Compact HEIDI at the LHC

—

Investigating new Physics with WHIZARD

Christian Speckner

in collaboration with
N. Christensen, C. Duhr, B. Fuks, J. Reuter

[arXiv:1010.3251]

May 27th, 2011
IPHC Strasbourg

Outline:

Why should we expect new physics at the LHC?

- S matrix unitarity: $S^\dagger S = 1$ (conservation of propability)
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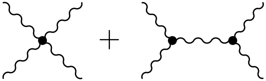

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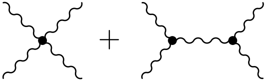

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Tree-Level Unitarity

Either the Higgs or new physics appear below 1 TeV and restore Tree-Level Unitarity, or the SM becomes strongly interacting at this scale!

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We cannot fail to find something new there!

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However:

Can the new physics be such that we cannot see it even if it exists?

$\longrightarrow$ example: (compact) HEIDI [J. v. d. Bij]

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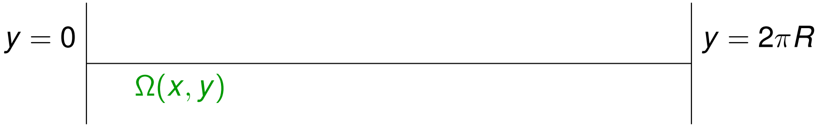

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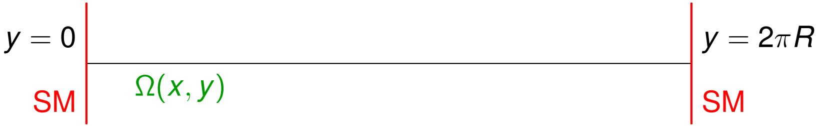

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- 5D mass shell condition: $k^2 = k_{4\text{D}}^2 - k_5^2 = m_\Omega^2$
- Compactification $\rightarrow$ k_5 discretized: $k_{5,n} = nR^{-1}$
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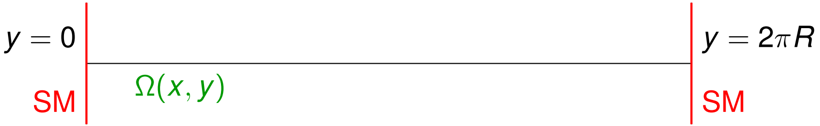

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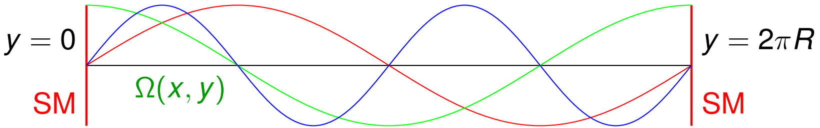
$$\Omega(x, y) = \sum_{n=0}^{\infty} \frac{1}{N_n}\Omega_n(x)\cos k_n y$$



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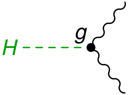
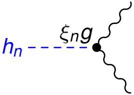


Insert expansion into $\mathcal{L}_{SM}$ $\longrightarrow$



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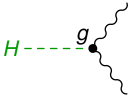

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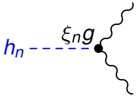
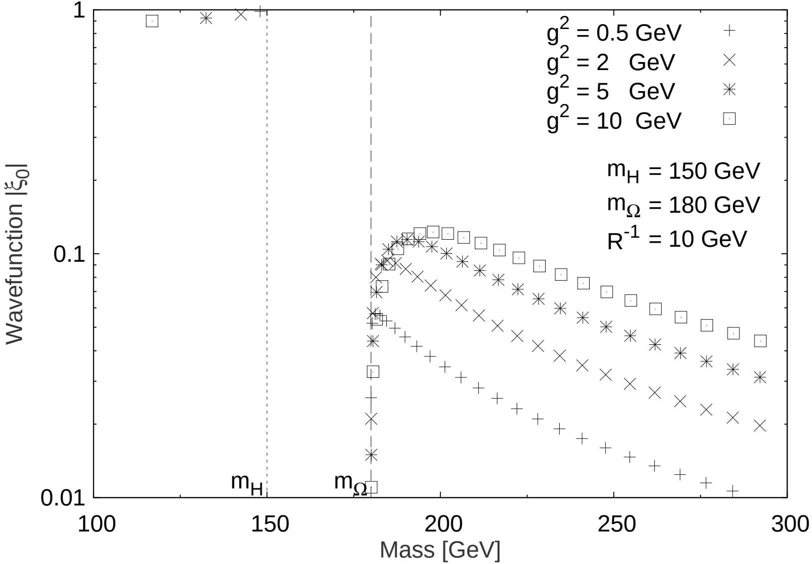


- Widths: $\Gamma_{h_n} \approx \xi_n^2 \Gamma_H(m_n^2)$

Spectrum I ($m_H < m_\Omega$):



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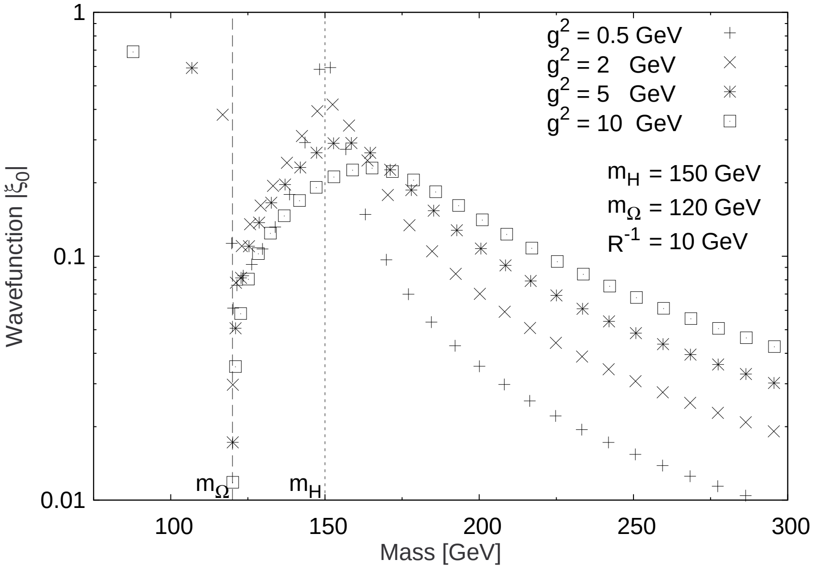
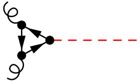
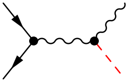
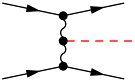


Impact on LHC Phenomenology

- Same production (and decay) channels for the h_n as for the Higgs...

gluon fusion	HEIDI strahlung	gauge boson fusion
		

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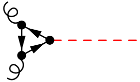	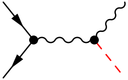	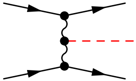

- ...BUT: couplings of the individual h_n can be very small
- Considerable dependence on the parameter space

General feature

Higgs resonance gets dilluted to a series of weak resonances.

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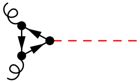	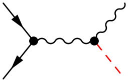	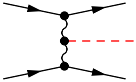

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$\longrightarrow$ Could this be invisible at the LHC? Simulate!

What is WHIZARD?

Verbatim from the website:

“WHIZARD is a program system designed for the efficient calculation of multi-particle scattering cross sections and simulated event samples.”

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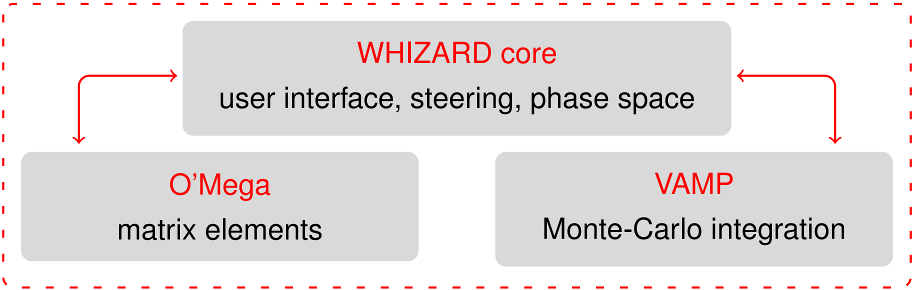

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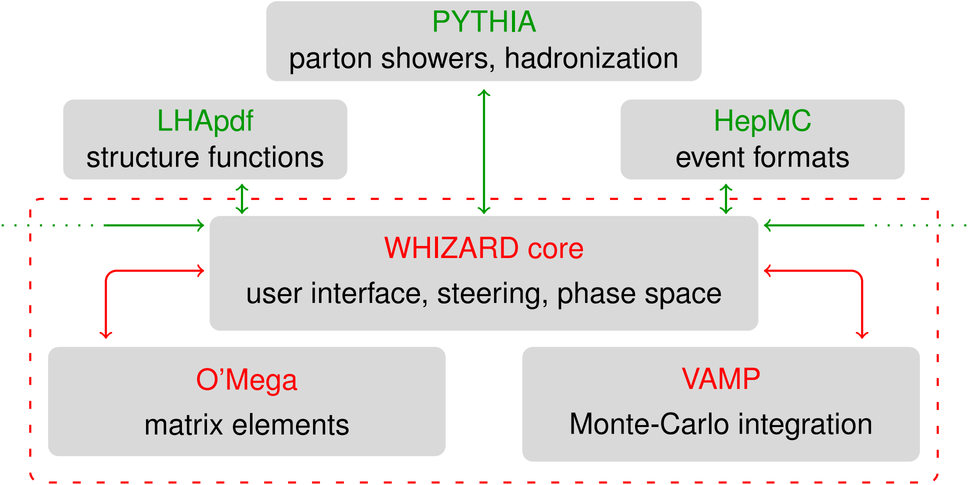

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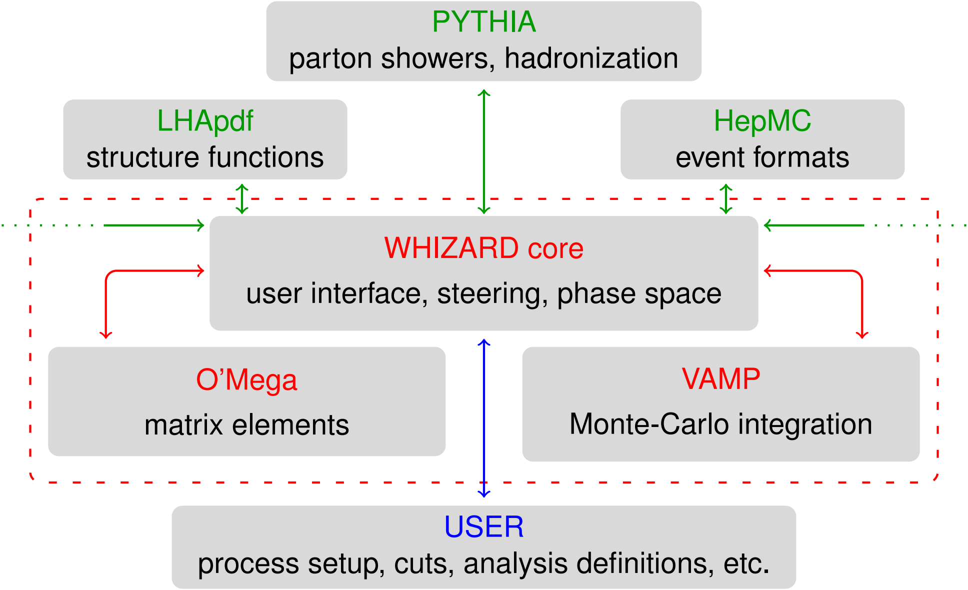
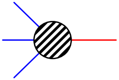


A few words on O'Mega

Physics and algorithm:

- 1-particle off-shell wavefunction (1POW):

$$\langle \text{in} | \phi(x) | 0 \rangle \quad =$$



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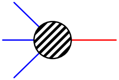

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Implementation:

- Written in O'Caml (impure functional language)
- Graph of 1POWs transformed into FORTRAN 95 code
- Numerical calculation of helicity amplitudes
- Generated code is human readable, can be easily modified

WHIZARD core

Structure:

- Self-contained FORTRAN 2003 program
- Control language “SINDARIN” for steering all aspects of the run:
  1. Call O’Mega to generate process library of matrix elements
  2. Compile and dynamically load process library
  3. Generate phasespace maps adapted to process
  4. Adaptive multichannel Monte-Carlo integration
  5. Generate and analyze either weighted or unweighted events

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Features:

- Speed (unweighted events for to up to 8 final state particles)
- Integrated analysis facilities
- Many BSM models implemented: (N)MSSM, Little Higgs, UED, Three-Site model and more
- SINDARIN: Powerful command language for elaborate cut and analysis variables, possibility to scan over parameters

References:

- References: arXiv:0708.4233 , arXiv:hep-ph/0102195
- Web page: `http://projects.hepforge.org/whizard/`

The WHIZARD team

Bach, Boschmann, Kilian, Ohl, Reuter, Schmidt, Schwertfeger, CS, Wiesler, Wirtz

FeynRules — New models in WHIZARD

Implementing a new model into any Monte-Carlo is

- Cumbersome: each code has its own conventions and quirks
- Error-prone: for exactly the same reasons

$\longrightarrow$ usually takes a fair amount of testing and debugging

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Enter FeynRules [N. Christensen, C. Duhr, B. Fuks]

- Mathematica package
- Calculates Feynman rules from Lagrangian
- Generates model files for: CalcHEP, Sherpa, Madgraph, FeynArts, WHIZARD [CS], ...
- Reference: arXiv:1010.3251 / CPC 180, 1614 (2009)

Simulation setup

Model implementation:

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  $\longrightarrow$ variable number of fields in model!

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- Top loop described by effective operator (top integrated out)
- Full simulation of $pp \to 4l$ at $\int \mathcal{L} = 100\ \text{fb}^{-1}$ with 14 TeV ($l \in \{e, \mu\}$)

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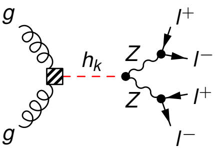	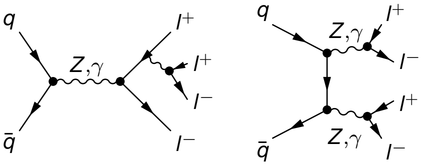

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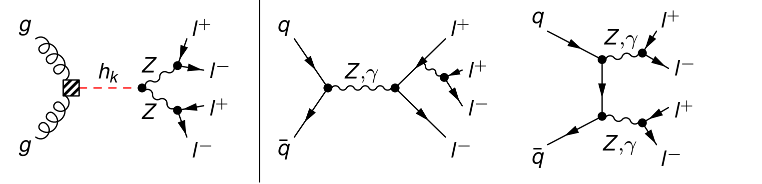
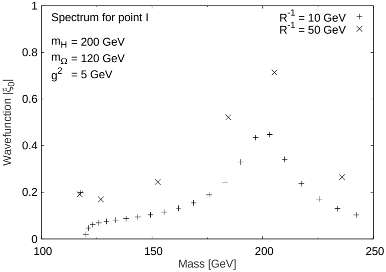


- Cuts: $|y| \leq 2$ $\quad$ $10\ \text{GeV} \leq m_{ll} \leq 100\ \text{GeV}$ $\quad$ $5\ \text{GeV} \leq p_t$

Simulation results, point I



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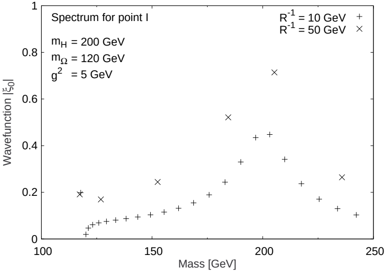

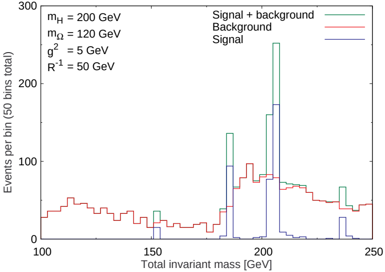

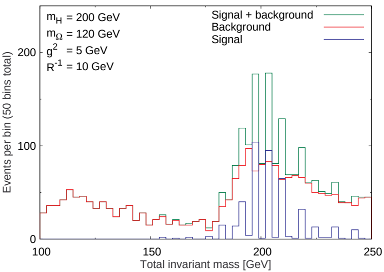

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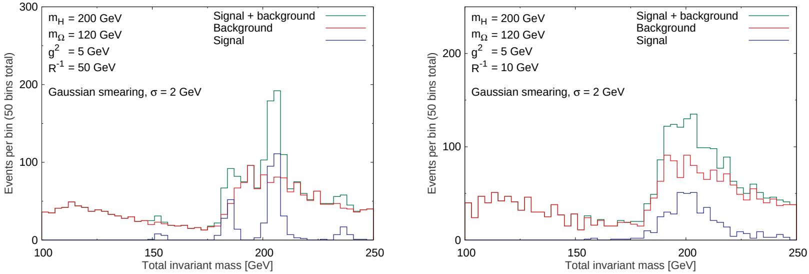
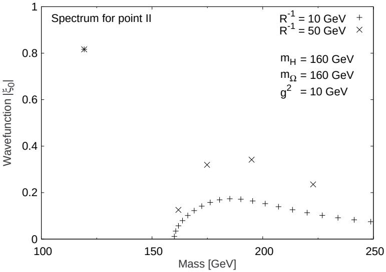
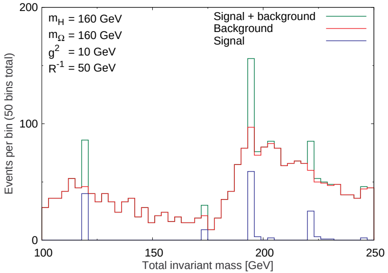


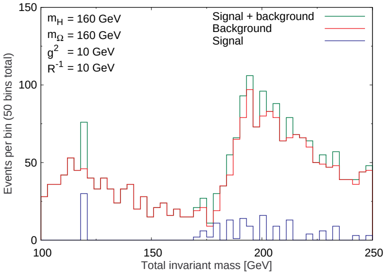
Results for point I after smearing:

- $R^{-1} = 50$ GeV: peaks get smeared, remain visible
- $R^{-1} = 10$ GeV: only a pronounced, broad excess remains!

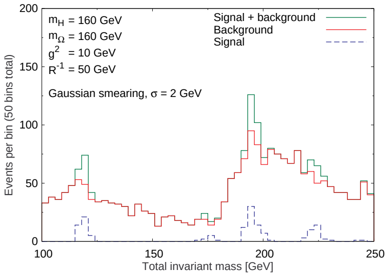
Simulation results, point II

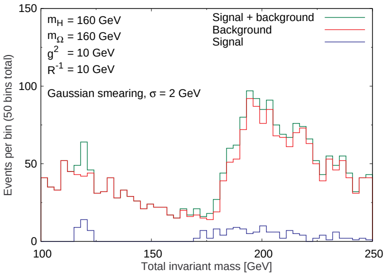






Simulation results, point II — smearing





Results for point II after smearing:

- Peaks much less pronounced as for point I
- Might be resolved for $R^{-1} = 50$ GeV...
- ...but only diffuse excess remains for $R^{-1} = 10$ GeV — might well be hopeless!

What happens in the limit of large R?

1) Choose invariant mass interval $\mathcal{I} = [s - \Delta s; s + \Delta s]$ such that

- Many modes $h_i, \ldots, h_{i+N}$ lie within $\mathcal{I}$
- The ξ_n do not vary considerably: $\xi_n \approx \xi_{\mathcal{I}}$

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2) Estimate the integral of the cross section over $\mathcal{I}$:

- Contribution of a single mode (narrow width)
  - to the differential cross section: $d\sigma_n \propto \frac{\xi_n^4}{\left|s - m_n^2 + i m_n \Gamma_n\right|^2}$
  - to the integrated cross section: $\sigma_n = \int_{\mathcal{I}} ds\, \sigma_n \propto \frac{\xi_n^4}{\Gamma_n} \approx \frac{\xi_{\mathcal{I}}^2}{\Gamma_H(s)}$
- Sum over all modes: $\sigma = \sum_n \sigma_n \propto \frac{N \xi_{\mathcal{I}}^2}{\Gamma_H(s)}$
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$\longrightarrow$ Integrated cross section asymptotically independent of R!

What can we conclude from the simulations?

$\longrightarrow$ General features of HEIDI in the golden channel

- SM Higgs resonance gets distributed to a “comb”
- Total cross section is largely independent of R...
- ...but: for large R, individual peaks cannot be resolved, forming a diffuse excess
- Shape depends on g, m_H and m_Ω

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However, several things remain to be done:

- Full one loop calculation
- Analyze constraints on parameter space (Higgs searches, EWPT)
- Study other channels: HEIDI strahlung, gauge boson fusion, etc.
- Generalizations of the model
- ...

⟶ onging work in collaboration with J. v. d. Bij

HiggsBounds

HiggsBounds [Bechtle, Brein et al. 2010]

- Software package for the easy application of LEP / Tevatron limits to a broad class of models
- Especially easy to use for copies of the SM Higgs $\longrightarrow$ cHEIDI!!!

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1. Feed scalar spectrum + couplings to HB (cutoff @ 300 GeV)
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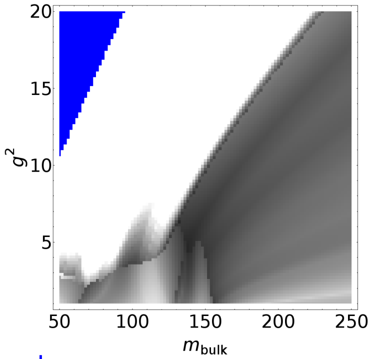
Caveat: cHEIDI modes are currently not combined in observables

Preliminary results ($m_H = 160$ GeV , $R^{-1} = 50$ GeV)



Legend:

- Both axes in GeV
- Shading: ΔR — white: excluded, black: unconstrained
- Blue area: no symmetry breaking

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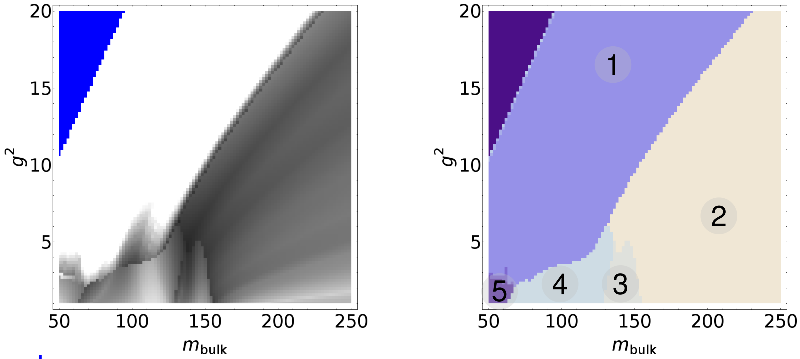

Legend:

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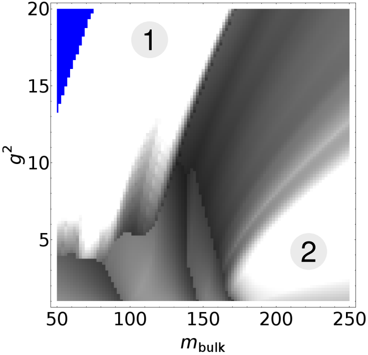
- 1 : Higgsstrahlung @ LEP, 2 – 5 : Tevatron combined

Preliminary results ($m_H = 180$ GeV)

$R^{-1} = 50$ GeV



- 1 : Lowest mode in Higgstrahlung @ LEP
- 2 : Lowest mode pushed into the Tevatron exclusion window!!!

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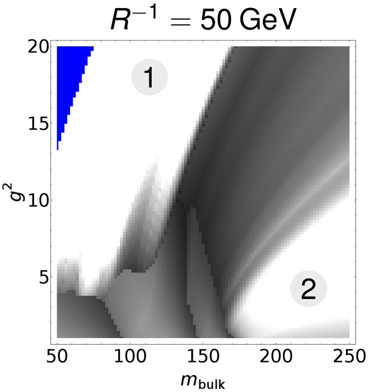

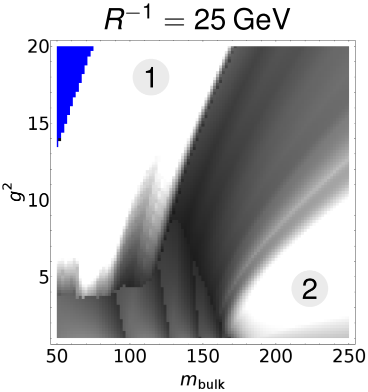

- 1 : Lowest mode in Higgstrahlung @ LEP
- 2 : Lowest mode pushed into the Tevatron exclusion window!!!
- Changing R^{-1} does not influence the exclusion?!
  - No suprise: lowest mode not sensitive to R^{-1}

Conclusions

WHIZARD:

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Compact HEIDI (in the golden channel):

- Could come in a variety of shapes
- Potentially hard to discover in distributions (shapeless excess!)
- Grain of salt: our simulation is not adequate for significance estimates $\longrightarrow$ improve (ongoing work)