC was first published in Balazs, P.N., Schmidt, Yuan, PLB 489, 157 (2000) (hep-ph/9905551). Compared to that calculation, the latest ResBos predicts

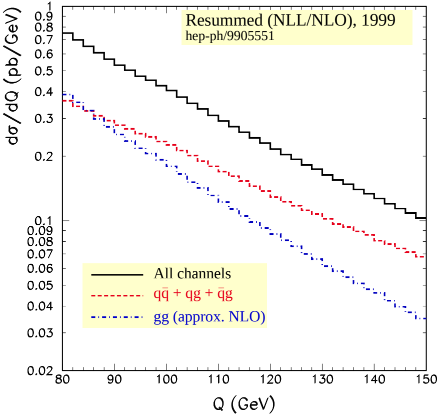
- a 30% increase in the $q\bar{q} + qg$ cross section (due to the enhanced $qg$ contribution, different isolation prescription)
- a 25% decrease in the $gg + gq$ cross section (due to virtual two-loop and $gq$ contributions)
- a 10-15% increase in the combined cross section
- more realistic isolation (reject events with hadronic $E_T > E_T^{iso}$ in the isolation cone $\Delta R$)
- improved matching of resummed and fixed-order cross sections at large $Q_T$
- positive distributions in $\gamma\gamma$ decay angles $\theta_*, \varphi_*$ in the $\gamma\gamma$ rest frame (Collins-Soper frame)

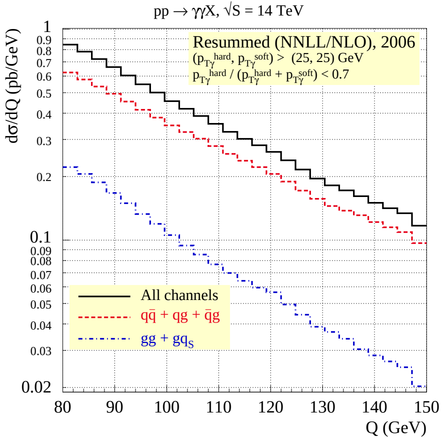
# Numerical comparison

I will compare the new and old distributions for the same cuts as in hep-ph/9905551:

- $80 < M_{\gamma\gamma} \equiv Q < 150$ GeV, $|y_\gamma| < 2.5$, $\Delta R_{\gamma\gamma} > 0.4$
- $p_{T\gamma}^{hard}, p_{T\gamma}^{soft} > 25$ GeV
- $p_{T\gamma}^{hard}/(p_{T\gamma}^{hard} + p_{T\gamma}^{soft}) < 0.7$ (if specified)
- The new calculation removes the FS $qg$ singularity by quasi-experimental isolation at $\Delta R_{\gamma q_5} < 0.4$, $E_{Tq_5} = Q_T > E_T^{iso}$; subtraction of the FS collinear contribution at $E_{Tq_5} = Q_T \leq E_T^{iso}$
- The old calculation subtracts the FS contribution for $\Delta R_{\gamma q_5} < 0.4$, $(p_\gamma + p_{q_5})^2 < 25 - 100$ GeV$^2$, and any $E_{Tq_5}$ *(as in Balazs, Berger, Mrenna, Yuan, 1998)*

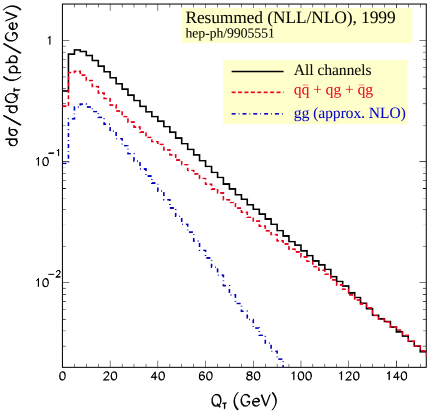
# Invariant mass ($Q$) distributions

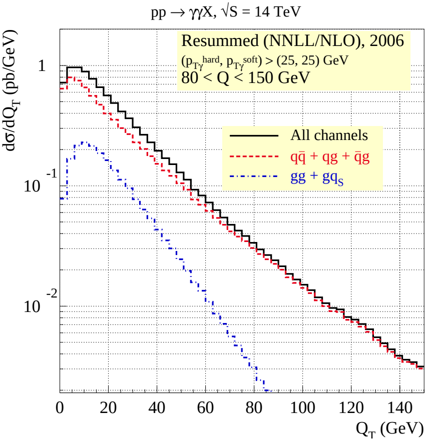



New ResBos: increased $q\bar{q} + qg$ contribution; reduced $gg + gq_S$ contribution; larger total cross section

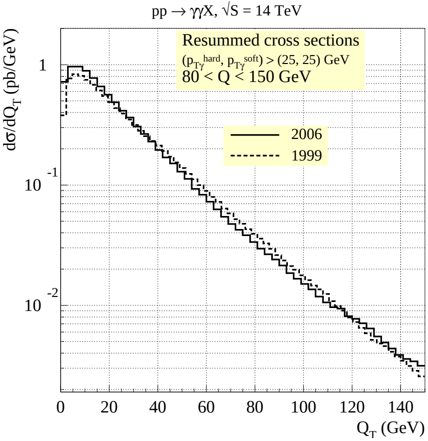
# Transverse momentum ($Q_T$) distributions



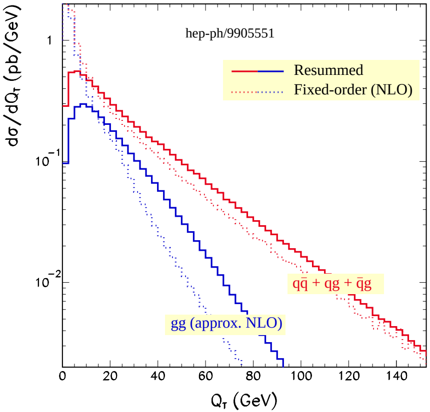



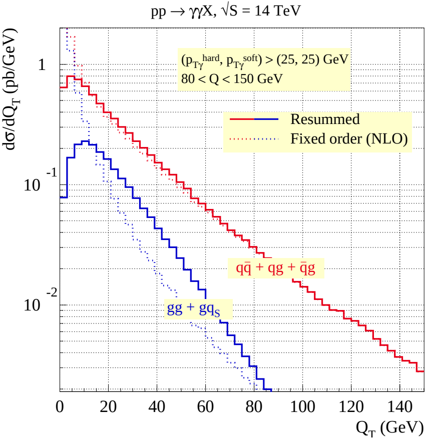

$q\bar{q} + qg$ cross section dominates at large $Q_T$, where it is about the same for the chosen cuts in both calculations

# $Q_T$ distributions in all channels



# Matching at large $Q_T$




At large $Q_T$, the new resummed cross sections reduce to fixed-order cross sections (more reliable theory predictions)