Scaling of the CKM Matrix in the 5D MSSM

Ahmad TARHINI
Institut de Physique Nucléaire de Lyon (IPNL)

Based on: *arXiv:1110.1942 [hep-ph]* :
Scaling of the CKM Matrix in the 5D MSSM

Collaborators : Aldo Deandrea, Alan Cornell, Lu-Xin Liu

GDR Terascale, CPPM Marseille, 12 October 2011

Outline

Introduction

- We discuss a five dimensional $\mathcal{N} = 1$ supersymmetric model compactified on the S^1/Z_2 orbifold to test effects of extra-dimension on the quark Yukawa couplings and the CKM matrix observables.
-

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- Different possibilities for the matter fields are discussed, where they are in the bulk or localised to the brane. The two possibilities give rise to quite different behaviours.
- β-function can be derived more easily in the superfield formalism.

Introduction

- We discuss a five dimensional $\mathcal{N} = 1$ supersymmetric model compactified on the S^1/Z_2 orbifold to test effects of extra-dimension on the quark Yukawa couplings and the CKM matrix observables.
- $$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$
- Different possibilities for the matter fields are discussed, where they are in the bulk or localised to the brane. The two possibilities give rise to quite different behaviours.
- β-function can be derived more easily in the superfield formalism.

Introduction

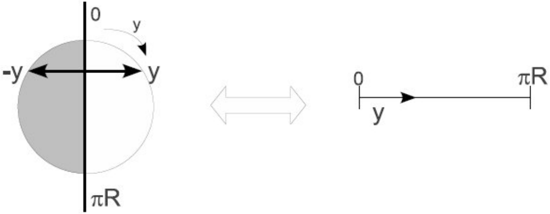
- We discuss a five dimensional $\mathcal{N} = 1$ supersymmetric model compactified on the S^1/Z_2 orbifold to test effects of extra-dimension on the quark Yukawa couplings and the CKM matrix observables.
- $$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$
- Different possibilities for the matter fields are discussed, where they are in the bulk or localised to the brane. The two possibilities give rise to quite different behaviours.
- β-function can be derived more easily in the superfield formalism.

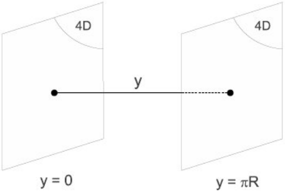
Introduction

- We discuss a five dimensional $\mathcal{N} = 1$ supersymmetric model compactified on the S^1/Z_2 orbifold to test effects of extra-dimension on the quark Yukawa couplings and the CKM matrix observables.
- $$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$
- Different possibilities for the matter fields are discussed, where they are in the bulk or localised to the brane. The two possibilities give rise to quite different behaviours.
- β-function can be derived more easily in the superfield formalism.

orbifold & brane

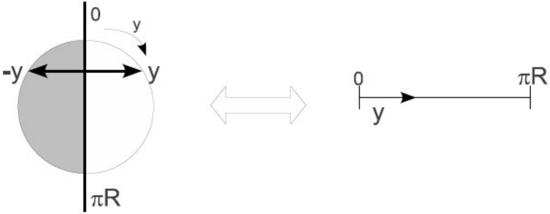
- To recover MSSM at low energy, we need **chiral zero modes for fermions** so we compactify the fifth dimention on the orbifold S_1/Z_2.
- Z_2 symmetry identifies $y \longrightarrow -y$ and we have two orbifold fixed points invariant under Z_2 called branes.

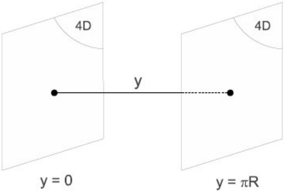




orbifold & brane

- To recover MSSM at low energy, we need **chiral zero modes for fermions** so we compactify the fifth dimention on the orbifold S_1/Z_2.
- Z_2 symmetry identifies $y \to -y$ and we have two orbifold fixed points invariant under Z_2 called branes.





5D MSSM (gauge sector)

- Described by a 5D $\mathcal{N} = 1$ vector supermultiplet which consists (on-shell) of a 5D vector field A^M, a real scalar S and two gauginos λ and λ'.
- 5D $\mathcal{N} = 1$ supersymmetric action :

$$S_g = \int d^5x \frac{1}{2kg^2} \mathrm{Tr} \left[-\frac{1}{2} F^{MN} F_{MN} - D^M S D_M S - i\bar{\lambda}\Gamma^M D_M \lambda \right.$$

$$\left. - i\bar{\lambda}'\Gamma^M D_M \lambda' + (\bar{\lambda} + \bar{\lambda}')[S, \lambda + \lambda'] \right]$$

$D_M = \partial_M + iA_M$ and $\Gamma^M = (\gamma^\mu, i\gamma^5)$. $F^{MN} = -\frac{i}{g}[D^M, D^N]$ and k normalises the trace over the generators of the gauge groups.

- Decomposition of the 5D supercharge (which is a Dirac spinor) into two Majorana-type supercharges which constitute a $\mathcal{N} = 2$ superalgebra in 4D.

5D MSSM (gauge sector)

- Described by a 5D $\mathcal{N} = 1$ vector supermultiplet which consists (on-shell) of a 5D vector field A^M, a real scalar S and two gauginos λ and λ'.
- 5D $\mathcal{N} = 1$ supersymmetric action :

$$S_g = \int d^5x \frac{1}{2kg^2} \mathrm{Tr} \left[-\frac{1}{2} F^{MN} F_{MN} - D^M S D_M S - i\bar{\lambda}\Gamma^M D_M \lambda \right.$$

$$\left. - \, i\bar{\lambda}'\Gamma^M D_M \lambda' + (\bar{\lambda} + \bar{\lambda}')[S, \lambda + \lambda'] \right]$$

$D_M = \partial_M + iA_M$ and $\Gamma^M = (\gamma^\mu, i\gamma^5)$. $F^{MN} = -\frac{i}{g}[D^M, D^N]$ and k normalises the trace over the generators of the gauge groups.

- Decomposition of the 5D supercharge (which is a Dirac spinor) into two Majorana-type supercharges which constitute a $\mathcal{N} = 2$ superalgebra in 4D.

5D MSSM (gauge sector)

- Described by a 5D $\mathcal{N} = 1$ vector supermultiplet which consists (on-shell) of a 5D vector field A^M, a real scalar S and two gauginos λ and λ'.
- 5D $\mathcal{N} = 1$ supersymmetric action :

$$S_g = \int d^5x \frac{1}{2kg^2} \mathrm{Tr} \left[-\frac{1}{2} F^{MN} F_{MN} - D^M S D_M S - i\bar{\lambda}\Gamma^M D_M \lambda \right.$$

$$\left. - i\bar{\lambda}'\Gamma^M D_M \lambda' + (\bar{\lambda} + \bar{\lambda}')[S, \lambda + \lambda'] \right]$$

$D_M = \partial_M + iA_M$ and $\Gamma^M = (\gamma^\mu, i\gamma^5)$. $F^{MN} = -\frac{i}{g}[D^M, D^N]$ and k normalises the trace over the generators of the gauge groups.

- Decomposition of the 5D supercharge (which is a Dirac spinor) into two Majorana-type supercharges which constitute a $\mathcal{N} = 2$ superalgebra in 4D.

5D MSSM

- One can rearrange these fields in terms of a $\mathcal{N} = 2$ **4D vector supermultiplet**, $\Omega = (V\,, \chi)$:
  - $V : \mathcal{N} = 1$ vector supermultiplet containing A^μ and λ.
  - $\chi : \mathcal{N} = 1$ chiral supermultiplet containing λ' and $S' = S + iA^5$.
- 4D $\mathcal{N} = 1$ action :

$$S_g = \int \mathrm{d}^5x\mathrm{d}^2\theta\mathrm{d}^2\overline{\theta}\frac{1}{4kg^2}\mathrm{Tr}\left[\frac{1}{4}(W^\alpha W_\alpha \delta(\overline{\theta}^2) + h.c) + (e^{-2gV}\nabla_y e^{2gV})^2\right]$$

$W^\alpha = -\frac{1}{4}\overline{D}^2 e^{-2gV} D_\alpha e^{2gV}$. D_α is the covariant derivative in the 4D $\mathcal{N} = 1$ superspace and $\nabla_y = \partial_y + \chi$.

- Action of Matter sector and its coupling to gauge sector :
  $S = \int \mathrm{d}^5x\mathrm{d}^2\theta\mathrm{d}^2\overline{\theta}\left[\overline{\Phi}e^{2gV}\Phi + \Phi^c e^{-2gV}\overline{\Phi}^c + (\Phi^c(\nabla_5 + m)\Phi\delta(\overline{\theta}^2) + h.c)\right]$

5D MSSM

- One can rearrange these fields in terms of a $\mathcal{N} = 2$ **4D vector supermultiplet**, $\Omega = (V, \chi)$:
  • $V : \mathcal{N} = 1$ vector supermultiplet containing A^μ and λ.
  • $\chi : \mathcal{N} = 1$ chiral supermultiplet containing λ' and $S' = S + iA^5$.
- 4D $\mathcal{N} = 1$ action :

$$S_g = \int \mathrm{d}^5x\mathrm{d}^2\theta\mathrm{d}^2\overline{\theta}\frac{1}{4kg^2}\mathrm{Tr}\left[\frac{1}{4}(W^\alpha W_\alpha\delta(\overline{\theta}^2) + h.c) + (e^{-2gV}\nabla_y e^{2gV})^2\right]$$

  $W^\alpha = -\frac{1}{4}\overline{D}^2 e^{-2gV}D_\alpha e^{2gV}$. D_α is the covariant derivative in the 4D $\mathcal{N} = 1$ superspace and $\nabla_y = \partial_y + \chi$.
- Action of Matter sector and its coupling to gauge sector :
  $S = \int \mathrm{d}^5x\mathrm{d}^2\theta\mathrm{d}^2\overline{\theta}\left[\overline{\Phi}e^{2gV}\Phi + \Phi^c e^{-2gV}\overline{\Phi}^c + (\Phi^c(\nabla_5 + m)\Phi\delta(\overline{\theta}^2) + h.c)\right]$

5D MSSM

- One can rearrange these fields in terms of a $\mathcal{N}=2$ **4D vector supermultiplet**, $\Omega=(V\,,\chi)$:
  • V : $\mathcal{N}=1$ vector supermultiplet containing A^μ and λ.
  • χ : $\mathcal{N}=1$ chiral supermultiplet containing λ' and $S'=S+iA^5$.
- 4D $\mathcal{N}=1$ action :

$$S_g=\int \mathrm{d}^5x\mathrm{d}^2\theta\mathrm{d}^2\overline{\theta}\frac{1}{4kg^2}\mathrm{Tr}\left[\frac{1}{4}(W^\alpha W_\alpha\delta(\overline{\theta}^2)+\,h.c)+(e^{-2gV}\nabla_y e^{2gV})^2\right]$$

  $W^\alpha=-\frac{1}{4}\overline{D}^2e^{-2gV}D_\alpha e^{2gV}$. D_α is the covariant derivative in the 4D $\mathcal{N}=1$ superspace and $\nabla_y=\partial_y+\chi$.
- Action of Matter sector and its coupling to gauge sector :
  $S=\int \mathrm{d}^5x\mathrm{d}^2\theta\mathrm{d}^2\overline{\theta}\left[\overline{\Phi}e^{2gV}\Phi+\Phi^c e^{-2gV}\overline{\Phi}^c+(\Phi^c(\nabla_5+m)\Phi\delta(\overline{\theta}^2)+\,h.c)\right]$

Fields

- The χ-field should be odd under Z_2 symmetry because it appears togother with a derivative ∂_y, whereas V is even.
- For the two matter superfields, we choose Φ to be even and the conjugate Φ^c to be odd. Only the even fields have zero modes.
- The Fourier decomposition of the fields :

$$V(x,y) = \frac{1}{\sqrt{\pi R}}\left[V^{(0)}(x) + \sqrt{2}\sum_{n\geq 1} V^{(n)}(x)\cos\left(\frac{ny}{R}\right)\right]$$

$$\chi(x,y) = \sqrt{\frac{2}{\pi R}}\sum_{n\geq 1}\chi^{(n)}(x)\sin\left(\frac{ny}{R}\right)$$

$$\Phi(x,y) = \frac{1}{\sqrt{\pi R}}\left[\Phi^{(0)}(x) + \sqrt{2}\sum_{n\geq 1}\Phi^{(n)}(x)\cos\left(\frac{ny}{R}\right)\right]$$

$$\Phi^c(x,y) = \sqrt{\frac{2}{\pi R}}\sum_{n\geq 1}\Phi^{c(n)}\sin\left(\frac{ny}{R}\right) \quad ,$$

Fields

- The χ-field should be odd under Z_2 symmetry because it appears togother with a derivative ∂_y, whereas V is even.
- For the two matter superfields, we choose Φ to be even and the conjugate Φ^c to be odd. Only the even fields have zero modes.
- The Fourier decomposition of the fields :

$$
\begin{aligned}
V(x,y) &= \frac{1}{\sqrt{\pi R}}\left[V^{(0)}(x) + \sqrt{2}\sum_{n\geq 1} V^{(n)}(x)\cos\left(\frac{ny}{R}\right)\right] \\
\chi(x,y) &= \sqrt{\frac{2}{\pi R}}\sum_{n\geq 1}\chi^{(n)}(x)\sin\left(\frac{ny}{R}\right) \\
\Phi(x,y) &= \frac{1}{\sqrt{\pi R}}\left[\Phi^{(0)}(x) + \sqrt{2}\sum_{n\geq 1}\Phi^{(n)}(x)\cos\left(\frac{ny}{R}\right)\right] \\
\Phi^c(x,y) &= \sqrt{\frac{2}{\pi R}}\sum_{n\geq 1}\Phi^{c(n)}\sin\left(\frac{ny}{R}\right) ,
\end{aligned}
$$

Fields

- The χ-field should be odd under Z_2 symmetry because it appears togother with a derivative ∂_y, whereas V is even.
- For the two matter superfields, we choose Φ to be even and the conjugate Φ^c to be odd. Only the even fields have zero modes.
- The Fourier decomposition of the fields :

$$
\begin{aligned}
V(x,y) &= \frac{1}{\sqrt{\pi R}}\left[V^{(0)}(x) + \sqrt{2}\sum_{n\geq 1} V^{(n)}(x)\cos\left(\frac{ny}{R}\right)\right] \\
\chi(x,y) &= \sqrt{\frac{2}{\pi R}}\sum_{n\geq 1}\chi^{(n)}(x)\sin\left(\frac{ny}{R}\right) \\
\Phi(x,y) &= \frac{1}{\sqrt{\pi R}}\left[\Phi^{(0)}(x) + \sqrt{2}\sum_{n\geq 1}\Phi^{(n)}(x)\cos\left(\frac{ny}{R}\right)\right] \\
\Phi^c(x,y) &= \sqrt{\frac{2}{\pi R}}\sum_{n\geq 1}\Phi^{c(n)}\sin\left(\frac{ny}{R}\right) \quad ,
\end{aligned}
$$

Low energy spectrum

- At energies well below the scale R^{-1}, where the massive Kaluza-Klein states decouple, only the zero modes remain in the spectrum and we assume that physics is described by the usual MSSM.
- The matter superfields (and Higgs superfields) of the MSSM will be identified with a Φ^0 superfield and the gauge fields with a V^0 mode.
- Brane interactions contain Yukawa-type couplings :

$$S_{brane} = \int \mathrm{d}^8 z \mathrm{d}y \delta(y) \left[(\frac{1}{6} \tilde{\lambda}_{ijk} \Phi_i \Phi_j \Phi_k + ..) \delta(\bar{\theta}) + h.c. \right]$$

Low energy spectrum

- At energies well below the scale R^{-1}, where the massive Kaluza-Klein states decouple, only the zero modes remain in the spectrum and we assume that physics is described by the usual MSSM.
- The matter superfields (and Higgs superfields) of the MSSM will be identified with a Φ^0 superfield and the gauge fields with a V^0 mode.
- Brane interactions contain Yukawa-type couplings :

$$S_{brane} = \int d^8z dy \delta(y) \left[(\frac{1}{6}\tilde{\lambda}_{ijk}\Phi_i\Phi_j\Phi_k + ..)\delta(\bar{\theta}) + h.c. \right]$$

Low energy spectrum

- At energies well below the scale R^{-1}, where the massive Kaluza-Klein states decouple, only the zero modes remain in the spectrum and we assume that physics is described by the usual MSSM.
- The matter superfields (and Higgs superfields) of the MSSM will be identified with a Φ^0 superfield and the gauge fields with a V^0 mode.
- Brane interactions contain Yukawa-type couplings :

$$S_{brane} = \int \mathrm{d}^8 z \mathrm{d}y \delta(y) \left[(\frac{1}{6} \tilde{\lambda}_{ijk} \Phi_i \Phi_j \Phi_k + ..) \delta(\overline{\theta}) + h.c. \right]$$

Equations of gauge couplings

- The evolution of g_i in 4D :

$$16\pi^2 \frac{dg_i}{dt} = b_i {g_i}^3$$

in 4D MSSM b_i read $(b_1, b_2, b_3) = (\frac{33}{5}, 1, -3)$

- If we consider our 5D theory as effective up to a scale Λ, then between the compactification scale R^{-1} (first KK states are excited) and the cut-off scale Λ, there are finite quantum corrections from the ΛR number of KK states.
- As a result, once the KK states are excited, these couplings exhibit power law dependencies on Λ.

$$\beta^{4D} \rightarrow \beta^{4D} + (S(\mu) - 1)\,\tilde{\beta}$$

$\tilde{\beta}$ **is a generic contribution from a single KK level**. $S(\mu) = \mu R$.

Equations of gauge couplings

- The evolution of g_i in 4D :

$$16\pi^2 \frac{dg_i}{dt} = b_i {g_i}^3$$

in 4D MSSM b_i read $(b_1, b_2, b_3) = (\frac{33}{5}, 1, -3)$

- If we consider our 5D theory as effective up to a scale Λ, then between the compactification scale R^{-1} (first KK states are excited) and the cut-off scale Λ, there are finite quantum corrections from the ΛR number of KK states.
- As a result, once the KK states are excited, these couplings exhibit power law dependencies on Λ.

$$\beta^{4D} \rightarrow \beta^{4D} + (S(\mu) - 1)\,\tilde{\beta}$$

$\tilde{\beta}$ **is a generic contribution from a single KK level**. $S(\mu) = \mu R$.

Equations of gauge couplings

- The evolution of g_i in 4D :

$$16\pi^2 \frac{dg_i}{dt} = b_i {g_i}^3$$

in 4D MSSM b_i read $(b_1, b_2, b_3) = (\frac{33}{5}, 1, -3)$

- If we consider our 5D theory as effective up to a scale Λ, then between the compactification scale R^{-1} (first KK states are excited) and the cut-off scale Λ, there are finite quantum corrections from the ΛR number of KK states.
- As a result, once the KK states are excited, these couplings exhibit power law dependencies on Λ.

$$\beta^{4D} \rightarrow \beta^{4D} + (S(\mu) - 1)\,\tilde{\beta}$$

$\tilde{\beta}$ **is a generic contribution from a single KK level**. $S(\mu) = \mu R$.

Gauge couplings

- We use $d(\ln \frac{\mu}{M_Z}) = dt$, for $\mu = M_Z e^t$ where M_Z is the renormalisation point, and $S(t) = e^t M_Z R$.
- In terms of the scale parameter t :

$$16\pi^2 \frac{dg_i}{dt} = [b_i + (S(t) - 1)\tilde{b}_i]{g_i}^3$$

- $(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3) = (\frac{6}{5}, -2, -6) + 4\eta$;
  η represents the number of generations of fermions in the bulk.
- If all fields propagating in the bulk, i.e. $\eta = 3$, $\tilde{b}_i = (\frac{66}{5}, 10, 6)$
  For all fields on the brane, i.e. $\eta = 0$, $\tilde{b}_i = (\frac{6}{5}, -2, -6)$

Gauge couplings

- We use $d(\ln \frac{\mu}{M_Z}) = dt$, for $\mu = M_Z e^t$ where M_Z is the renormalisation point, and $S(t) = e^t M_Z R$.
- In terms of the scale parameter t :

$$16\pi^2 \frac{dg_i}{dt} \quad = \quad [b_i + (S(t) - 1)\tilde{b}_i]{g_i}^3$$

- $(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3) = (\frac{6}{5}, -2, -6) + 4\eta$;
  η represents the number of generations of fermions in the bulk.
- If all fields propagating in the bulk, i.e. $\eta = 3$, $\tilde{b}_i = (\frac{66}{5}, 10, 6)$
  For all fields on the brane, i.e. $\eta = 0$, $\tilde{b}_i = (\frac{6}{5}, -2, -6)$

Gauge couplings

- We use $d(\ln \frac{\mu}{M_Z}) = dt$, for $\mu = M_Z e^t$ where M_Z is the renormalisation point, and $S(t) = e^t M_Z R$.
- In terms of the scale parameter t :

$$16\pi^2 \frac{dg_i}{dt} = [b_i + (S(t) - 1)\tilde{b}_i] {g_i}^3$$

- $(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3) = (\frac{6}{5}, -2, -6) + 4\eta$;
  η represents the number of generations of fermions in the bulk.
- If all fields propagating in the bulk, i.e. $\eta = 3$, $\tilde{b}_i = (\frac{66}{5}, 10, 6)$
  For all fields on the brane, i.e. $\eta = 0$, $\tilde{b}_i = (\frac{6}{5}, -2, -6)$

Gauge couplings

- We use $d(\ln \frac{\mu}{M_Z}) = dt$, for $\mu = M_Z e^t$ where M_Z is the renormalisation point, and $S(t) = e^t M_Z R$.
- In terms of the scale parameter t :

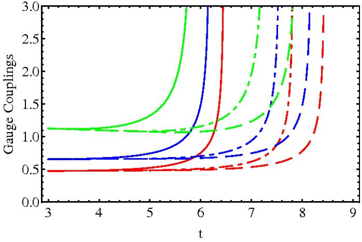
$$16\pi^2 \frac{dg_i}{dt} \quad = \quad [b_i + (S(t) - 1)\tilde{b}_i] {g_i}^3$$

- $(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3) = (\frac{6}{5}, -2, -6) + 4\eta$;
  η represents the number of generations of fermions in the bulk.
- If all fields propagating in the bulk, i.e. $\eta = 3$, $\tilde{b}_i = (\frac{66}{5}, 10, 6)$
  For all fields on the brane, i.e. $\eta = 0$, $\tilde{b}_i = (\frac{6}{5}, -2, -6)$

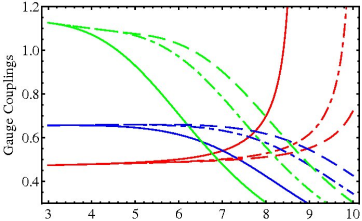
Gauge couplings as a function of the scale parameter t

g_1, g_2, g_3 for 3 different values of the compactification scales : 2 TeV (solid line), 8 TeV (dot-dashed line), 15 TeV (dashed line)

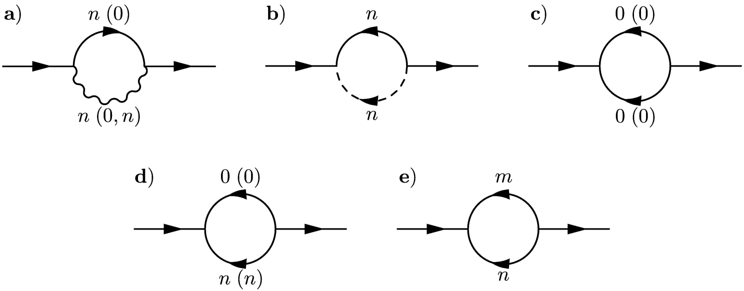
bulk



brane

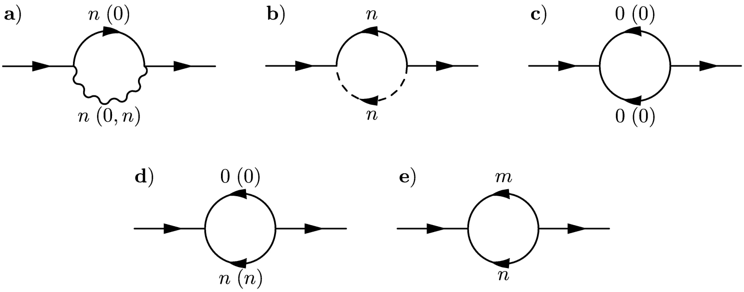


One-loop diagrams



- **The one-loop diagrams related to the wave-function renormalisation of the matter superfields**, in which Figs.a-e refer to the case where all the matter fields are in the bulk, and the excited KK states are labeled by the number without the bracket ; whereas Figs.a,c,d are related to the brane localised matter fields case, in which the KK states are labeled by the number inside the bracket.

One-loop diagrams



- **The one-loop diagrams related to the wave-function renormalisation of the matter superfields**, in which Figs.a-e refer to the case where all the matter fields are in the bulk, and the excited KK states are labeled by the number without the bracket ; whereas Figs.a,c,d are related to the brane localised matter fields case, in which the KK states are labeled by the number inside the bracket.

RGE in 5D MSSM for 3 generations propagating in the bulk

- $$\begin{aligned}
16\pi^2 \frac{dY_d}{dt} &= Y_d(3Tr(Y_d^\dagger Y_d) + Tr(Y_e^\dagger Y_e) + 3Y_d^\dagger Y_d + Y_u^\dagger Y_u)\pi S(t)^2 \\
&- Y_d\left(\frac{7}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2\right) S(t), \\
16\pi^2 \frac{dY_u}{dt} &= Y_u(3Tr(Y_u^\dagger Y_u) + 3Y_u^\dagger Y_u + Y_d^\dagger Y_d)\pi S(t)^2 \\
&- Y_u\left(\frac{13}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2\right) S(t)
\end{aligned}$$

- When $E < 1/R$ or $t < \ln(\frac{1}{M_Z R})$ β-functions become those for the usual 4D MSSM.

RGE in 5D MSSM for 3 generations propagating in the bulk

- $$\begin{aligned} 16\pi^2 \frac{dY_d}{dt} &= Y_d(3Tr(Y_d^\dagger Y_d) + Tr(Y_e^\dagger Y_e) + 3Y_d^\dagger Y_d + Y_u^\dagger Y_u)\pi S(t)^2 \\ &- Y_d\left(\frac{7}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2\right)S(t), \\ 16\pi^2 \frac{dY_u}{dt} &= Y_u(3Tr(Y_u^\dagger Y_u) + 3Y_u^\dagger Y_u + Y_d^\dagger Y_d)\pi S(t)^2 \\ &- Y_u\left(\frac{13}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2\right)S(t) \end{aligned}$$
- When $E < 1/R$ or $t < \ln(\frac{1}{M_Z R})$ β-functions become those for the usual 4D MSSM.

RGE on the brane

- $$\begin{aligned} 16\pi^2 \frac{dY_d}{dt} &= Y_d(3Tr(Y_d^\dagger Y_d) + Tr(Y_e^\dagger Y_e) + (6Y_d^\dagger Y_d + 2Y_u^\dagger Y_u)S(t)) \\ &- Y_d \left(\frac{19}{30} g_1^2 + \frac{9}{2} g_2^2 + \frac{32}{3} g_3^2 \right) S(t) \ , \\ 16\pi^2 \frac{dY_u}{dt} &= Y_u(3Tr(Y_u^\dagger Y_u) + (6Y_u^\dagger Y_u + 2Y_d^\dagger Y_d)S(t)) \\ &- Y_u \left(\frac{43}{30} g_1^2 + \frac{9}{2} g_2^2 + \frac{32}{3} g_3^2 \right) S(t) \end{aligned}$$
- We also found the RGE for leptonic sector $16\pi^2 \frac{dY_e}{dt} = ...$ refer to the paper.

CKM matrix (bulk case)

- The square of the quark Yukawa coupling matrices can be diagonalized by using two unitary matrices U and V :
  $\mathrm{diag}\left(f_u^2, f_c^2, f_t^2\right) = U Y_u^\dagger Y_u U^\dagger$; $\mathrm{diag}\left(h_d^2, h_s^2, h_b^2\right) = V Y_d^\dagger Y_d V^\dagger$
- CKM matrix appears as a result of the transition from the quark flavor eigenstates to the quark mass eigenstates : $V_{CKM} = U V^\dagger$
- We obtain after calculation in the bulk case :

$$16\pi^2 \frac{df_i^2}{dt} = f_i^2 [2(T_u \pi S^2 - G_u) + 6\pi S^2 f_i^2 + 2\pi S^2 \sum_j h_j^2 |V_{ij}|^2]$$

$$16\pi^2 \frac{dh_j^2}{dt} = h_j^2 [2(T_d \pi S^2 - G_d) + 6\pi S^2 h_j^2 + 2\pi S^2 \sum_i f_i^2 |V_{ij}|^2]$$

- Variation of CKM matrix and its evolution equation when the energy scale is beyond the threshold R^{-1} :

$$16\pi^2 \frac{dV_{ik}}{dt} = \pi S^2 \left[\sum_{m,j\neq i} \frac{f_i^2 + f_j^2}{f_i^2 - f_j^2} h_m^2 V_{im} V_{jm}^* V_{jk} + \sum_{j,m\neq k} \frac{h_k^2 + h_m^2}{h_k^2 - h_m^2} f_j^2 V_{jm}^* V_{jk} V_{im} \right]$$

CKM matrix (bulk case)

- The square of the quark Yukawa coupling matrices can be diagonalized by using two unitary matrices U and V :
  $\mathrm{diag}\left(f_u^2, f_c^2, f_t^2\right) = U Y_u^\dagger Y_u U^\dagger$; $\mathrm{diag}\left(h_d^2, h_s^2, h_b^2\right) = V Y_d^\dagger Y_d V^\dagger$
- CKM matrix appears as a result of the transition from the quark flavor eigenstates to the quark mass eigenstates : $V_{CKM} = U V^\dagger$
- We obtain after calculation in the bulk case :

$$16\pi^2 \frac{df_i^2}{dt} = f_i^2 [2(T_u \pi S^2 - G_u) + 6\pi S^2 f_i^2 + 2\pi S^2 \sum_j h_j^2 |V_{ij}|^2]$$

$$16\pi^2 \frac{dh_j^2}{dt} = h_j^2 [2(T_d \pi S^2 - G_d) + 6\pi S^2 h_j^2 + 2\pi S^2 \sum_i f_i^2 |V_{ij}|^2]$$

- Variation of CKM matrix and its evolution equation when the energy scale is beyond the threshold R^{-1} :

$$16\pi^2 \frac{dV_{ik}}{dt} = \pi S^2 \left[\sum_{m,j\neq i} \frac{f_i^2 + f_j^2}{f_i^2 - f_j^2} h_m^2 V_{im} V_{jm}^* V_{jk} + \sum_{j,m\neq k} \frac{h_k^2 + h_m^2}{h_k^2 - h_m^2} f_j^2 V_{jm}^* V_{jk} V_{im} \right]$$

CKM matrix (bulk case)

- The square of the quark Yukawa coupling matrices can be diagonalized by using two unitary matrices U and V :
  $\mathrm{diag}\left(f_u^2, f_c^2, f_t^2\right) = UY_u^\dagger Y_u U^\dagger$; $\mathrm{diag}\left(h_d^2, h_s^2, h_b^2\right) = VY_d^\dagger Y_d V^\dagger$
- CKM matrix appears as a result of the transition from the quark flavor eigenstates to the quark mass eigenstates : $V_{CKM} = UV^\dagger$
- We obtain after calculation in the bulk case :

$$16\pi^2 \frac{df_i^2}{dt} = f_i^2[2(T_u \pi S^2 - G_u) + 6\pi S^2 f_i^2 + 2\pi S^2 \sum_j h_j^2 |V_{ij}|^2]$$

$$16\pi^2 \frac{dh_j^2}{dt} = h_j^2[2(T_d \pi S^2 - G_d) + 6\pi S^2 h_j^2 + 2\pi S^2 \sum_i f_i^2 |V_{ij}|^2]$$

- Variation of CKM matrix and its evolution equation when the energy scale is beyond the threshold R^{-1} :

$$16\pi^2 \frac{dV_{ik}}{dt} = \pi S^2 \left[\sum_{m,j\neq i} \frac{f_i^2 + f_j^2}{f_i^2 - f_j^2} h_m^2 V_{im} V_{jm}^* V_{jk} + \sum_{j,m\neq k} \frac{h_k^2 + h_m^2}{h_k^2 - h_m^2} f_j^2 V_{jm}^* V_{jk} V_{im} \right]$$

CKM matrix (bulk case)

- The square of the quark Yukawa coupling matrices can be diagonalized by using two unitary matrices U and V :
  $\text{diag}\left(f_u^2, f_c^2, f_t^2\right) = U Y_u^\dagger Y_u U^\dagger$; $\text{diag}\left(h_d^2, h_s^2, h_b^2\right) = V Y_d^\dagger Y_d V^\dagger$
- CKM matrix appears as a result of the transition from the quark flavor eigenstates to the quark mass eigenstates : $V_{CKM} = U V^\dagger$
- We obtain after calculation in the bulk case :

$$16\pi^2 \frac{df_i^2}{dt} = f_i^2[2(T_u \pi S^2 - G_u) + 6\pi S^2 f_i^2 + 2\pi S^2 \sum_j h_j^2 |V_{ij}|^2]$$

$$16\pi^2 \frac{dh_j^2}{dt} = h_j^2[2(T_d \pi S^2 - G_d) + 6\pi S^2 h_j^2 + 2\pi S^2 \sum_i f_i^2 |V_{ij}|^2]$$

- Variation of CKM matrix and its evolution equation when the energy scale is beyond the threshold R^{-1} :

$$16\pi^2 \frac{dV_{ik}}{dt} = \pi S^2 \left[\sum_{m,j\neq i} \frac{f_i^2 + f_j^2}{f_i^2 - f_j^2} h_m^2 V_{im} V_{jm}^* V_{jk} + \sum_{j,m\neq k} \frac{h_k^2 + h_m^2}{h_k^2 - h_m^2} f_j^2 V_{jm}^* V_{jk} V_{im} \right]$$

CKM matrix (brane case)

-
$$16\pi^2 \frac{df_i^2}{dt} = f_i^2[2(T_u - G_u) + 12Sf_i^2 + 4S\sum_j h_j^2|V_{ij}|^2] \,,$$
$$16\pi^2 \frac{dh_j^2}{dt} = h_j^2[2(T_d - G_d) + 12Sh_j^2 + 4S\sum_i f_i^2|V_{ij}|^2]$$

- $T_d = 3Tr(Y_d^\dagger Y_d) + Tr(Y_e^\dagger Y_e), G_d = \left(\frac{19}{30}g_1^2 + \frac{9}{2}g_2^2 + \frac{32}{3}g_3^2\right) S(t),$
  $T_u = 3Tr(Y_u^\dagger Y_u), G_u = \left(\frac{43}{30}g_1^2 + \frac{9}{2}g_2^2 + \frac{32}{3}g_3^2\right) S(t).$
- The evolution equation of CKM matrix elements on the brane :

$$16\pi^2 \frac{dV_{ik}}{dt} = 2S\left[\sum_{m,j\neq i} \frac{f_i^2+f_j^2}{f_i^2-f_j^2} h_m^2 V_{im}V_{jm}^* V_{jk} + \sum_{j,m\neq k} \frac{h_k^2+h_m^2}{h_k^2-h_m^2} f_j^2 V_{jm}^* V_{jk} V_{im}\right]$$

CKM matrix (brane case)

- $$16\pi^2 \frac{df_i^2}{dt} = f_i^2[2(T_u - G_u) + 12Sf_i^2 + 4S\sum_j h_j^2|V_{ij}|^2] \,,$$
  $$16\pi^2 \frac{dh_j^2}{dt} = h_j^2[2(T_d - G_d) + 12Sh_j^2 + 4S\sum_i f_i^2|V_{ij}|^2]$$

- $T_d = 3Tr(Y_d^\dagger Y_d) + Tr(Y_e^\dagger Y_e), G_d = \left(\frac{19}{30}g_1^2 + \frac{9}{2}g_2^2 + \frac{32}{3}g_3^2\right) S(t),$
  $T_u = 3Tr(Y_u^\dagger Y_u), G_u = \left(\frac{43}{30}g_1^2 + \frac{9}{2}g_2^2 + \frac{32}{3}g_3^2\right) S(t).$

- The evolution equation of CKM matrix elements on the brane :

$$16\pi^2 \frac{dV_{ik}}{dt} = 2S\left[\sum_{m,j\neq i} \frac{f_i^2 + f_j^2}{f_i^2 - f_j^2} h_m^2 V_{im} V_{jm}^* V_{jk} + \sum_{j,m\neq k} \frac{h_k^2 + h_m^2}{h_k^2 - h_m^2} f_j^2 V_{jm}^* V_{jk} V_{im}\right]$$

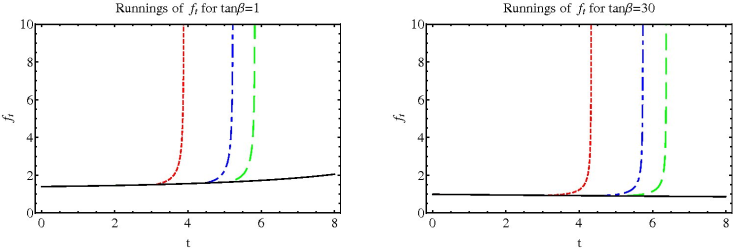
CKM matrix (brane case)

- $$\begin{aligned} 16\pi^2 \frac{df_i^2}{dt} &= f_i^2[2(T_u - G_u) + 12Sf_i^2 + 4S\sum_j h_j^2 |V_{ij}|^2] \, , \\ 16\pi^2 \frac{dh_j^2}{dt} &= h_j^2[2(T_d - G_d) + 12Sh_j^2 + 4S\sum_i f_i^2 |V_{ij}|^2] \end{aligned}$$

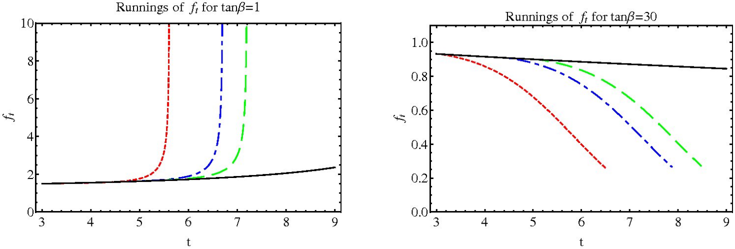
- $T_d = 3Tr(Y_d^\dagger Y_d) + Tr(Y_e^\dagger Y_e), G_d = \left(\frac{19}{30}g_1^2 + \frac{9}{2}g_2^2 + \frac{32}{3}g_3^2\right) S(t),$
  $T_u = 3Tr(Y_u^\dagger Y_u), G_u = \left(\frac{43}{30}g_1^2 + \frac{9}{2}g_2^2 + \frac{32}{3}g_3^2\right) S(t).$
- The evolution equation of CKM matrix elements on the brane :

$$16\pi^2 \frac{dV_{ik}}{dt} = 2S\left[\sum_{m,j\neq i} \frac{f_i^2 + f_j^2}{f_i^2 - f_j^2} h_m^2 V_{im} V_{jm}^* V_{jk} + \sum_{j,m\neq k} \frac{h_k^2 + h_m^2}{h_k^2 - h_m^2} f_j^2 V_{jm}^* V_{jk} V_{im}\right]$$

Top Yukawa for R^{-1} :2 TeV (dotted line), 8 TeV (dot-dashed line), 15 TeV (dashed line)



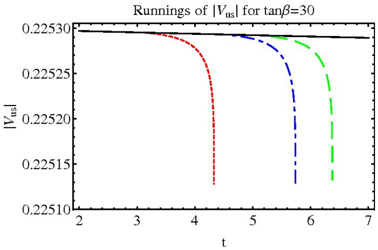
bulk



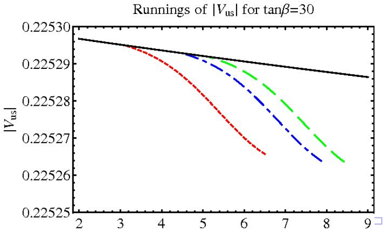
brane

Vus for R^{-1} :2 TeV (dotted line), 8 TeV (dot-dashed line), 15 TeV (dashed line)

bulk

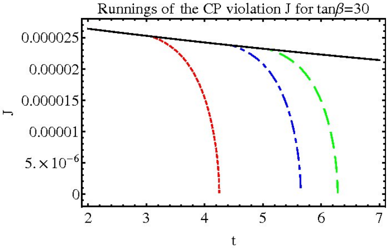


brane

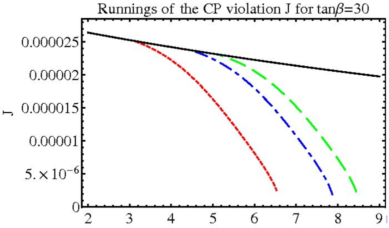


Jarlskog Parameter for R^{-1} :2 TeV (dotted line), 8 TeV (dot-dashed line), 15 TeV (dashed line)

bulk



brane



Conclusion

- In the first case, all matter superfields are allowed to propagate in the fifth dimension whereas in the second they are restricted to the brane.
- In the bulk case, there is a quadratic running for yukawa couplings. y_t becomes non-perturbative already at rather low energies. This strongly limits the range of validity of the model.
- In the brane case, the dependence on the energy scale is only linear and Yukawa couplings remain perturbative until gauge coupling unification.
- In the numerical analysis of the evolution of the CKM parameters, both cases give us a scenario with
small or no quark flavour mixings at high energies.
- In the universal 5D MSSM model, the evolution of these CKM parameters have a rapid variation till reaching a cut-off scale where the top Yukawa coupling develops a singularity point and the model breaks down.

Conclusion

- In the first case, all matter superfields are allowed to propagate in the fifth dimension whereas in the second they are restricted to the brane.
- In the bulk case, there is a quadratic running for yukawa couplings. y_t becomes non-perturbative already at rather low energies. This strongly limits the range of validity of the model.
- In the brane case, the dependence on the energy scale is only linear and Yukawa couplings remain perturbative until gauge coupling unification.
- In the numerical analysis of the evolution of the CKM parameters, both cases give us a scenario with small or no quark flavour mixings at high energies.
- In the universal 5D MSSM model, the evolution of these CKM parameters have a rapid variation till reaching a cut-off scale where the top Yukawa coupling develops a singularity point and the model breaks down.

Conclusion

- In the first case, all matter superfields are allowed to propagate in the fifth dimension whereas in the second they are restricted to the brane.
- In the bulk case, there is a quadratic running for yukawa couplings. y_t becomes non-perturbative already at rather low energies. This strongly limits the range of validity of the model.
- In the brane case, the dependence on the energy scale is only linear and Yukawa couplings remain perturbative until gauge coupling unification.
- In the numerical analysis of the evolution of the CKM parameters, both cases give us a scenario with small or no quark flavour mixings at high energies.
- In the universal 5D MSSM model, the evolution of these CKM parameters have a rapid variation till reaching a cut-off scale where the top Yukawa coupling develops a singularity point and the model breaks down.

Conclusion

- In the first case, all matter superfields are allowed to propagate in the fifth dimension whereas in the second they are restricted to the brane.
- In the bulk case, there is a quadratic running for yukawa couplings. y_t becomes non-perturbative already at rather low energies. This strongly limits the range of validity of the model.
- In the brane case, the dependence on the energy scale is only linear and Yukawa couplings remain perturbative until gauge coupling unification.
- In the numerical analysis of the evolution of the CKM parameters, both cases give us a scenario with
small or no quark flavour mixings at high energies.
- In the universal 5D MSSM model, the evolution of these CKM parameters have a rapid variation till reaching a cut-off scale where the top Yukawa coupling develops a singularity point and the model breaks down.

Conclusion

- In the first case, all matter superfields are allowed to propagate in the fifth dimension whereas in the second they are restricted to the brane.
- In the bulk case, there is a quadratic running for yukawa couplings. y_t becomes non-perturbative already at rather low energies. This strongly limits the range of validity of the model.
- In the brane case, the dependence on the energy scale is only linear and Yukawa couplings remain perturbative until gauge coupling unification.
- In the numerical analysis of the evolution of the CKM parameters, both cases give us a scenario with small or no quark flavour mixings at high energies.
- In the universal 5D MSSM model, the evolution of these CKM parameters have a rapid variation till reaching a cut-off scale where the top Yukawa coupling develops a singularity point and the model breaks down.