The proton charge radius: a persistent mystery

Paul Indelicato

CREMA: Muonic Hydrogen Collaboration

F.D Amaro, A. Antognini, F.Biraben, J.M.R. Cardoso, D.S. Covita, A. Dax, S. Dhawan, L.M.P. Fernandes, A. Giesen, T. Graf, T.W. Hänsch, P. Indelicato, L.Julien, C.-Y. Kao, P.E. Knowles, F. Kottmann, J.A.M. Lopes, E. Le Bigot, Y.-W. Liu, L. Ludhova, C.M.B. Monteiro, F. Mulhauser, T. Nebel, F. Nez, R. Pohl, P. Rabinowitz, J.M.F. dos Santos, L.A. Schaller, K. Schuhmann, C. Schwob, D. Taqqu, J.F.C.A. Veloso

http://muhy.web.psi.ch

http://www.lkb.ens.fr/-Metrology-of-simple-sistems-and-

8 juillet 2010

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History of the proton size

From collisions to atomic physics

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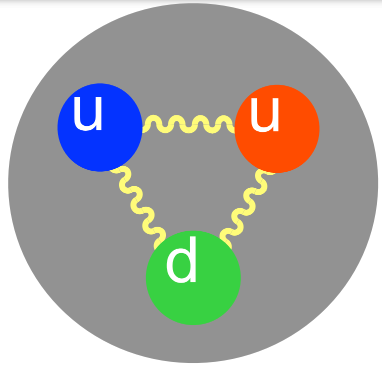
Proton form factor

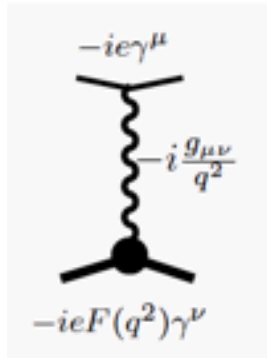
2 quarks up (2/3 e) + 1 quark down (-1/3 e) + strong interaction (gluons)



Vertex EM interaction: Dirac and Pauli Form factors
(S, P: spin and 4-momentum of nucleon, f: quark flavor)

$$\langle P', S' | V^{\mu}_{(f)} | P, S \rangle = \bar{U}(P', S') \Big[\gamma^{\mu} F_1^{(f)}(Q^2) + i\sigma^{\mu\nu} \frac{q_\nu}{2M_N} F_2^{(f)}(Q^2) \Big] U(P, S),$$

$$V^{\mu}_{(f)} = \bar{\psi}_{(f)} \gamma^{\mu} \psi_{(f)},$$



Physical charge density are derived from the Sachs Form factors

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{(2M_N)^2} F_2(Q^2),$$

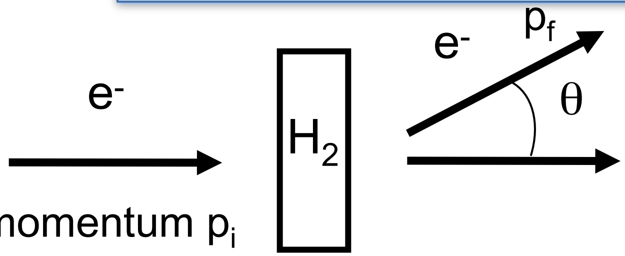
$$G_N(q^2) = \int d\boldsymbol{r} e^{-i\boldsymbol{q}\cdot\boldsymbol{r}} \frac{\rho_N(\boldsymbol{r})}{4\pi},$$

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2).$$

Measure the moments of the charge distribution:

$$< r^n > = \int_0^{\infty} r^{2+n} \rho(r) dr,$$

Electron-proton scattering



momentum p_i
energy E_i

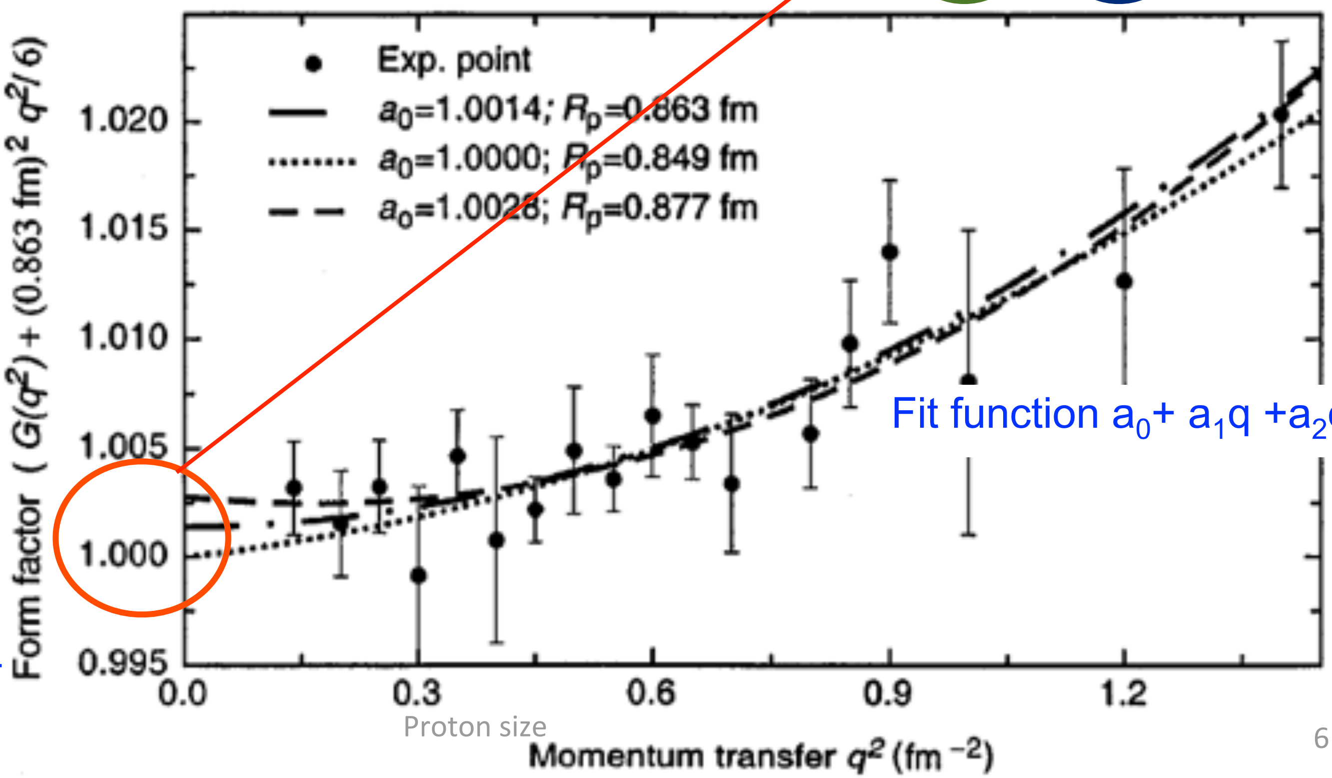
$\vec{q} = \vec{p_f} - \vec{p_i}$

$$q = 2p_f \sin(\frac{\theta}{2})$$

$$G_N(q^2) = \frac{1}{\left(1+\frac{R^2q^2}{12}\right)^2} \approx 1 - \frac{R^2}{6}q^2 + \frac{R^4}{48}q^4 + \cdots$$

$$\frac{d\sigma(E_i,\theta)}{d\omega} = \frac{d\sigma_{\mathrm{Rut.}}(E_i,\theta)}{d\omega} G_E(q^2)$$

$$G_N(q^2) = e^{-\frac{1}{6}R^2q^2} \approx 1 - \frac{R^2}{6}q^2 + \frac{R^4}{72}q^4 + \cdots$$



Fit function $a_0 + a_1q + a_2q^4$

see S. Karshenboim in Can. J. Phys. 77, 241-266 (1999) and refs therein

vendredi 18 novembre 2011

Metrology in hydrogen

From collisions to muonic atoms

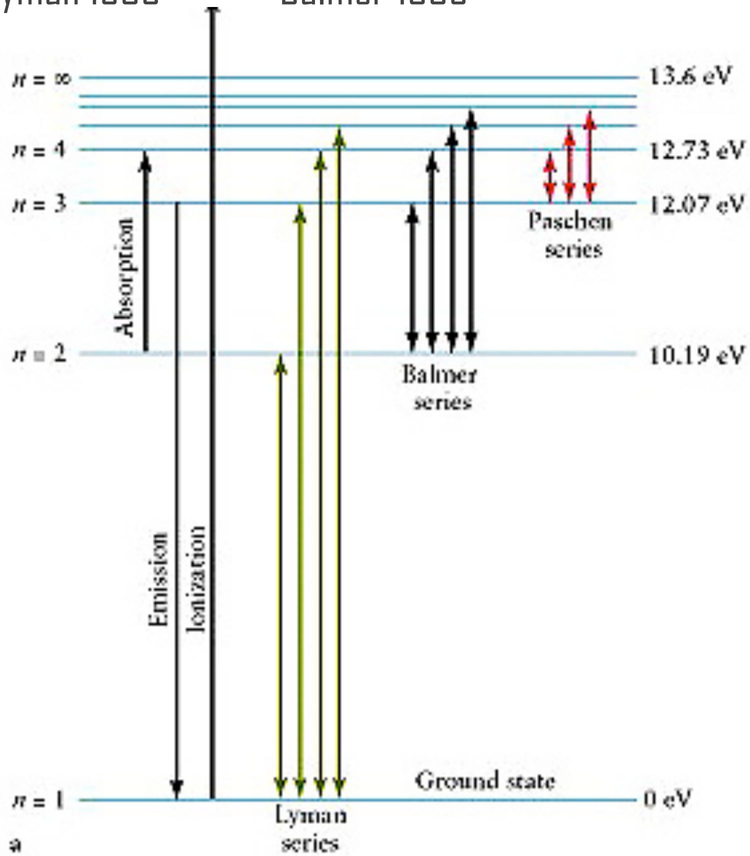
Hydrogen

Rydberg 1888

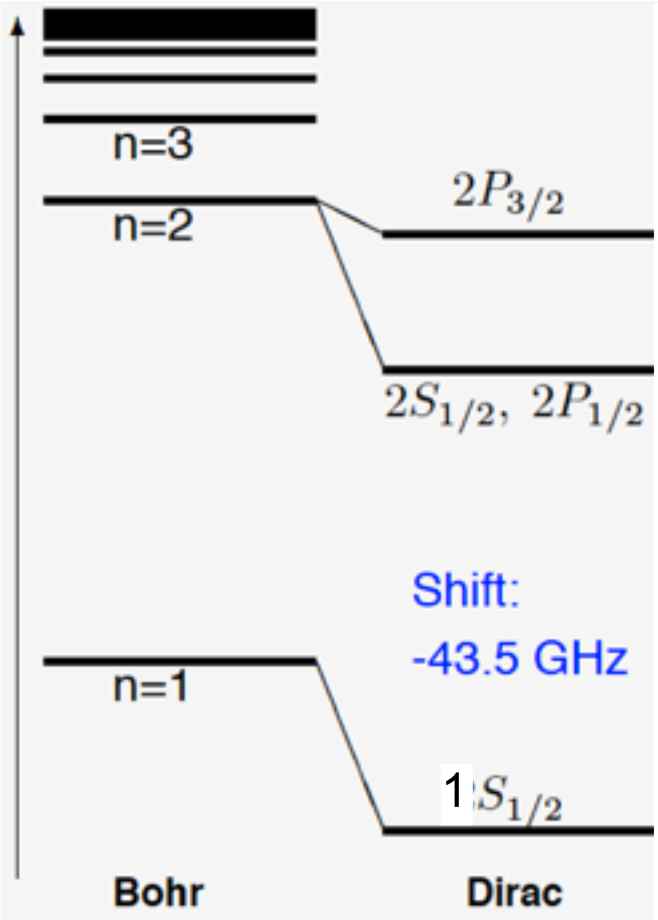
Lyman 1906

Balmer 1885

Model of the atom 1913



Hydrogen



Dirac

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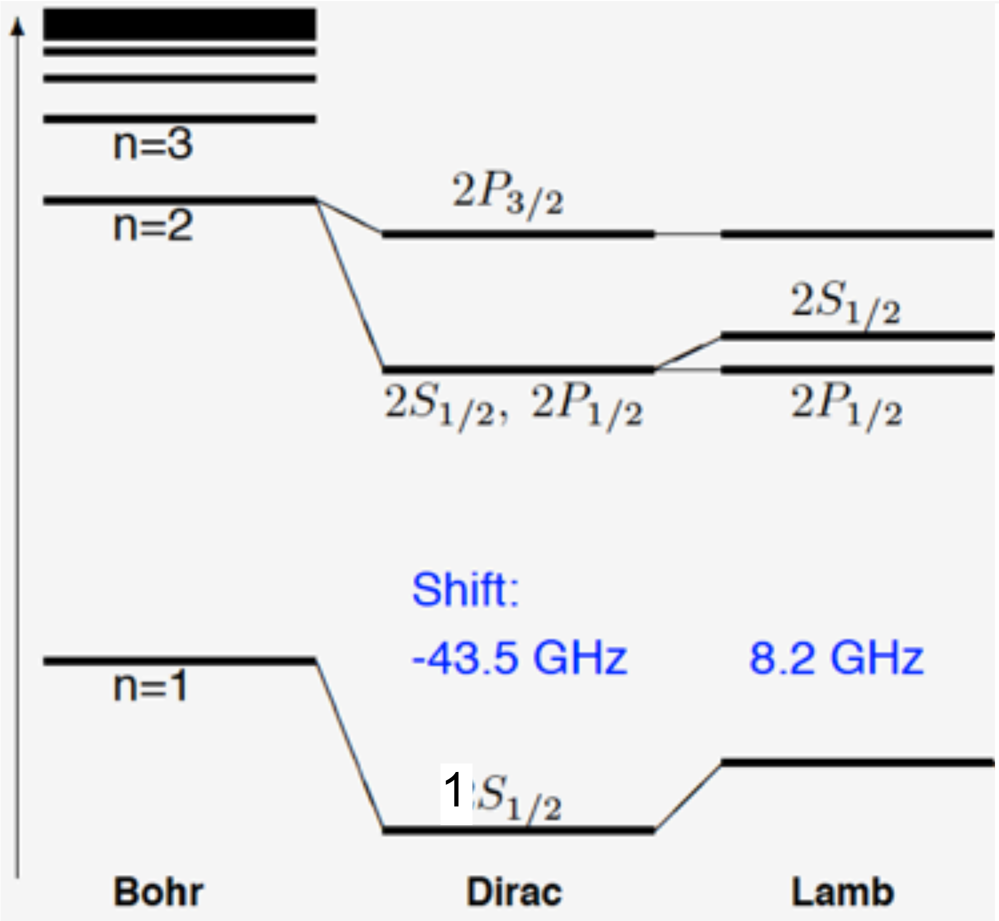
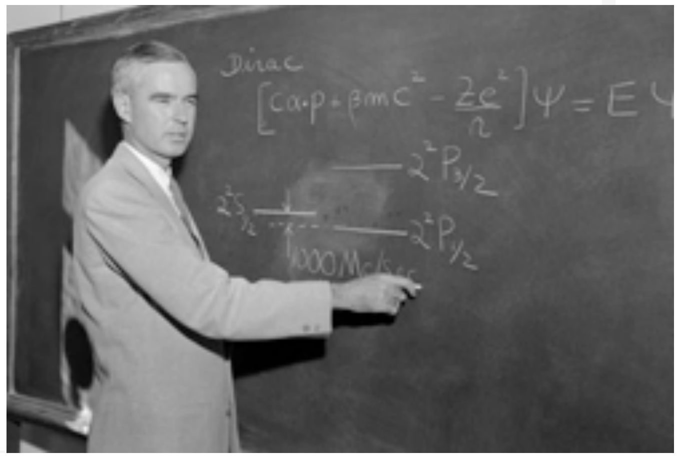
L'hydrogène



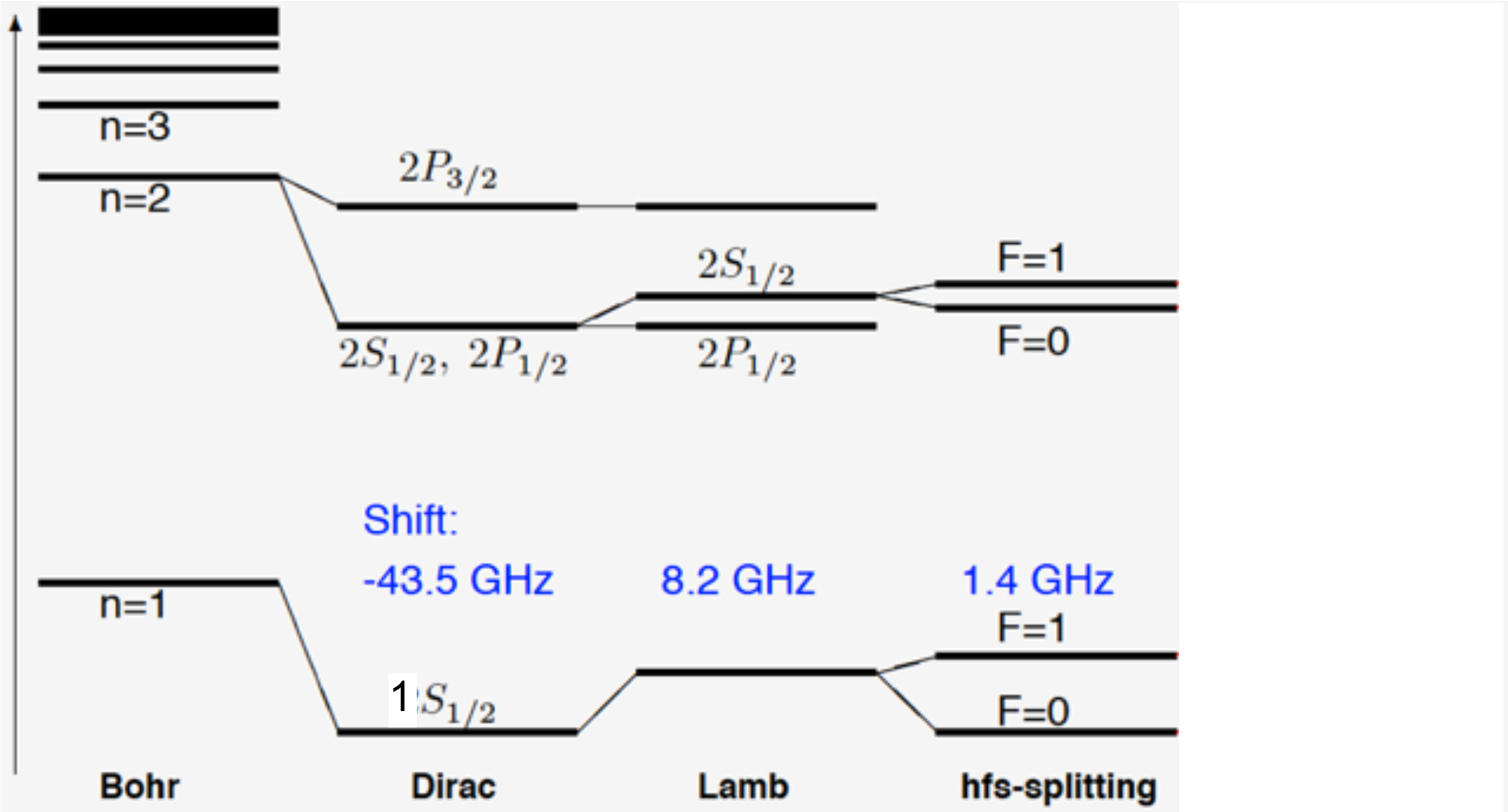
H. Bethe

R.P. Feynman



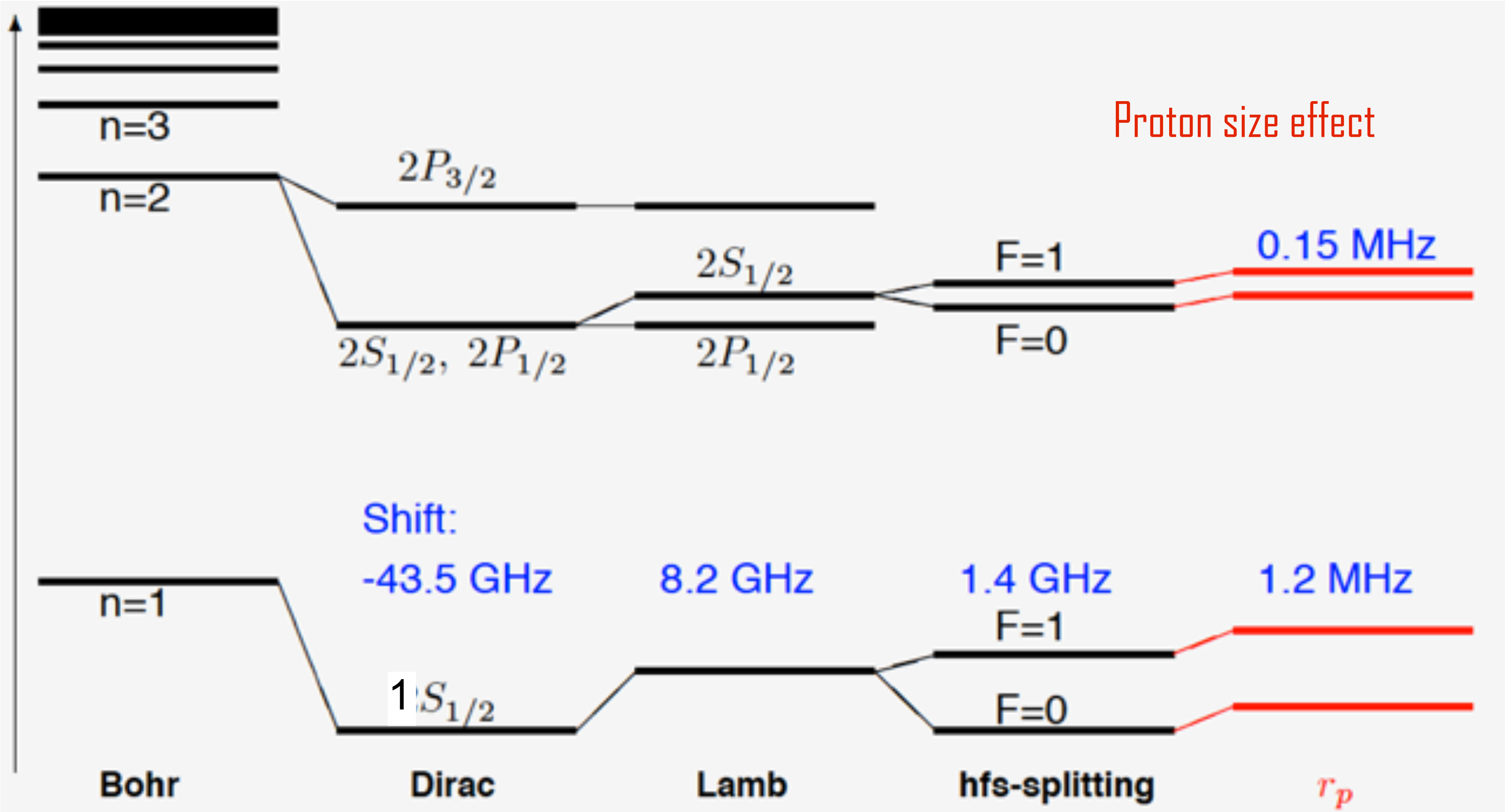
Lamb 1948

L'hydrogène

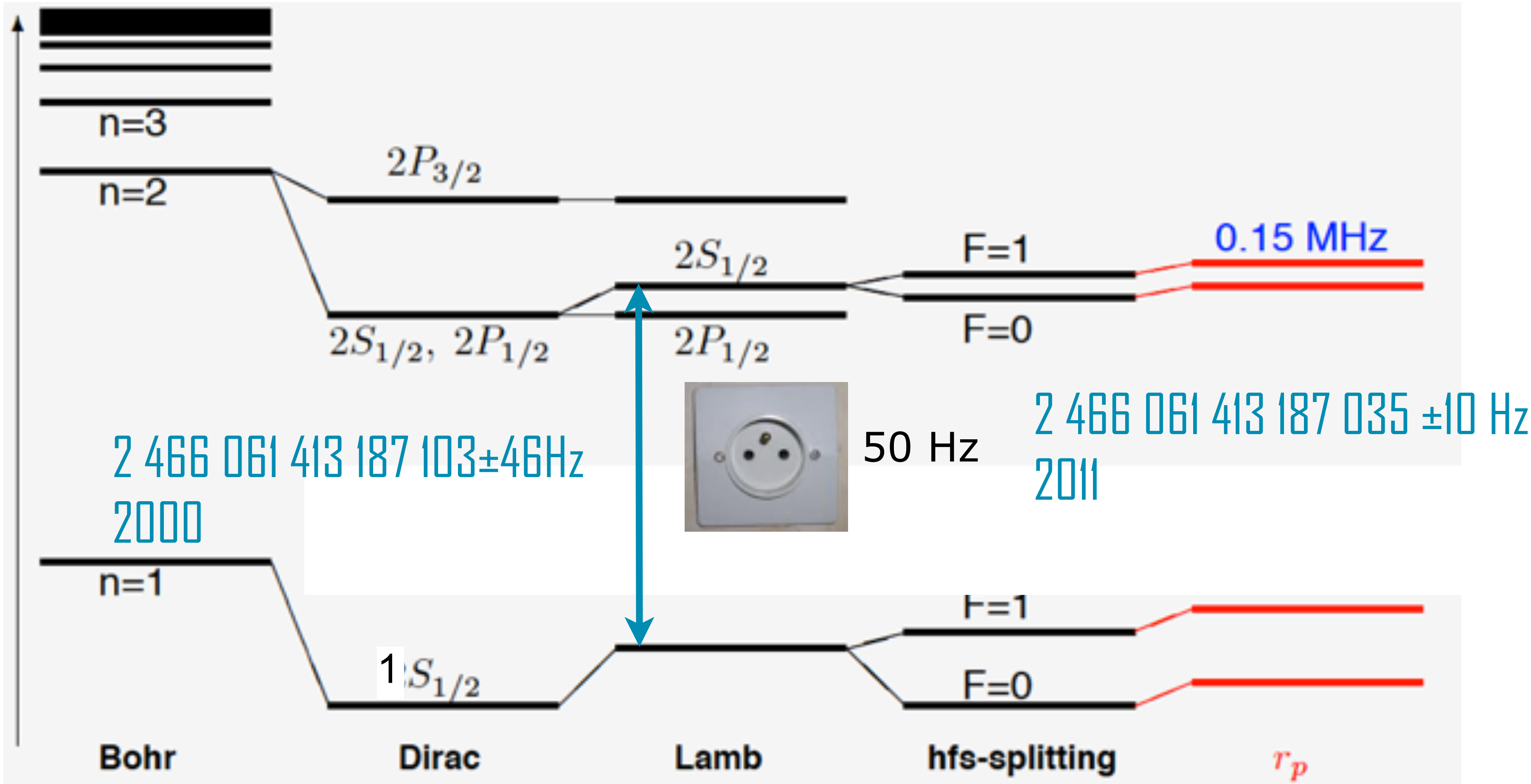


n=3
n=2
$2P_{3/2}$
$2S_{1/2}$
F=1
F=0
$2S_{1/2},\ 2P_{1/2}$
$2P_{1/2}$
Shift:
-43.5 GHz
8.2 GHz
1.4 GHz
n=1
F=1
$1S_{1/2}$
F=0
Bohr
Dirac
Lamb
hfs-splitting

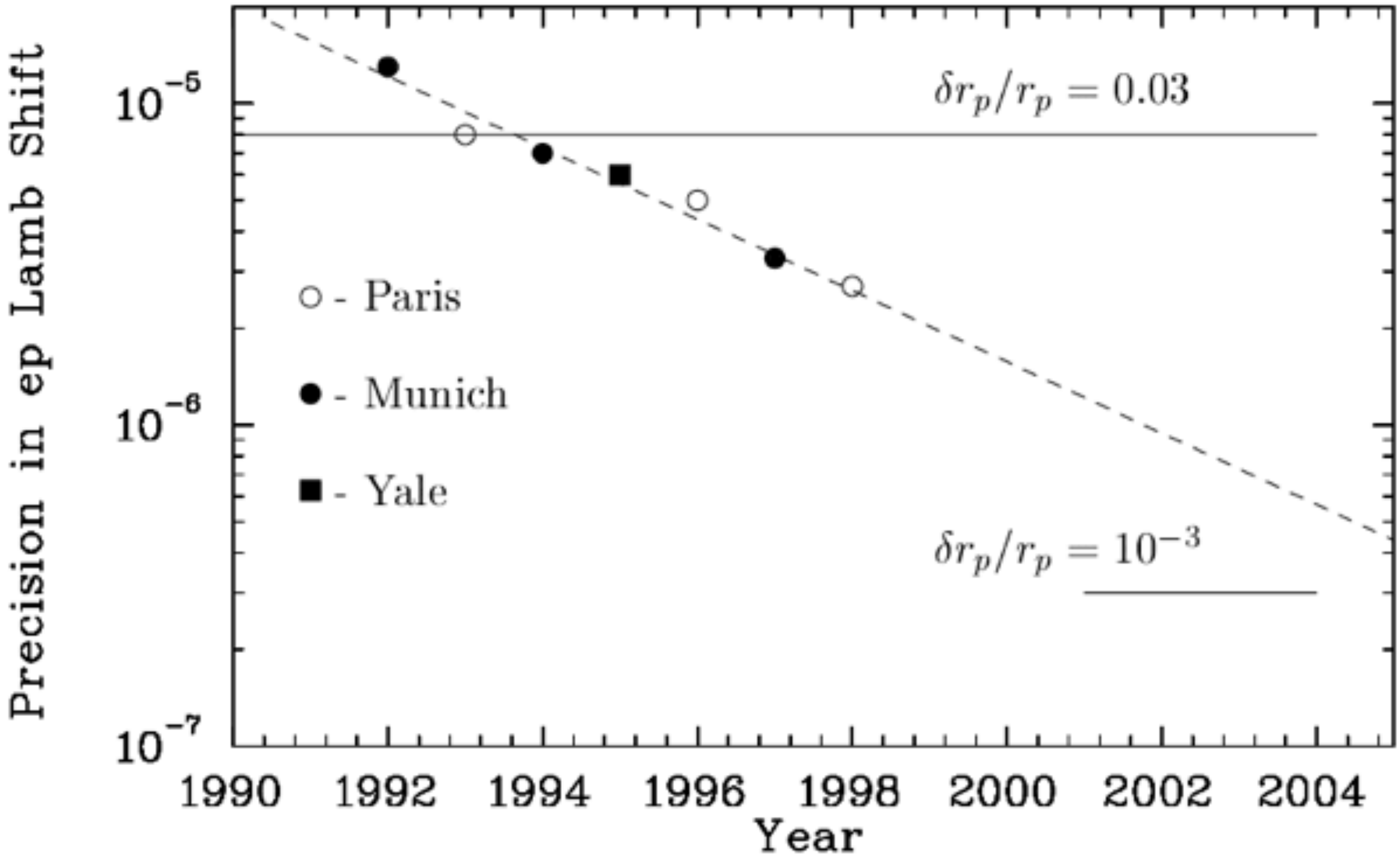
Zoom sur l'hydrogène



L'hydrogène


n=3
n=2
n=1
$2P_{3/2}$
$2S_{1/2}$, $2P_{1/2}$
$2S_{1/2}$
$2P_{1/2}$
$1S_{1/2}$
F=1
F=0
F=1
F=0
0.15 MHz
50 Hz
2 466 061 413 187 103±46Hz
2000
2 466 061 413 187 035 ±10 Hz
2011
Bohr
Dirac
Lamb
hfs-splitting
r_p

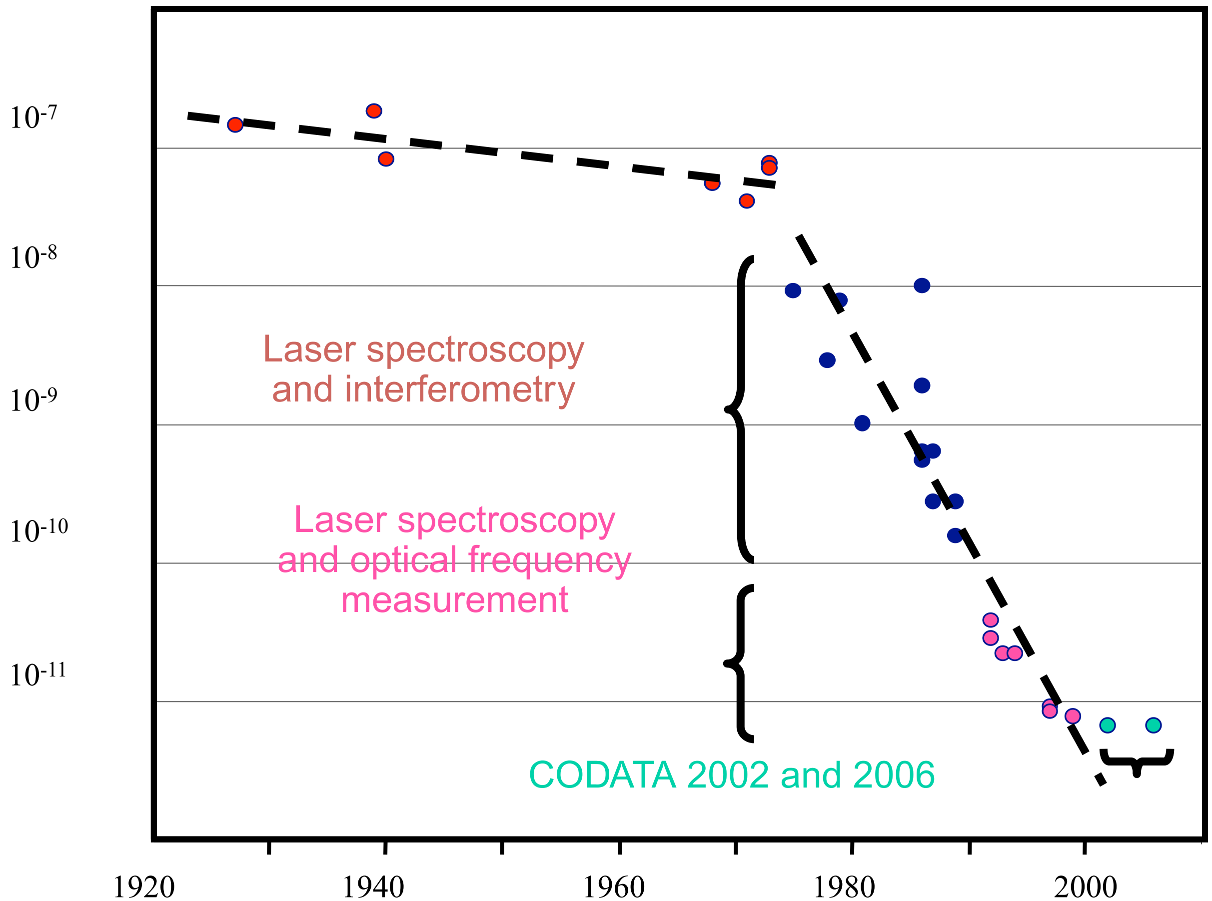
Why re-measure the proton charge radius?



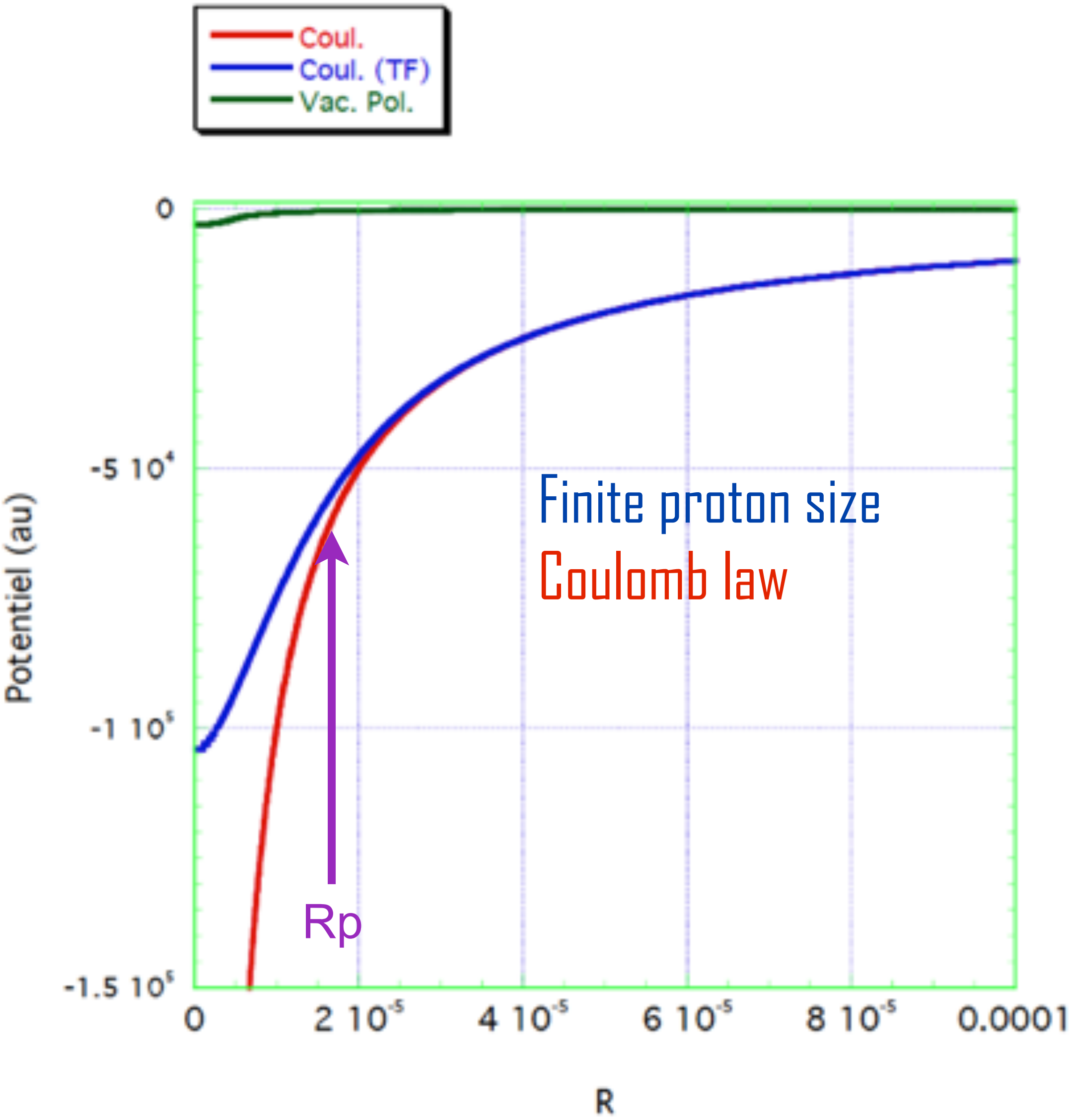
The Rydberg constant

The Rydberg constant is deduced from the measurements of the 1S-2S and 2S-nD transitions and from the scaling law $1/n^3$ of the Lamb shift. The limitation is due to the uncertainty in the 2S-nD measurement.

Relative uncertainty in R_∞



Proton size effect and VP potential



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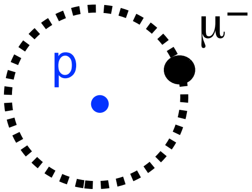
Muonic hydrogen spectroscopy

Exotic atom



$\frac{m_\mu}{m_e} = 207$

radius : ~ $a_0/200$

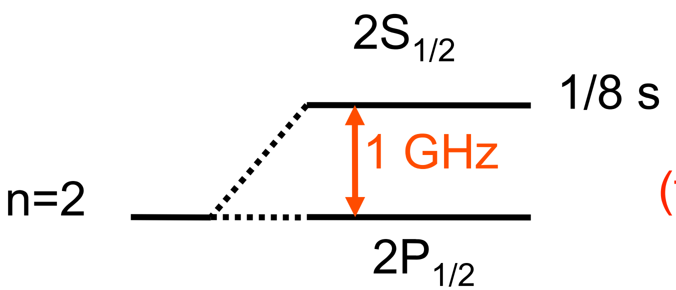
$$\mathcal{E}_{NS} = \frac{2}{3}\left(\frac{m_r}{m_e}\right)^3 \frac{(Z\alpha)^2}{n^3} m_e c^2 \left(\frac{Z\alpha R_N}{\lambda\!\!\!^-_C}\right)^2$$

Lamb shift = self-energy + vacuum polarization + proton radius

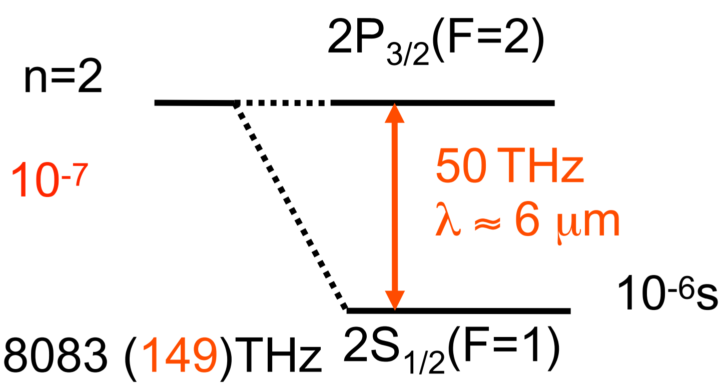
2S-2P	self-energy	vacuum pol.	r_p	total
e-p	1084.1 MHz	-26.9 MHz	0.146 MHz	1057.8 MHz
μ-p	0.17 THz	-50.94 THz	0.96 THz	-49,81THz

electronic hydrogen : e-p

muonic hydrogen : μ-p



$(f_{\mu p}/f_{ep}) \propto 1/(200)^3 \quad 10^{-7}$

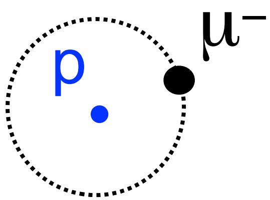


$\Delta E_{2S\text{-}2P} = 50.7724(12) - 1.2637\, r_p^2 - 0.0084\, r_p^3 = 49.8083\,(149)\,\text{THz}$

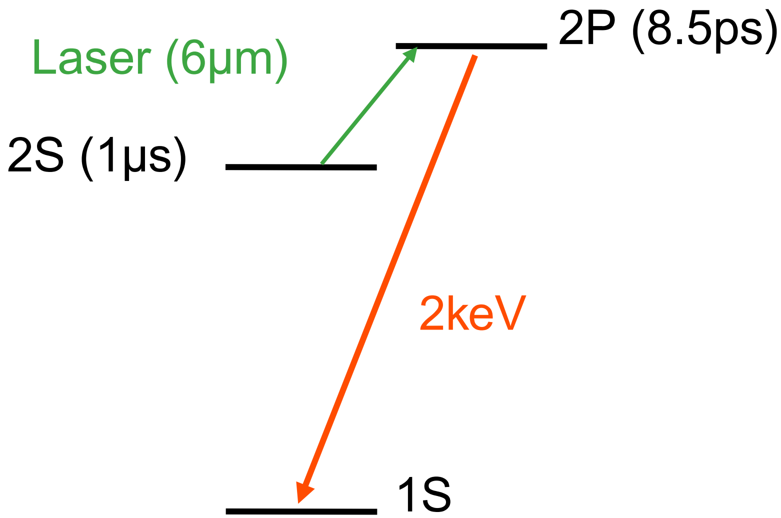
with $r_p = 0.8760(68)$ fm

muonic hydrogen 2S Lamb shift determination of the "proton radius"

Exotic atom



Experiment

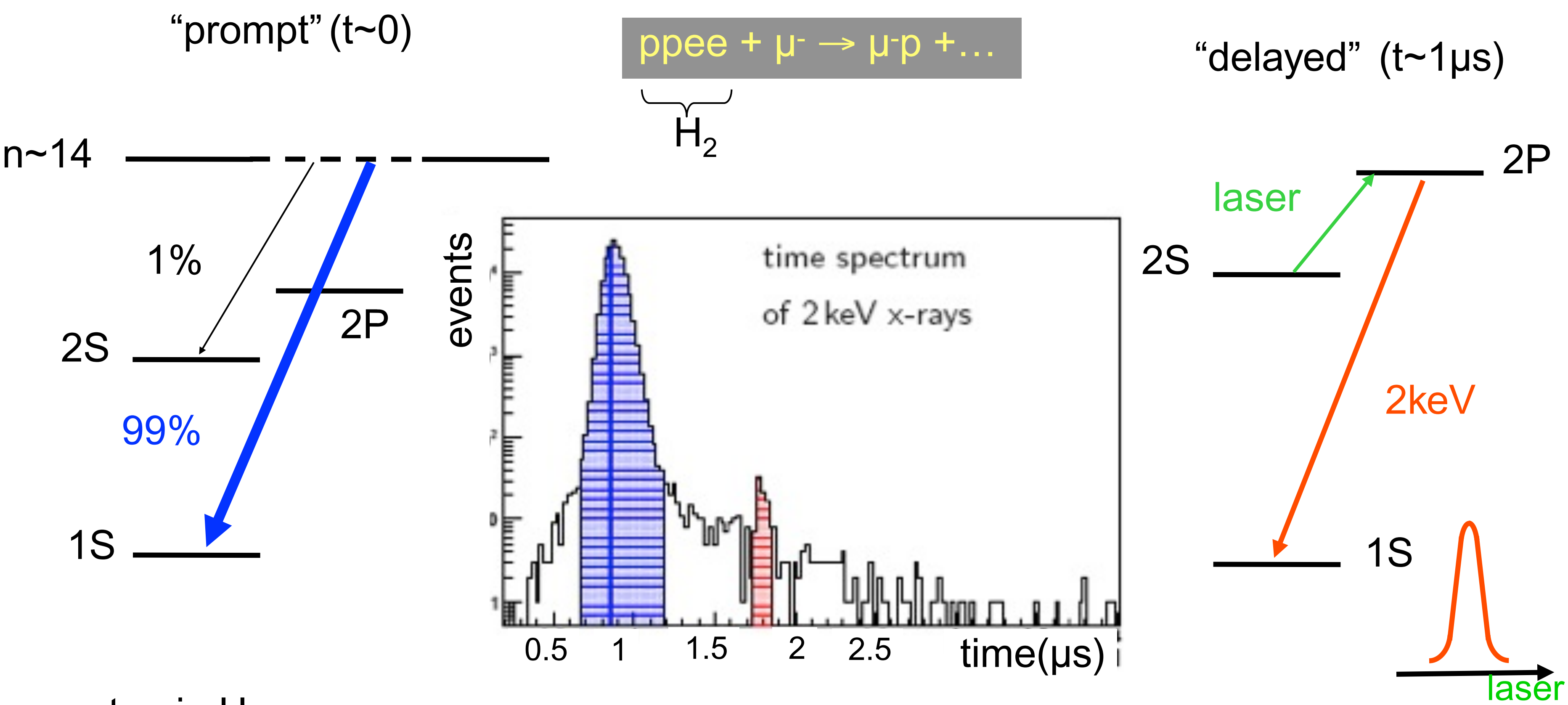


Challenges

- production of muonic hydrogen in 2S
- powerful triggerable 6μm laser
- small signal analysis

Aim : better determination of proton radius r_p

Principle of the experiment



μ^- stop in H_2 gas

⇒ μp* atoms formed (n~14)

99%: cascade to 1S emitting
prompt Kα,Kβ,…

1%: long lived 2S state (τ ~ 1μs at 1mbar)

Fire laser (λ~6μm, ΔE~0.2eV)

⇒ induce μp(2S-2P)

⇒ observe delayed Kα x-rays

⇒ normalize $\frac{\text{delayed K}\alpha}{\text{prompt K}\alpha}$ x-rays

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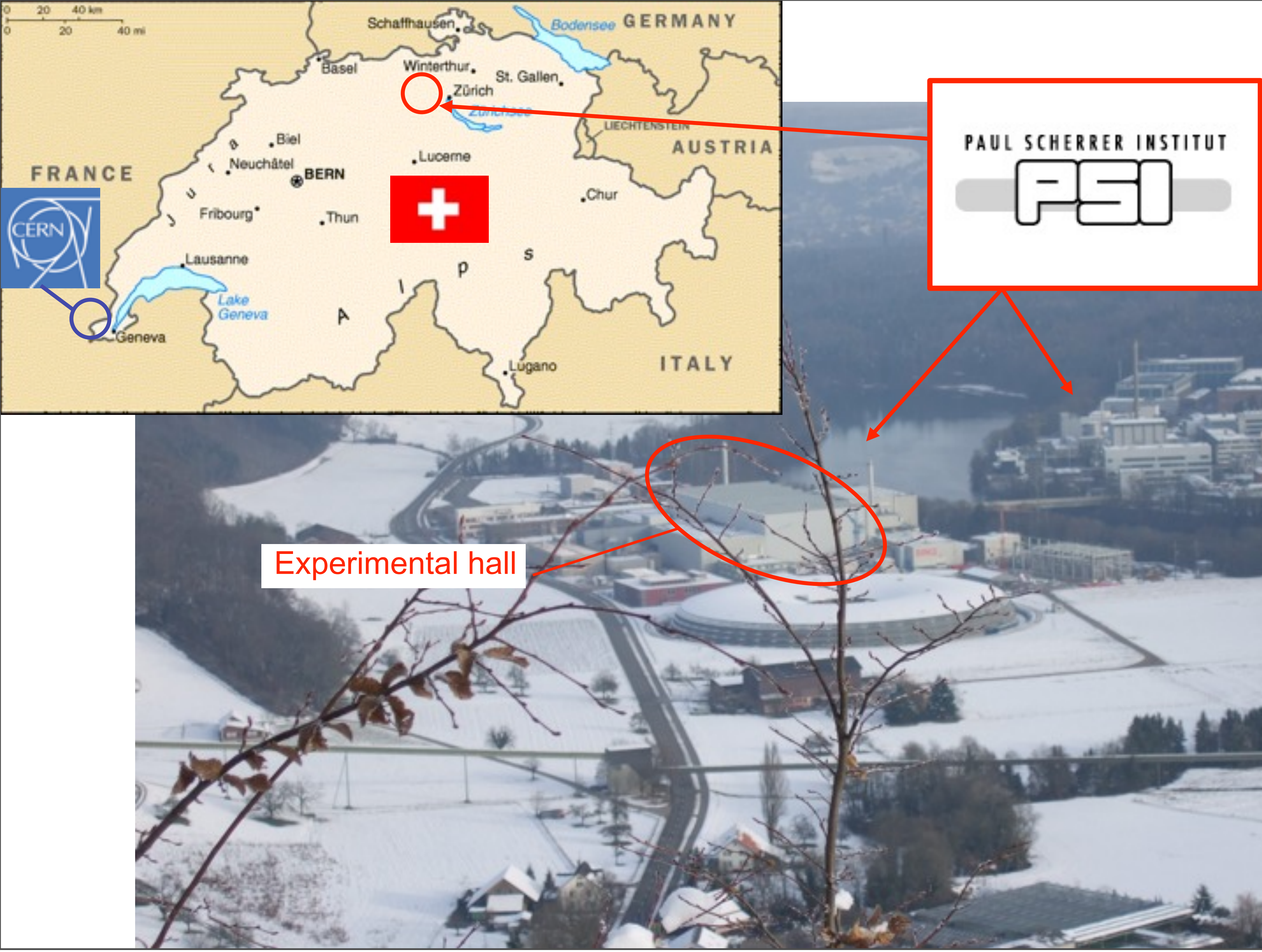
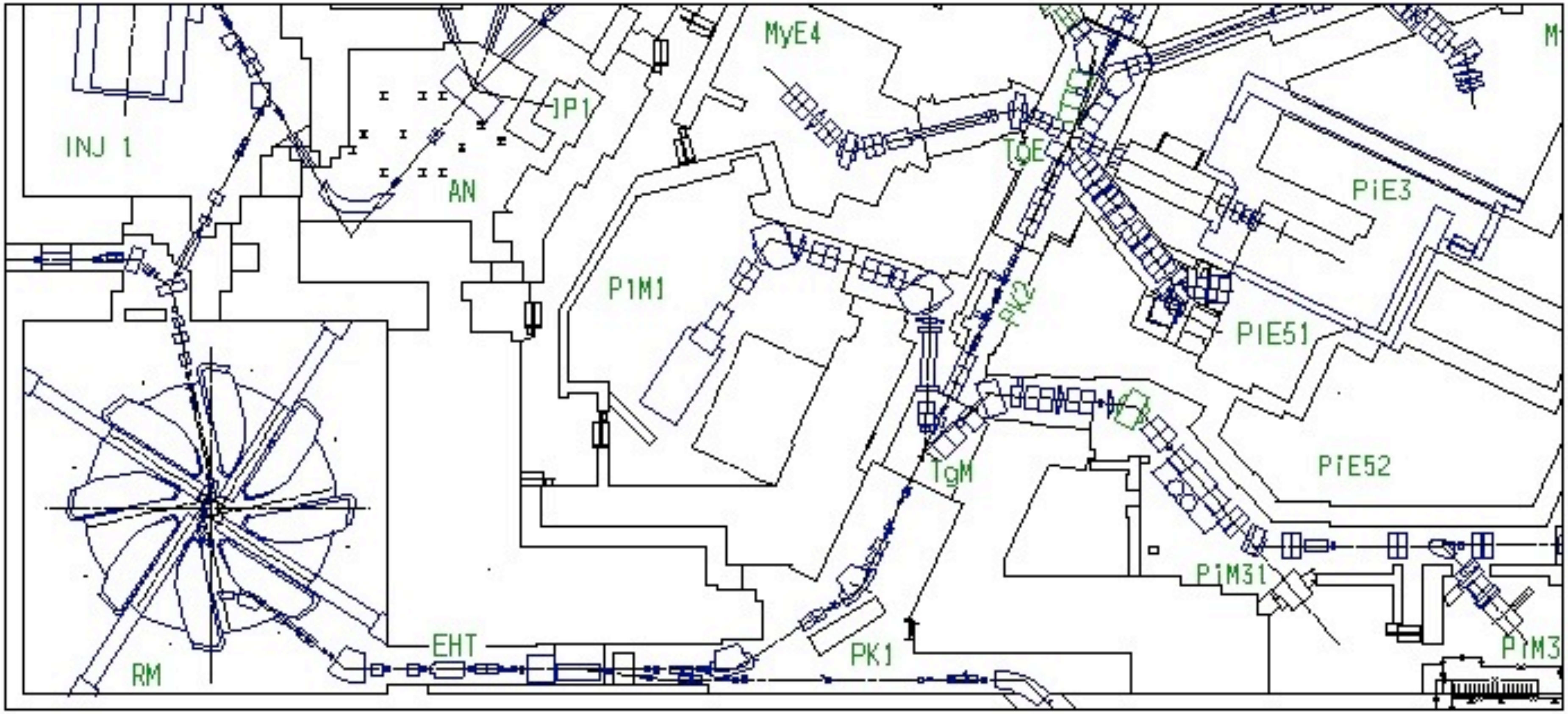
The muonic hydrogen experiment

Getting up close and personal with the proton!

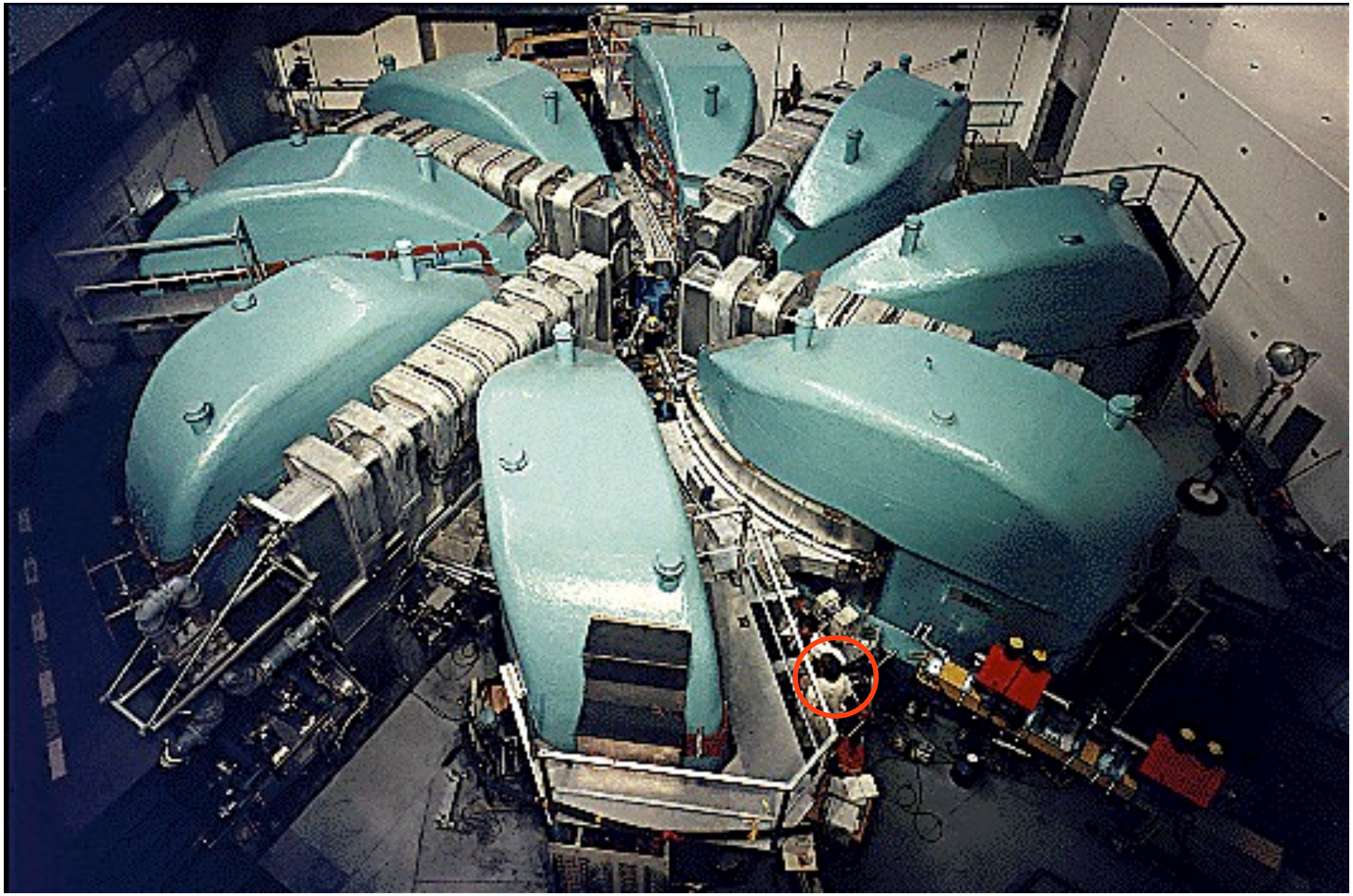

GERMANY
Schaffhausen
Bodensee
Basel
Winterthur
St. Gallen
Zürich
LIECHTENSTEIN
AUSTRIA
FRANCE
Biel
Neuchâtel
BERN
Lucerne
Chur
Fribourg
Thun
Jura
Alps
Lausanne
Lake Geneva
Geneva
Lugano
ITALY
CERN
PAUL SCHERRER INSTITUT
PSI
Experimental hall

The PSI accelerator facility

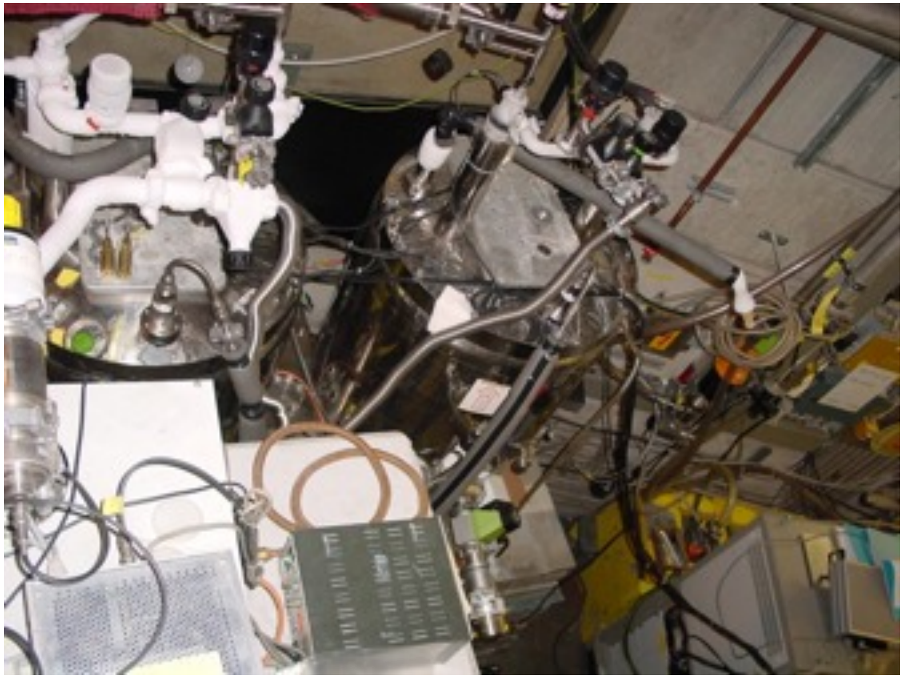
Proton beam, 570 MeV, 2 mA, 1.14 MW



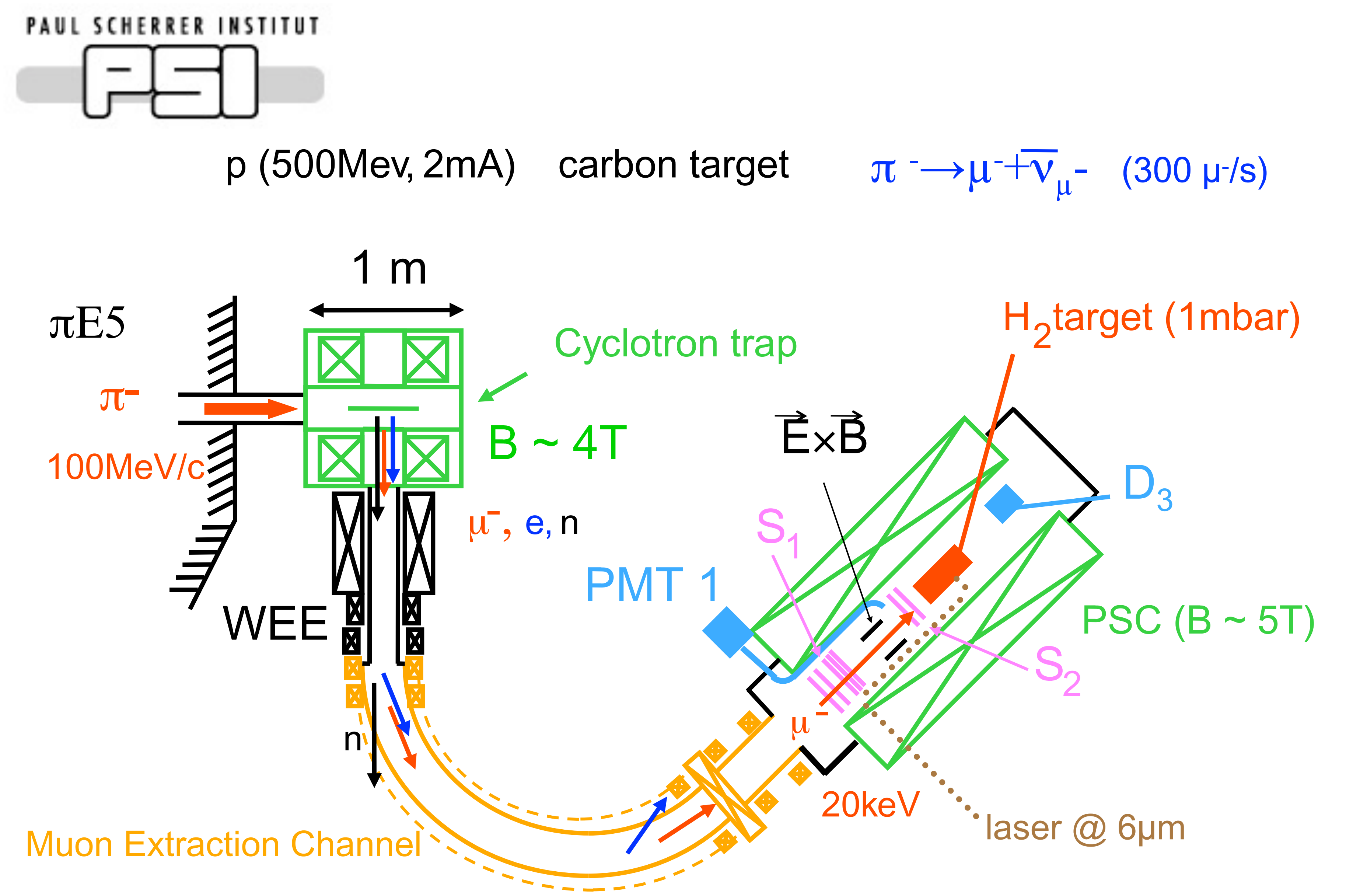
SINQ



The cyclotron trap



muonic hydrogen source

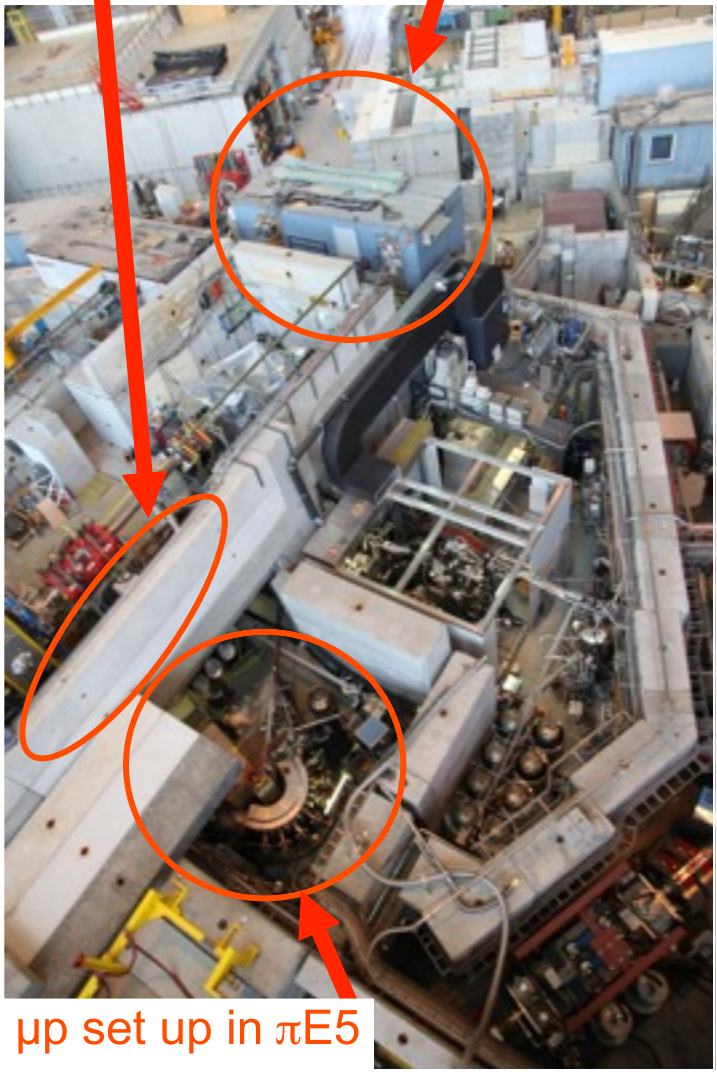


S_1 S_2 → T.O.F. → laser trigger

muon beam apparatus

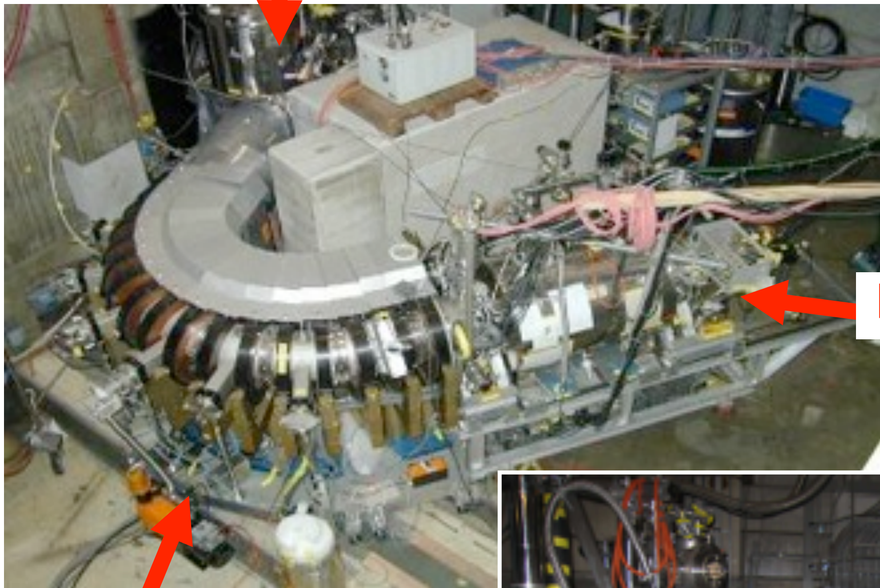
laser hut
below
concrete blocks

counting room



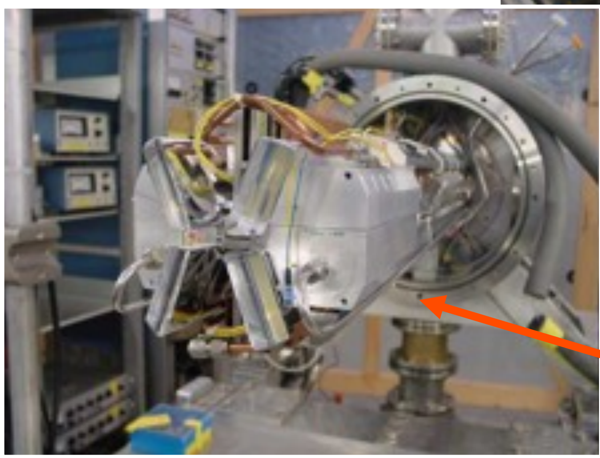
μp set up in πE5

$\pi^- \rightarrow \mu^- + \nu_\mu$



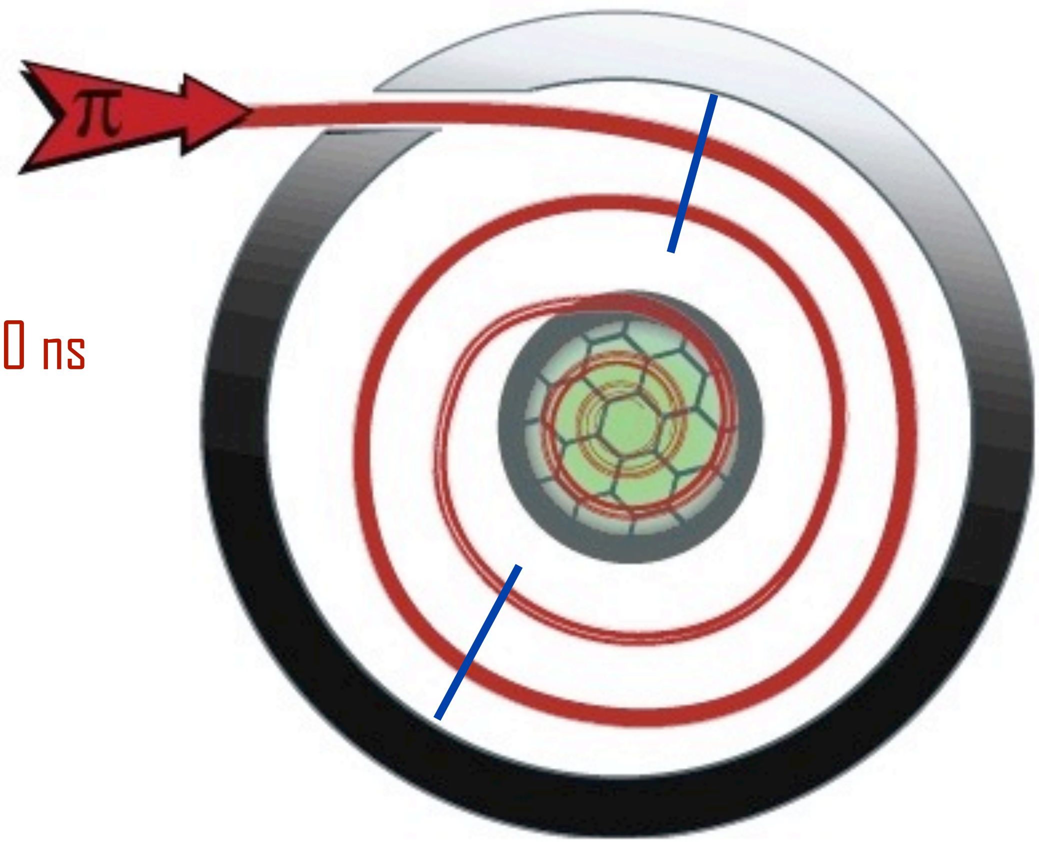
PSC solenoid,

Muon extraction
channel



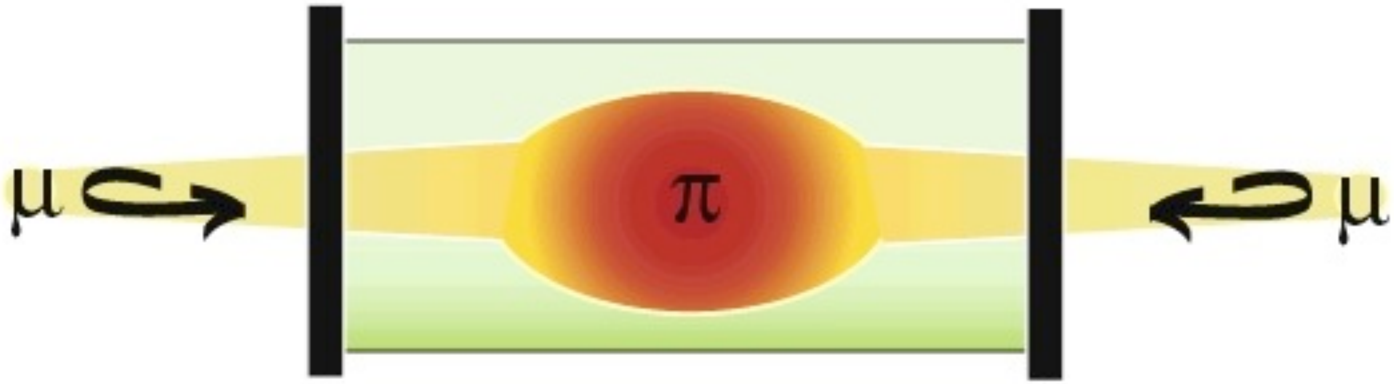
H2 target, laser cavity,
detectors

Slowing down pions

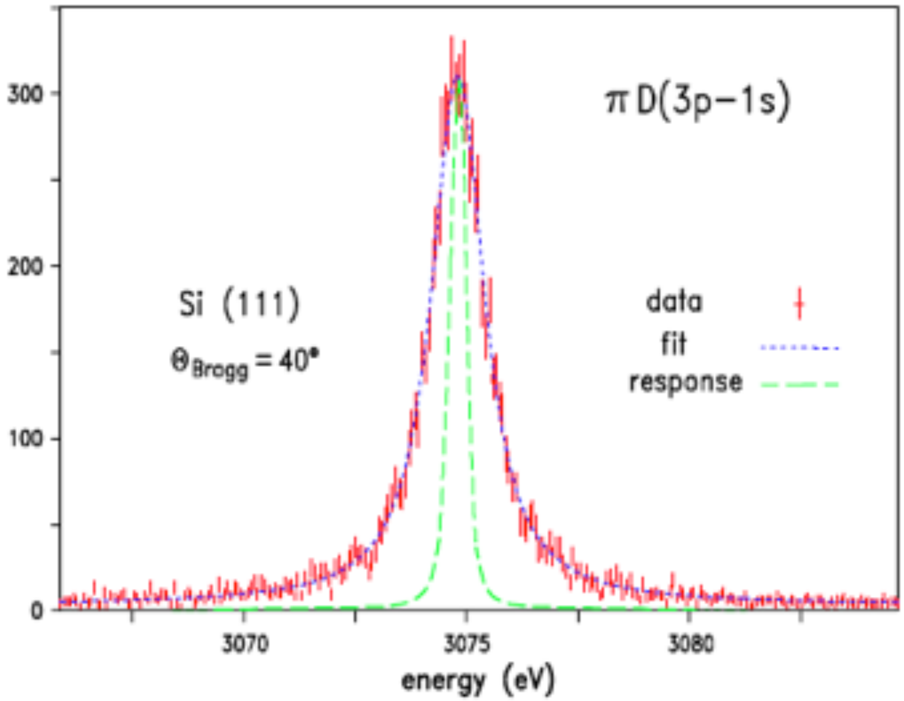


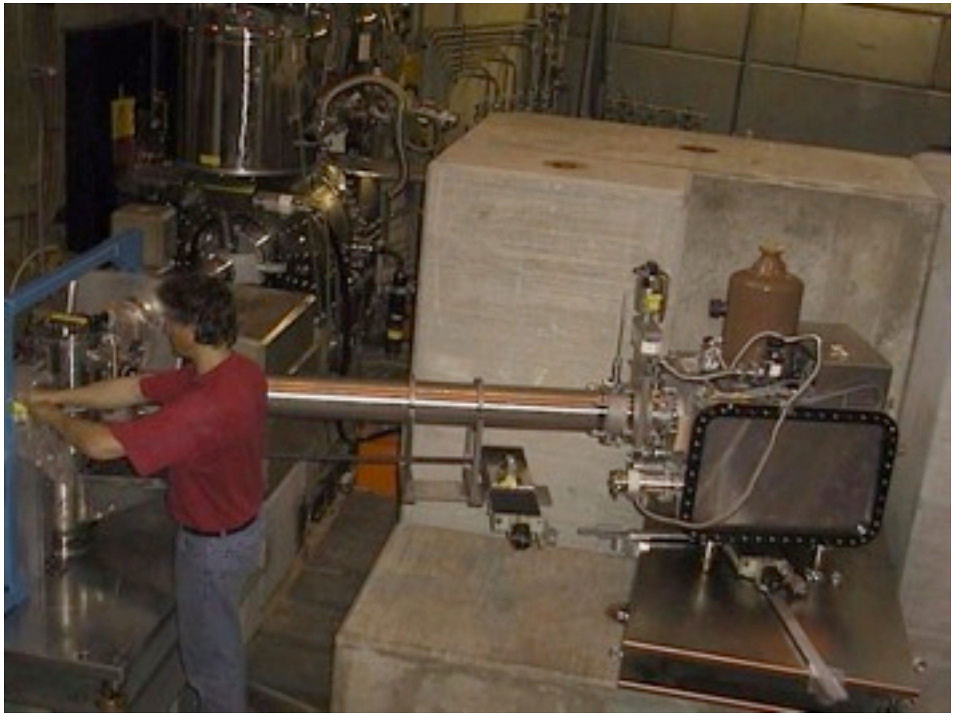
Pion lifetime is 80 ns

Slowing down pions



Pionic and muonic Hydrogens X-ray spectroscopy

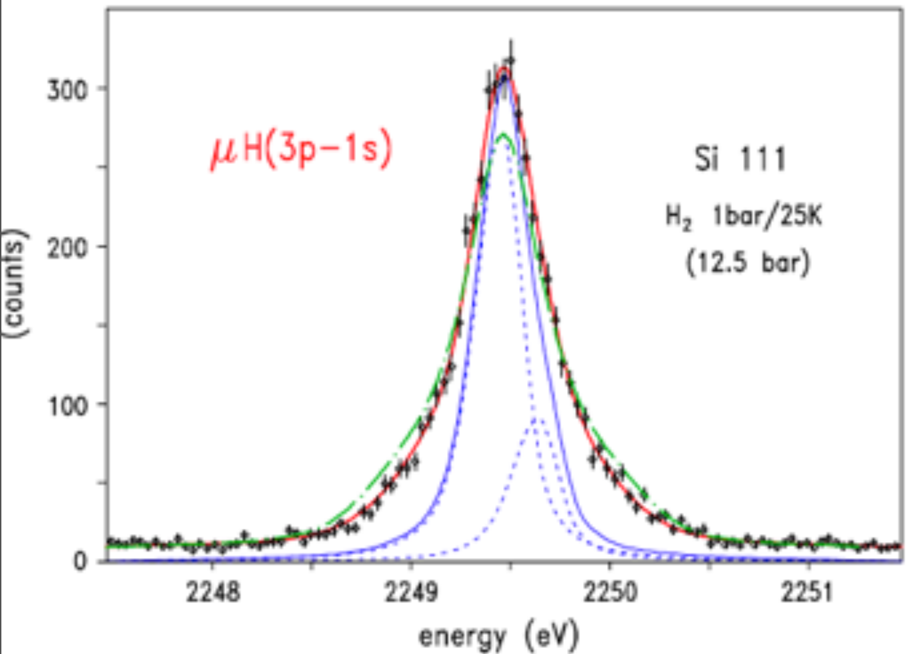




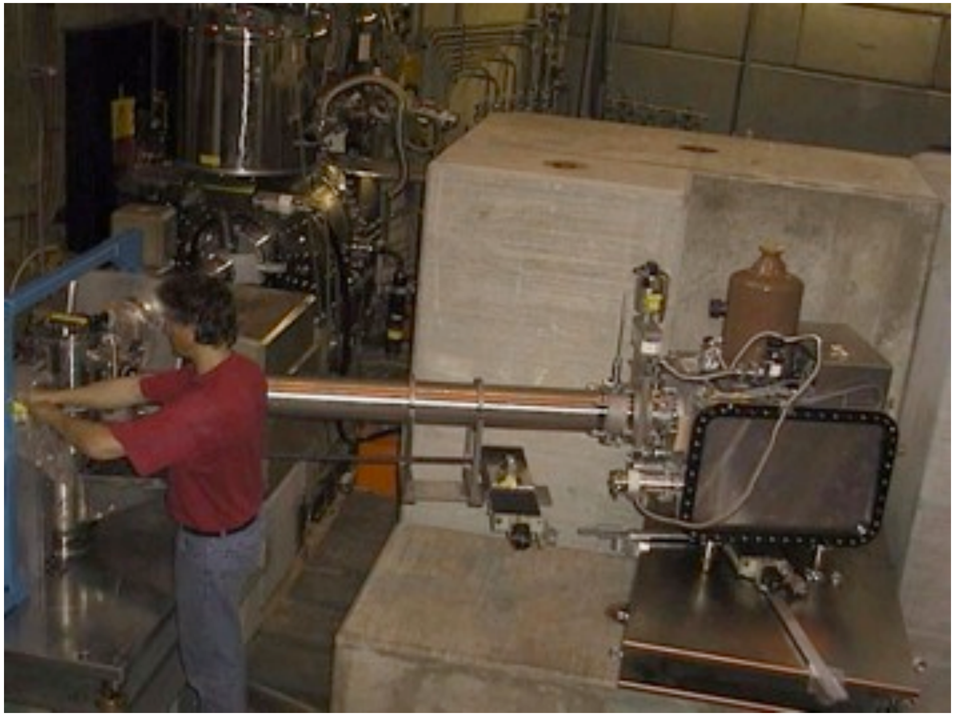
Precision Determination of the d π↔ NN Transition

Strength at Threshold, T. Strauch, F.D. Amaro, D.F. Anagnostopoulos, P. B¸hler, D.S. Covita, H. Gorke, D. Gotta, A. Gruber, A. Hirtl, P. Indelicato, E.O. Le Bigot, M. Nekipelov, J.M.F. dos Santos, S. Schlesser, P. Schmid, L.M. Simons, M. Trassinelli, J.F.C.A. Veloso and J. Zmeskal. Phys. Rev. Lett. 104, 142503 (2010).

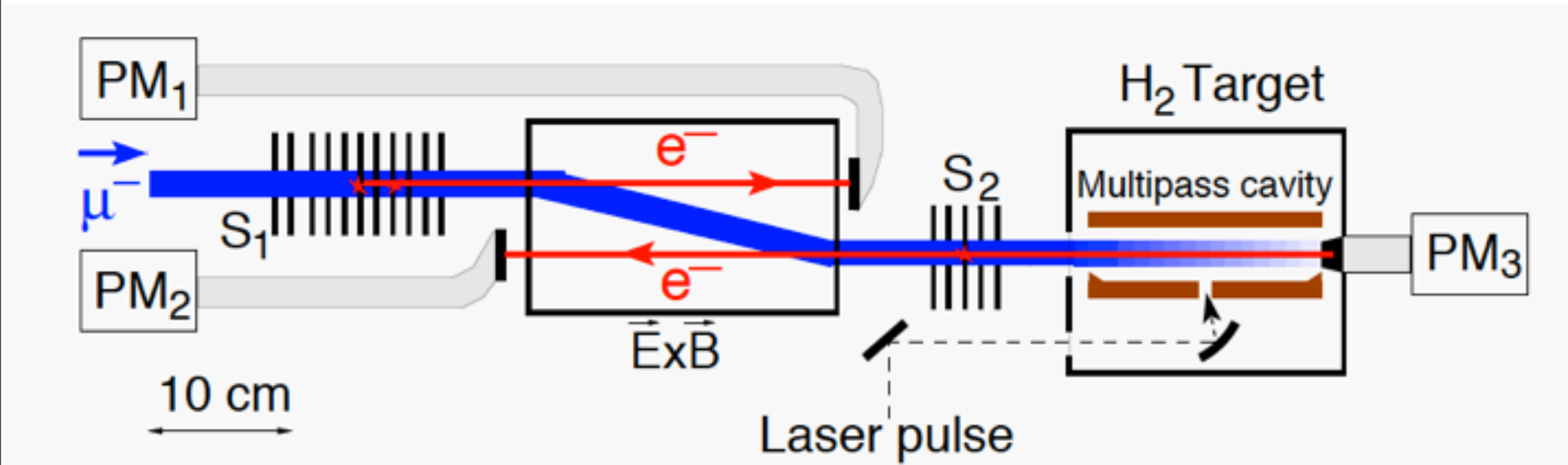
Pionic and muonic Hydrogens X-ray spectroscopy



Line Shape of the mu H(3p-1s) Hyperfine Transitions, D.S. Covita, D.F. Anagnostopoulos, H. Gorke, D. Gotta, A. Gruber, A. Hirtl, T. Ishiwatari, P. Indelicato, E.-O.L. Bigot, M. Nekipelov, J.M.F.d. Santos, P. Schmid, L.M. Simons, M. Trassinelli, J.F.C.A. Veloso and J. Zmeskal. Phys. Rev. Lett. 102, 023401 (2009).

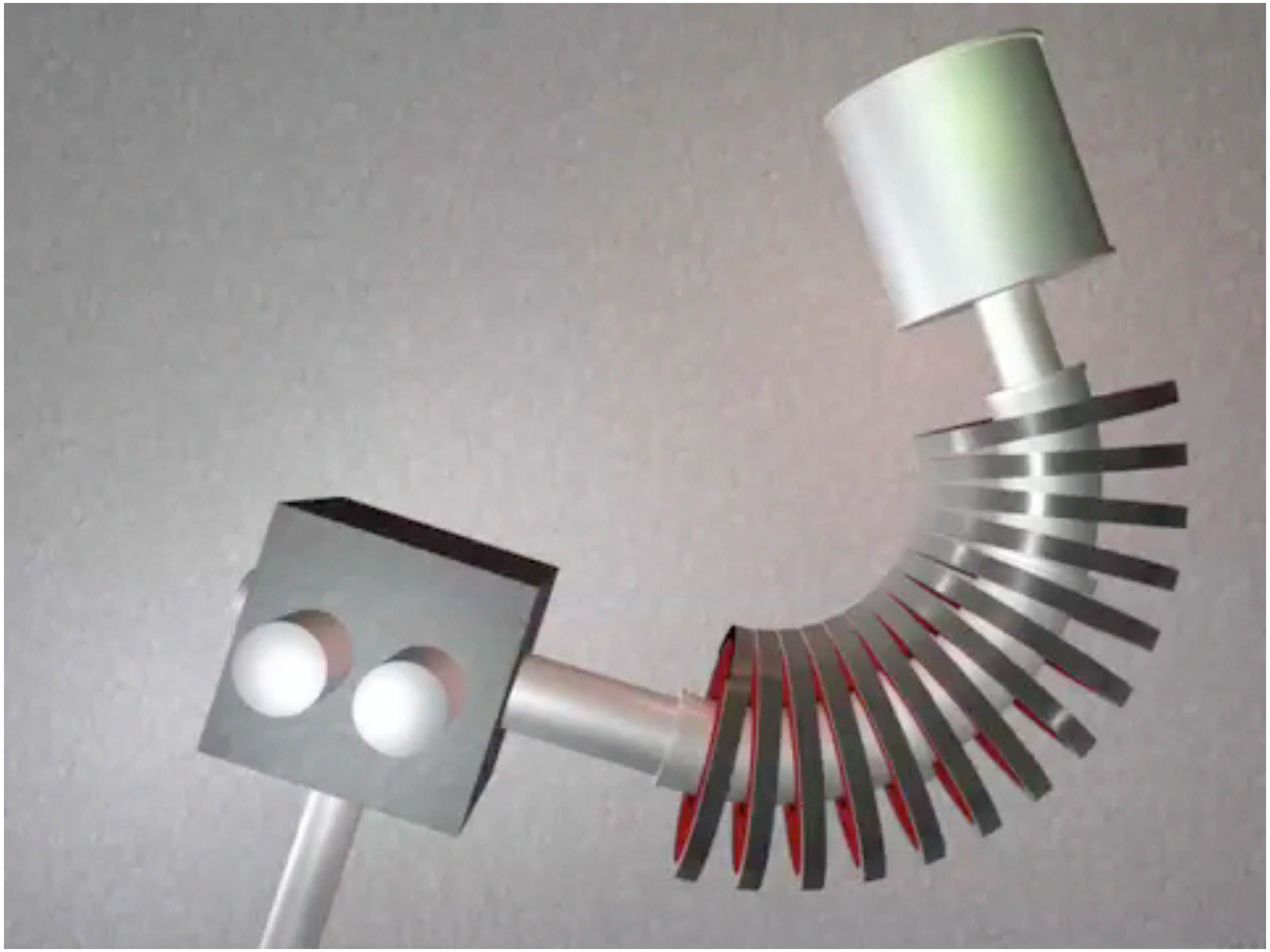


The laser trigger signal

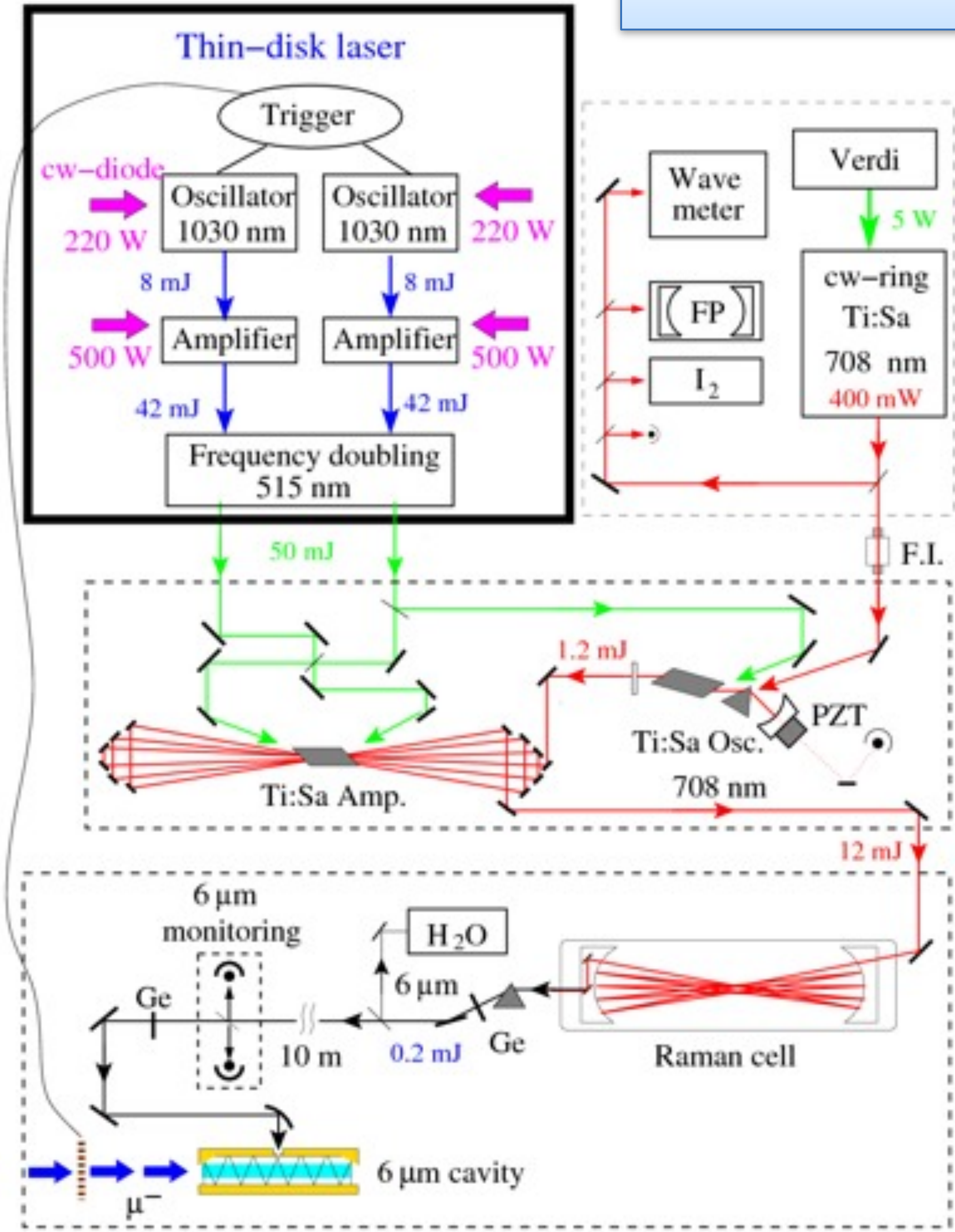


A muon's Odyssey

A muon’s Odyssey



Laser chain



- Each single muon triggers the laser system (random trigger)
- 2S lifetime ~1µs → short laser delay (disk laser)
- 6 µm tunable laser pulse (0.2mJ)

µ⁻ trigger

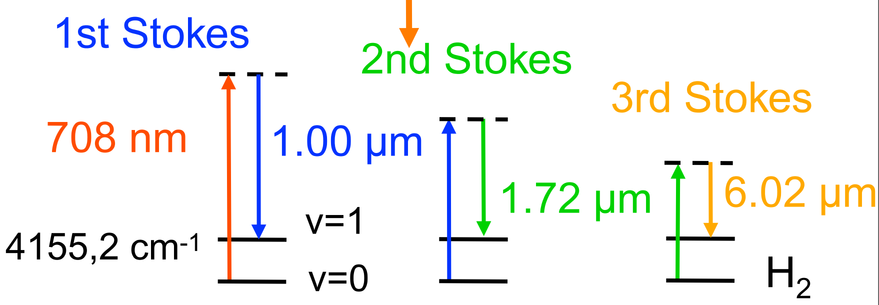
↓

Thin disk laser (1030 nm) + LBO (515 nm)

↓

pulsed TiSa oscillator + amplifier
(708 nm determined by cw TiSa seeding)

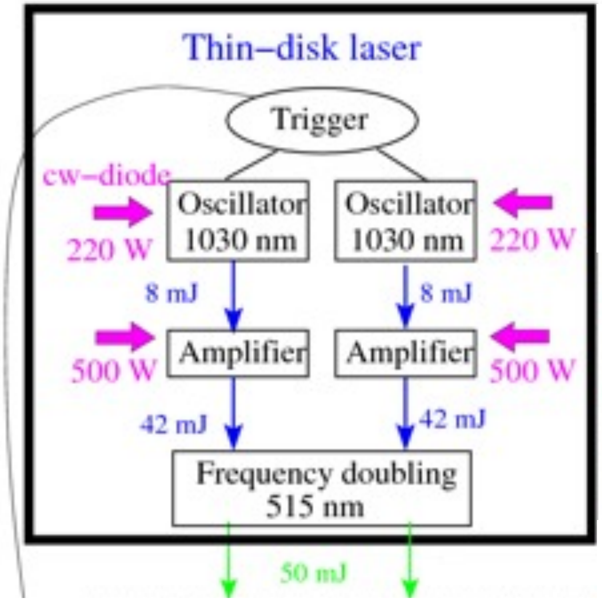
↓



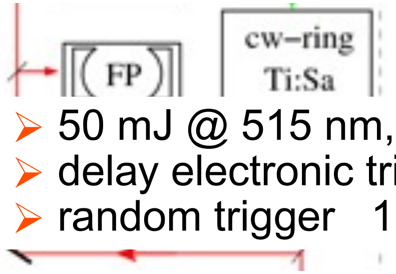
↓

Multipass cavity at 6µm
surrounding the H2 target

Laser chain : thin-disk Yb:YAG laser

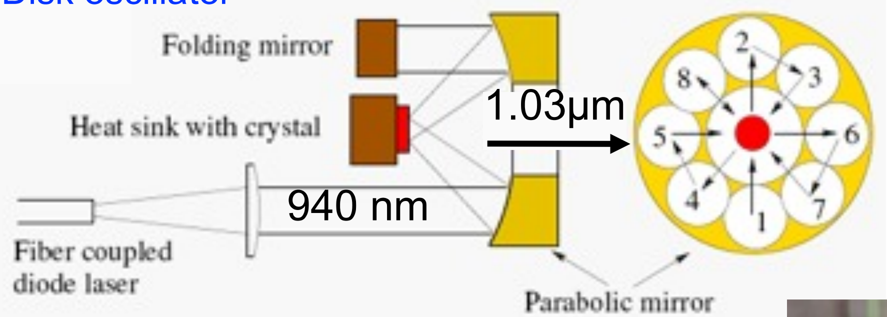


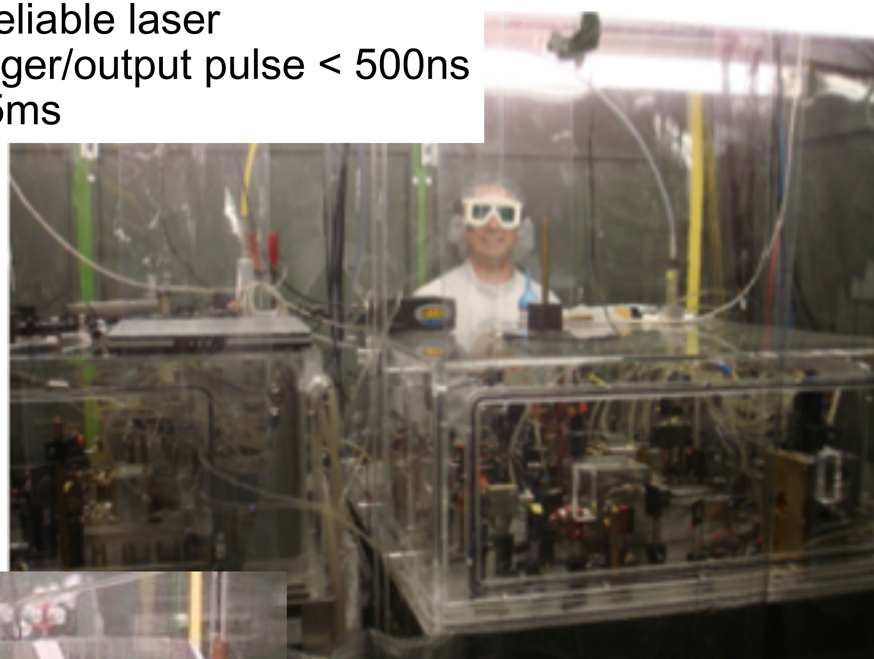
- Q-switch oscillator (cw prelasing mode → short delay between electronic trigger and opt. output (8mJ, xxx ns, delay 250ns)
- 12 pass amplifier (20m propagation) (42mJ)
- LBO frequency doubling 25mJ @ 515nm

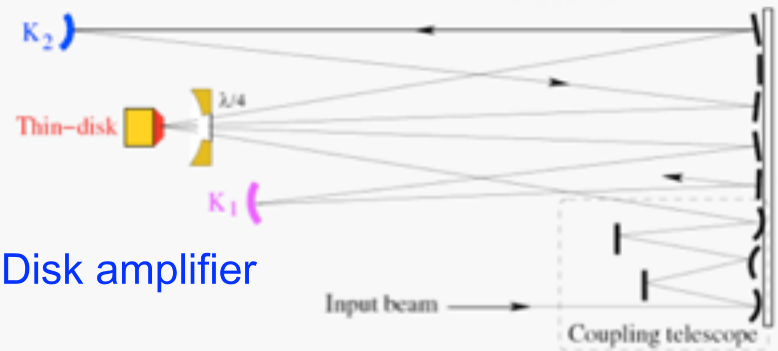


- 50 mJ @ 515 nm, reliable laser
- delay electronic trigger/output pulse < 500ns
- random trigger 1.5ms

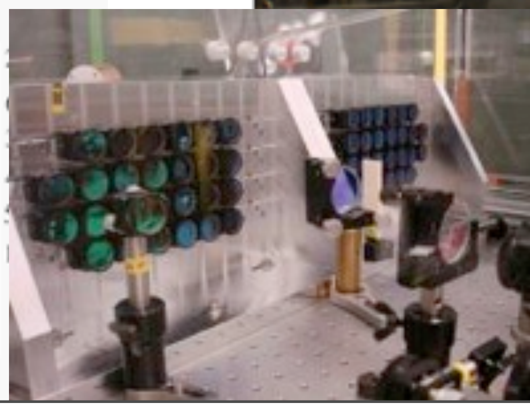
Disk oscillator







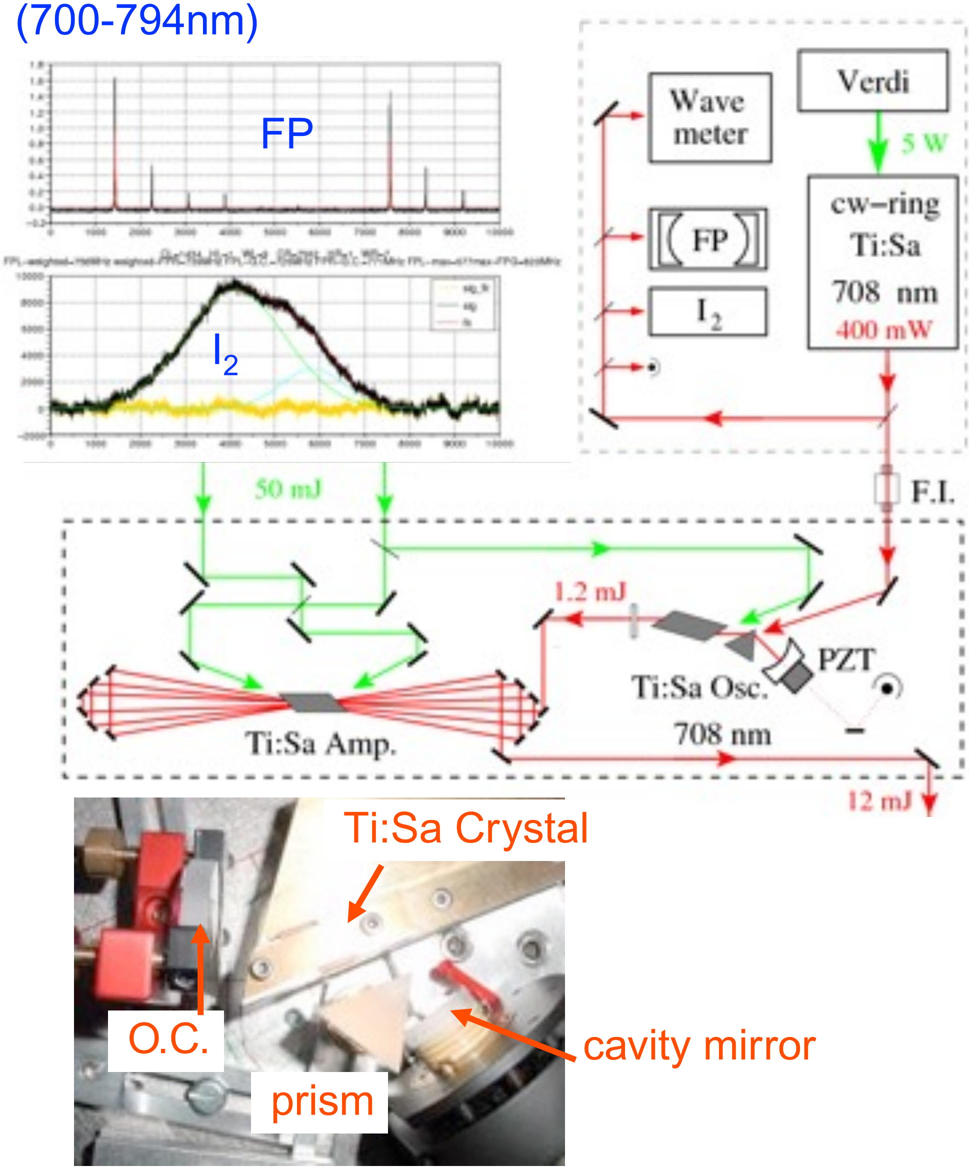
Disk amplifier

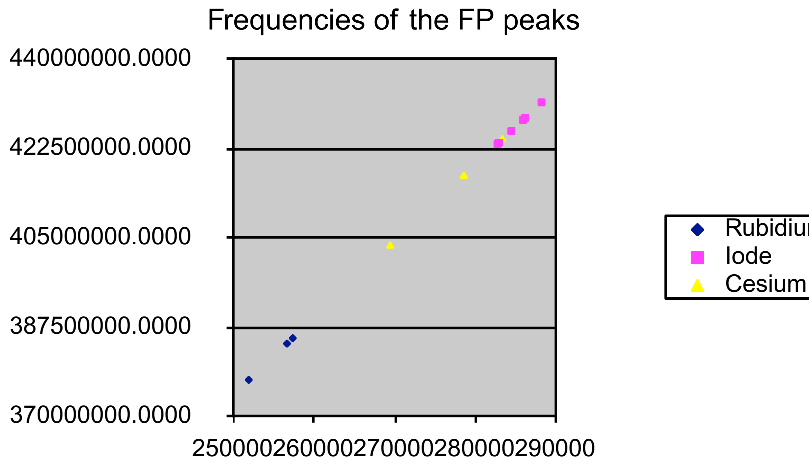


A. Antognini et al,
IEEE J. of Q. Electronics
vol 45, n 8, 2009

Laser chain : Ti:Sa

Calibration FP/ I_2, Cs, Rb
(700-794nm)

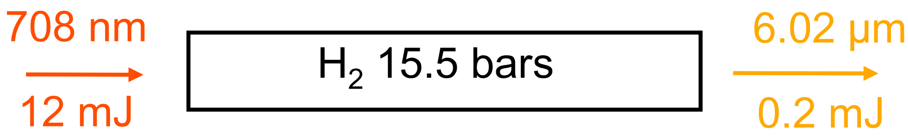


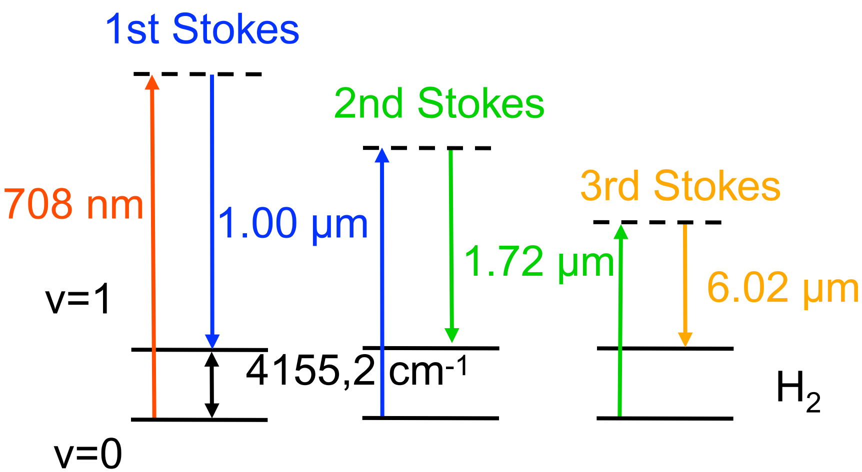


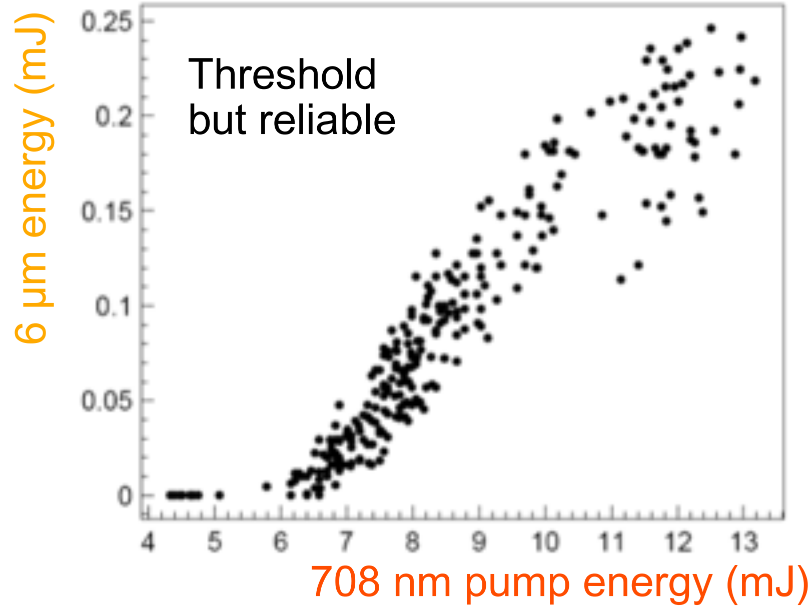
- home made Ti:Sa lasers
- cw-Ti:Sa frequency controlled with FP, atom, molecule (abs. freq. <50MHz)
- short length pulsed oscillator seeded with Cw-Ti:Sa (1.2 mJ, 6 ns, delay 50 ns, $\Delta\nu$ = 200MHz)
- multipass amplifier ´ 6 (12 mJ, 5 ns)

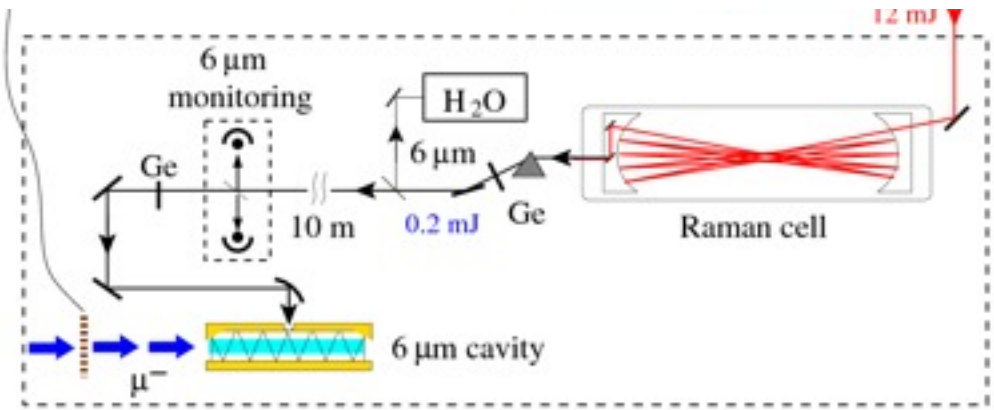
Antognini et al, Opt. Comm. 253 (2005) p.362

Laser chain : Raman cell

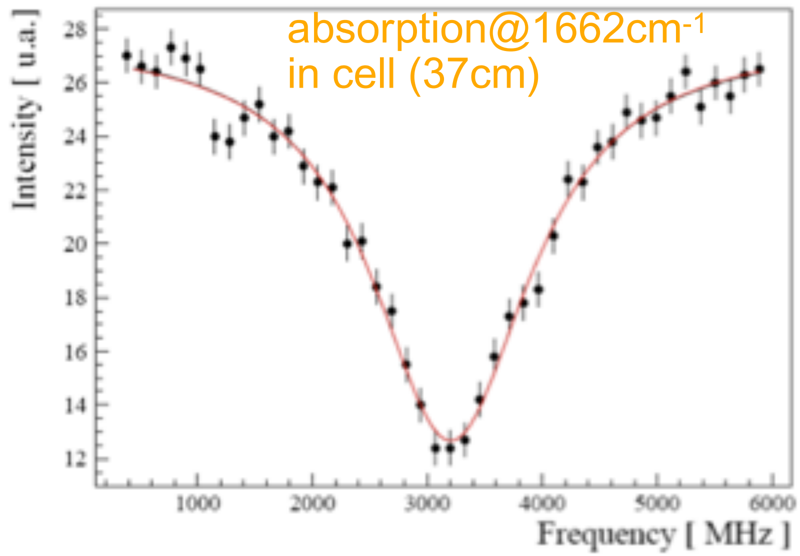






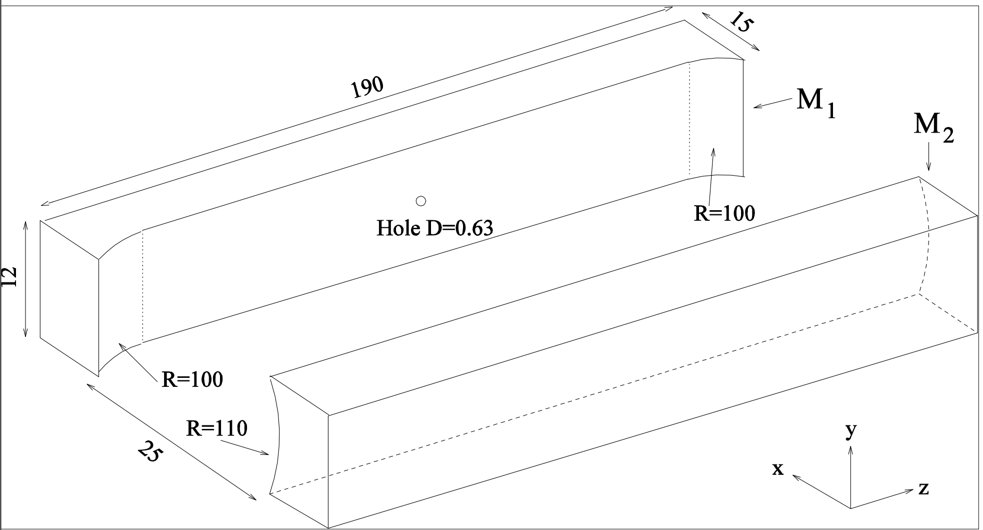


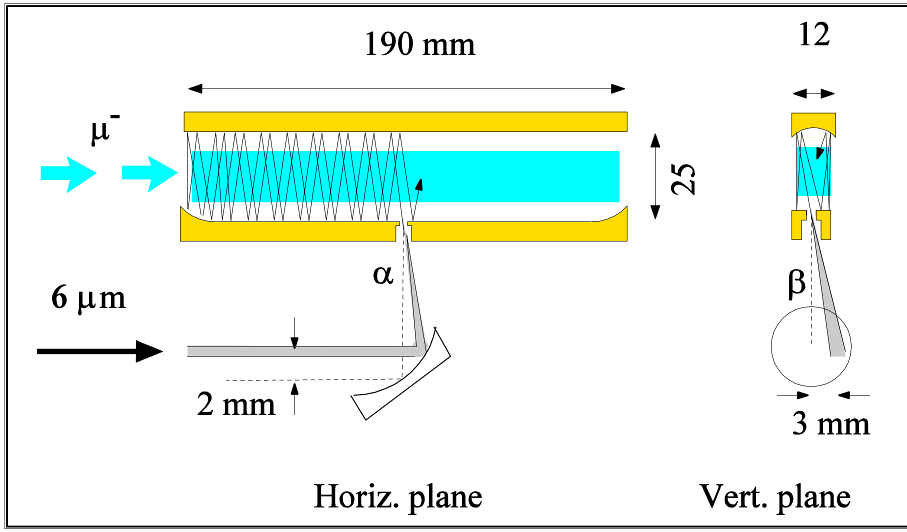
6 µm frequency calibration : H_20 lines



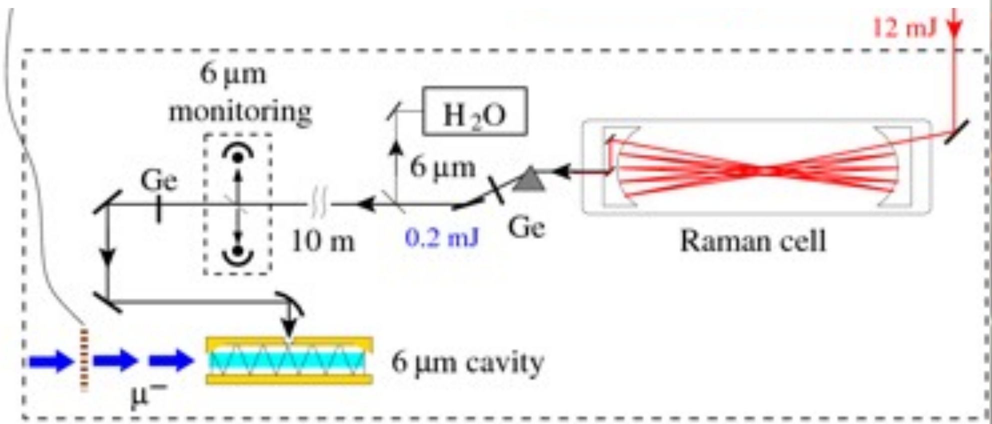
Laser chain : multipass cavity

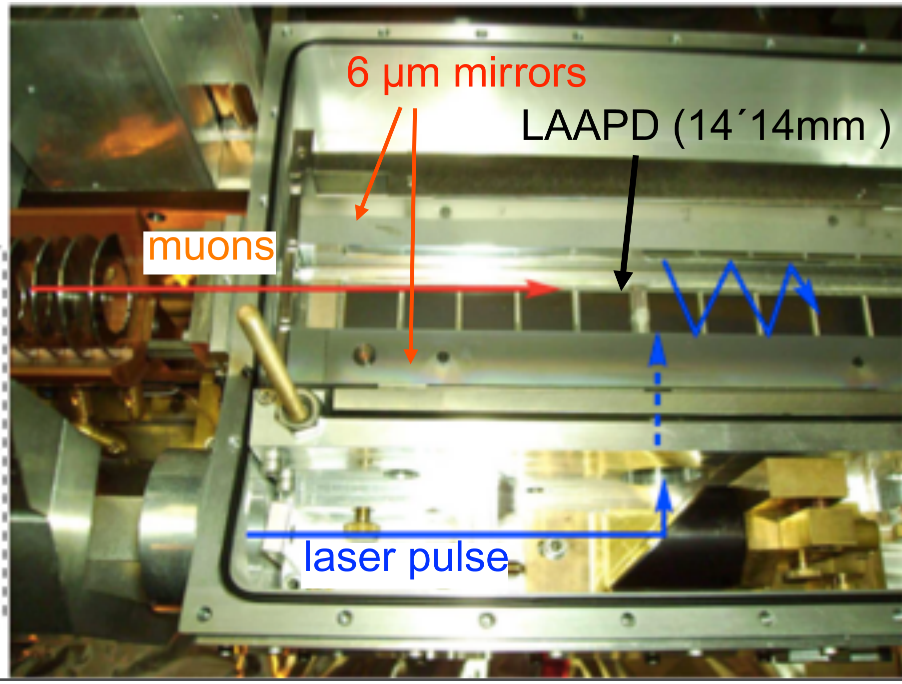
→ illuminate at 6 μm all the muon stopping volume (5´15´190 mm^3)

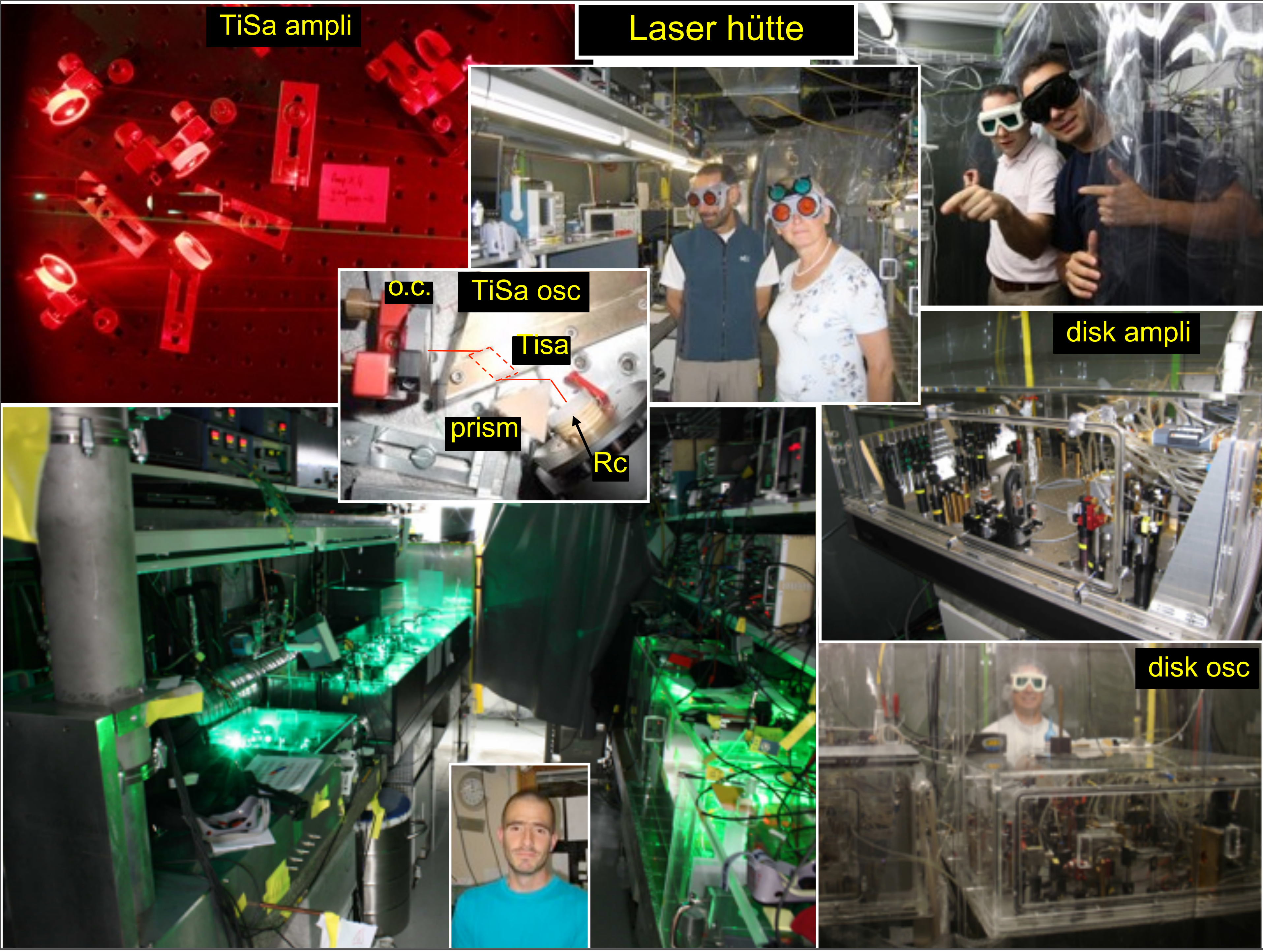




- coupling through a 0.63mm diameter hole
- R=99.90% at 6 μm
- 1000 reflections
- 0.15mJ injected → 2S-2P saturated





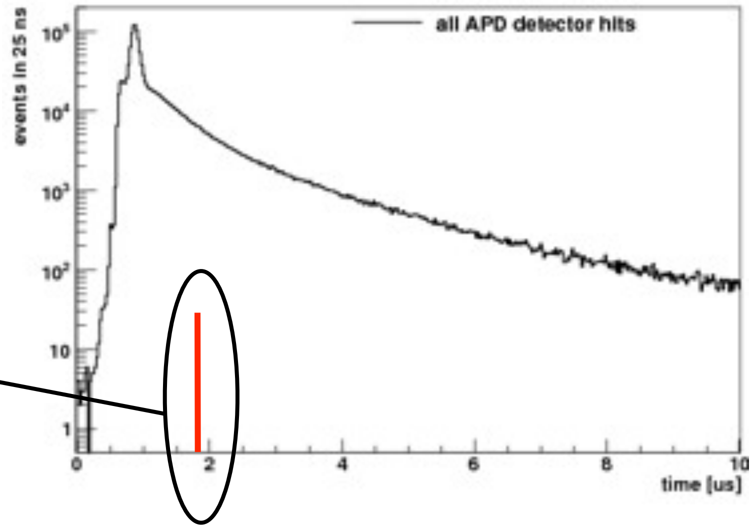

TiSa ampli
Laser hütte
o.c.
TiSa osc
Tisa
prism
Rc
disk ampli
disk osc

X-rays analysis → event gate sorting → noise rejection

Example : FP 900 - 11 hrs meas.

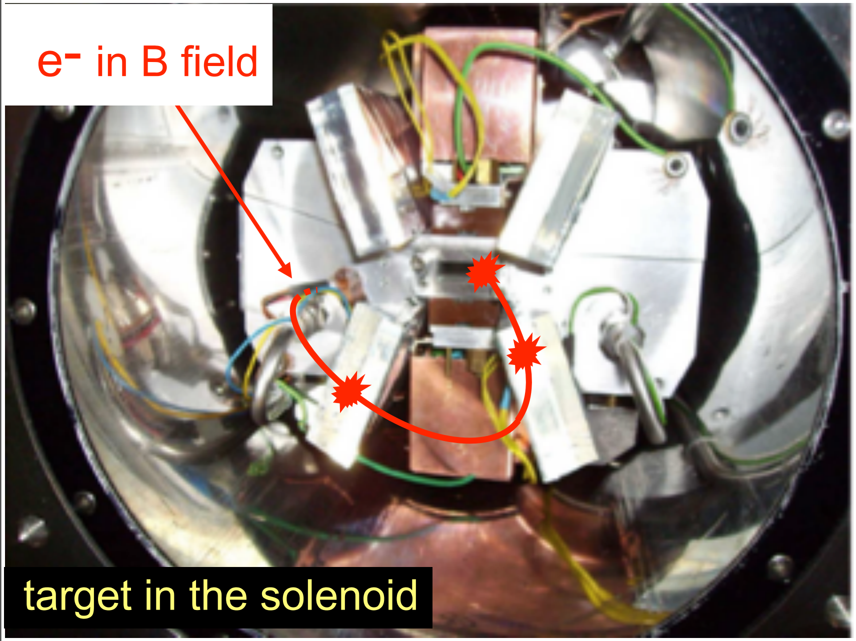
1.56 million detector events

expected 2-3 laser induced events/hour !



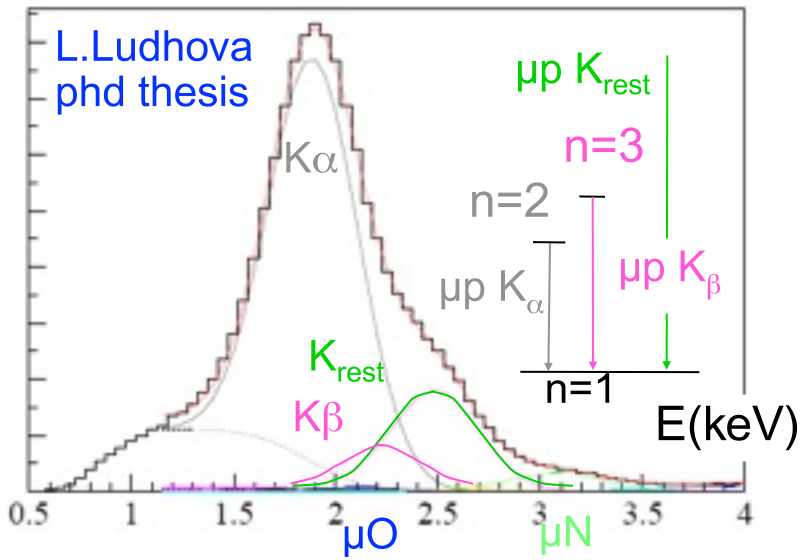
time signature in LAAPD

- photon < 10keV → 1 shot in the LAAPD
- e$^-$ in B = 5T → many counts in detectors



energy signature in LAAPD

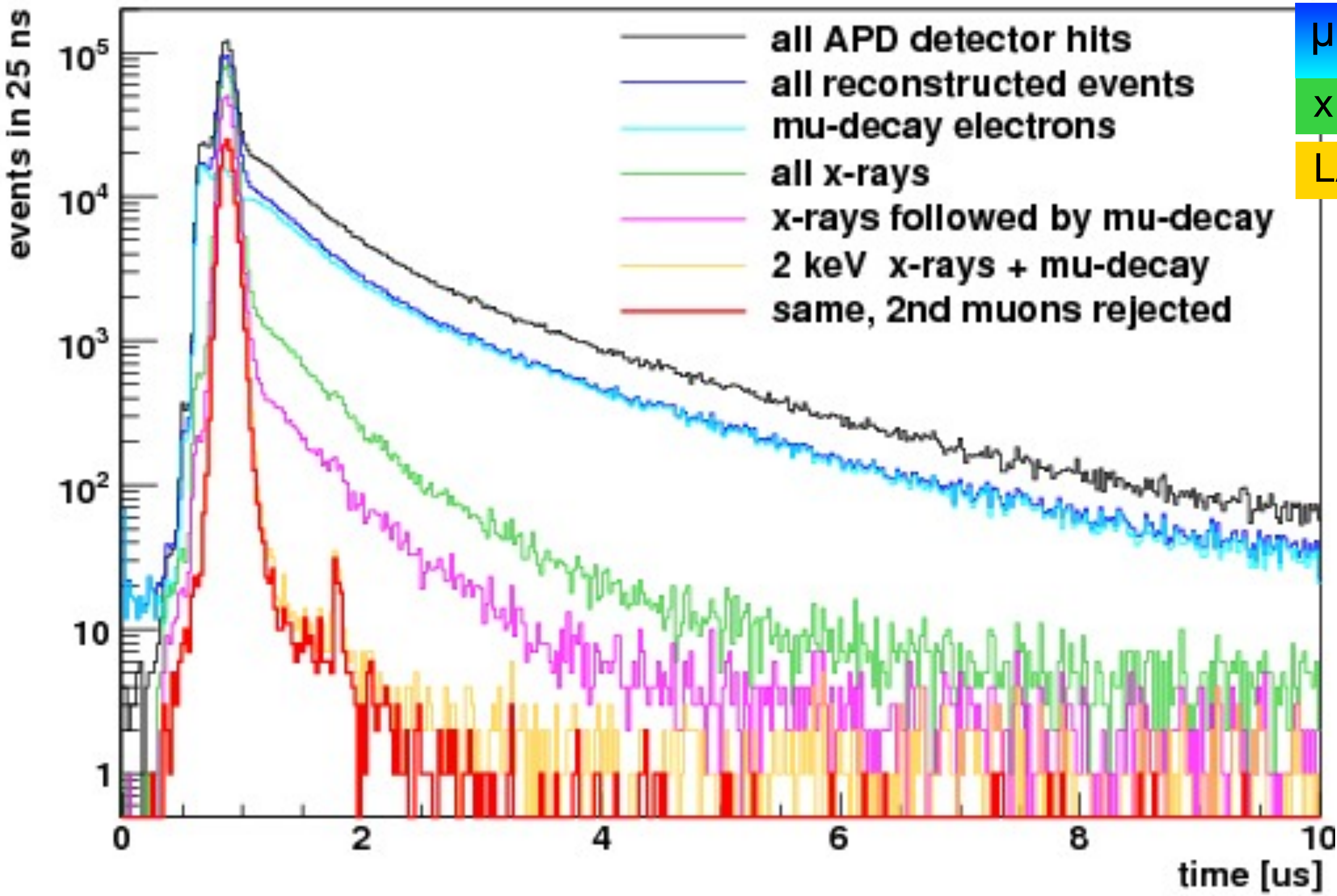
- E > 8keV ⇔ electron
- 1keV < E < 8keV ⇔ X ray
- E<1keV ⇔ neutron



X-rays analysis → noise rejection

Example : FP 900 - 11 hrs meas.

- 400 μ^-/s
- 240 laser shot/s
- 860 000 laser shot/hour
- 1.56 million detector clicks
- 19600 clicks in the laser region
- expected 2-3 laser induced events/hour !

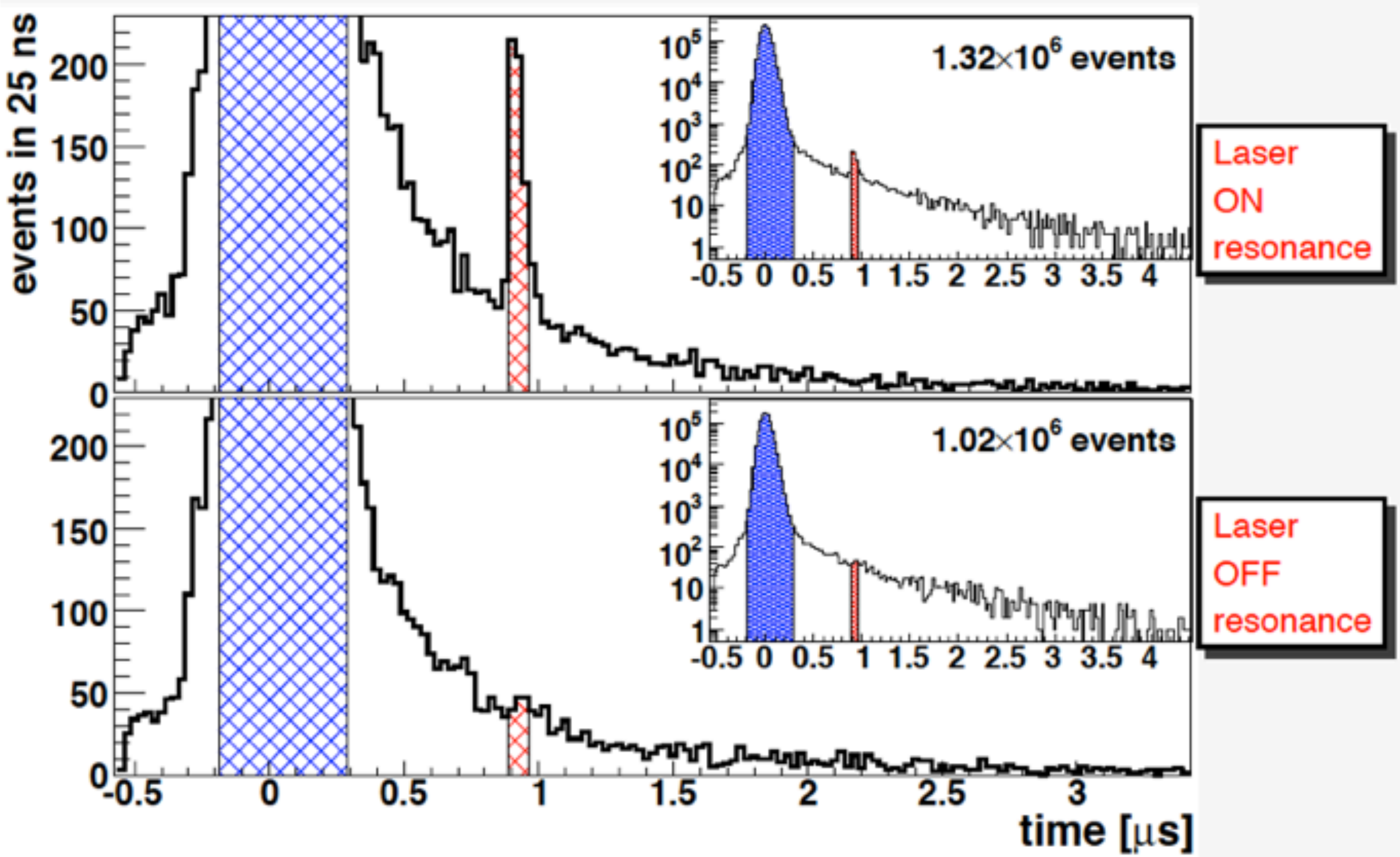


$\mu \to e\, \nu_\mu\, \bar{\nu}_e$

x rays multiplicity 1

LAAPD energy resolution

Time spectra

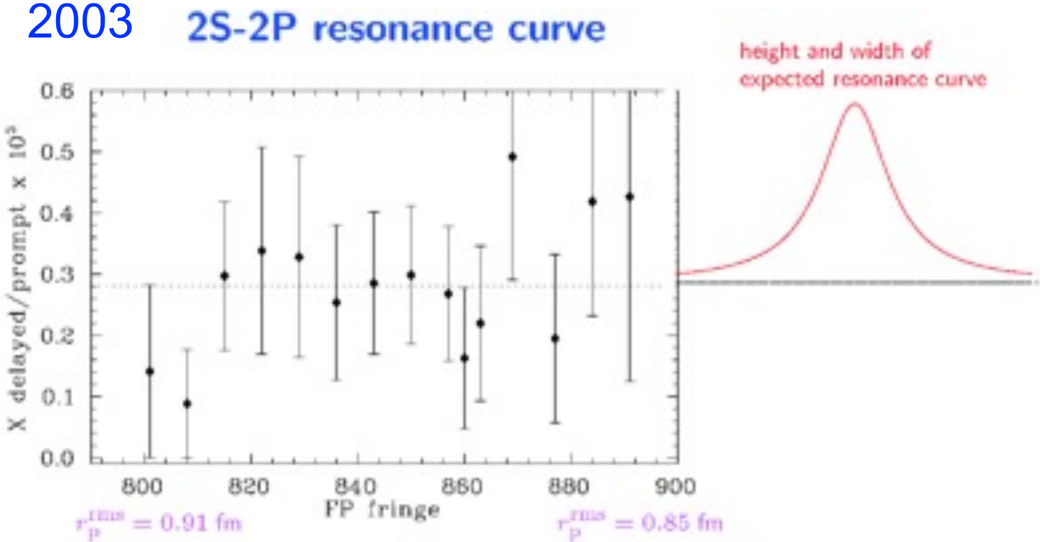


vendredi 18 novembre 2011

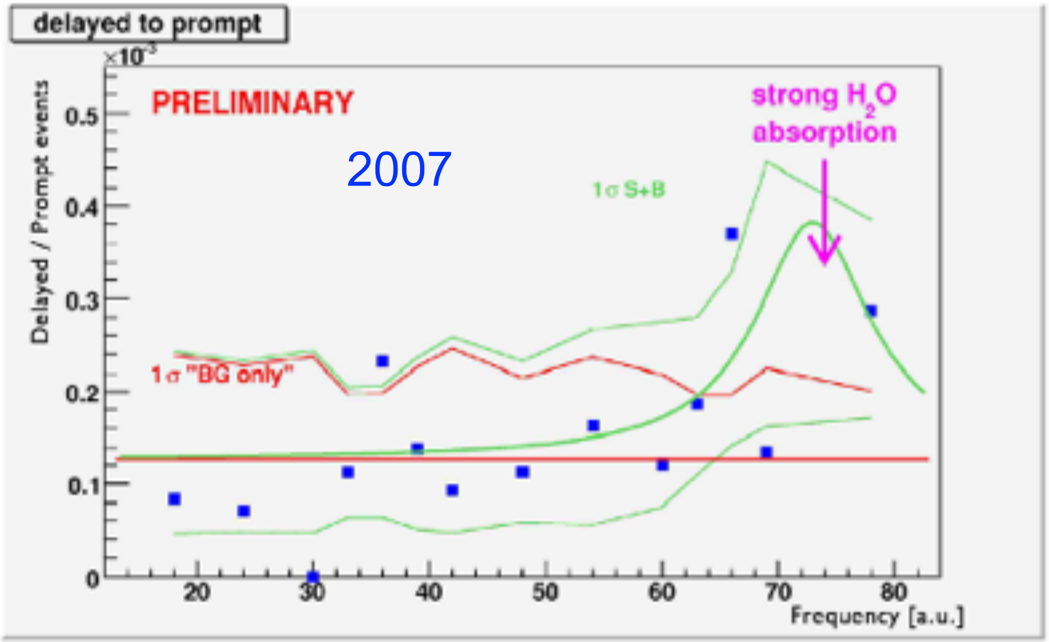
Resonance searches (2002, 2003, 2007)

2002 : Run with all the parts at the same time (search for resonance 10 hrs).

2003

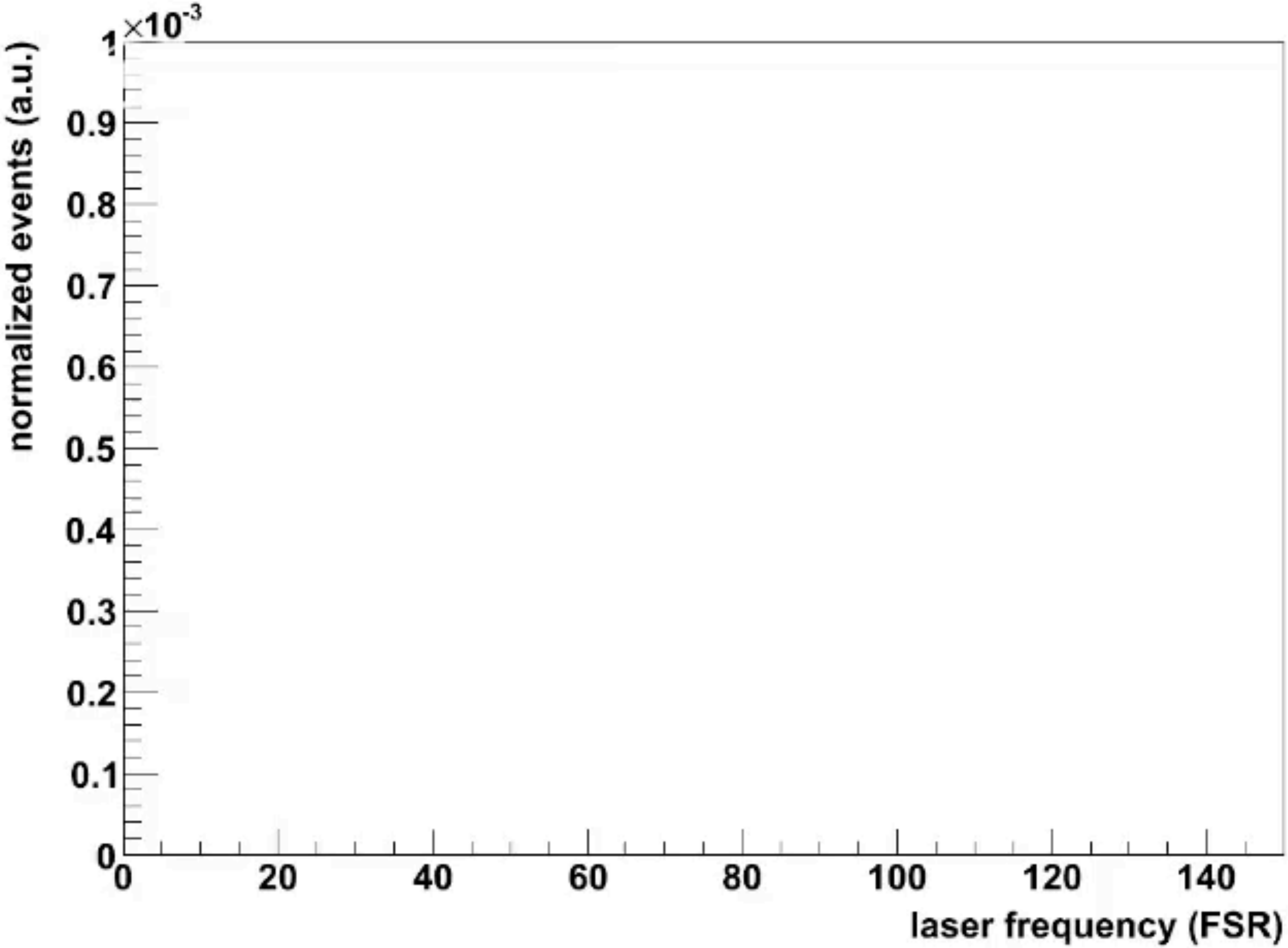


2 “new” XeCl lasers

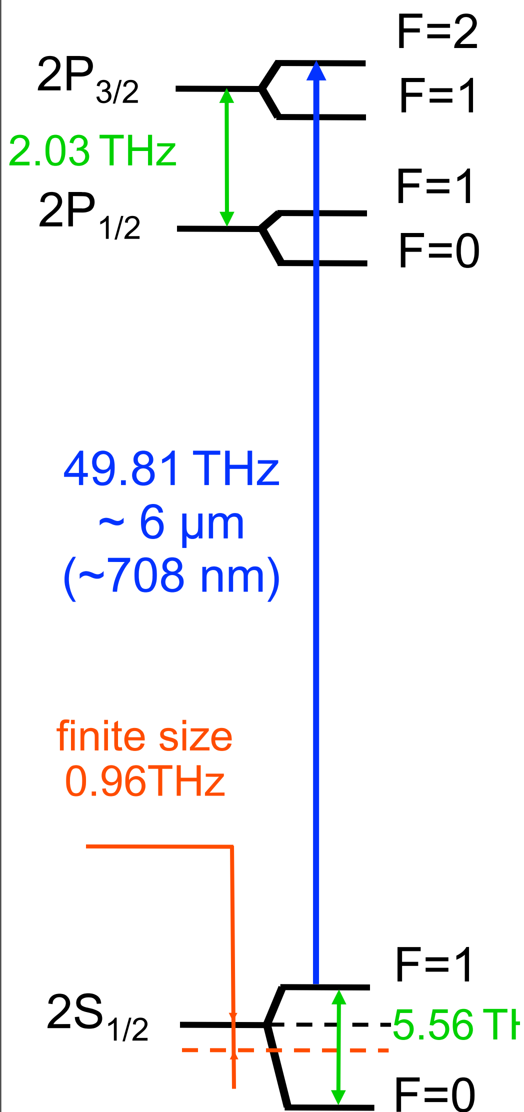


new disk laser

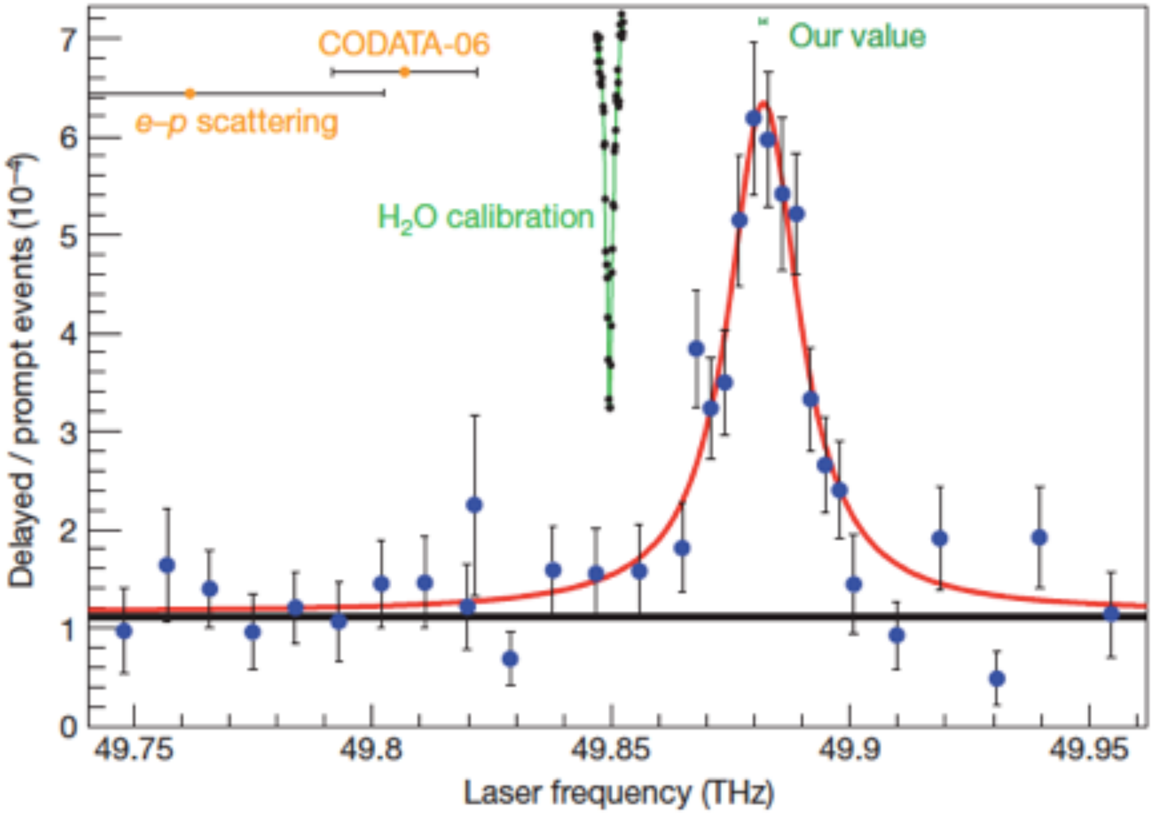
μp: $2S_{1/2}(F=1)$- $2P_{3/2}(F=2)$ movie resonance search



muonic hydrogen : $2S_{1/2}(F=1)$ - $2P_{3/2}(F=2)$



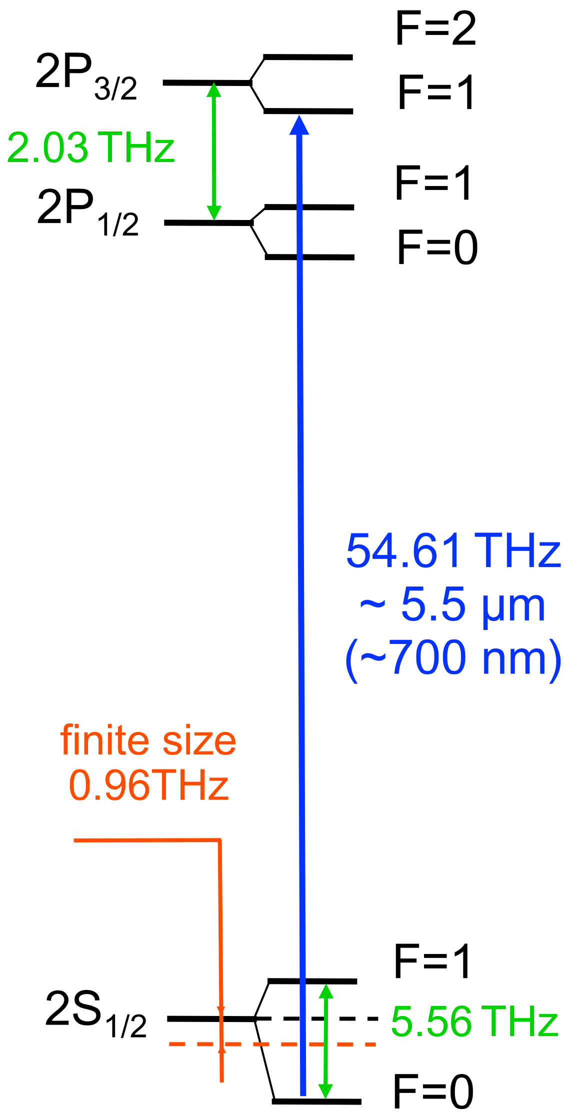
- 550 events measured
- 155 backgrounds
- 31 FP fringes
- 250 hours



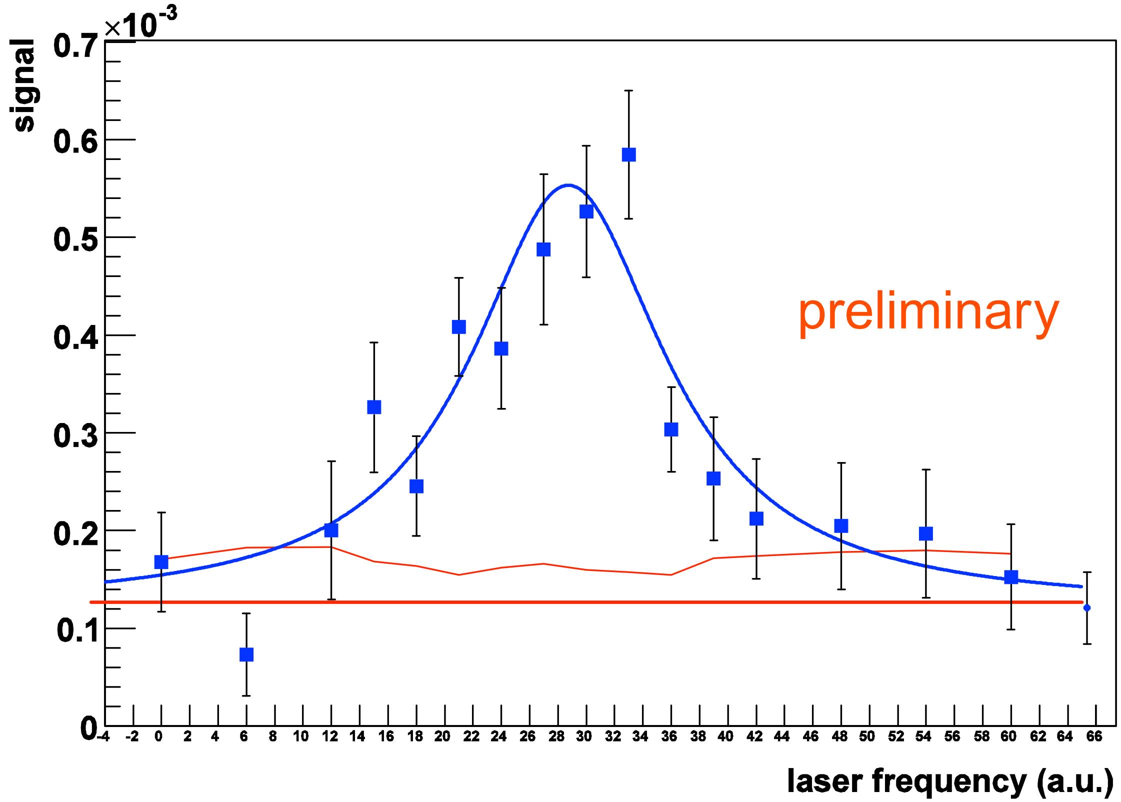
R. Pohl, A. Antognini, F. Nez, et al., Nature 466, 213 (2010).

→ proton charge radius (~0.1%)

muonic hydrogen: $2S_{1/2}$(F=0)- $2P_{3/2}$(F=1)

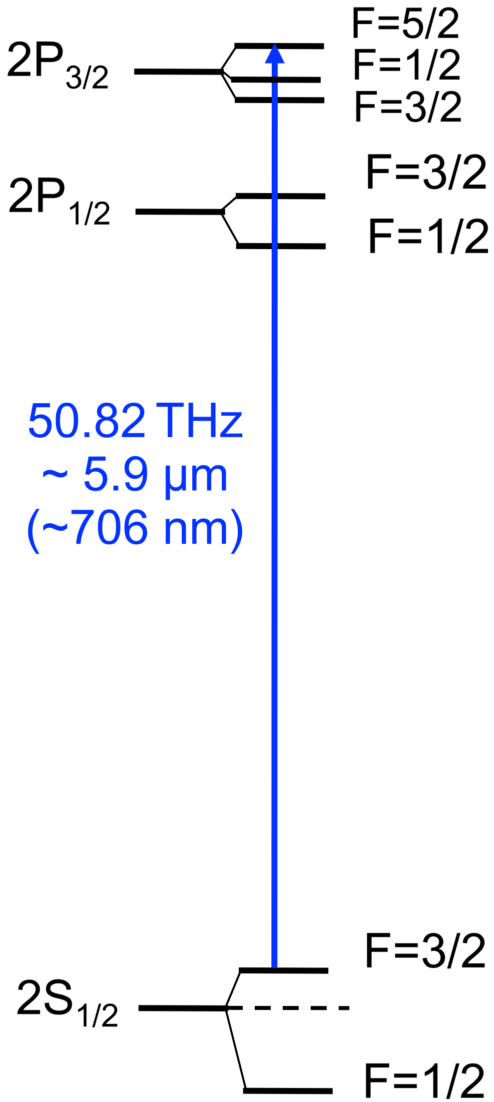


- measured position fits with our proton radius (preliminary)
- laser worked even better at 5.5 μm

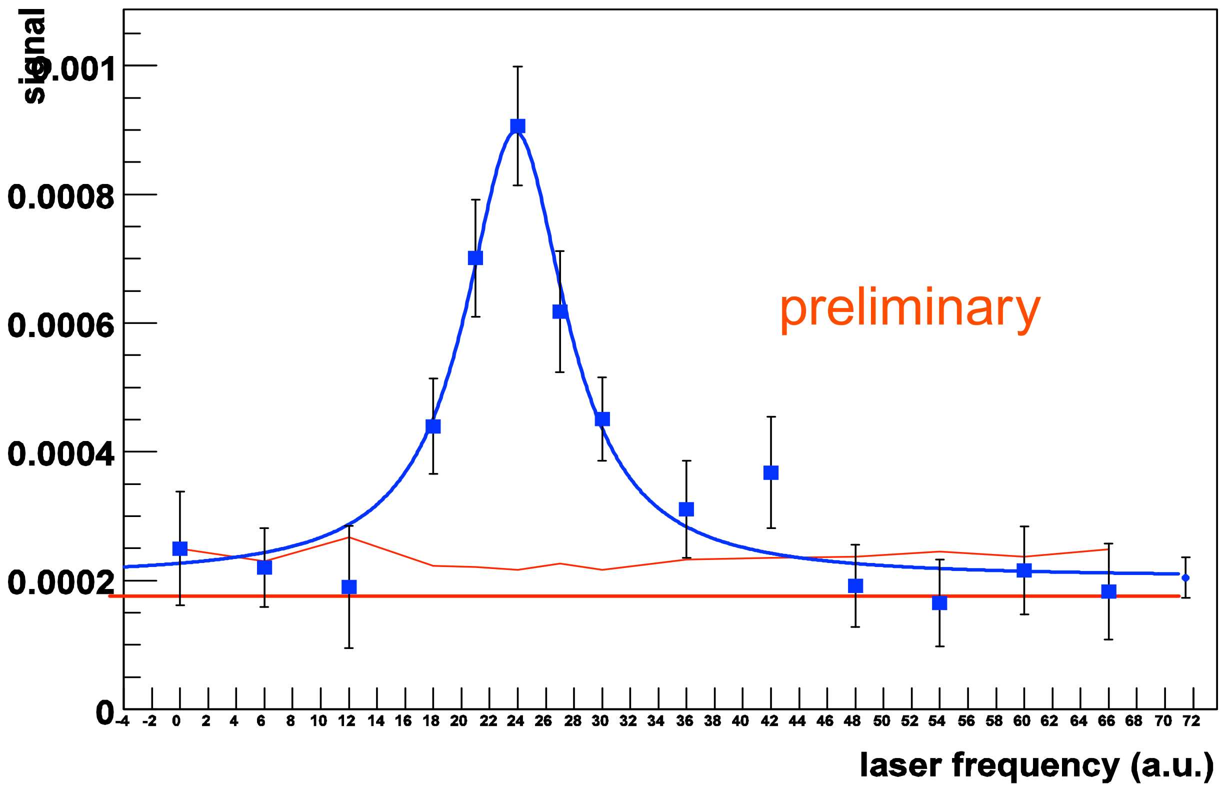


~ at the position deduced with new r_p → hfs : Zeemach radius (few %)

muonic deuterium : $2S_{1/2}(F=3/2)$- $2P_{3/2}(F=5/2)$

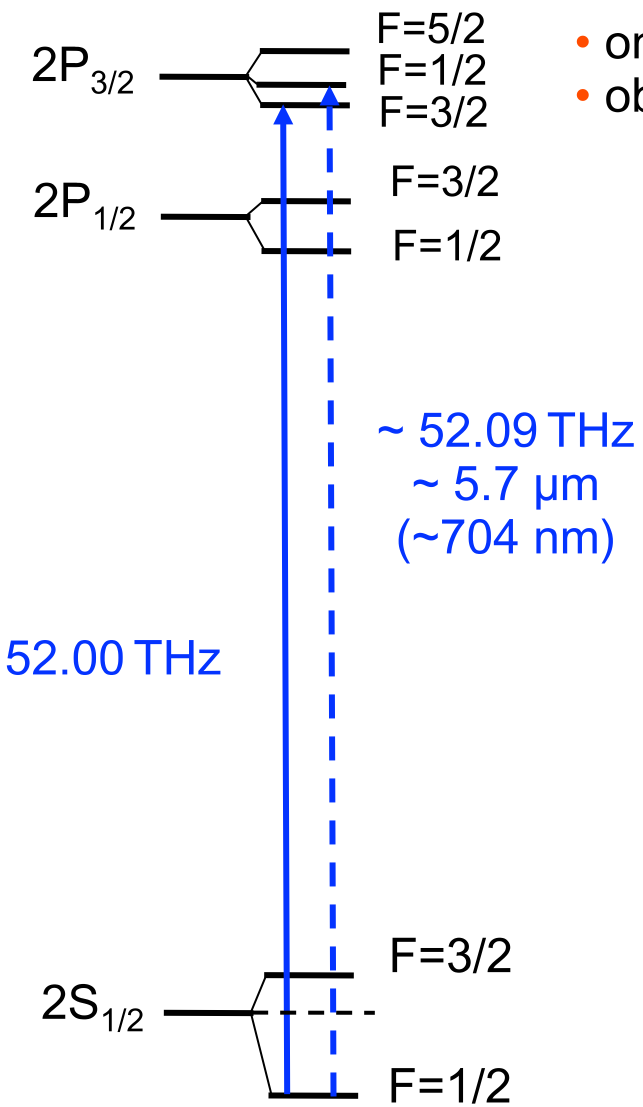


- on line signal, preliminary
- frequency position again off

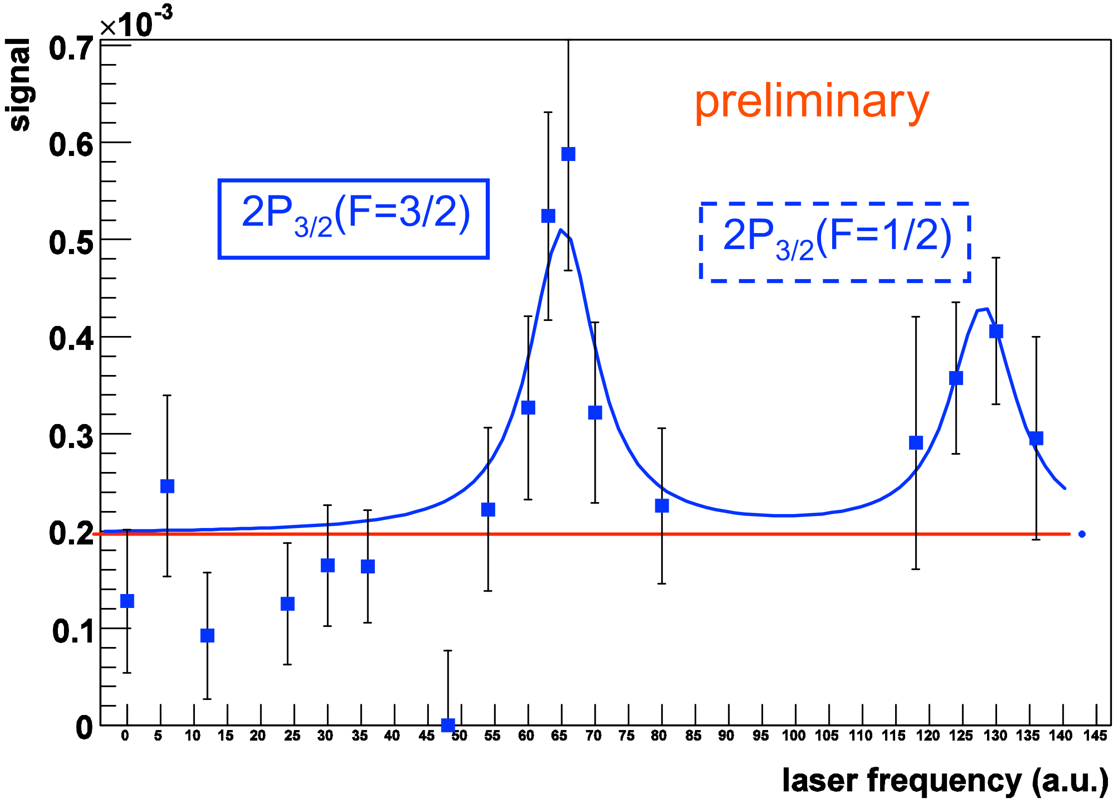


Not at the position estimated with new r_p and isotopic shift → deuteron polarizability

muonic deuterium : $2S_{1/2}(F=1/2)$- $2P_{3/2}(F=3/2)$ and $2S_{1/2}(F=1/2)$- $2P_{3/2}(F=1/2)$



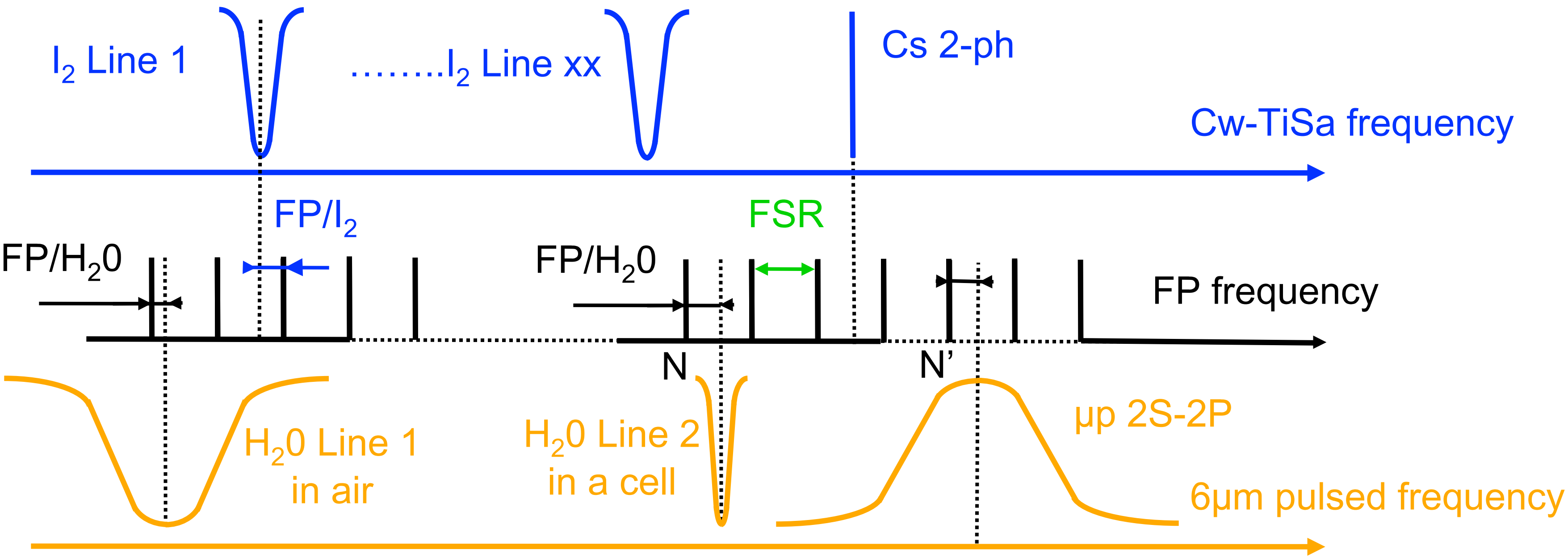
- on line signal, preliminary
- observation of 2 more lines in the last 2 days of beam line



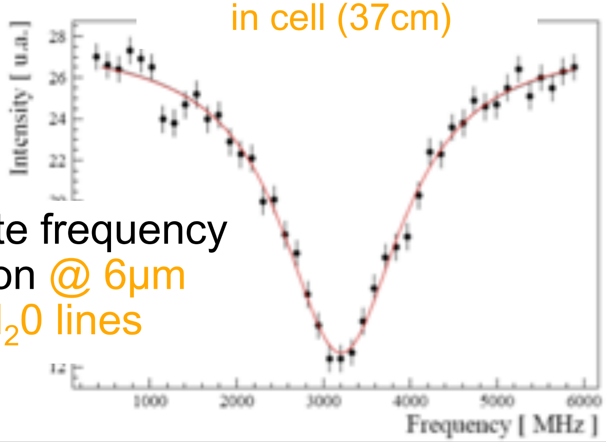
"observation" → check calculation in µd

Laser chain : frequency calibration

FSR measured/controlled in cw with I_2 (1 ph abs), Cs (2 ph fluo), Rb (2 ph fluo), lines



absorption@1662cm-1
in cell (37cm)



FP absolute frequency calibration @ 6µm with H_20 lines

ν(µp:2S-2P) = ν(H_20 Line 2) + (N-N') FSR

$\mu p\ (2S_{1/2}(F=1)\text{-}2P_{3/2}(F=2))$: uncertainty budget

Statistics

- uncertainty on position (fit) — 541 MHz (~ 4 % of Γ_{nat})

$\Delta\nu_{experimental}$ = 20 (1) GHz (Γ_{nat} = 18.6 GHz)

Sources :

• Laser frequency (H_20 calibration)	300 MHz
• AC and DC stark shift	< 1 MHz
• Zeeman shift (5 Telsa)	< 30 MHz
• Doppler shift	< 1 MHz
• Collisional shift	2 MHz

TOTAL UNCERTAINTY ON FREQUENCY — 618 MHz

Broadening :

• 6 µm laser line width	~ 2 GHz
• Doppler Broadening	< 1 GHz
• Collisional broadening	2.4 MHz

Updated: ν (µp : $2S_{1/2}(F=1)$- $2P_{3/2}(F=2)$) = 49 881.16 (62) GHz (12.5 ppm)

Nature: ν (µp : $2S_{1/2}(F=1)$- $2P_{3/2}(F=2)$) = 49 881.88 (76) GHz (16 ppm)

μp ($2S_{1/2}(F=0)-2P_{3/2}(F=1)$) : uncertainty budget

Statistics	
• uncertainty on position (fit)	960 MHz
Sources :	
• Laser frequency (H_20 calibration)	300 MHz
• AC and DC stark shift	< 1 MHz
• Zeeman shift (5 Telsa)	< 30 MHz
• Doppler shift	< 1 MHz
• Collisional shift	2 MHz
TOTAL UNCERTAINTY ON FREQUENCY	1006 MHz
Broadening :	
• 6 μm laser line width	~ 2 GHz
• Doppler Broadening	< 1 GHz
• Collisional broadening	2.4 MHz

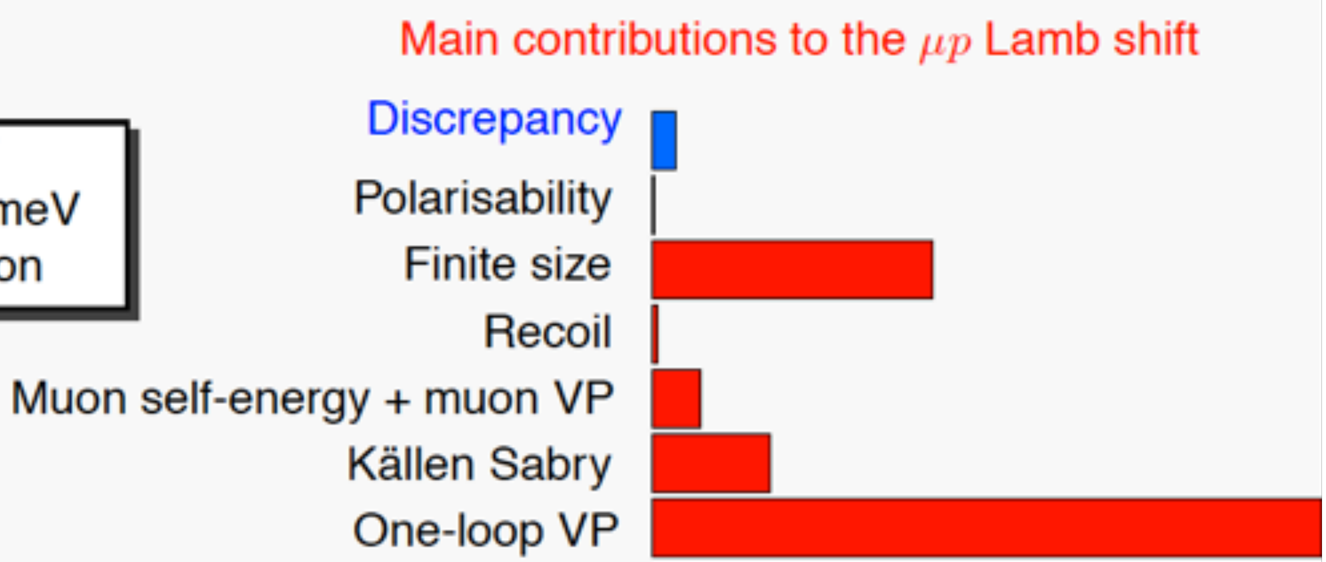
ν (μp : $2S_{1/2}(F=0)$- $2P_{3/2}(F=1)$) = 54611.87 (1.0) GHz (18.5ppm)

Extraction of the radii

Charge, magnetic and Zemach's radii

μP theory

Discrepancy=0.31 meV
Th. uncertainty=0.005 meV
$\Longrightarrow 60\delta(\text{theory})$ deviation



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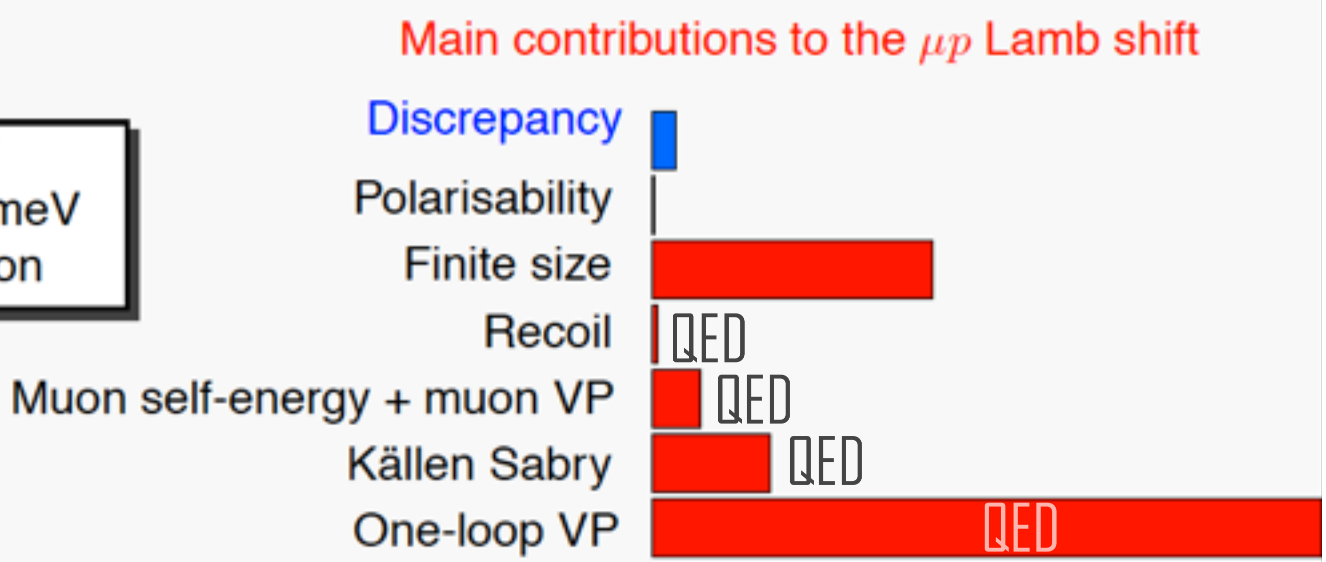

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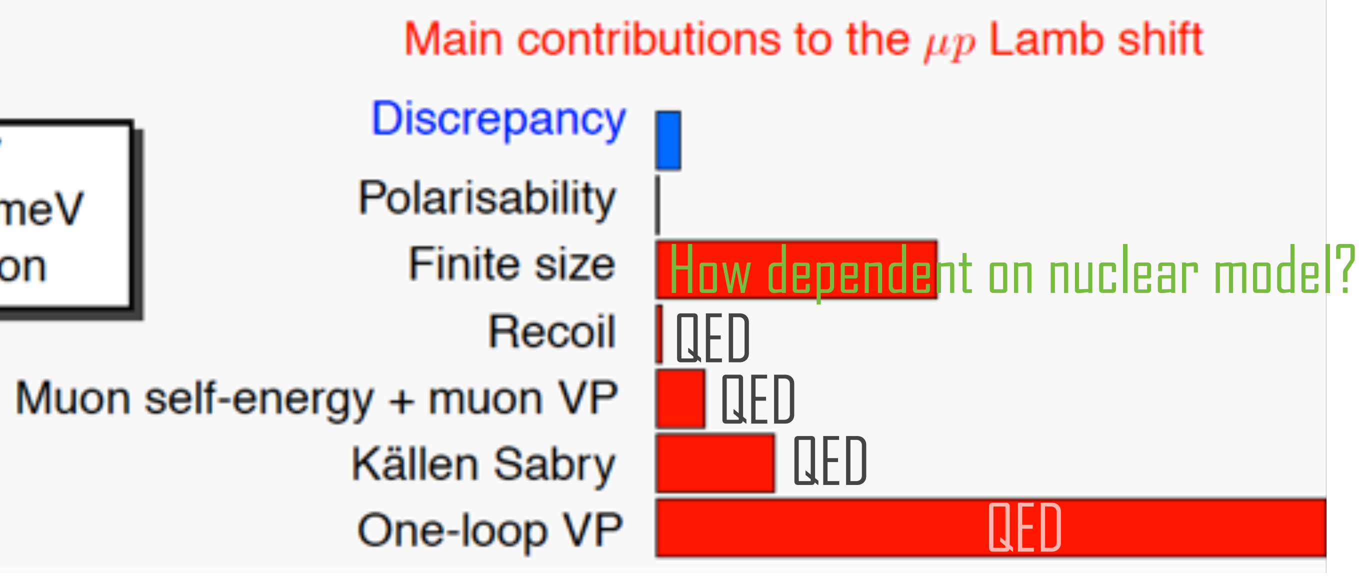

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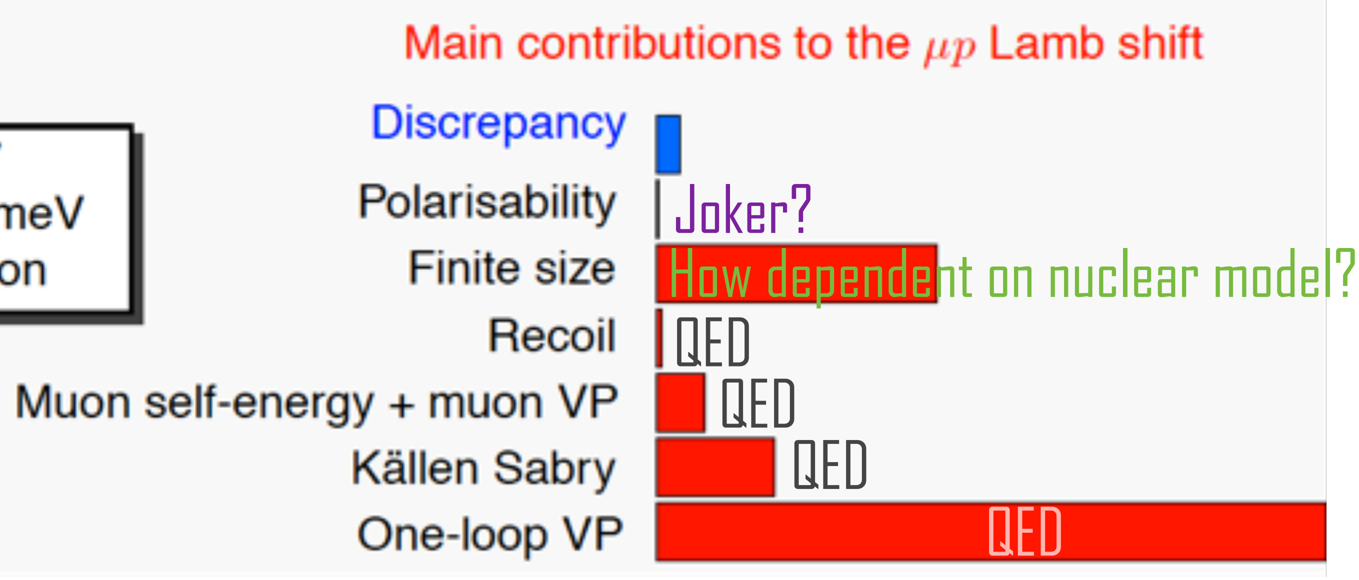

QED and Hypefine energy

- The two lines obey to:

$$E_{^5P_{3/2}} - E_{^3S_{1/2}} = \Delta E_{\rm LS} + \Delta E_{\rm FS} + \tfrac{3}{8}\Delta E_{\rm HFS}(2p_{3/2}) - \tfrac{1}{4}\Delta E_{\rm HFS}(2s)$$
$$E_{^3P_{3/2}} - E_{^1S_{1/2}} = \Delta E_{\rm LS} + \Delta E_{\rm FS} - \tfrac{5}{8}\Delta E_{\rm HFS}(2p_{3/2}) + \tfrac{3}{4}\Delta E_{\rm HFS}(2s)$$

$$\Delta E_{\rm HFS}(2s)^{\rm Exp.} = 22.8082 \pm 0.0078\,{\rm meV} = 5514.99\,{\rm GHz}$$

$$\Delta E_{\rm HFS}(2s)^{\rm Mart.} = 22.8148 \pm 0.0078\,{\rm meV}$$

$$\Delta E_{\rm HFS}(2s)^{\rm Carlson} = 22.8146 \pm 0.0049\,{\rm meV}$$

Proton charge Radius from muonic hydrogen

$$\Delta\tilde{E} = 209.9779(49) - 5.2262\ r_p^2 + 0.0347\ r_p^3\ \text{meV}$$

R. Pohl, A. Antognini, F. Nez, et al., Nature **466**, 213 (2010).

$$E_{^5P_{3/2}} - E_{^3S_{1/2}} = 209.9759 - 5.22888R^2 + 0.0357376R^3 - 0.000045R^4$$

P. Indelicato, P. Mohr, to be published

Proton charge Radius from muonic hydrogen

$$\Delta\tilde{E} = 209.9779(49) - 5.2262\ r_p^2 + 0.0347\ r_p^3\ \text{meV}$$

0.84184(67) fm

R. Pohl, A. Antognini, F. Nez, et al., Nature **466**, 213 (2010).

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R. Pohl, A. Antognini, F. Nez, et al., Nature **466**, 213 (2010).

$$E_{\mathrm{th}} = \left(209.9974(48) - 5.2262\frac{r_p^2}{\mathrm{fm}^2}\right)\mathrm{meV}$$

U. D. Jentschura, Annals of Physics **326**, 500 (2011).

$$E_{^5P_{3/2}} - E_{^3S_{1/2}} = 209.9759 - 5.22888R^2 + 0.0357376R^3 - 0.000045R^4$$

P. Indelicato, P. Mohr, to be published

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: 0.84169(66) fm.

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$$E_{^5P_{3/2}} - E_{^3S_{1/2}} = 209.9759 - 5.22888R^2 + 0.0357376R^3 - 0.000045R^4$$

P. Indelicato, P. Mohr, to be published

Proton charge Radius from muonic hydrogen

$$\Delta\tilde{E} = 209.9779(49) - 5.2262\ r_p^2 + 0.0347\ r_p^3\ \text{meV}$$

0.84184(67) fm

R. Pohl, A. Antognini, F. Nez, et al., Nature **466**, 213 (2010).

$$E_{\text{th}} = \left(209.9974(48) - 5.2262\,\frac{r_p^2}{\text{fm}^2}\right)\text{meV}$$

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U. D. Jentschura, Annals of Physics **326**, 500 (2011).

$$E_{^5P_{3/2}} - E_{^3S_{1/2}} = 209.9759 - 5.22888R^2 + 0.0357376R^3 - 0.000045R^4$$

0.84145(66) fm

P. Indelicato, P. Mohr, to be published

Zemach and magnetic radius

$$\Delta\tilde{E} = 209.9779(49) - 5.2262\ r_p^2 + \boxed{0.0347\ r_p^3}\ \text{meV}$$

R. Pohl, A. Antognini, F. Nez, et al., Nature **466**, 213 (2010).

Model dependent

Martynenko: 2s Hyperfine structure: 24.1819- 0.16018 R_z meV

$$E_{^5P_{3/2}} - E_{^3S_{1/2}} = 209.6341 - 5.22888 Rp^2 + 0.0357376 Rp^3 - 0.000045 Rp^4 + 0.32036 Rz$$

$$E_{^3P_{3/2}} - E_{^1S_{1/2}} = 230.5679 - 5.22888 Rp^2 + 0.0357376 Rp^3 - 0.000045 Rp^4 - 0.96108 Rz$$

$$R_{Z,e,e}^{\mathrm{Exp.},1} = \frac{3R^4 + 9R^3 R_\mathrm{M} + 11R^2 R_\mathrm{M}^2 + 9RR_\mathrm{M}^3 + 3R_\mathrm{M}^4}{2\sqrt{3}(R + R_\mathrm{M})^3}$$

Zemach and magnetic radius

$$\Delta\tilde{E} = 209.9779(49) - 5.2262\ r_p^2 + \boxed{0.0347\ r_p^3}\ \text{meV}$$

0.84184(67) fm

R. Pohl, A. Antognini, F. Nez, et al., Nature **466**, 213 (2010).

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$$R_{Z,e,e}^{\text{Exp.},1} = \frac{3R^4+9R^3R_{\text{M}}+11R^2R_{\text{M}}^2+9RR_{\text{M}}^3+3R_{\text{M}}^4}{2\sqrt{3}(R+R_{\text{M}})^3}$$

Zemach radius

- Perturbation theory on HFS leads to a combined finite charge correction and magnetic moment distribution distribution (Zemach, 1958):

$$\rho_{em}(\boldsymbol{r}) = \int \rho(\boldsymbol{r} - \boldsymbol{u})\mu(\boldsymbol{u})d\boldsymbol{u}$$

$$R_{\mathbf{Z}} = \langle r_{\mathbf{Z}} \rangle = \int r \rho_{em}(\boldsymbol{r})d\boldsymbol{r}.$$

-

LKB

Hyperfine structure and Zemach's moments

$$\langle R^n_{Z,a,b} \rangle = \int d^3 r_1 d^3 r_2 \rho_a (r_1) \rho_b (r_2) \left| \boldsymbol{r}_1 - \boldsymbol{r}_2 \right|^n$$

$$\Delta E_{\mathrm{HFS}} = E_{\mathrm{F}} \left(1 - 2 \alpha m_\mu \left\langle r^1_{Z,e,m} \right\rangle \right)$$

Using the Dipole model for the charge distribution

$$\langle R^3_{Z,e,e} \rangle^2 = \frac{3675}{256} \left[\langle r^2 \rangle \right]^3$$

Magnetic moment distribution

- From the muonic hydrogen LS:
  - Zemach radius: Charge radius: 0.84185 fm
  - Zemach radius: 1.079 (41) fm
  - Magnetic radius (assuming a dipole model) distribution: 0.867 (66) fm
- Mainz experiment:
  - Magnetic moment distribution: 0:777(13)$_{stat}$(9)$_{syst}$(5)$_{model}$(2)$_{group}$ fm
  - Zemach radius: 1.085 (3) fm cited as 1.045 in [1]
  - [1] Proton-structure corrections to hyperfine splitting in muonic hydrogen, C.E. Carlson, V. Nazaryan et K. Griffioen. Phys. Rev. A 83, 042509 (2011).

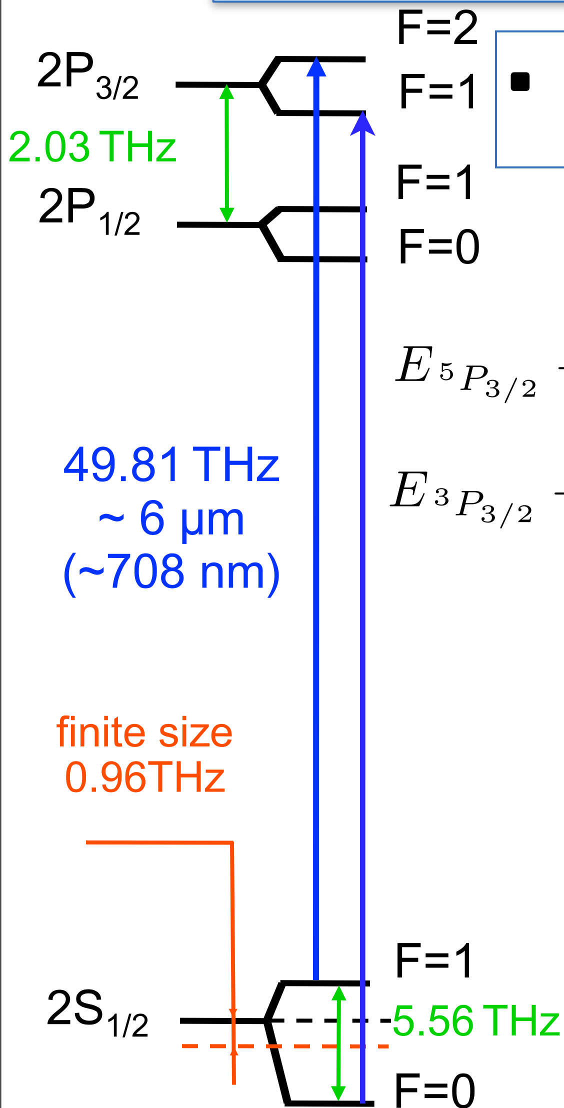
Zemach radius

TABLE I. Hyperfine splitting for the $2S$ state of muonic hydrogen, using different modern analytic fits in the terms that involve elastic form factors.

Form factor fit	E_{HFS}^{2S} (meV)	r_Z (fm)
AMT [18]	22.8123	1.080
Kelly [19]	22.8141	1.069
AS [20]	22.8105	1.091
Mainz 2010 [21–23]	22.8187	1.045

Muonic Hydrogen	1.079 (41)
Hydrogen (Volotka et al. 2005)	1.045 (16)

QED and Hyperfine energy



- The two measured lines obey to:

$$E_{^5P_{3/2}} - E_{^3S_{1/2}} = \Delta E_{\mathrm{LS}} + \Delta E_{\mathrm{FS}} + \tfrac{3}{8}\Delta E_{\mathrm{HFS}}(2p_{3/2}) - \tfrac{1}{4}\Delta E_{\mathrm{HFS}}(2s)$$

$$E_{^3P_{3/2}} - E_{^1S_{1/2}} = \Delta E_{\mathrm{LS}} + \Delta E_{\mathrm{FS}} - \tfrac{5}{8}\Delta E_{\mathrm{HFS}}(2p_{3/2}) + \tfrac{3}{4}\Delta E_{\mathrm{HFS}}(2s)$$

Hyperfine structure and Zemach's moments

$$\langle R^n_{Z,a,b} \rangle = \int d^3 r_1 d^3 r_2 \rho_a (r_1) \rho_b (r_2) \left| \boldsymbol{r}_1 - \boldsymbol{r}_2 \right|^n$$

$$\Delta E_{\mathrm{HFS}} = E_{\mathrm{F}} \left(1 - 2\alpha m_\mu \langle r^1_{Z,e,m} \rangle \right)$$

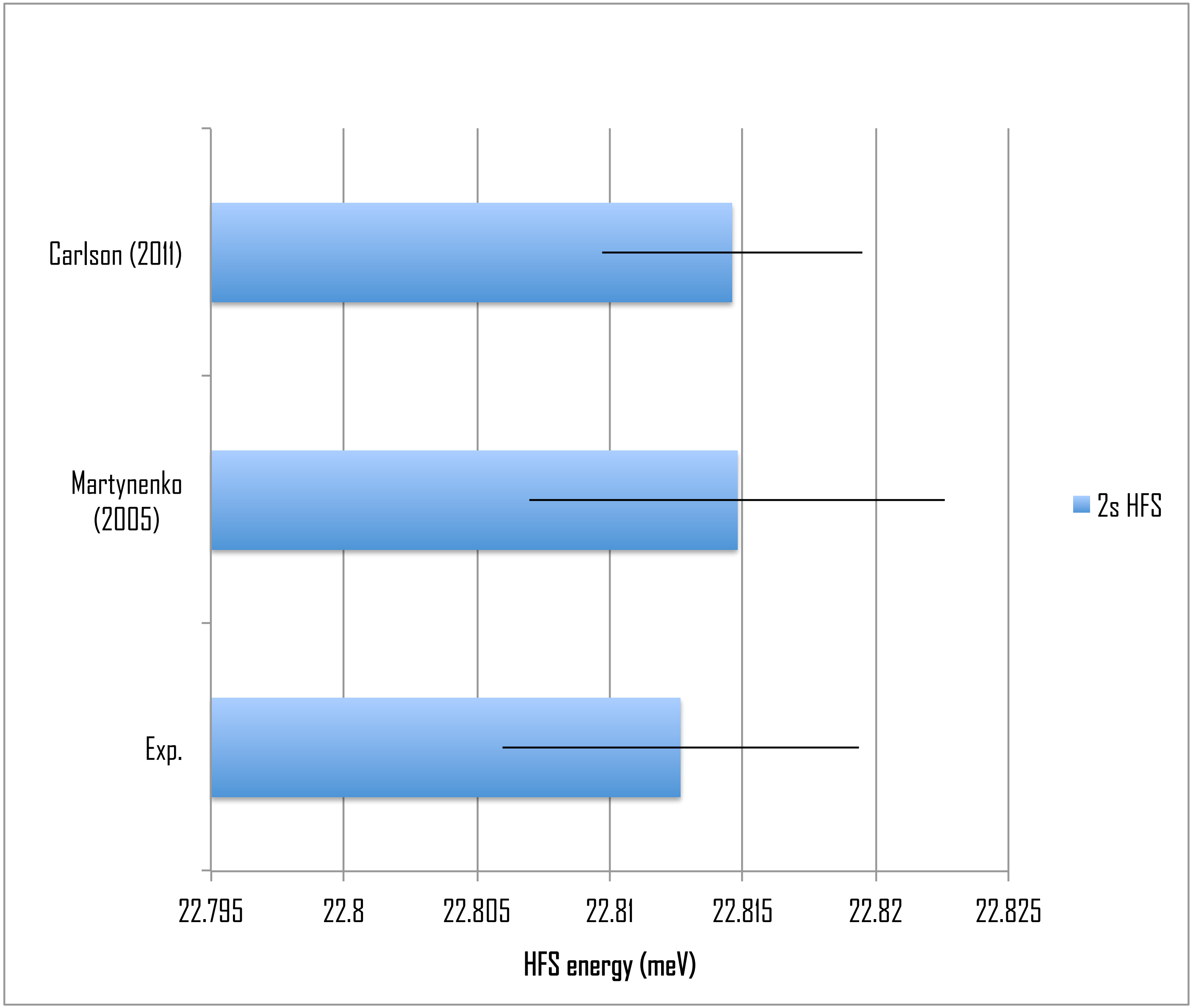
$$\rho_{em}(\boldsymbol{r}) = \int \rho(\boldsymbol{r} - \boldsymbol{u}) \mu(\boldsymbol{u}) d\boldsymbol{u}$$

$$R_{\mathbf{Z}} = \langle r_{\mathbf{Z}} \rangle = \int r \rho_{em}(\boldsymbol{r}) d\boldsymbol{r}.$$

Using the Dipole model for the charge distribution

$$\langle R^3_{Z,e,e} \rangle^2 = \frac{3675}{256} \left[\langle r^2 \rangle \right]^3$$

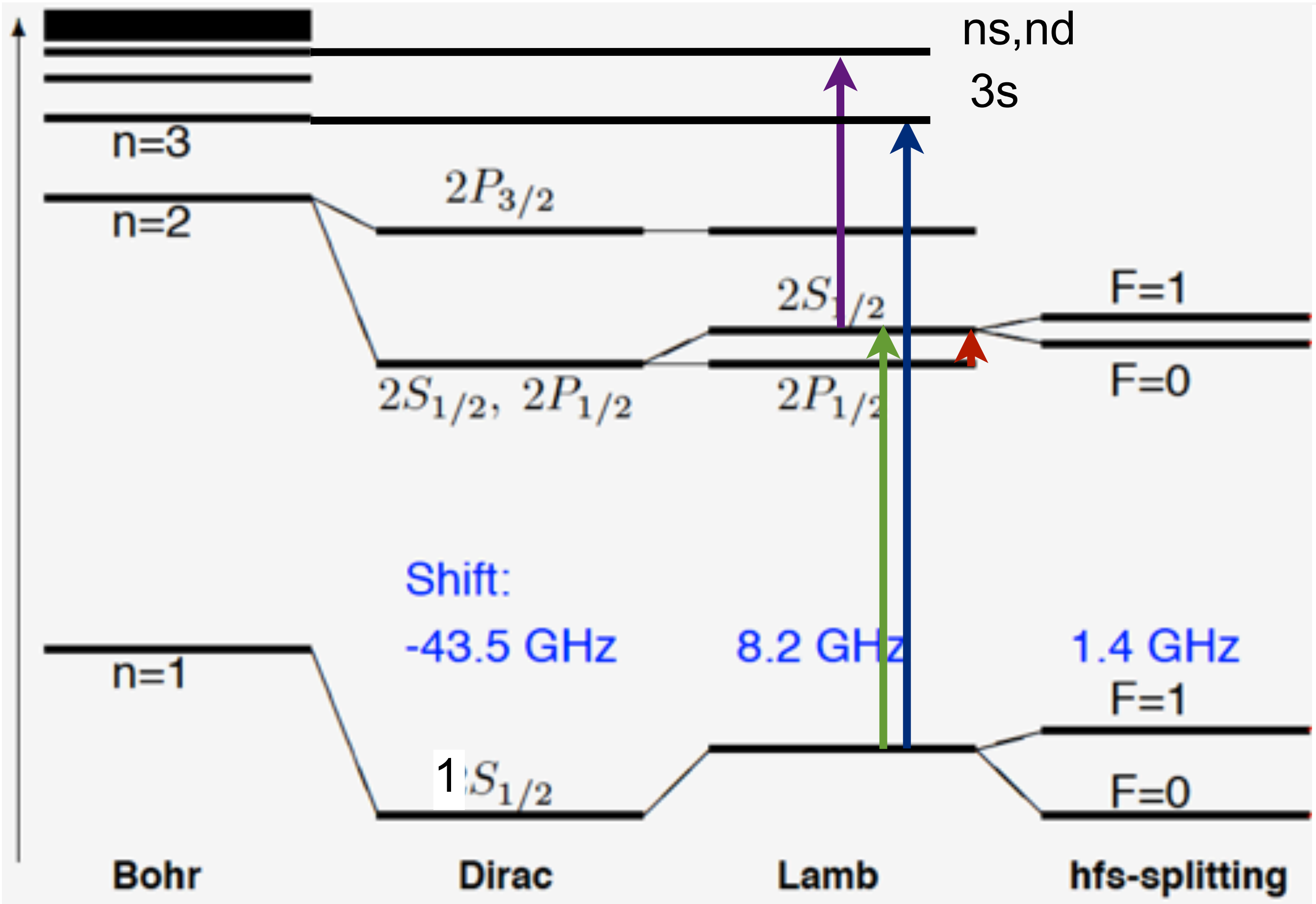
2s Hyperfine structure



Hydrogen

How to get the radius from hydrogen

Hydrogen

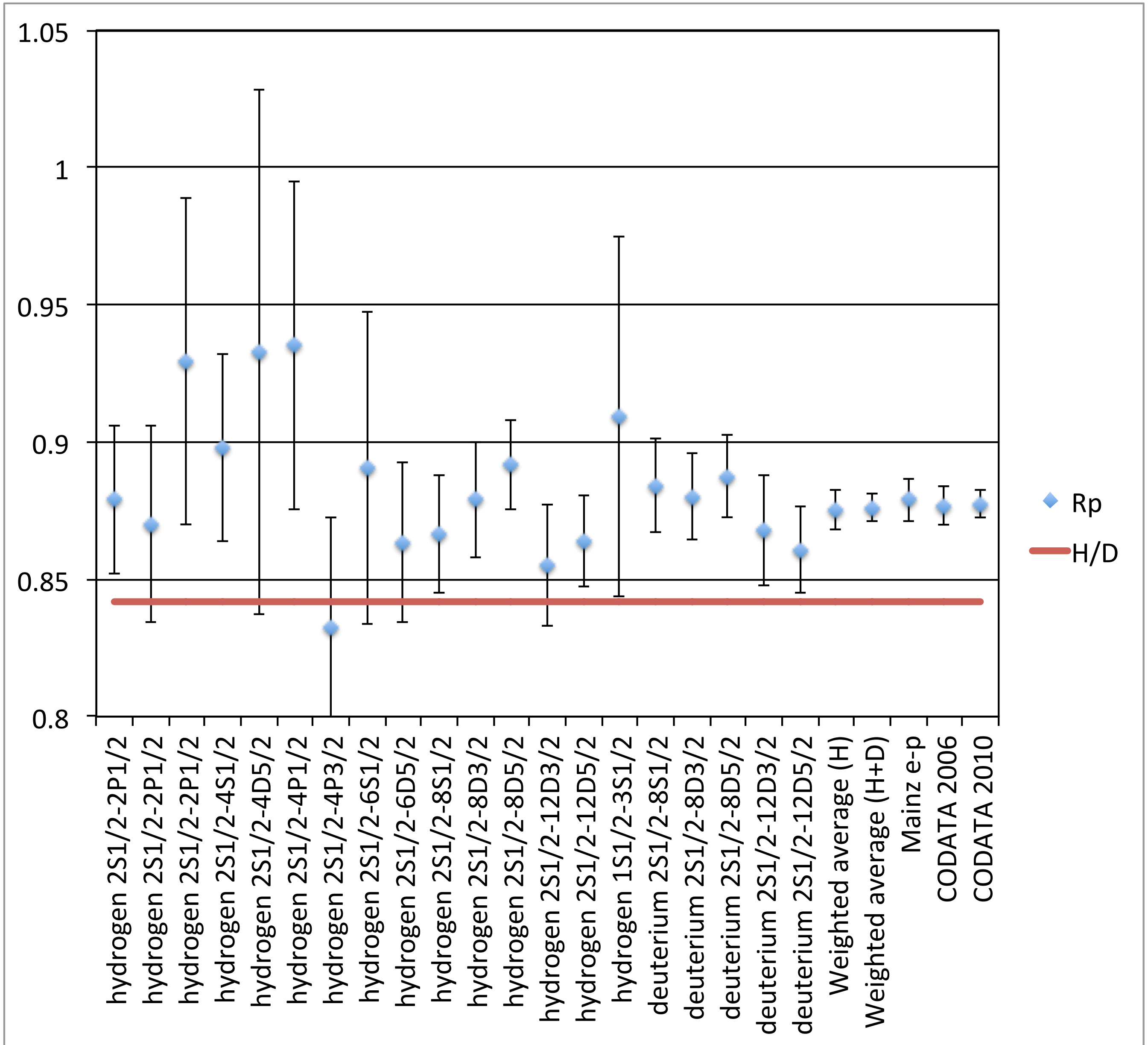


Two lines needed:
Rydberg constant
Proton radius

![LKB]

Hydrogen+Deuterium

Analysis by F. Biraben



Hydrogen+Deuterium

Hydrogen+Deuterium

- Muonic hydrogen: 0.84184(67) fm
- Hydrogen mean value: 0.8752(71) fm: 4.7 σ
- Combined Hydrogen+Deuterium mean value: 0.861(16) fm: 6.6 σ
- Mainz electron-proton scattering: 0.879(8) fm: 4.7 σ
- CODATA 2006: 0.8768(69) fm: 5.0 σ
- CODATA 2011: 0.8775(51) fm (uses latest scattering data): 6.9 σ

LKB

What could it be?

Douter de tout ou tout croire sont deux solutions également commodes, qui l'une et l'autre nous dispensent de réfléchir.
[H. Poincaré]

Doubting everything or believing everything are two easy solutions, which both allow not to think

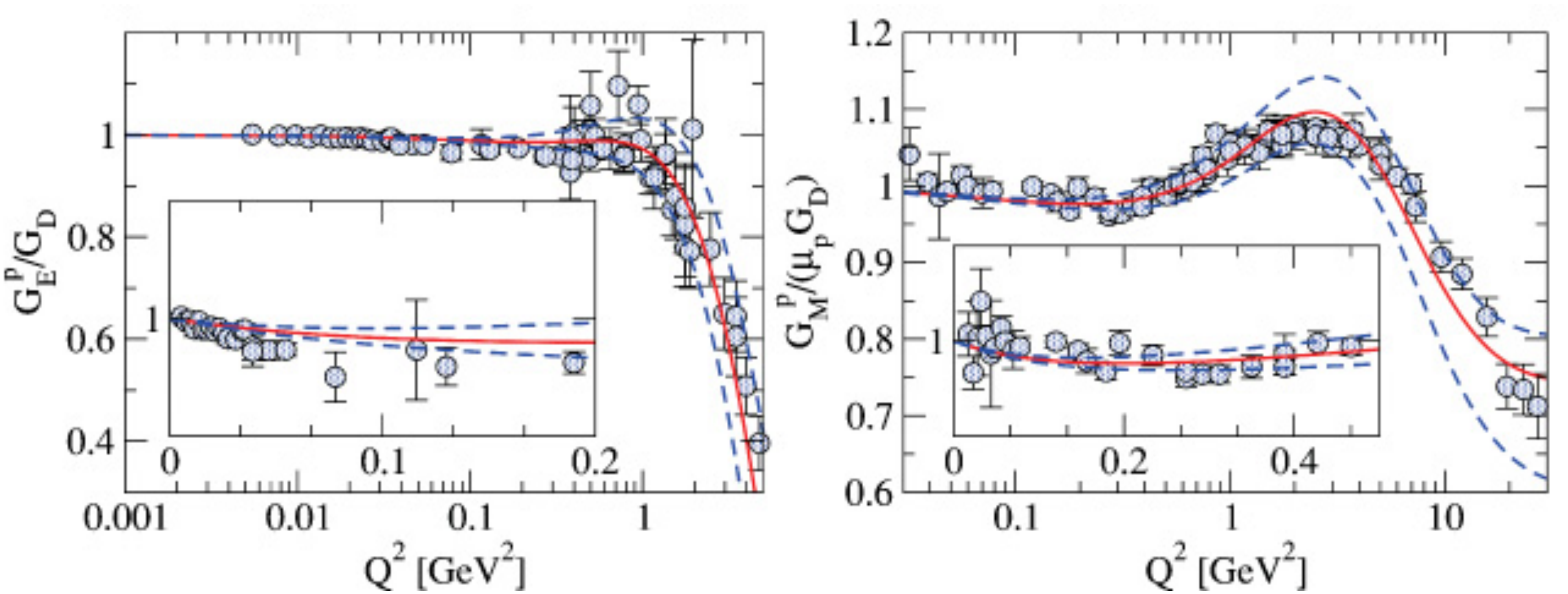
Possible origin for the discrepancy

- QED
- Electron-proton elastic scattering data analysis
- Under-estimated systematic errors in some hydrogen measurements
  - possible, but many different kind of experiments (microwave, 1s-3s, 2s-ns and 2s-nd)
- New physics

Electron-proton scattering

New, very accurate experiments, but...

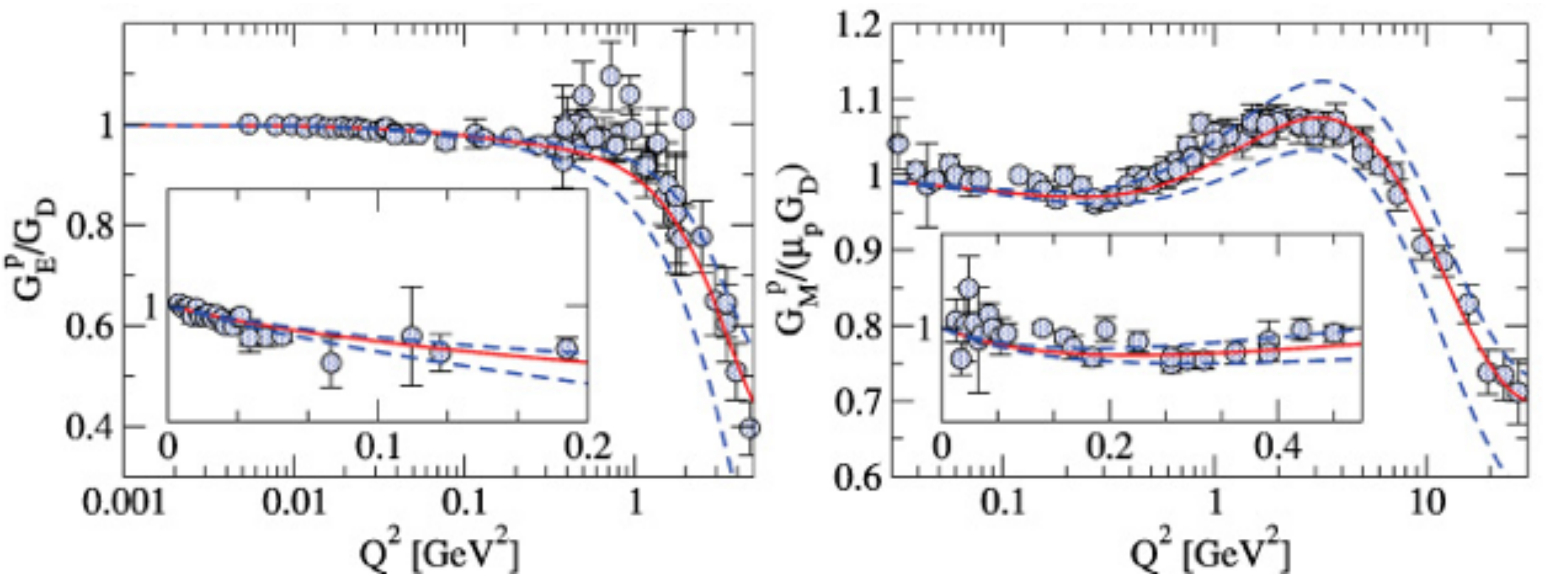
Using experimental proton form factors



Dispersion analysis of the nucleon form factors including meson continua, M.A. Belushkin, H.W. Hammer et U.-G. Meißner. Phys. Rev. C 75, 035202 (2007).

p-QCD approach 0.830 (0.822 . . . 0.835) fm

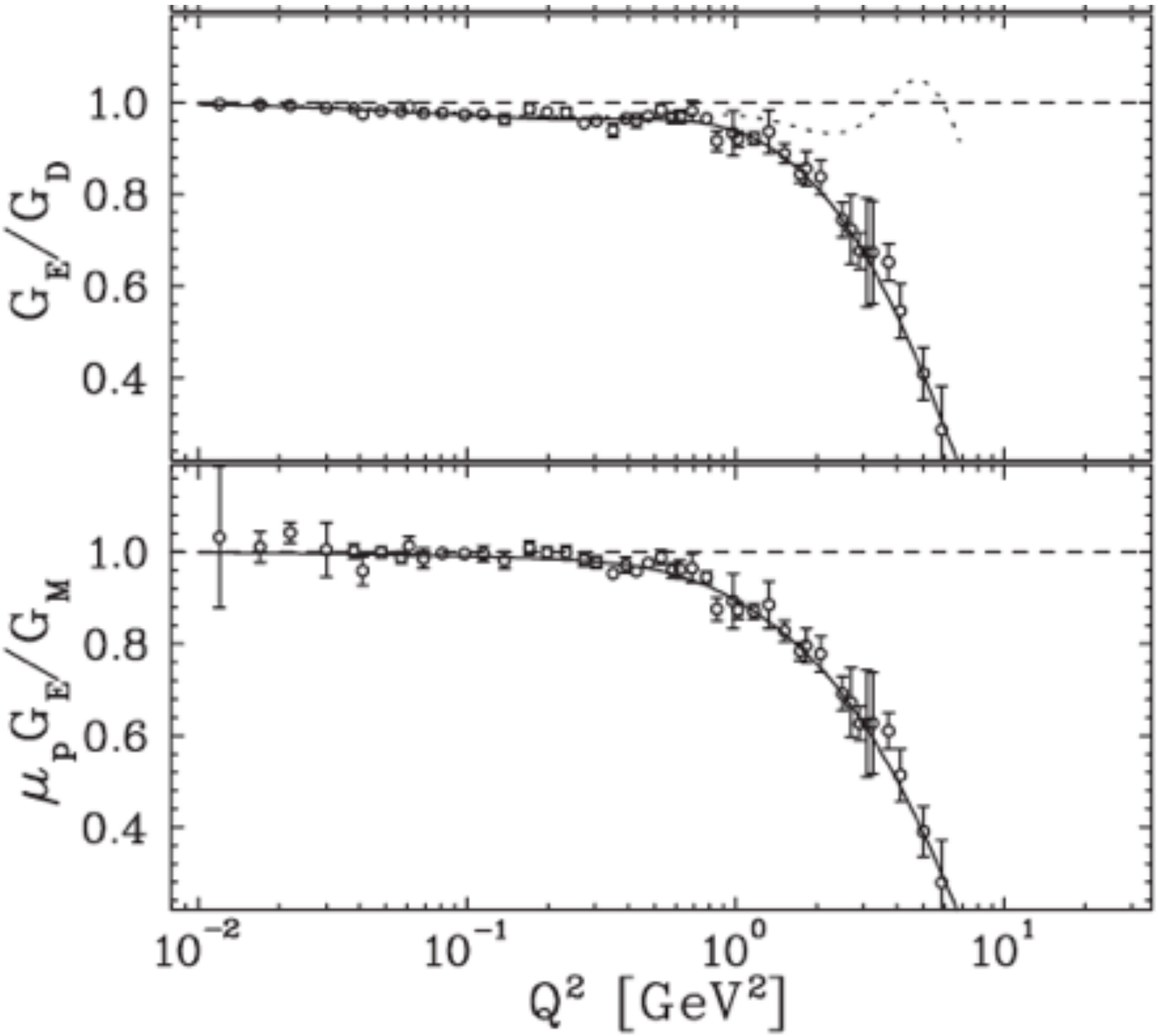
Using experimental proton form factors



Dispersion analysis of the nucleon form factors including meson continua, M.A. Belushkin, H.W. Hammer et U.-G. Meißner. Phys. Rev. C 75, 035202 (2007).

Super-convergence approach 0.844 (0.840 . . . 0.852) fm

Using experimental shapes



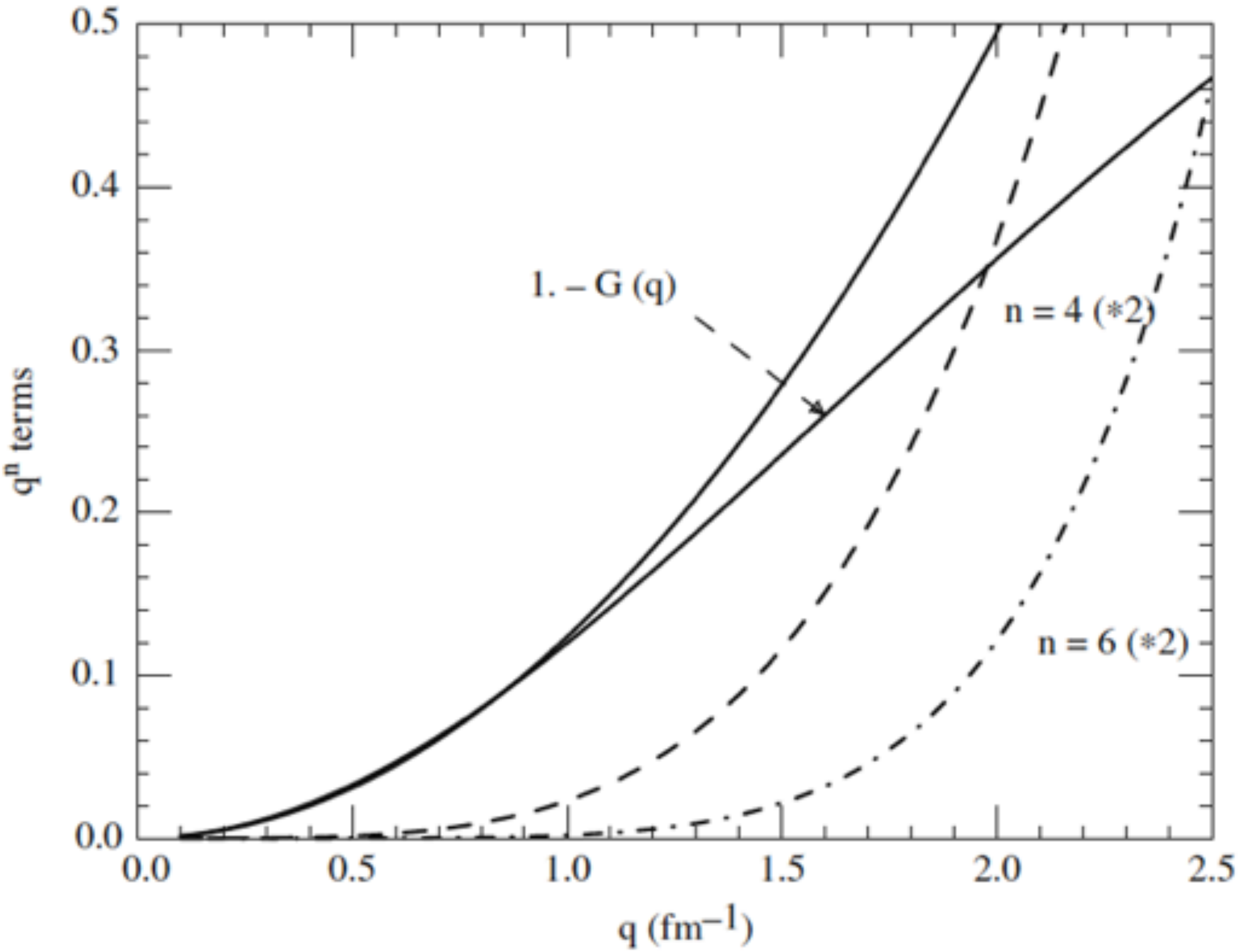
Global analysis of proton elastic form factor data with two-photon exchange corrections, J. Arrington, W. Melnitchouk et J.A. Tjon. Phys. Rev. C 76, 035205 (2007).

Extracted radius: 0.850 fm

Why is it so hard for protons?

- Dipole model lead to exponential charge density. The high moment of the distribution contributes a lot at high q.
- There is no region of q where the <r^2> dominates the finite size effect to >98%, and the finite size effect is large enough to get an accuracy <2%
- People have been letting float the F(0) value in fits (should be exactly 1), which spoils exact behavior where it is important for getting <r^2>.

Why is it so hard for protons?



(Re)analysis of electron-proton scattering

Model-independent extraction of the proton charge radius from electron scattering, R.J. Hill and G. Paz. Physical Review D **82,** 113005 (2010).

*Third Zemach moment of the proton, I.C. Cloët and G.A. Miller. Phys. Rev. C **83,** 012201 (2011).*

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QED is not endangered by the proton's size, A. De Rújula. Physics Letters B **693,** 555-558 (2010).
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Precision Measurement of μG_{E}^{p}/G_{M}^p} at Low Q2, S. Gilad. Few-Body Systems 1-3 (2011).

High-Precision Determination of the Electric and Magnetic Form Factors of the Proton, A1 Collaboration, J.C. Bernauer, P. Achenbach, C. Ayerbe Gayoso, R. Böhm, D. Bosnar, L. Debenjak, M.O. Distler, L. Doria, A. Esser, H. Fonvieille, J.M. Friedrich, J. Friedrich, M. Gómez Rodríguez de la Paz, M. Makek, H. Merkel, D.G. Middleton, U. Müller, L. Nungesser, J. Pochodzalla, M. Potokar, S. Sánchez Majos, B.S. Schlimme, S. Širca, T. Walcher and M. Weinriefer. Phys. Rev. Lett. **105,** 242001 (2010).

Troubles with the Proton rms-Radius, I. Sick. Few-Body Systems 1-3 (2011).

High Precision Measurement of the Proton Elastic Form Factor Ratio pGE=GM at Low Q2 (arXiv:1102.0318v1 [nucl-ex] 1 Feb 2011), X. Zhan et al (Jefferson Lab)

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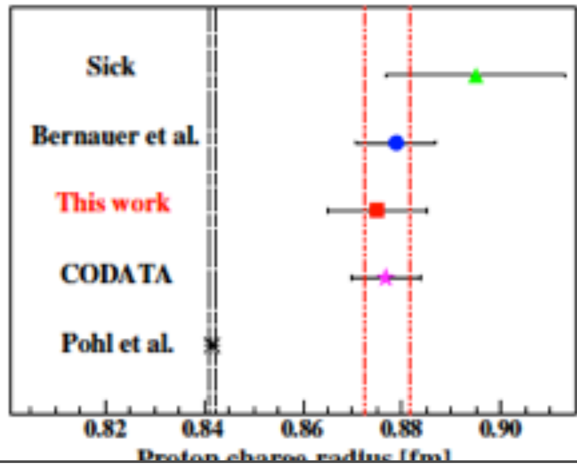

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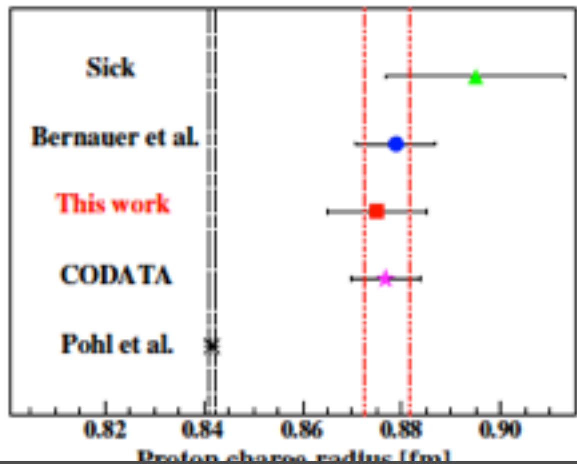


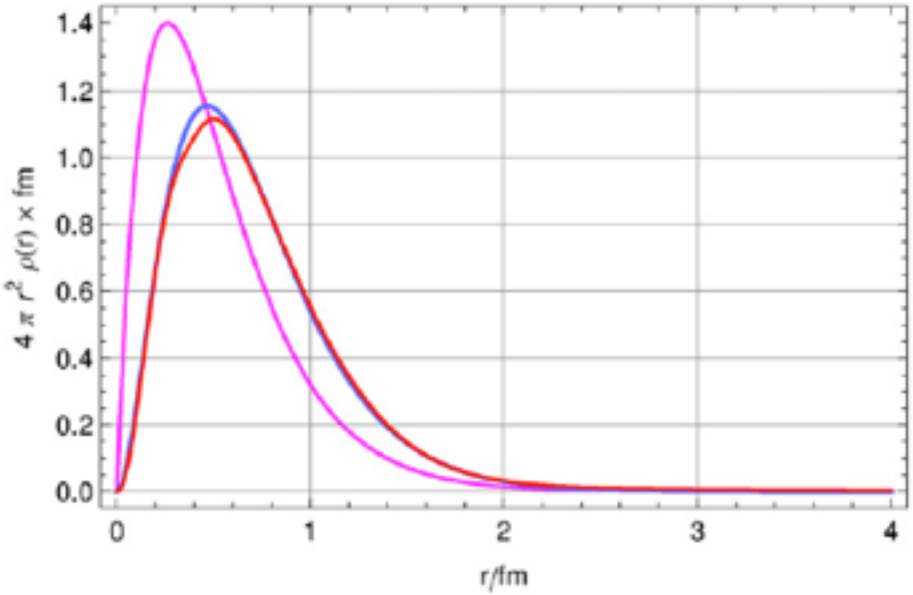
Analysis based on physical models

	SC approach	Explicit pQCD app.	Ref. [23]	Recent determ.
r_E^p (fm)	0.844 (0.840 . . . 0.852)	0.830 (0.822 . . . 0.835)	0.848	0.886(15) [72–74]
r_M^p (fm)	0.854 (0.849 . . . 0.859)	0.850 (0.843 . . . 0.852)	0.857	0.855(35) [73,75]
$(r_E^n)^2$ (fm^2)	−0.117 (−0.11 . . . −0.128)	−0.119 (−0.108 . . . − 0.13)	−0.12	−0.115(4) [52]
r_M^n (fm)	0.862 (0.854 . . . 0.871)	0.863 (0.859 . . . 0.871)	0.879	0.873(11) [76]

Resonance	Mass (GeV)	a_1 (GeV^2)	a_2 (GeV^2)	Γ (GeV)
ω	0.782	0.755960	0.370592	–
ϕ	1.019	−0.776537	−2.913229	–
s_1	1.124860	0.902379	2.484859	–
s_2	2.019536	0.022798	−0.130622	5.158635
v_1	1.062128	−0.127290	−2.162533	–
v_2	1.300946	−1.243412	3.704233	–
v_3	1.493630	4.191380	−7.091021	–
v_4	1.668522	−3.176013	3.723858	–
v_5	2.915451	0.048987	0.075965	19.088297

Dispersion analysis of the nucleon form factors including meson continua, M.A. Belushkin, H.W. Hammer and U.-G. Meißner. Phys. Rev. C **75,** 035202 (2007).

Comparison of Zemach's 3rd moments



[1] QED is not endangered by the proton's size, A. De Rújula. Physics Letters B 693, 555-558 (2010)

[2] QED confronts the radius of the proton, A. De Rújula. Physics Letters B 697, 26-31 (2011)

[3] The RMS charge radius of the proton and Zemach moments, M.O. Distler, J.C. Bernauer et T. Walcher. Physics Letters B 696, 343-347 (2011).

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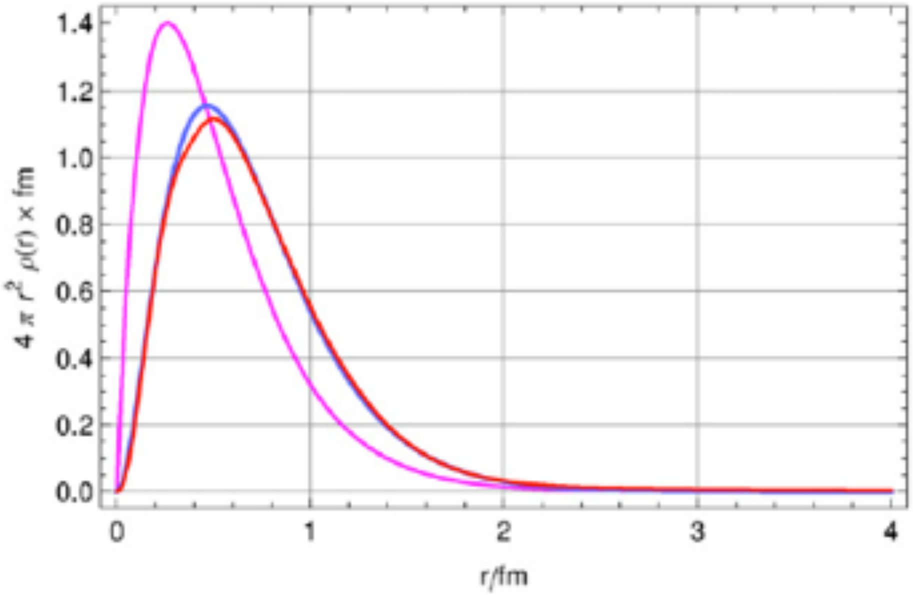
Comparison of Zemach's 3rd moments



- $\langle r^3 \rangle_{(2)} = 3.789 \ \langle r^2 \rangle^{3/2}$ Dipole
- $\langle r^3 \rangle_{(2)} = 1.960 \ \langle r^2 \rangle^{3/2}$ Gauss
- $\langle r^3 \rangle_{(2)} = 3.91 \ \langle r^2 \rangle^{3/2}$ Arrington (2007)
- $\langle r^3 \rangle_{(2)} = 3.78(13) \ \langle r^2 \rangle^{3/2}$ Friar & Sick (2005)
- $\langle r^3 \rangle_{(2)} = 4.18(13) \ \langle r^2 \rangle^{3/2}$ Distler, Bernauer, Walcher (2011)
- $\langle r^3 \rangle_{(2)} = 36.6 \pm 7.3 = 51 \ \langle r^2 \rangle^{3/2}$ De Rujula (2010), retracted (?) in 2011...

[1] QED is not endangered by the proton's size, A. De Rújula. Physics Letters B 693, 555-558 (2010)

[2] QED confronts the radius of the proton, A. De Rújula. Physics Letters B 697, 26-31 (2011)

[3] The RMS charge radius of the proton and Zemach moments, M.O. Distler, J.C. Bernauer et T. Walcher. Physics Letters B 696, 343-347 (2011).

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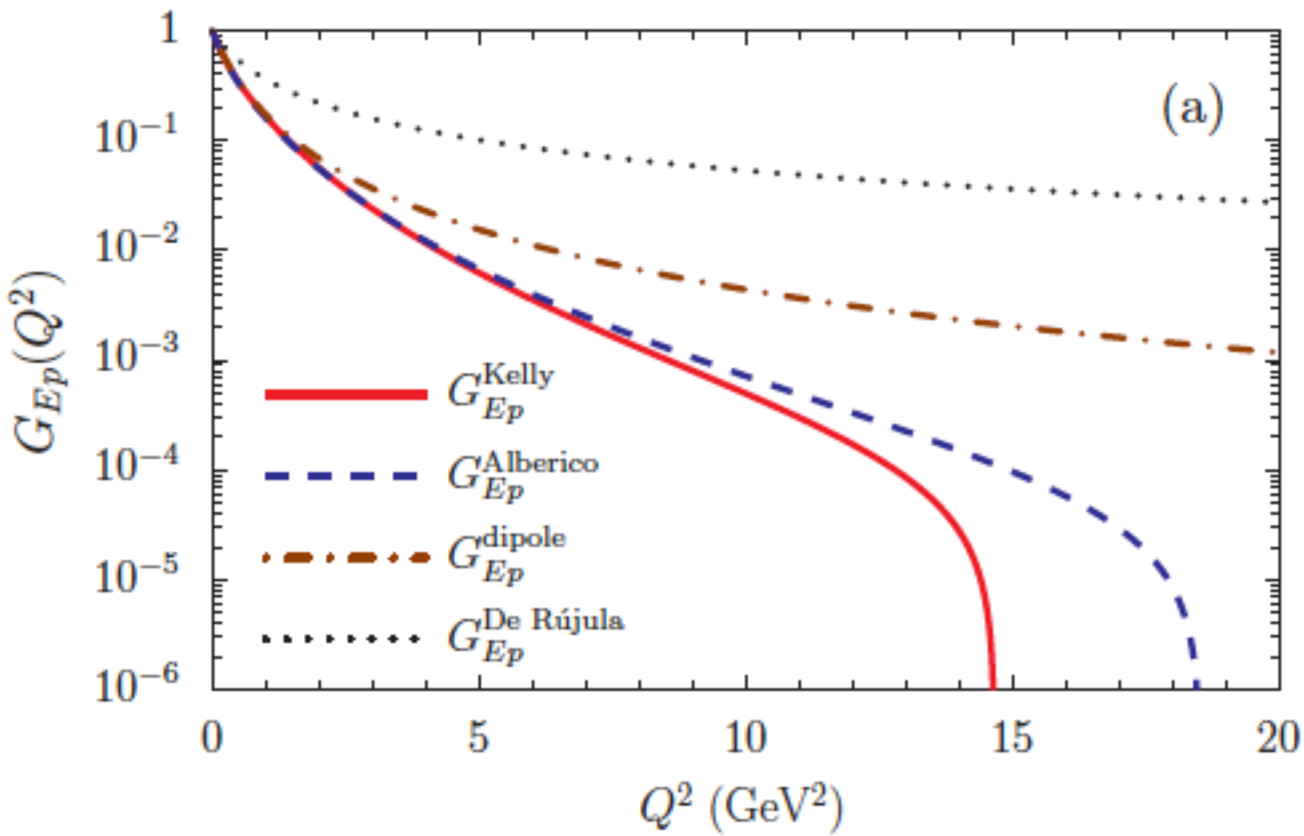
Third Zemach radius

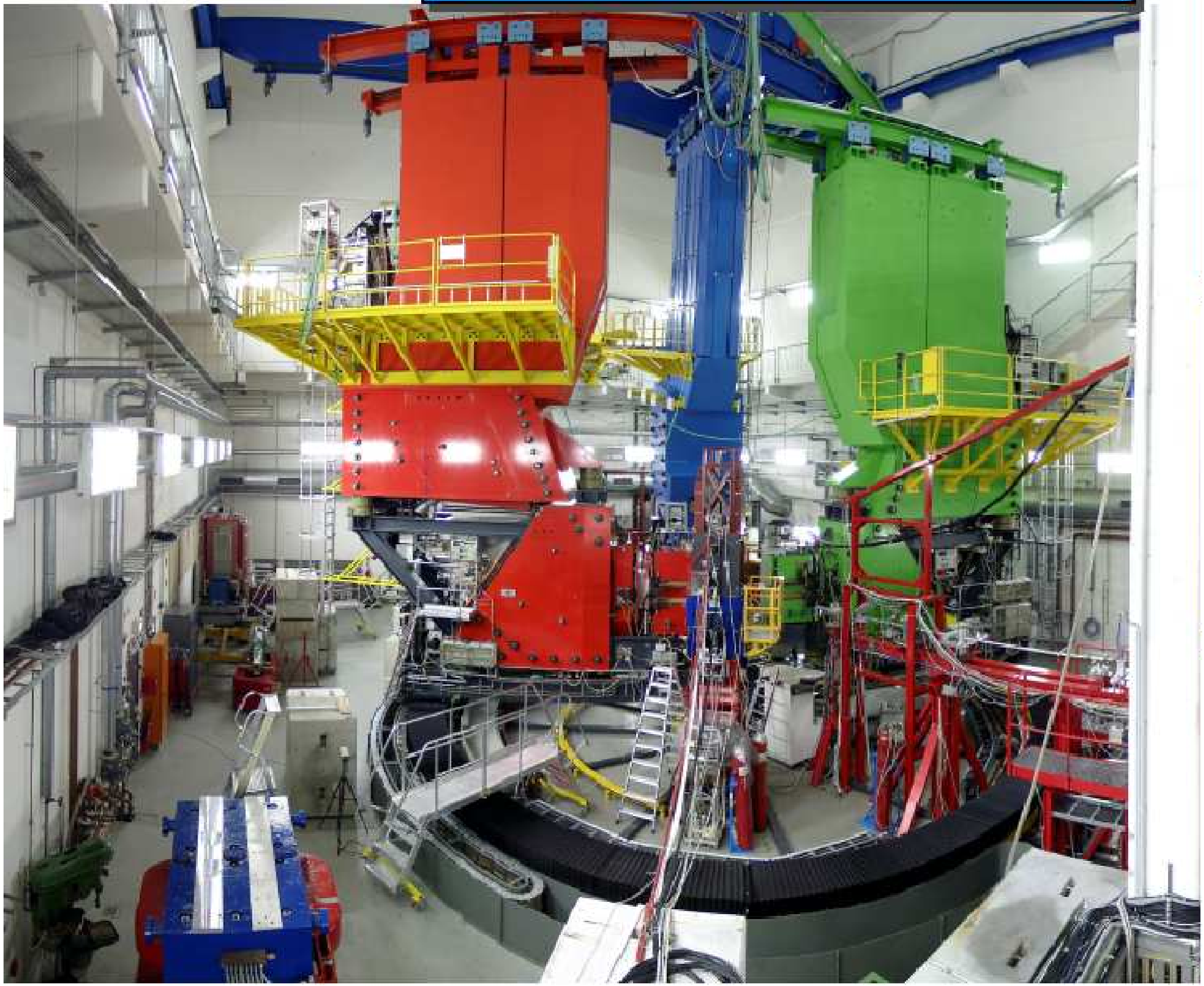
$$\Delta\tilde{E} = 209.9779(49) - 5.2262\ r_p^2 + 0.0347\ r_p^3\ \text{meV}$$



Third Zemach moment of the proton, I.C. Cloët and G.A. Miller. Phys. Rev. C **83,** 012201 (2011).

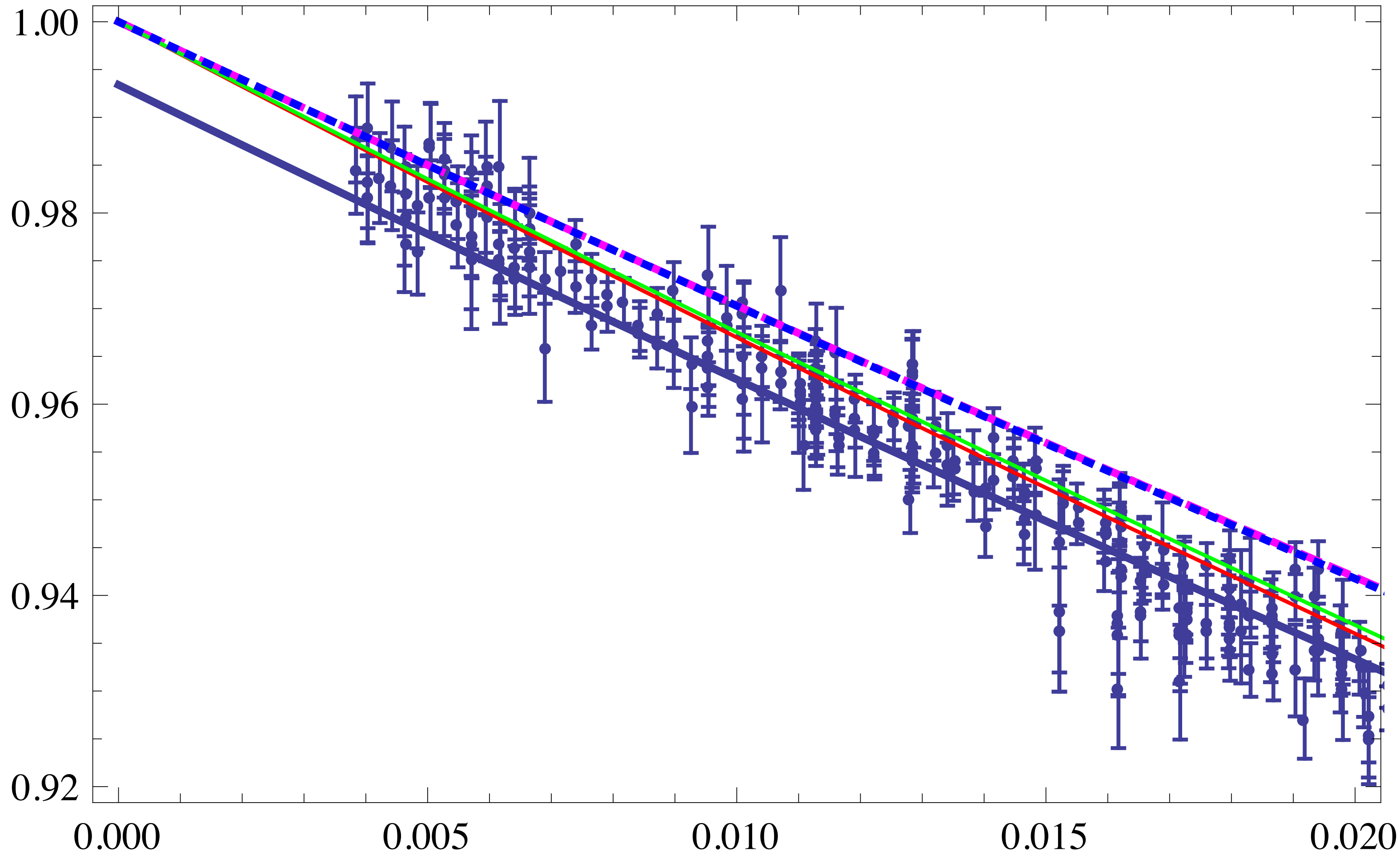
MAMI A1 experiment

A1: **The »three« spectrometer facility**



MAMI A1 experiment

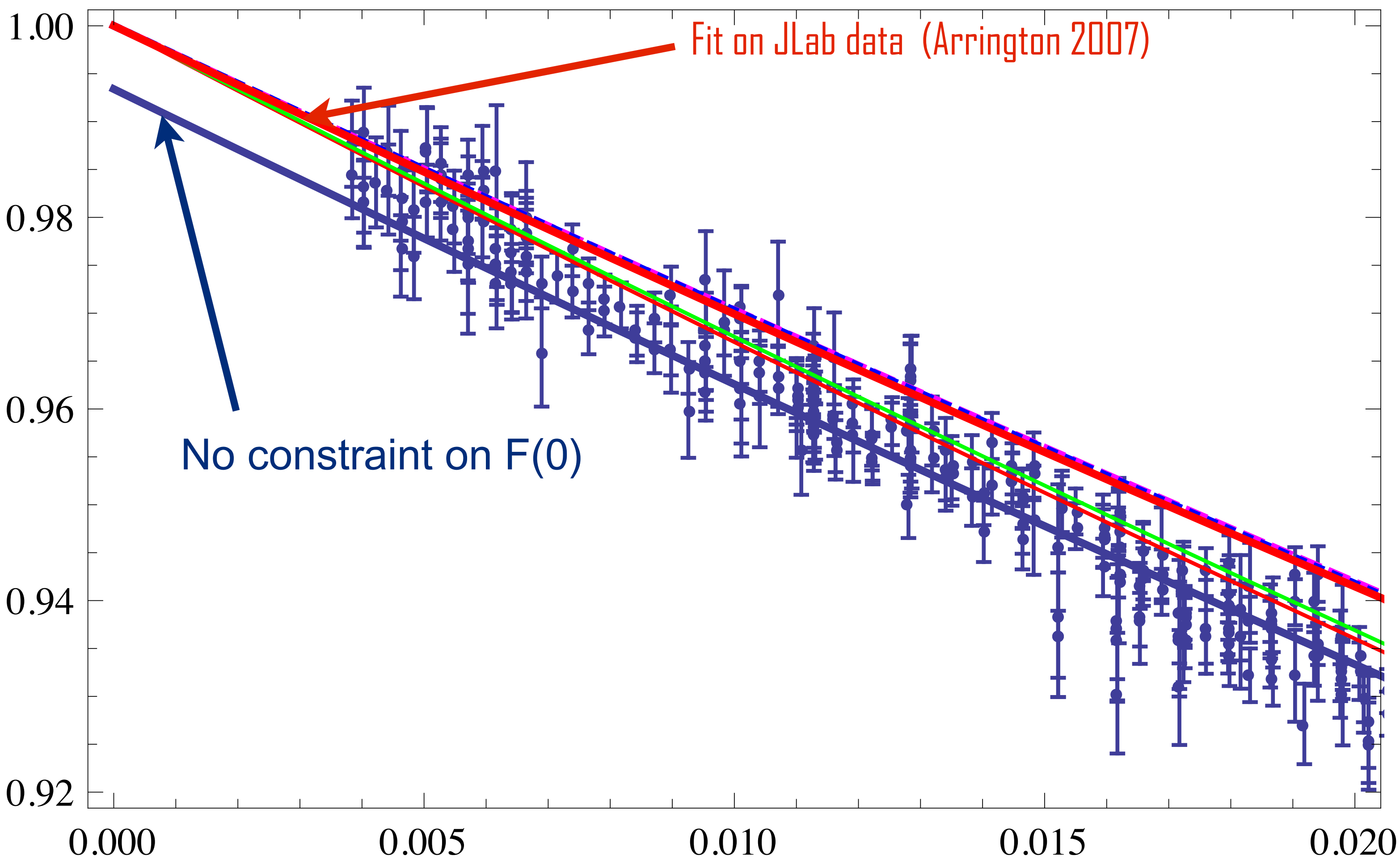
Are scattering data analysis reliable?



Using data from the A1 Collaboration, J. C. Bernauer, P. Achenbach, et al., Phys. Rev. Lett. **105**, 242001 (2010).

LKB

Are scattering data analysis reliable?



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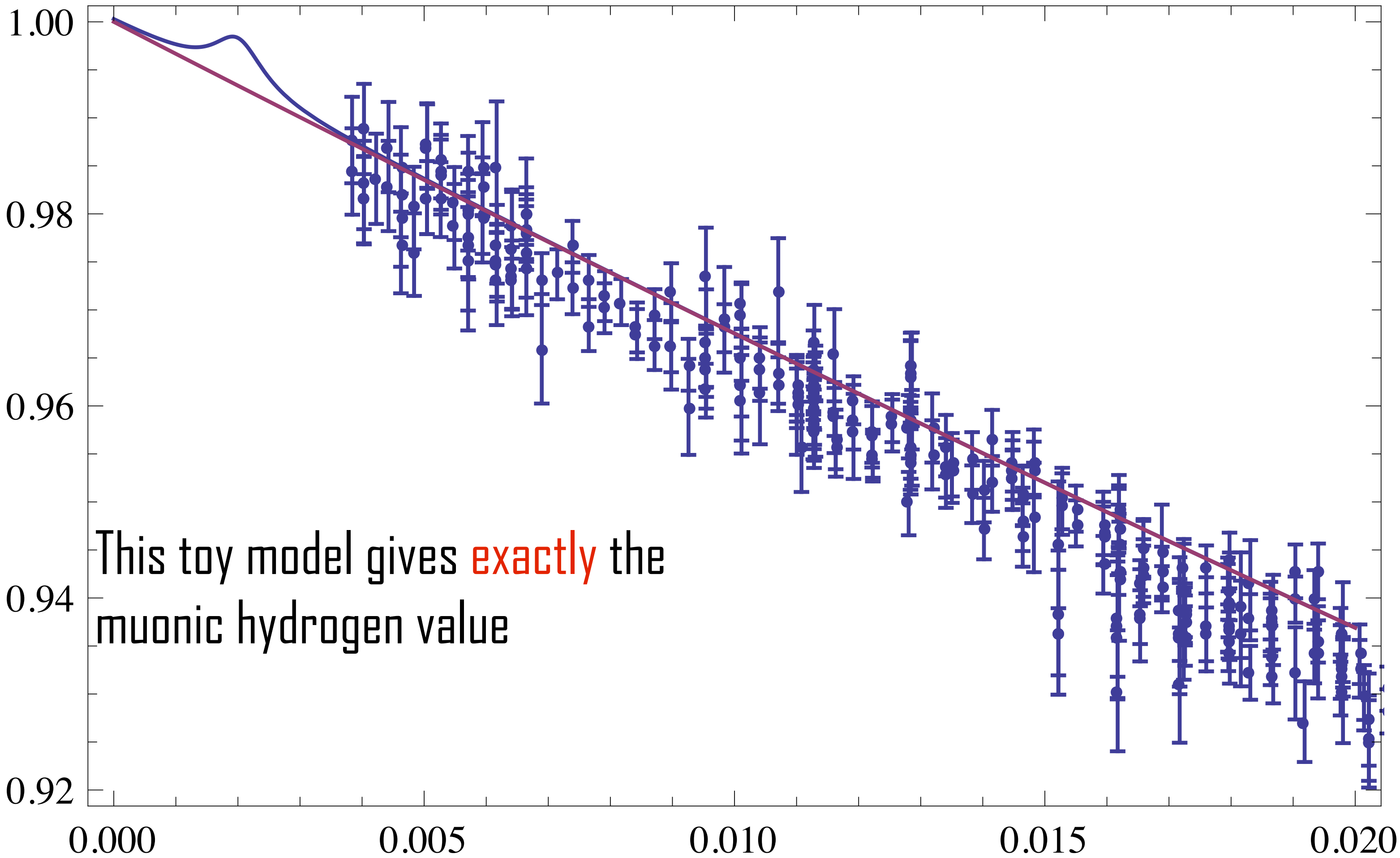

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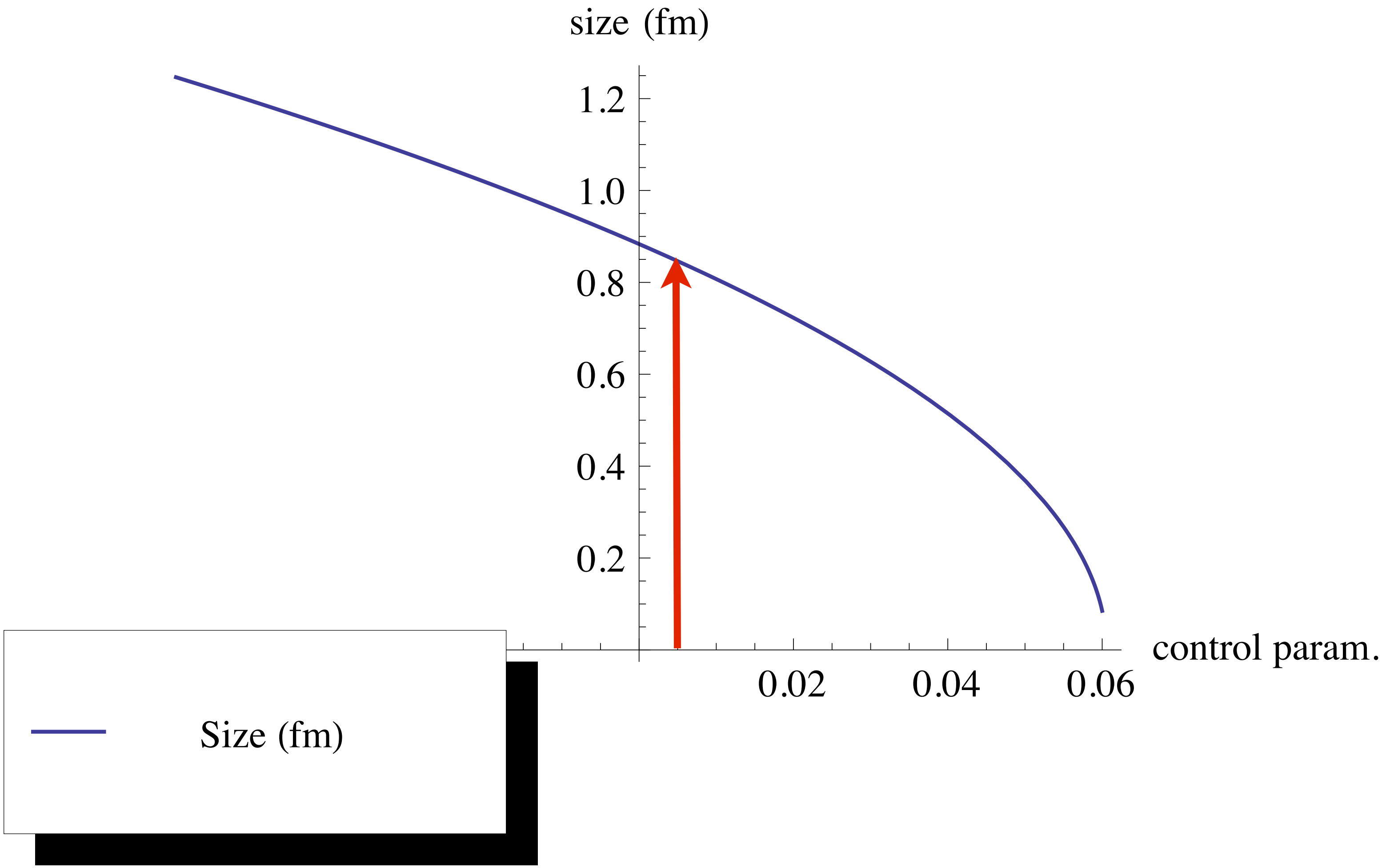
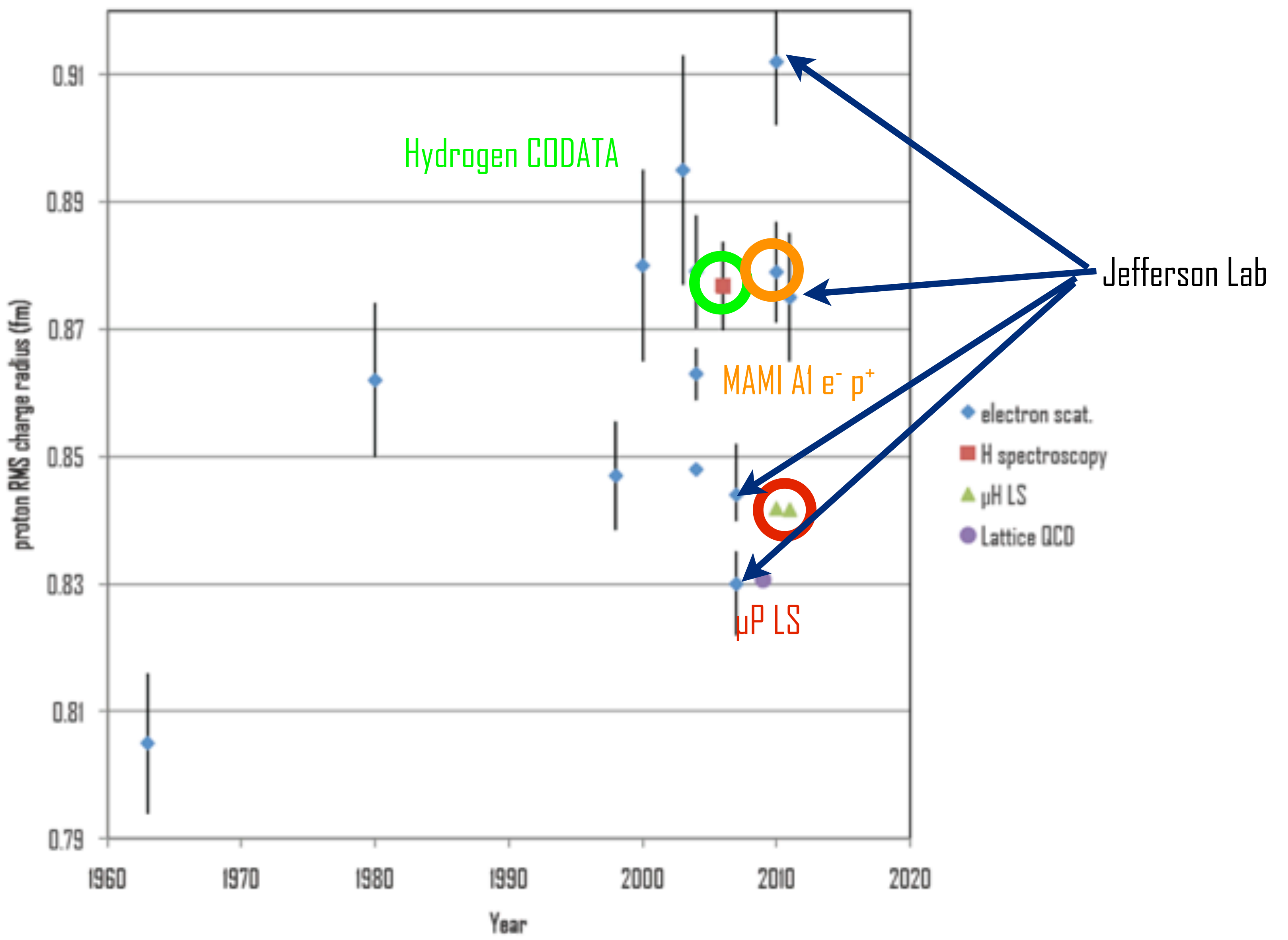


Proton size results summary

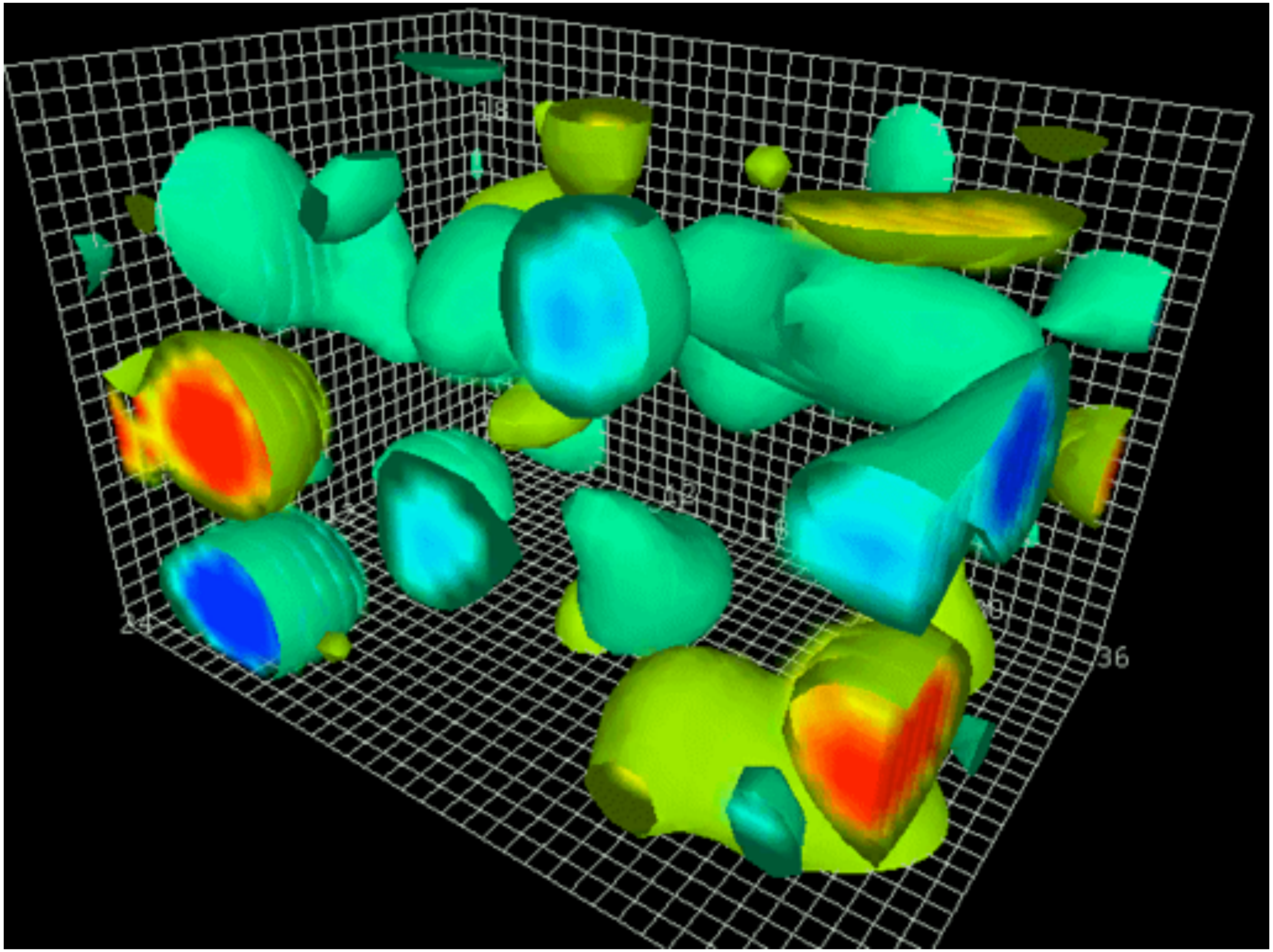


vendredi 18 novembre 2011

QCD+Chiral perturbation theory?

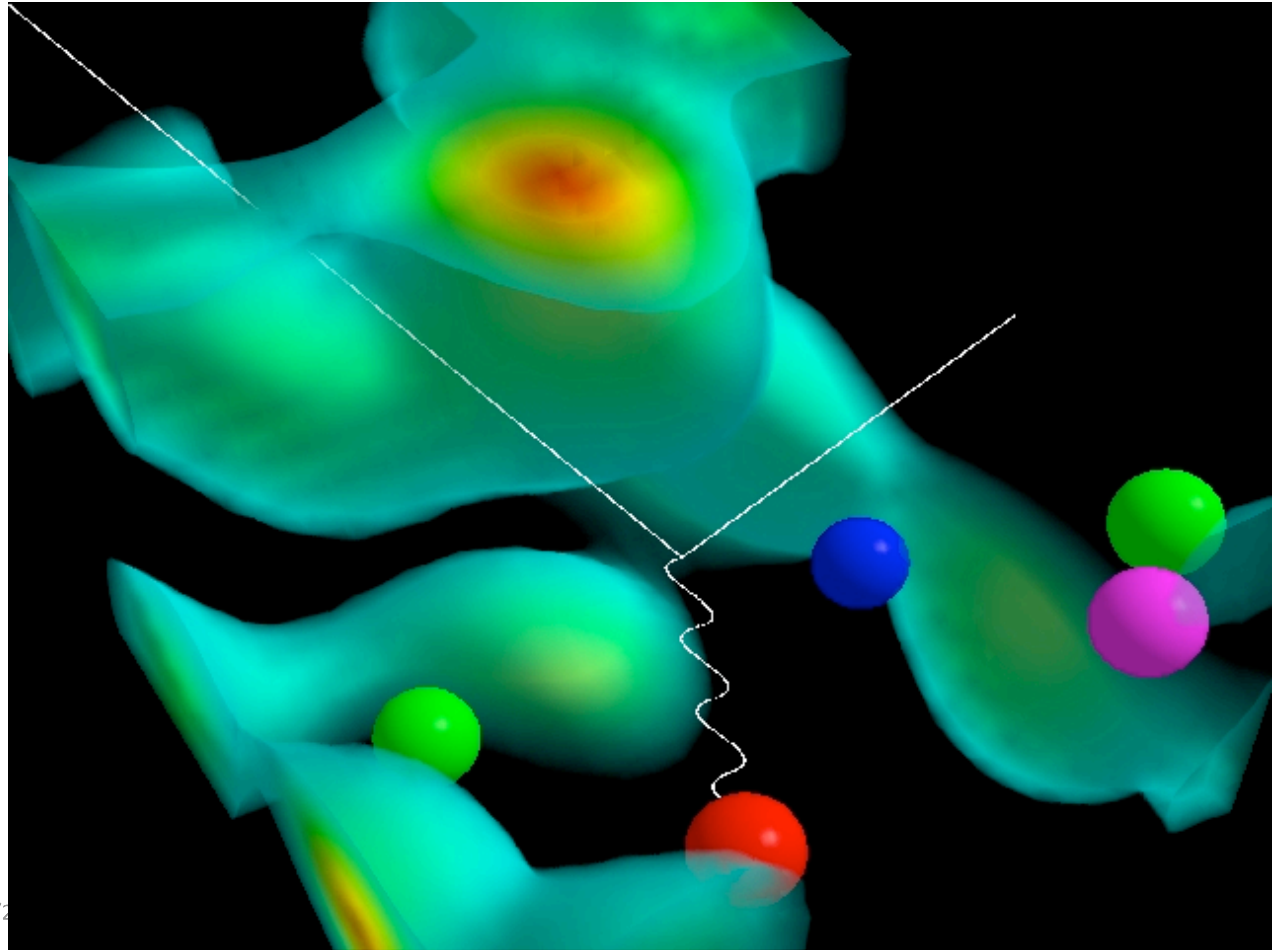
Predicted radius much smaller, but not accurate

QCD vacuum and the proton...

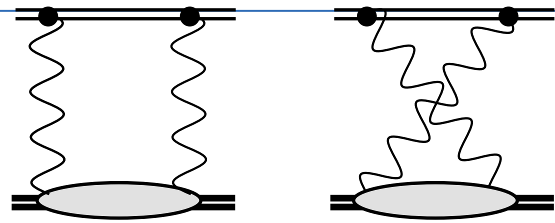


http://www.physics.adelaide.edu.au/~dleinweb/VisualQCD/QCDvacuum/welcome.html

Should we go beyond charge distributions?



Proton polarization



- Several calculations
  - Rosenfelder (1999)

$$\Delta E_{2s}^{\mathrm{p.pol}} = -\frac{136 \pm 30}{n^3}\,\mu\mathrm{eV} = -0.017 \pm 0.004\,\mathrm{meV}.$$

  - Pachucki (1999) $\Delta E_{2s}^{\mathrm{p.pol}} = -0.012 \pm 0.002\,\mathrm{meV},$

  - Martynenko (2006)

$$\begin{aligned}\Delta E_{2s}^{\mathrm{p.pol}} &= \Delta E^{\mathrm{subt.}} + \Delta E^{\mathrm{inel.}} \\ &= 0.0023 - 0.01613\,\mathrm{meV} \\ &= -0.0138(29)\,\mathrm{meV},\end{aligned}$$

  - Carlson and Vanderhaeghen (2011)

$$\begin{aligned}\Delta E_{2s}^{\mathrm{p.pol}} &= \Delta E^{\mathrm{subt.}} + \Delta E^{\mathrm{inel.}} + \Delta E^{\mathrm{el.}} \\ &= 0.0053(19) - 0.0127(5) - 0.0295(13)\,\mathrm{meV} \\ &= -0.0074(20) - 0.0295(13)\,\mathrm{meV}.\end{aligned}$$

  - Hill and Paz (2011+DPF 2011)

$$\begin{aligned}\Delta E_{2s}^{\mathrm{p.pol}} &= \Delta E^{\mathrm{subt.}} + \Delta E^{\mathrm{inel.}} \\ &= \left[\delta E^{W_1(0,Q^2)} + \delta E^{\mathrm{proton\ pole}}\right] + \delta E^{\mathrm{continuum}} \\ &= \left[\delta E^{W_1(0,Q^2)} + 0016\right] - 0.0127(5)\,\mathrm{meV},\end{aligned}$$

Could be wrong by 0.04 meV

A new polarization term

G. A. Miller, A. W. Thomas, J. D. Carroll, and J. Rafelski:
Natural Resolution of the Proton Size Puzzle
(arXiv:1101.4073v2 [physics.atom-ph], March 29th, 2011)

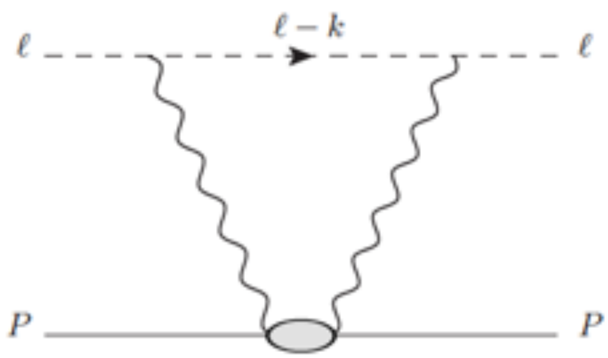


FIG. 1: Direct two-photon exchange graph corresponding to the hitherto neglected term. The dashed line denotes the lepton; the solid line, the nucleon; the wavy lines photons; and the ellipse the off-shell nucleon.

0.31 meV for μH and 9Hz for hydrogen
(with model dependent parametrization)

New physics

Predictions are always difficult, in particular about the Future
[N. Bohr]

Where could it come from?

- A muon edm? If $d_\mu = 2\times 10^{-19}$ e·cm would shifts the energy level < 200 MHz
- Charge equality between e^- and μ^- generation? Checked to $u_r = 10^{-8}$ (from $\mu^+ e^-$)

- Deviation from Coulomb's law: probe of hidden sector

  - Test of Coulomb law via spectroscopy is very clean and model independent probe of new particles
  It is independent on stability and decay channel.

$$V(r) = -\frac{Z\alpha}{r}(1+\alpha' e^{-mr}) \quad \text{or} \quad V(r) = -\frac{Z\alpha}{r}(1+\alpha''(\mathbf{s_1}\cdot\mathbf{s_2})e^{-mr})$$

  - From simple atoms there are constraints on light bosons with ultra-weak coupling:
  $m \in [1\mathrm{eV}, \mathrm{MeV}]$ and $\alpha' < 10^{-13}$, $\alpha'' < 10^{-17}$ [PRL 104,220406 (2010), arXiv:1008.3536v2]

- Minicharge particles? [Jaeckel and Roy (2010), Jentschura(2010)]

  Vacuum polarization with pair production of light fermions with $q = \varepsilon e$ and masses $m_\varepsilon < m_e$

No parameter found explaining r_p puzzle without contradicting, simple atoms spectroscopy, $g_{e/\mu} - 2$, α...

- New bound-state QED theory?

  - Non-local in time interaction...(R. K. Gainutdinov)?

Where could it come from?

Proton Size Anomaly, V. Barger, C.-W. Chiang, W.-Y. Keung et al. Phys. Rev. Lett. 106, 153001 (2011):

We explore the possibility that new scalar, pseudoscalar, vector, and tensor flavor-conserving nonuniversal interactions may be responsible for the discrepancy.We consider exotic particles that, among leptons, couple preferentially to muons and mediate an attractive nucleon-muon interaction. We find that the many constraints from low energy data disfavor new spin-0, spin-1, and spin-2 particles as an explanation.

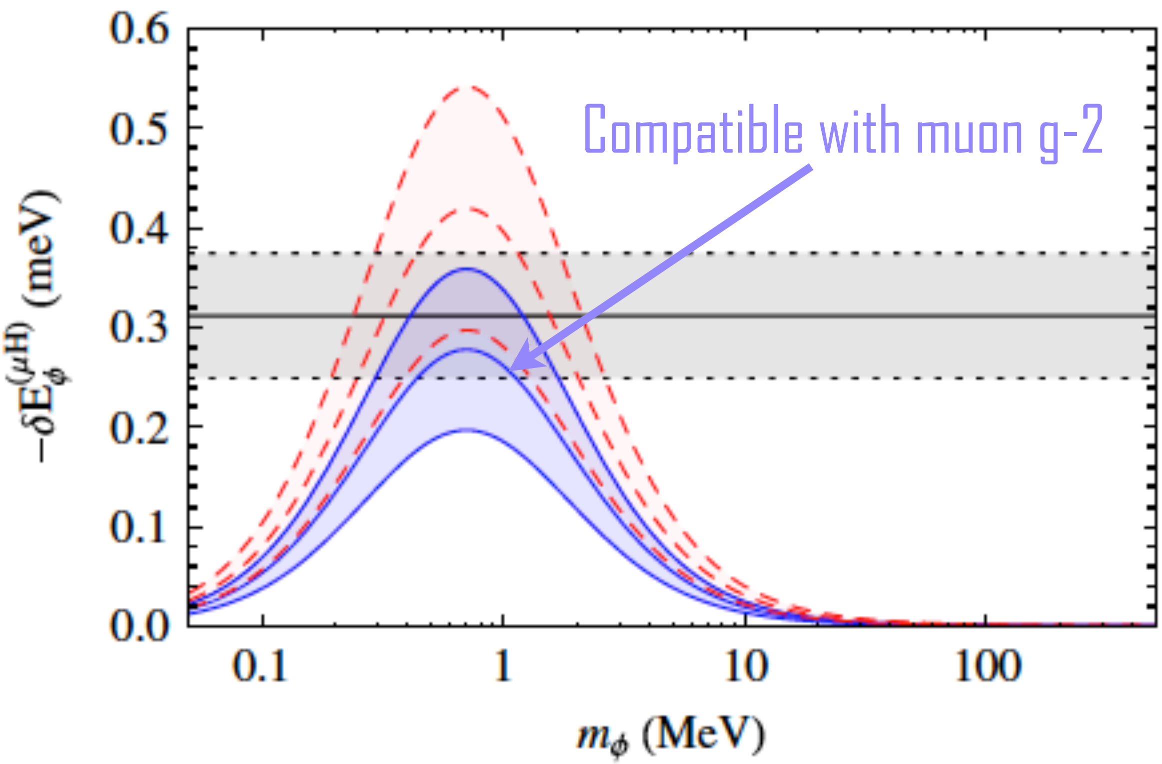
Where could it come from?

Muonic hydrogen and MeV forces, D. Tucker-Smith et I. Yavin. Physical Review D 83, 101702 (2011).

We explore the possibility that a new interaction between muons and protons is responsible for the discrepancy between the CODATA value of the proton-radius and the value deduced from the measurement of the Lamb shift in muonic hydrogen. We show that a new force carrier with roughly MeV-mass can account for the observed energy-shift as well as the discrepancy in the muon anomalous magnetic moment. However, measurements in other systems constrain the couplings to electrons and neutrons to be suppressed relative to the couplings to muons and protons, which seems challenging from a theoretical point of view. One can nevertheless make predictions for energy shifts in muonic deuterium, muonic helium, and true muonium under the assumption that the new particle couples dominantly to muons and protons.

Where could it come from?

Muonic hydrogen and MeV forces, D. Tucker-Smith et I. Yavin. Physical Review D 83, 101702 (2011).



Where could it come from?

New Parity-Violating Muonic Forces and the Proton Charge Radius, B. Batell, D. McKeen et M. Pospelov. Phys. Rev. Lett. 107, 011803 (2011).

The recent discrepancy between proton charge radius measurements extracted from electron-proton versus muon-proton systems is suggestive of a new force that differentiates between lepton species. We identify a class of models with gauged right-handed muon number, which contains new vector and scalar force carriers at the 100 MeV scale or lighter, that is consistent with observations. Such forces would lead to an enhancement by several orders-of-magnitude of the parity-violating asymmetries in the scattering of low-energy muons on nuclei. The relatively large size of such asymmetries, $O(10^{-4})$, opens up the possibility for new tests of parity violation in neutral currents with existing low-energy muon beams.

Where could it come from?

B. Batell, D. McKeen, and M. Pospelov, Phys. Rev. Lett. 107, 011803 (2011).

$$\Delta r_p^2|_{e\text{-H}} = -\frac{6\kappa^2}{m_V^2}; \quad \Delta r_p^2|_{\mu\text{-H}} = -\frac{6(\kappa^2+\eta)}{m_V^2} f(am_V) \quad (11)$$

where $a = (\alpha m_\mu m_p)^{-1}(m_\mu + m_p)$ is the μ-H Bohr radius, $f(\hat{x}) \equiv \hat{x}^4(1+\hat{x})^{-4}$, and $\eta \equiv \kappa g_R/(2e)$. The difference $\Delta r_p^2|_{e\text{-H}} - \Delta r_p^2|_{\mu\text{-H}}$ must be consistent with the observed pattern (5) and requires η to be positive. In the scaling regime of $am_V \gg 1$ one has

$$\frac{\eta}{m_V^2} \simeq \frac{\Delta r^2}{6} \simeq 0.01\ \text{fm}^2 \simeq \frac{2.5 \times 10^{-5}}{(10\ \text{MeV})^2}. \quad (12)$$

In the same regime, the model predicts that future experiments with μ-He would detect the effective charge radius of the helium nucleus shifted down by $\boxed{\Delta r_{\text{He}}^2 = -0.06\ \text{fm}^2.}$

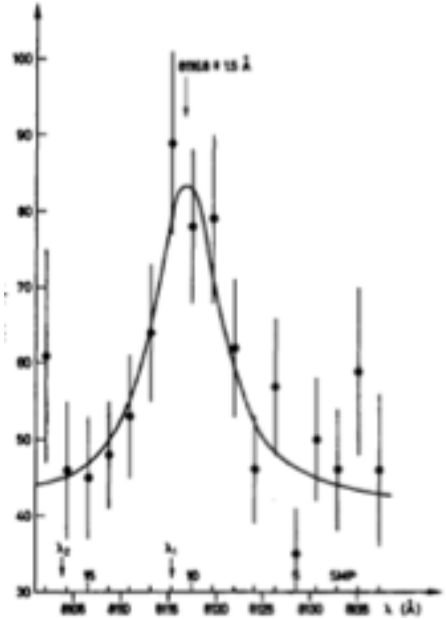
What's next

Deuterium: deuton polarization too large

Helium?

Prospect : muonic helium spectroscopy at PSI

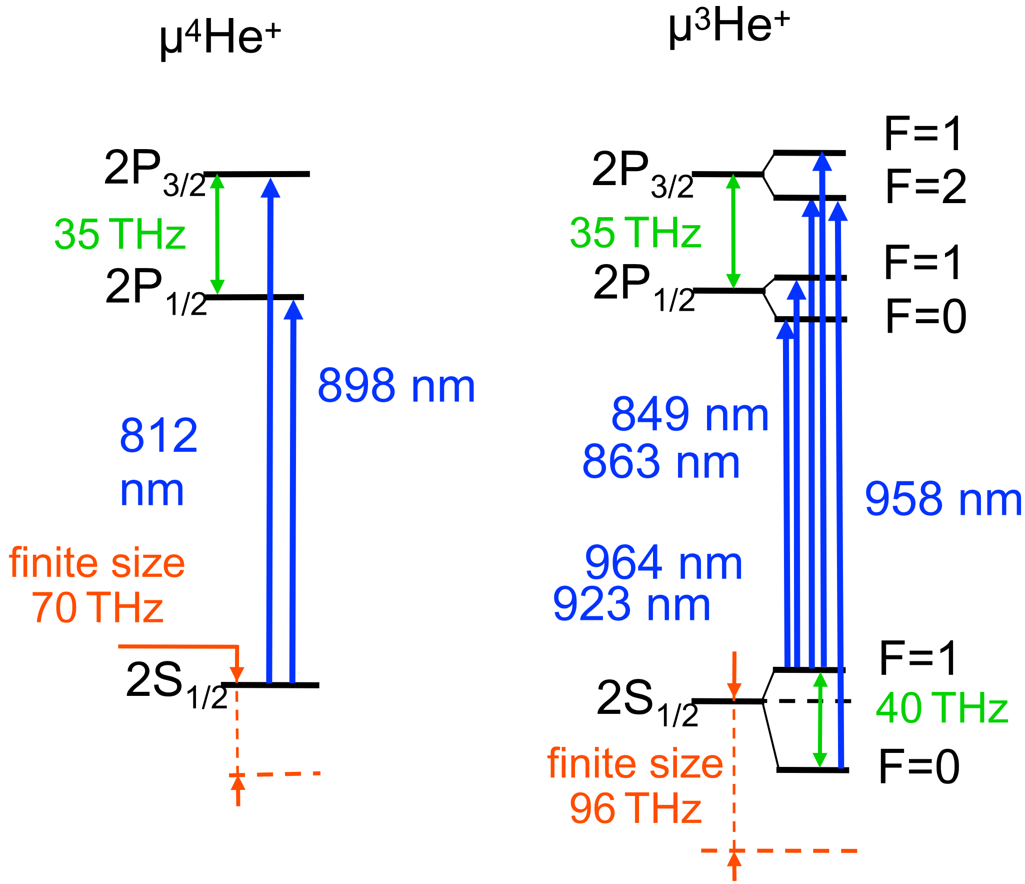
812 nm
p_{He} = 40 bars



Nuclear Physics A278 (1977) p. 381

but signal never reproduced (10 bars, 40 bars)

2011-2013→ muonic helium spectroscopy (4 mbar)



- μHe⁺ spectroscopy + He⁺ spectroscopy → QED test ($Z\alpha$)
- improve He spectroscopy

CREMA 2011

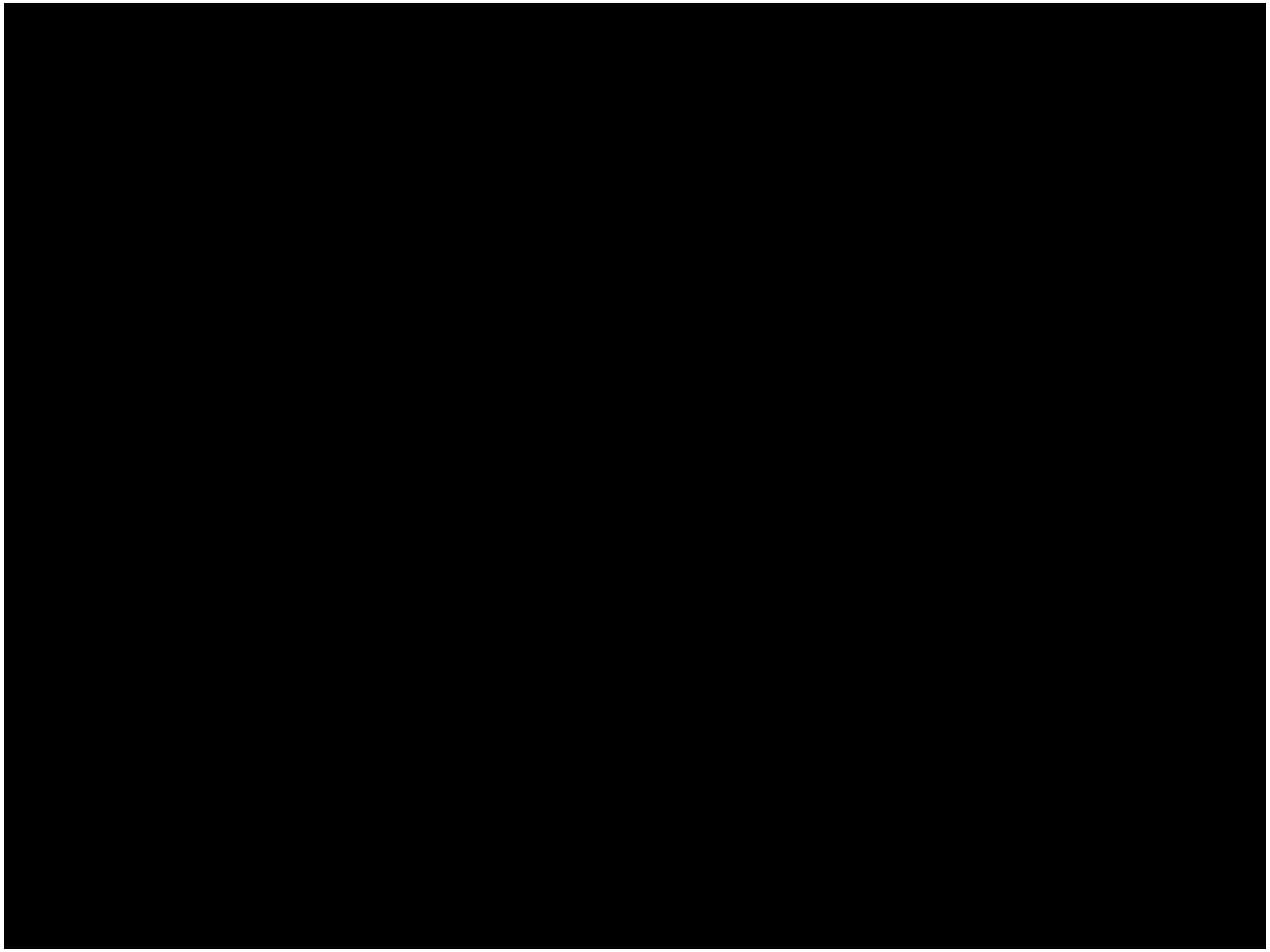
LKB

Conclusions

- We have performed a 15 ppm measurement of the Lamb-shift in muonic hydrogen
- The deduced proton radius using a Dipole model is 5 standard deviations away from the hydrogen and electron-proton elastic scattering data
- Better modeling of the proton form-factor required to confirm or reduce the disagreement
- Experiment checked with 2nd μH line and 3 μD lines
- Muonic He in 2013 (check of theory, different laser wavelength-in the red)

One day on the life of a Proton Size Investigator

One day on the life of a Proton Size Investigator



Proton Size Investigators thank you for your attention

Hydrogen spectroscopy

Transition energy : $\left(E_n - E_p\right) = \left(E_n^{Dirac} - E_p^{Dirac}\right) + \left(E_n^{Recoil} - E_p^{recoil}\right) + \left(L_n - L_p\right)$

in which : E_{Dirac} and E_{Recoil} have an exact formulation prop. to R_∞

$L_n = F(QED, R_p)$ Lamb shift

✓ Q.E.D. = series expansion of α, $Z\alpha$, $\ln(Z\alpha)$ which depends on n as $1/n^3$

$$\Delta E = \left(\frac{\alpha}{\pi}\right)^2 \frac{(Z\alpha)^4}{n^3} m_e c^2 \left[B_{40} + (Z\alpha) B_{50} + (Z\alpha)^2 \left(L^3 B_{63} + L^2 B_{62} + L B_{61} + B_{60}\right) + \dots\right]$$

$$\approx \quad \dots \quad \left[0{,}54 - 0{,}16 + 5 \times 10^{-5} \left(-280 - 50 + 500 - 100\right) + \dots\right]$$

$$avec \quad L = \ln\left[(Z\alpha)^{-2}\right]$$

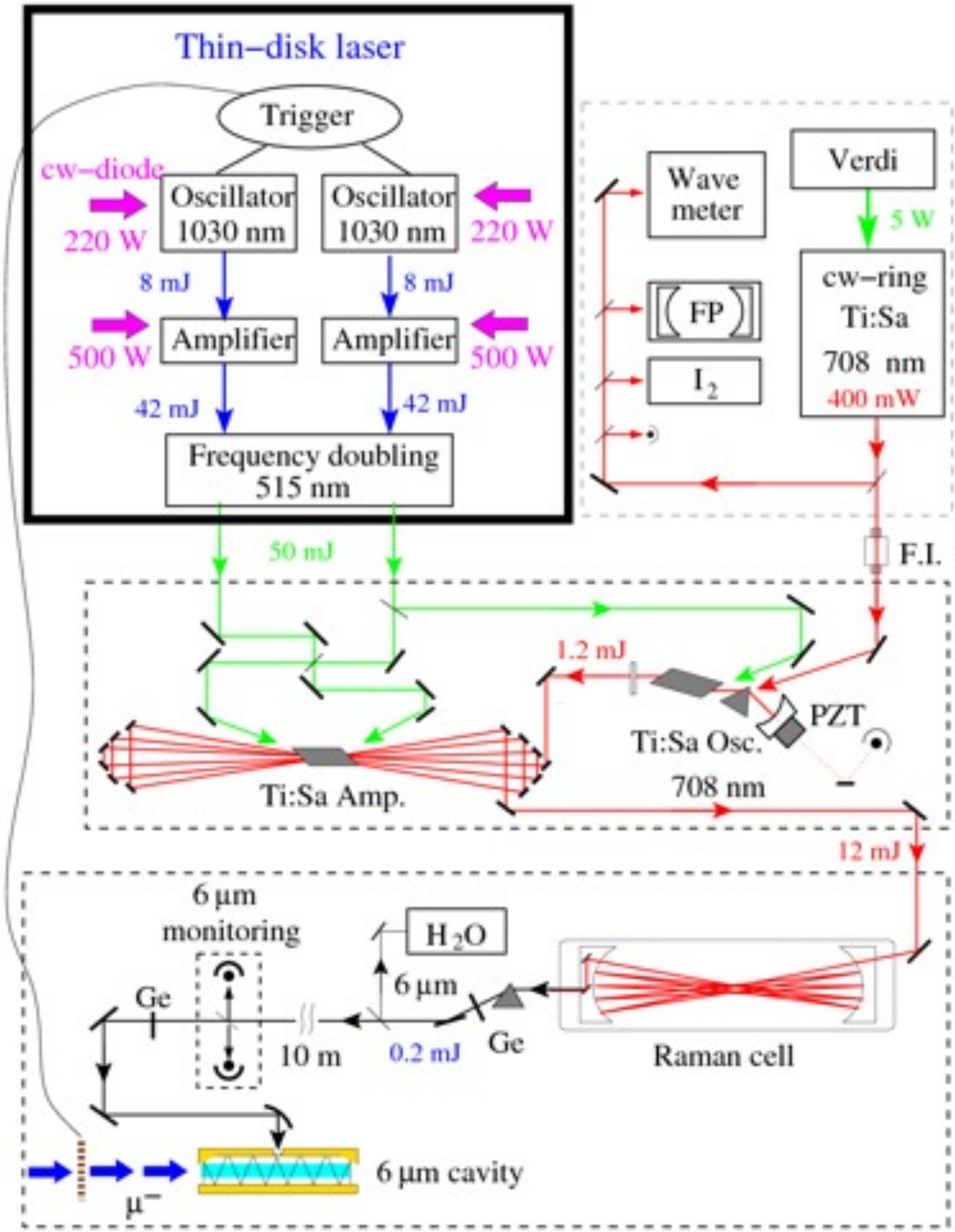
✓ Distribution charge radius contributions vary as $1/n^3$

$$E_{NS} = \frac{2}{3}\left(\frac{\mu}{m_e}\right)^3 \frac{(Z\alpha)^2}{n^3} m_e c^2 \left(\frac{Z\alpha R_p}{\bar{\lambda}_c}\right)^2 \delta_{l0}$$

- Q.E.D. calculations are right ! : 2 experimental data R , R_p
- are Q.E.D. calculations right ? : 3 experimental data R , R_p, Q.E.D.

The end!

Laser chain : principle



- Each single muon triggers the laser system (random trigger)
- 2S lifetime ~1µs → short laser delay
- 6 µm tunable laser pulse (0.2mJ)

μ^- trigger

↓

Thin disk laser (1030 nm) + LBO (515 nm)

↓

pulsed TiSa oscillator + amplifier
(708 nm determined by cw TiSa seeding)

↓

Raman cell (6 µm)

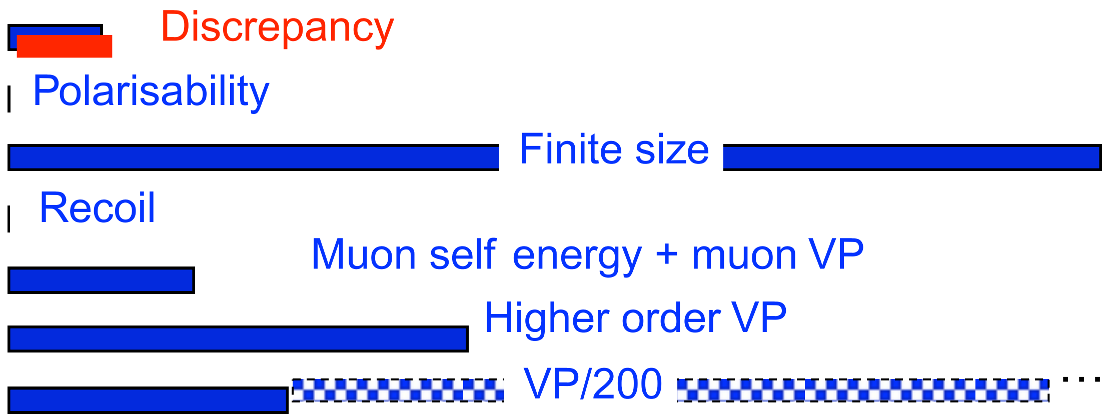
↓

Multipass cavity at 6µm
surrounding the H2 target

Lamb shift - form factor – polarizability

Muonic hydrogen

Lamb shift contribution



- Dipole form factor $$\tilde{n}(r) = e^{-\frac{r}{R}}$$
- Gaussian form factor $$\tilde{n}(r) = e^{-\frac{r^2}{R^2}}$$

	polarizability	form-factor
• 2S state electronic hydrogen (e-p)	8.7 Hz	
• 2S state electronic deuterium (e-d)	2.7 kHz	
• 2S state muonic hydrogen (μ-p)	3.6 GHz	500 MHz
• 2S state muonic deuterium (μ-d)	360 GHz	

A. Antognini thesis, P.Indelicato P.Mohr in preparation

Muonic hydrogen Lamb shift

Name	Reference	formula	value		Pachucki [1]	Borie [3]
NR One loop electron VP		$\frac{4\alpha(Z\alpha)^2}{3\pi n^3}Q^{(1)}_{nl}m_r$	205.00740		205.0074	
Relativistic one loop VP	[3]					205.0282
NR two-loop electron VP		$\frac{4\alpha(Z\alpha)^2}{\pi^2 n^3}Q^{(2)}_{nl}m_r$	1.50790		1.5079	1.5081
NR three-loop electron VP	[12]	$0.08353(1)\frac{\alpha^3(Z\alpha)^2}{\pi^3}m_r$	0.00530		check	
Polarization insertion in two coulomb lines	[4,1]	$0.01244\frac{\alpha^2(Z\alpha)^2}{\pi^2}m_r$	0.15090		0.1509	0.1510
Polarisation insertion in two and three coulomb lines	[12]	$0.036506(4)\frac{\alpha^3(Z\alpha)^2}{\pi^2}m_r$	0.00230			
Relativistic correction	[2,4,1,3]	$\alpha(Z\alpha)^4$	0.01690		0.0169	
Wichmann-Kroll	[13,14,3]	$\frac{4\alpha(Z\alpha)^4}{\pi n^3}Q^{WK}_{nl}m_r$	-0.00103			-0.00103
Radiative photon and electron polarization in the Coulomb line $\alpha^2(Z\alpha)^4$	[4,1]		-0.00500	0.0010	-0.006	
Electron loop in the radiative photon of order $\alpha^2(Z\alpha)^4$		$\left[1.09426\left(-\frac{m}{5m_e}-1\right)-4\times 2.21656\right] \times\frac{\alpha^2(Z\alpha)^4}{\pi^2 n^3}\left(\frac{m_r}{m}\right)^3 m$	-0.00150			
Mixed electron and muon loops	[15]	$\frac{64\alpha^2(Z\alpha)^2}{45\pi^2 n^3}\left(\frac{m_r}{m}\right)^2 Q^{(3)}_{nl}m_r$	0.00007			0.00007
Hadronic polarization $\alpha(Z\alpha)^4 m$	[16]	$-0.638(22)\frac{4\alpha(Z\alpha)^4}{15\pi^2 n^3}\left(\frac{m_r}{m}\right)^3 m\delta_{l0}$	0.01080	0.0004	0.0113	0.0110
Hadronic polarization $\alpha(Z\alpha)^5 m$			0.00005			
Hadronic polarization in the radiative photon $\alpha^2(Z\alpha)^4 m$			-0.00002			
Recoil contribution	[17]	$\frac{(Z\alpha)^4 m_r^3}{2n^3 M^2}\left(\frac{1}{j+\frac{1}{2}}-\frac{1}{l+\frac{1}{2}}\right)(1-\delta_{l0})$	0.05750		0.0575	0.0575
Radiative corrections of order $\alpha^n(Z\alpha)^k m$	[11]	Tables 3.2, 3.3, 3.7, 3.8, 3.9	-0.66770		-0.6677	-0.66788
Recoil corrections of order $\alpha(Z\alpha)^n\frac{m}{M}m$	[11]	Table 4.1	-0.04470		-0.045	-0.04497
Radiative Recoil corrections of order $\alpha(Z\alpha)^n\frac{m}{M}m$	[11]	Table 5.1	-0.00960		-0.0099	-0.0096
Nuclear structure correction of order $(Z\alpha)^5$	[1]	$-\frac{0.095(18)}{n^3}\delta_{l0}$ meV	0.01200		0.012	0.015
Light by light electron loop contribution			0.00135	0.00135		
Polarization operator induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m$		$\frac{-0.00152}{n^3}\delta_{l0}$ meV	0.00019			
Radiative photon induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m$		$\frac{0.00069}{n^3}\delta_{l0}$ meV	-0.00001			
Sum			206.04310	0.00275	206.0353	206.04739
terms dependent on the nuclear radius from Ref. [11] (contributions in meV)						
Leading nuclear size contribution		$\frac{2(Z\alpha)^4}{3n^3}m_r^3 < r^2 > \delta_{l0}$	-5.1975	$< r_p^2 >$		
Nuclear size correction of order $(Z\alpha)^5$	[4,1]	$-\frac{(Z\alpha)^5}{3n^3}m_r^4 < r^3 >_{(2)} \delta_{l0}$	0.0347	$< r_p^3 >$	$0.0363 < r_p^3 >$	$0.0347 < r_p^3 >$
Radiative corrections to nuclear finite size effect	[1]	$\alpha(Z\alpha)^4 m_r^3 < r_p^2 >$	-0.0275	$< r_p^2 >$		$-0.0273 < r_p^2 >$
Nuclear size correction of order $(Z\alpha)^6 < r_p^2 >$	[1]	$-\left(\ln\frac{Z\alpha}{2}\right)\frac{2(Z\alpha)^6}{3n^3}m_r^3 < r_p^2 >$		$< r_p^2 >$		
Nuclear size correction of order $(Z\alpha)^6 < r_p^4 >$	[1]	$\frac{2}{3}\frac{2(Z\alpha)^2}{3n^3}m_r^5(< r_p^2 >)^2$		$< r_p^2 >^2$		
Total $< r^2 >$ contribution			-5.225			

Table 7. All known contributions from Eides et al.[11])

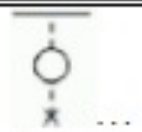
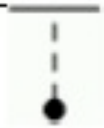
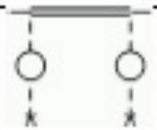
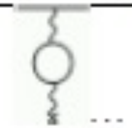
P. Indelicato et al.: Summary of QED calculations for muonic hydrogen and deuterium

Form factor (dipole, Gaussian, …)

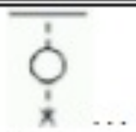
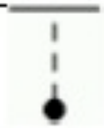
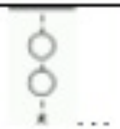
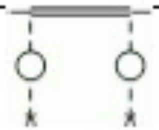
d:4, G:1

LKB

Most important contributions 1

Diagram	Value (in meV)	Name & order	References
	205.0074(1)	leading order VP $[\alpha(Z\alpha)^2]$	Galanin and Pomeranchuk [1952], Pachucki [1999, Table I], Eides et al. [2001, Eq. (208)]
	−3.862(108)	leading nuclear size contribution $[(Z\alpha)^4 m_r^3 \langle r^2\rangle]$	Eides et al. [2001, § 9.6], Pachucki [1999, Table I]
	1.5079(1)	two-loop EVP $[\alpha^2(Z\alpha)^2 m]$	Di Giacomo [1969], Pachucki [1999, Table I], Eides et al. [2001, Eq. (213)]
	−0.6677(1)	muon self-energy + muon VP	Pachucki [1999, Table I], Eides et al. [2001, § 9.5]
	0.1509(1)	double EVP $[\alpha^2(Z\alpha)^2]$	Pachucki [1999, Table I], -0.3268 Eides et al. [2001, Eq. (215)], Pachucki [1996, Eq. (31)]
	0.0594(1)	rel. corr. to EVP $[\alpha(Z\alpha)^4]$	Pachucki [1996], Pachucki [1999, Table I], Eides et al. [2001, Eq. (223)]
	0.0575(1)	recoil of order α^4 $[\alpha^4]$	Barker and Glover [1955], Pachucki [1999, Table I], Eides et al. [2001, Table 11]
	−0.0440(1)	recoil corrections of order $(Z\alpha)^n \frac{m}{M} m$	Eides et al. [2001, § 9.5], Pachucki [1999, Table I]

Most important contributions 1

Diagram	Value (in meV)	Name & order	References
	205.0074(1)	leading order VP $[\alpha(Z\alpha)^2]$	Galanin and Pomeranchuk [1952], Pachucki [1999, Table I], Eides et al. [2001, Eq. (208)]
	−3.862(108)	leading nuclear size contribution $[(Z\alpha)^4 m_r^3 \langle r^2 \rangle]$	Eides et al. [2001, § 9.6], Pachucki [1999, Table I]
	1.5079(1)	two-loop EVP $[\alpha^2(Z\alpha)^2 m]$	Di Giacomo [1969], Pachucki [1999, Table I], Eides et al. [2001, Eq. (213)]
	−0.6677(1)	muon self-energy + muon VP	Pachucki [1999, Table I], Eides et al. [2001, § 9.5]
	0.1509(1)	double EVP $[\alpha^2(Z\alpha)^2]$	Pachucki [1999, Table I], -0.3268 Eides et al. [2001, Eq. (215)], Pachucki [1996, Eq. (31)]
	0.0594(1)	rel. corr. to EVP $[\alpha(Z\alpha)^4]$	Pachucki [1996], Pachucki [1999, Table I], Eides et al. [2001, Eq. (223)]
	0.0575(1)	recoil of order α^4 $[\alpha^4]$	Barker and Glover [1955], Pachucki [1999, Table I], Eides et al. [2001, Table 11]
	−0.0440(1)	recoil corrections of order $(Z\alpha)^n \frac{m}{M} m$	Eides et al. [2001, § 9.5], Pachucki [1999, Table I]

Most important contributions 1

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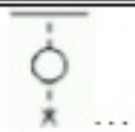
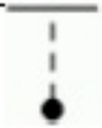
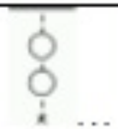
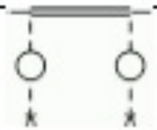
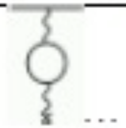
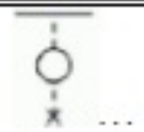
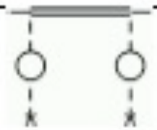
	205.0074(1)	leading order VP $[\alpha(Z\alpha)^2]$	Galanin and Pomeranchuk [1952], Pachucki [1999, Table I], Eides et al. [2001, Eq. (208)]
	−3.862(108)	leading nuclear size contribution $[(Z\alpha)^4 m_r^3 \langle r^2 \rangle]$	Eides et al. [2001, § 9.6], Pachucki [1999, Table I]
	1.5079(1)	two-loop EVP $[\alpha^2(Z\alpha)^2 m]$	Di Giacomo [1969], Pachucki [1999, Table I], Eides et al. [2001, Eq. (213)]
	−0.6677(1)	muon self-energy + muon VP	Pachucki [1999, Table I], Eides et al. [2001, § 9.5]
	0.1509(1)	double EVP $[\alpha^2(Z\alpha)^2]$	Pachucki [1999, Table I], -0.3268 Eides et al. [2001, Eq. (215)], Pachucki [1996, Eq. (31)]
	0.0594(1)	rel. corr. to EVP $[\alpha(Z\alpha)^4]$	Pachucki [1996], Pachucki [1999, Table I], Eides et al. [2001, Eq. (223)]
	0.0575(1)	recoil of order α^4 $[\alpha^4]$	Barker and Glover [1955], Pachucki [1999, Table I], Eides et al. [2001, Table 11]
	−0.0440(1)	recoil corrections of order $(Z\alpha)^n \frac{m}{M} m$	Eides et al. [2001, § 9.5], Pachucki [1999, Table I]

Most important contributions 1

Diagram	Value (in meV)	Name & order	References
	205.0074(1)	leading order VP $[\alpha(Z\alpha)^2]$	Galanin and Pomeranchuk [1952], Pachucki [1999, Table I], Eides et al. [2001, Eq. (208)]
	−3.862(108)	leading nuclear size contribution $[(Z\alpha)^4 m_r^3 \langle r^2 \rangle]$	Eides et al. [2001, § 9.6], Pachucki [1999, Table I]
	1.5079(1)	two-loop EVP $[\alpha^2(Z\alpha)^2 m]$	Di Giacomo [1969], Pachucki [1999, Table I], Eides et al. [2001, Eq. (213)]
	−0.6677(1)	muon self-energy + muon VP	Pachucki [1999, Table I], Eides et al. [2001, § 9.5]
	0.1509(1)	double EVP $[\alpha^2(Z\alpha)^2]$	Pachucki [1999, Table I], -0.3268 Eides et al. [2001, Eq. (215)], Pachucki [1996, Eq. (31)]
	0.0594(1)	rel. corr. to EVP $[\alpha(Z\alpha)^4]$	Pachucki [1996], Pachucki [1999, Table I], Eides et al. [2001, Eq. (223)]
	0.0575(1)	recoil of order α^4 $[\alpha^4]$	Barker and Glover [1955], Pachucki [1999, Table I], Eides et al. [2001, Table 11]
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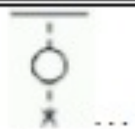
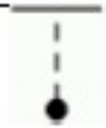
Most important contributions 1

Diagram	Value (in meV)	Name & order	References
	205.0074(1)	leading order VP $[\alpha(Z\alpha)^2]$	Galanin and Pomeranchuk [1952], Pachucki [1999, Table I], Eides et al. [2001, Eq. (208)]
	−3.862(108)	leading nuclear size contribution $[(Z\alpha)^4 m_r^3 \langle r^2 \rangle]$	Eides et al. [2001, § 9.6], Pachucki [1999, Table I]
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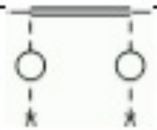
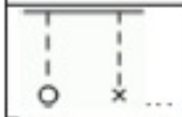
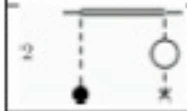
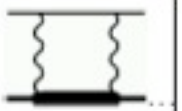
	−0.6677(1)	muon self-energy + muon VP	Pachucki [1999, Table I], Eides et al. [2001, § 9.5]
	0.1509(1)	double EVP $[\alpha^2(Z\alpha)^2]$	Pachucki [1999, Table I], -0.3268 Eides et al. [2001, Eq. (215)], Pachucki [1996, Eq. (31)]
	0.0594(1)	rel. corr. to EVP $[\alpha(Z\alpha)^4]$	Pachucki [1996], Pachucki [1999, Table I], Eides et al. [2001, Eq. (223)]
	0.0575(1)	recoil of order α^4 $[\alpha^4]$	Barker and Glover [1955], Pachucki [1999, Table I], Eides et al. [2001, Table 11]
	−0.0440(1)	recoil corrections of order $(Z\alpha)^n \frac{m}{M} m$	Eides et al. [2001, § 9.5], Pachucki [1999, Table I]

Most important contributions 2

	0.03(3)	light by light electron-loop contribution of order $\alpha^2(Z\alpha)^3 m$ $[\alpha^2(Z\alpha)^3 m]$	Pachucki [1996, p. 2095], Eides et al. [2001, § 9.3.2]
	0.0232(15)	nuclear size correction of order $(Z\alpha)^5$ $[(Z\alpha)^5 m_r^3 \langle r^3\rangle_{(2)} m]$	Pachucki [1996], Faustov and Martynenko [2000], Eides et al. [2001, Eq. (256)], Pachucki [1999]
	−0.0126(1)	(part of the) EVP with finite size $[\alpha(Z\alpha)^4 m_r^3 \langle r^2\rangle]$	Pachucki [1996], Pachucki [1999, Table I], Eides et al. [2001, Eq. (268)]
	0.012(2)	proton polarizability $[(Z\alpha)^5 m]$	Startsev et al. [1976], Rosenfelder [2000], Faustov and Martynenko [2000], Pachucki [1999, Table I], Eides et al. [2001, Eq. (261)]
	0.0108(4)	hadronic polarization, order $\alpha(Z\alpha)^4 m$ $[\alpha(Z\alpha)^4 m]$	Folomeshkin [1974], Friar et al. [1999], Faustov and Martynenko [1999], Eides et al. [2001, Eq. (252)], Pachucki [1999, Table I]
	−0.0099(1)	proton self-energy	Pachucki [1999, Table I]
	−0.0095(1)	radiative-recoil corrections of order $\alpha(Z\alpha)^n \frac{m}{M} m$	Eides et al. [2001, § 9.5]
	−0.0083(1)	(part of the) EVP with finite size $[\alpha(Z\alpha)^4 m_r^3 \langle r^2\rangle]$	Friar [1979a], Friar [1981], Pachucki [1996], Pachucki [1999, Table I], Eides et al. [2001, Eq. (266)]
	−0.006(1)	muon self-energy with electron VP $[\alpha^2(Z\alpha)^4]$	Pachucki [1996, Eqs. (40) and (45)], Pachucki [1999, Table I], Eides et al. [2001, Eq. (237)]
	0.0053(1)	three-loop electron polarization contribution, order $\alpha^3(Z\alpha)^2$ $[\alpha^3(Z\alpha)^2 m]$	Kinoshita and Nio [1999], Eides et al. [2001, Eq. (214)], Pachucki [1999, Table I]

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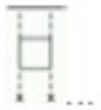
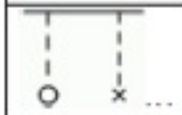
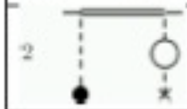
1

Most important contributions 2

	0.03(3)	light by light electron-loop contribution of order $\alpha^2(Z\alpha)^3 m$ $[\alpha^2(Z\alpha)^3 m]$	Pachucki [1996, p. 2095], Eides et al. [2001, § 9.3.2]
	0.0232(15)	nuclear size correction of order $(Z\alpha)^5$ $[(Z\alpha)^5 m_r^2 \langle r^3 \rangle_{(2)} m]$	Pachucki [1996], Faustov and Martynenko [2000], Eides et al. [2001, Eq. (256)], Pachucki [1999]
	−0.0126(1)	(part of the) EVP with finite size $[\alpha(Z\alpha)^4 m_r^3 \langle r^2 \rangle]$	Pachucki [1996], Pachucki [1999, Table I], Eides et al. [2001, Eq. (268)]
	0.012(2)	proton polarizability $[(Z\alpha)^5 m]$	Startsev et al. [1976], Rosenfelder [2000], Faustov and Martynenko [2000], Pachucki [1999, Table I], Eides et al. [2001, Eq. (261)]
	0.0108(4)	hadronic polarization, order $\alpha(Z\alpha)^4 m$ $[\alpha(Z\alpha)^4 m]$	Folomeshkin [1974], Friar et al. [1999], Faustov and Martynenko [1999], Eides et al. [2001, Eq. (252)], Pachucki [1999, Table I]
	−0.0099(1)	proton self-energy	Pachucki [1999, Table I]
	−0.0095(1)	radiative-recoil corrections of order $\alpha(Z\alpha)^n \frac{m}{M} m$	Eides et al. [2001, § 9.5]
	−0.0083(1)	(part of the) EVP with finite size $[\alpha(Z\alpha)^4 m_r^3 \langle r^2 \rangle]$	Friar [1979a], Friar [1981], Pachucki [1996], Pachucki [1999, Table I], Eides et al. [2001, Eq. (266)]
	−0.006(1)	muon self-energy with electron VP $[\alpha^2(Z\alpha)^4]$	Pachucki [1996, Eqs. (40) and (45)], Pachucki [1999, Table I], Eides et al. [2001, Eq. (237)]
	0.0053(1)	three-loop electron polarization contribution, order $\alpha^3(Z\alpha)^2$ $[\alpha^3(Z\alpha)^2 m]$	Kinoshita and Nio [1999], Eides et al. [2001, Eq. (214)], Pachucki [1999, Table I]

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1

New calculation: S. G. Karshenboim, V. G. Ivanov, E. Y. Korzinin, et al., Phys. Rev. A 81, 060501 (2010) and 1 S. Karshenboim, E. Korzinin, V. Ivanov, et al., JETP Letters 92, 8 (2010).
0.00115(1) meV

Check of QED calculations

- Non-perturbative effects due to proton size? Several papers mention «distortion of the wavefunction at the origin»...
  - Dirac + Vacuum polarization
  - Källén and Sabry
  - Self-energy
- Non-perturbative effects in all-order re-summation

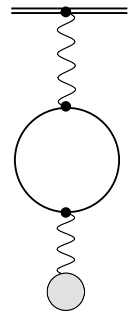
Muon Wavefunction Distortion?

- One question is, of course, whether the QED corrections are complete or whether some higher order contributions are not yet sufficiently well studied. The most recent, comprehensive discussion of these possible deficiencies can be found in the paper by Edith Borie [22]. One might add that it could be necessary that the distorted relativistic wave functions had to be treated in a perturbative sum over multiphoton exchanges. The convergence of such sums is not proven in QED. Such a calculation may imply the solution of a Bethe-Salpeter equation and appears hardly possible [24]. However, further theoretical studies may finally show that the approximations made so far are adequate
- In «The RMS charge radius of the proton and Zemach moments», M.O. Distler, J.C. Bernauer et T. Walcher. Physics Letters B 696, 343-347 (2011).

Finite nuclear size effect by direct numerical solution of Dirac Equation

Direct numerical solution of Dirac equation, numerical grid, 10000 points, ~4000 inside the proton

$$\left(c\boldsymbol{\alpha}\cdot\boldsymbol{p}+\beta\mu_{\mathrm{r}}c^2+V_{\mathrm{Nuc}}(\boldsymbol{r})\right)\Phi_{n\kappa\mu}(\boldsymbol{r})=\mathcal{E}_{n\kappa\mu}\Phi_{n\kappa\mu}(\boldsymbol{r}),$$



$$V_{11}^{pn}(r)=-\frac{\alpha(Z\alpha)}{3\pi}\int_1^\infty dz\,\sqrt{z^2-1}\left(\frac{2}{z^2}+\frac{1}{z^4}\right)\frac{e^{-2m_erz}}{r}$$
$$=-\frac{2\alpha(Z\alpha)}{3\pi}\frac{1}{r}\chi_1\left(\frac{2}{\lambda_e}r\right)$$

$$\chi_n(x)=\int_1^\infty dze^{-xz}\frac{1}{z^n}\left(\frac{1}{z}+\frac{1}{2z^3}\right)\sqrt{z^2-1}.$$

$$V_{11}(r)=-\frac{2\alpha(Z\alpha)}{12\pi}\frac{1}{r}\int_0^\infty dr'\,r'\rho(r')$$
$$\times\left[\chi_2\left(\frac{2}{\lambda_e}\mid r-r'\mid\right)-\chi_2\left(\frac{2}{\lambda_e}\mid r+r'\mid\right)\right].$$

Analytical expression for the evaluation of vacuum-polarization potentials in muonic atoms, S. Klarsfeld. Physics Letters 66B, 86-88 (1977).

LKB

Check point nucleus

Non relativistic

Pachucki 1996 calculation	205.006
Pachucki 1999 calculation	205.007 4
Numerical calc with Pachucki data	205.007 385
Numerical calc with CODATA 2006 data	205.007 359

Relativistic

Borie calc with ? data	205.028 2
PI calc with CODATA 2006 data	205.028 20
PM calc with CODATA 2006 data	205.028 201

Energies in meV

Charge radius dependence Coul.+VP

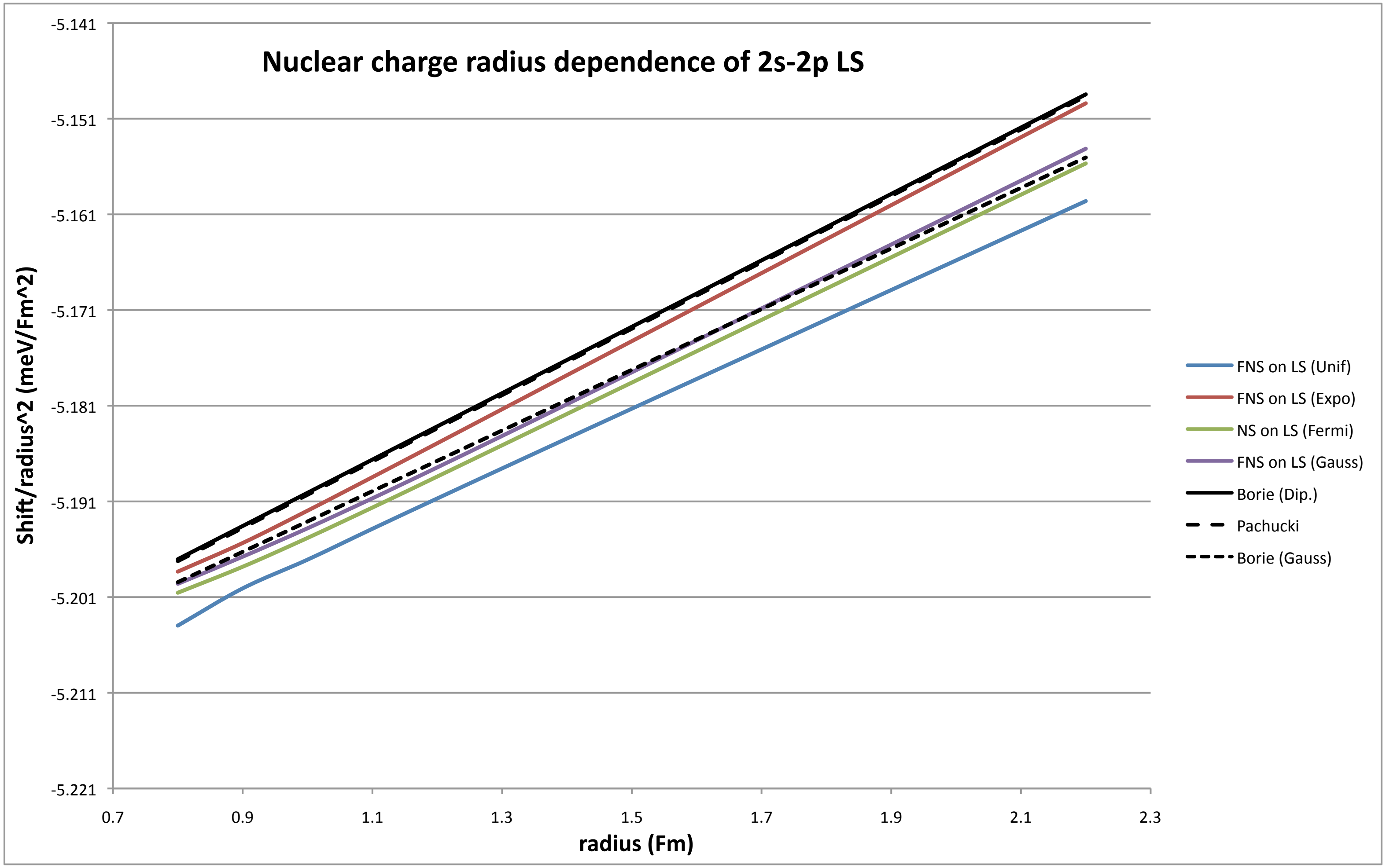
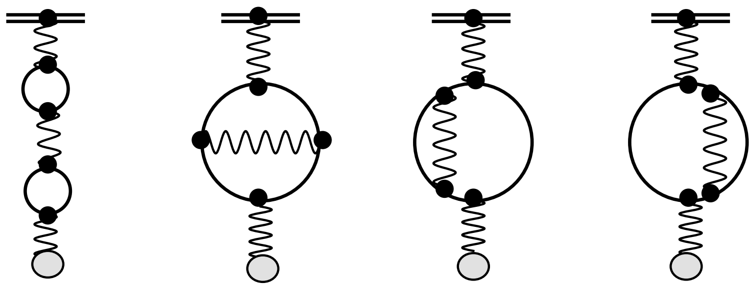


Fig. 3. Dependence of $\frac{\Delta E_{\mathrm{V11FN}}}{R^2}$ as a function of R in meV/fm^2 for different charge distribution models.

Charge radius dependence Coul.+VP

Model	a (meV/fm^2)	b (meV/fm^3)
Uniform	-5.2284	0.0313
Dipole	-5.2271	0.0353
Fermi	-5.2271	0.0324
Gauss	-5.2265	0.0328
Ref.[34], Dip.	-5.2248	0.0347
Ref.[32]	-5.225	0.0347
Ref.[34], Gauss	-5.2248	0.0317

Källen and Sabry



$$\Delta E = aR^2 + bR^3 + cR^4$$

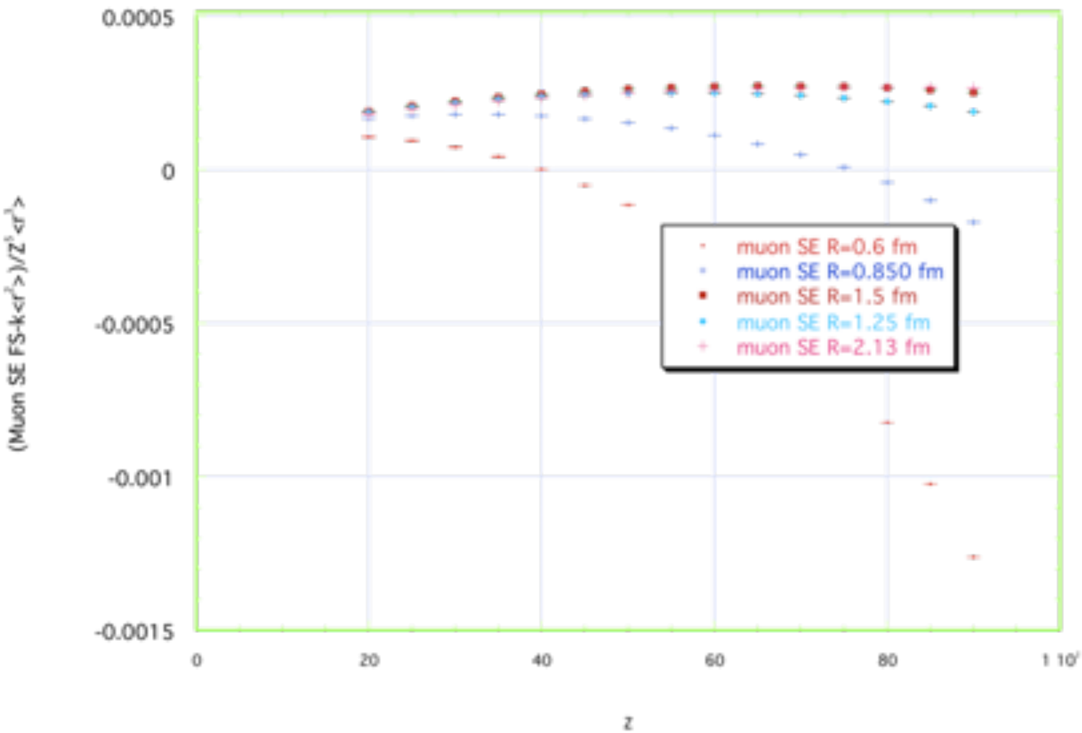
Model	Uniform	Exponential	Fermi	Gaussian
a	-0.0002145	-0.0002145	-0.0002146	-0.0002145
b	0.0000078	0.0000086	0.0000082	0.0000083
c	-0.0000008	-0.0000009	-0.0000008	-0.0000009

Finite size correction on muon self-energy

All-orders calculations

$$E_{SE-NS} = \left(4\ln 2 - \frac{23}{4}\right)\alpha(Z\alpha)\mathcal{E}_{NS}$$

$$\mathcal{E}_{NS} = \frac{2}{3}\left(\frac{\mu_r}{m_\mu}\right)^3 \frac{(Z\alpha)^2}{n^3} m_\mu \left(\frac{Z\alpha < r >}{\lambda\!\!\!{}^-_C}\right)^2$$



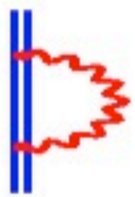
$$E_{SE-NS} = -0.000824 < r^2 >$$

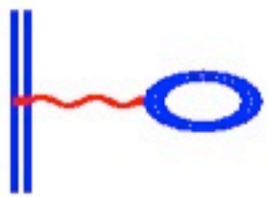
(All-orders calculations-E_{SE-NS}<r^2>)/Z^5 <r^3>

$1.8 \pm 1 \times 10^{-5}$

QED at order α and α^2

Self Energy





Vacuum Polarization

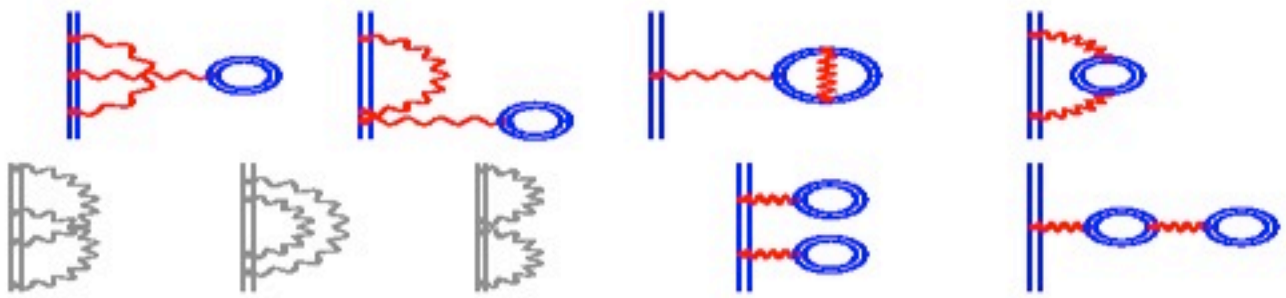
H-like "One Photon" order (α/π)

QED at order α and α^2

Self Energy

Vacuum Polarization

H-like "One Photon" order (α/π)

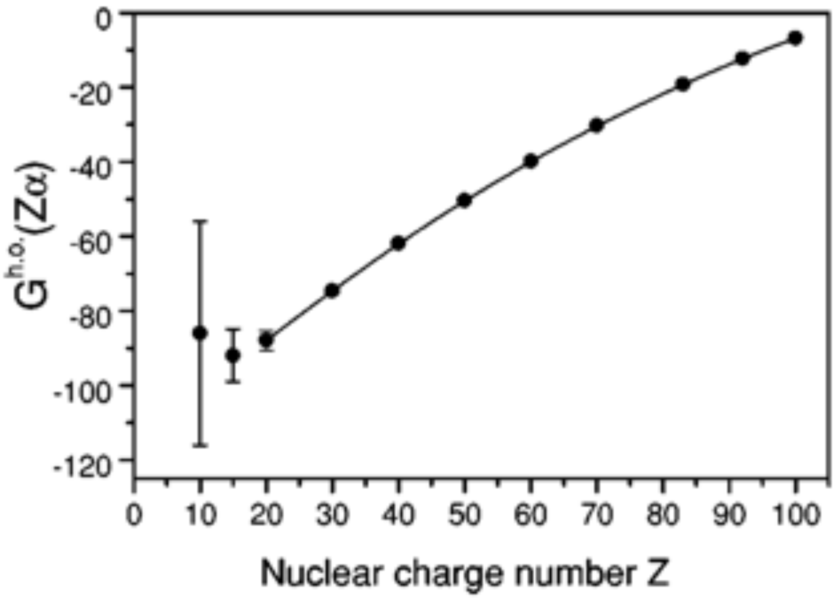


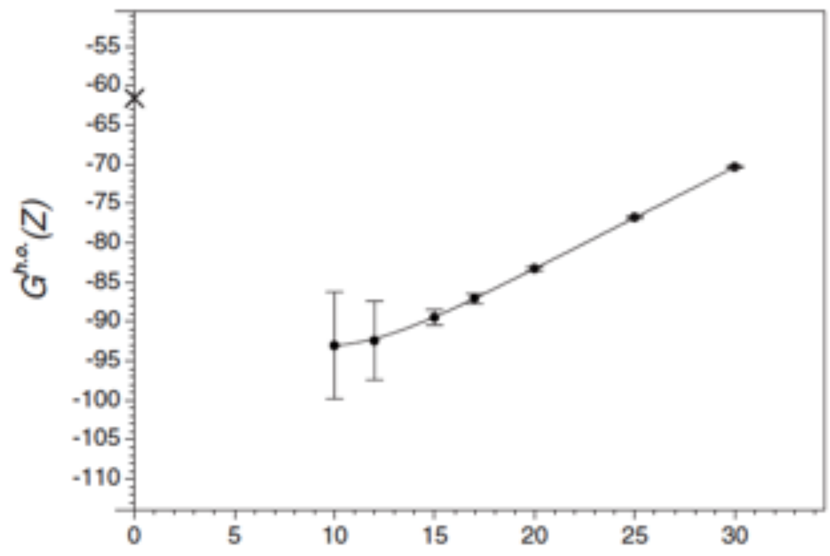
H-like "Two Photon" order $(\alpha/\pi)^2$

Two-loop self-energy (1s)

V. A. Yerokhin, P. Indelicato, and V. M. Shabaev, Phys. rev. A 71, 040101(R) (2005).

$$\Delta E_{\mathrm{SESE}} = m\left(\frac{\alpha}{\pi}\right)^2 (Z\alpha)^4 \{B_{40} + (Z\alpha)B_{50} + (Z\alpha)^2 \times [L^3 B_{63} + L^2 B_{62} + L B_{61} + G^{\mathrm{h.o.}}_{\mathrm{SESE}}(Z)]\},$$





V. A. Yerokhin, Physical Review A 80, 040501 (2009)

Evolution at low-Z

All order numerical calculations

V. A. Yerokhin, Phys. Rev. A **80**, 040501 (2009) $G_{\mathrm{SESE}}^{\mathrm{h.o.}}(Z=0) \equiv B_{60} = -84(15)$,

V. A. Yerokhin, P. Indelicato, and V. M. Shabaev, Phys. Rev. A 71, 040101(R) (2005). $-127(42)$

Analytic calculations

$$B_{40}=1.409244, \quad B_{50}=-24.2668(31),$$

$$B_{63}=-8/27, \quad B_{62}=16/27-(16/9)\ln 2,$$

K. Pachucki and U. D. Jentschura, Phys. Rev. Lett. **91**, 113005 (4) (2003). $B_{60}=-61.6(9.2)$.

U. D. Jentschura, A. Czarnecki, and K. Pachucki, Physical Review A **72**, 062102 (2005). $B_{61}=48.388913$, $\delta B_{61}=-1.4494\ldots$

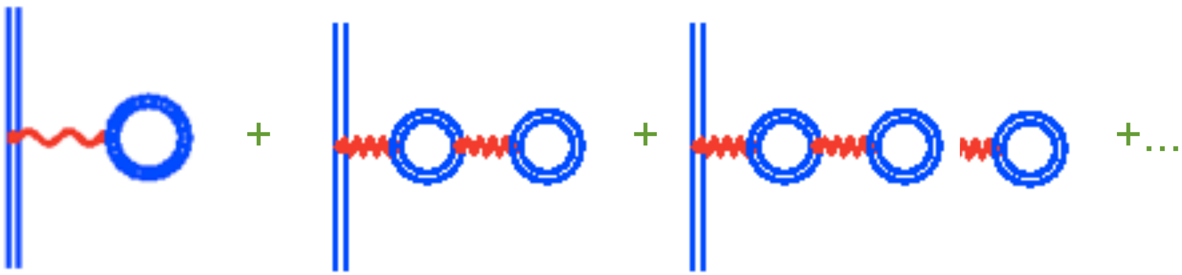
Example of unsolved problems

Dyson: the expansion in α of QED has zero convergence radius… e→-e, plus d'état lié!

Divergence of Perturbation Theory in Quantum Electrodynamics, F.J. Dyson. Physical Review 85, 631–632 (1952).

Example of unsolved problems

Example of unsolved problems



Vacuum Polarization re-summation

$$V_{\mathrm{VP}\infty}\left(k^2\right) = \frac{1}{k^2}\left[1+\Pi\left(k^2\right)+\Pi\left(k^2\right)\frac{1}{k^2}\Pi\left(k^2\right)+\Pi\left(k^2\right)\frac{1}{k^2}\Pi\left(k^2\right)\frac{1}{k^2}\Pi\left(k^2\right)+\cdots\right]$$

$$= \frac{\Pi\left(k^2\right)}{1-\Pi\left(k^2\right)}$$

$$\Pi\left(k^2\right) = \frac{2\alpha k^2}{3\pi}\int_1^\infty dz\frac{1}{z}\left(\frac{1}{z}+\frac{1}{2z^3}\right)\frac{\sqrt{z^2-1}}{4m_e^2z^2+k^2}.$$

Singularity at $k_0 = e^{\frac{3\pi}{2\alpha}+\frac{5}{6}} \approx 6.53\times 10^{280}$ **huge momenta = very short distances**

S. Brodsky, P. J. Mohr, P. Indelicato