

## Antimatter and Gravitation 2011

# ATOM INTERFEROMETRY AND THE GRAVITATIONAL REDSHIFT

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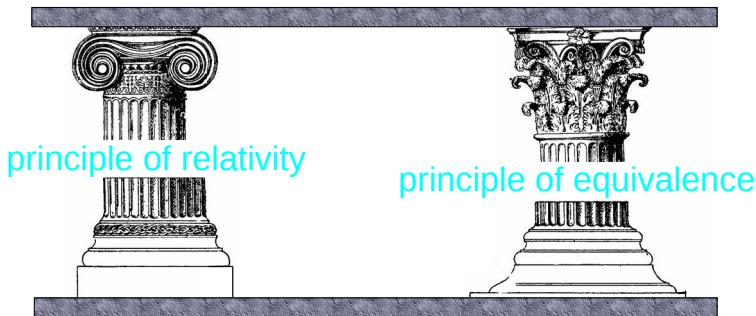
*Based on a collaboration with*

Peter Wolf, Christian Bordé, Serge Reynaud,  
Christophe Salomon & Claude Cohen-Tannoudji

10 octobre 2011

# Two fundamental principles

## GENERAL RELATIVITY



## EXPERIMENTS

# Weak version of the equivalence principle

WEP [Philipponus V<sup>th</sup> century, Galileo 1610, Newton 1687, Laplace 1780, Bessel 1850, Eötvös 1898]

All test bodies follow the same universal trajectory in a gravitational field, independently of their mass, detailed internal structure and composition

For all test bodies,  $m_i = m_g$  where

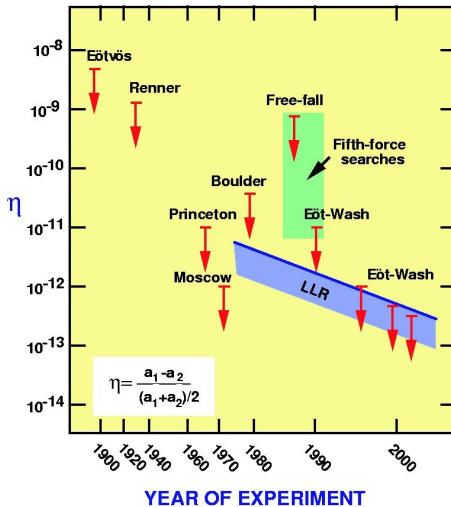
$$\begin{aligned}\mathbf{F} &= m_i \mathbf{a} & (m_i = \text{inertial mass}) \\ \mathbf{F}_g &= m_g \mathbf{g} & (m_g = \text{passive gravitational mass})\end{aligned}$$

Precision is measured in terms of the Eötvös ratio

$$\eta_{AB} = \left| \left( \frac{m_g}{m_i} \right)_A - \left( \frac{m_g}{m_i} \right)_B \right|$$

# Experimental limits on the weak equivalence principle

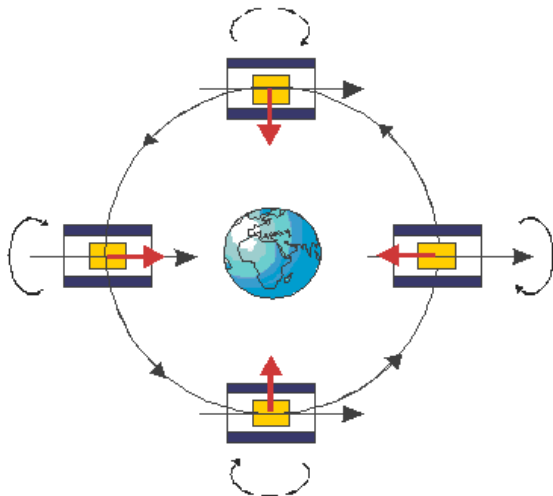
## TESTS OF THE WEAK EQUIVALENCE PRINCIPLE



2013  
 $\mu$ -SCOPE  $10^{-15}$

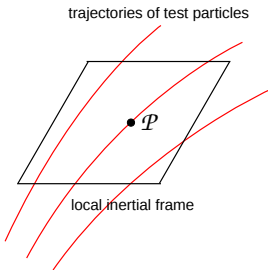


# $\mu$ -SCOPE experiment



Expected accuracy  $10^{-15}$

# Einstein equivalence principle



- 1 Weak equivalence principle (WEP). Guarantees existence of a family of trajectories followed by all test bodies
- 2 Local Lorentz invariance (LLI). Implies existence of fields  $\Phi^{(A)}$  reducing in local inertial frames to

$$\Phi_{\alpha\beta}^{(A)} = \Phi^{(A)}(\mathcal{P}) \eta_{\alpha\beta}$$

- 3 Local position invariance (LPI). Implies independence of results of experiments on position

$$\Phi^{(A)}(\mathcal{P}) = \underbrace{c^{(A)}}_{\substack{\text{set to one} \\ \text{by change of units}}} \Phi(\mathcal{P})$$

Einstein equivalence principle equivalent to coupling to a universal field [Will 1993]

$$g_{\mu\nu} = \Phi^{-1} \Phi_{\mu\nu}$$

reducing to Minkowski metric  $\eta_{\alpha\beta}$  in local inertial frames

# Metric theories of gravity

EEP  $\iff$  Gravitation must be a **curved-space-time** phenomenon

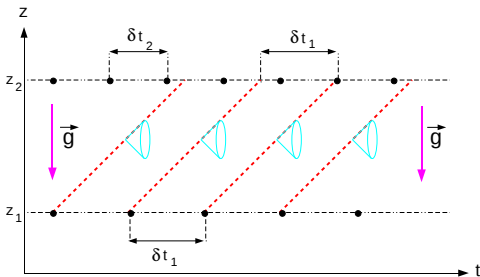
Postulates of metric theories [Thorne, Lee & Lightman 1973]

- Space-time is endowed with a metric  $g_{\mu\nu}$
- The world-lines of test bodies are geodesics of that metric
- The non-gravitational laws of physics are those of special relativity

Some metric theories

- Nordström-Einstein-Fokker  $g_{\mu\nu} = \Omega^2 \eta_{\mu\nu}$
- General relativity  $g_{\mu\nu}$
- Jordan-Brans-Dicke  $g_{\mu\nu}, \phi$
- Rosen  $g_{\mu\nu}, \eta_{\mu\nu}$
- Ni  $g_{\mu\nu}, \psi, t$

# Gravitational redshift experiment



- Two identical clocks at different elevations in a static gravitational field

$$\delta\tau = \sqrt{-g_{00}(z_1)} \delta t_1 = \sqrt{-g_{00}(z_2)} \delta t_2$$

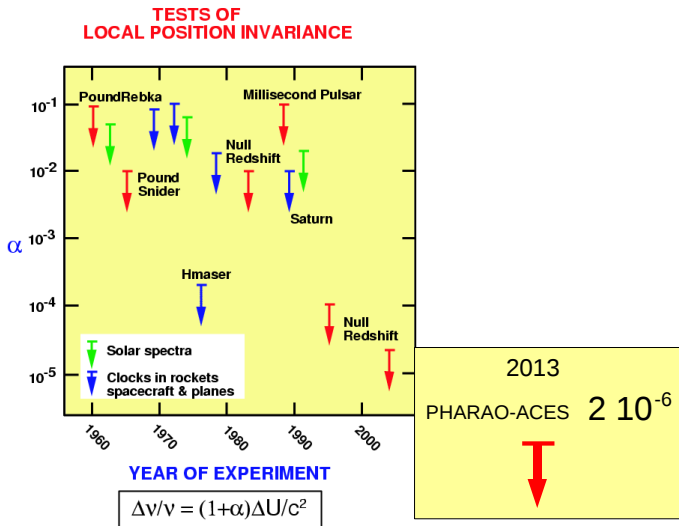
- The clocks are compared by continuous exchanges of light signals

$$\frac{\Delta\nu}{\nu} = \frac{\delta t_1}{\delta t_2} - 1 = \frac{g \Delta z}{c^2}$$

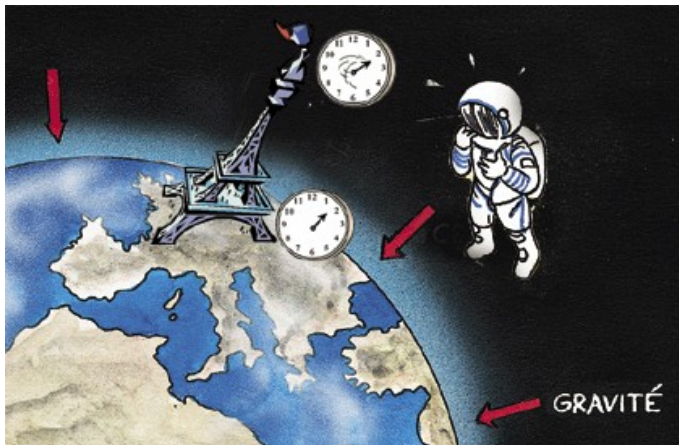
- With  $g$  and  $\Delta z$  known from independent measurements one can check the GR prediction for the redshift



# Experimental limits on the local position invariance



# Atomic Clock Ensemble in Space [Cacciapuoti & Salomon 2009]



## LETTERS

# A precision measurement of the gravitational redshift by the interference of matter waves

Holger Müller<sup>1,2</sup>, Achim Peters<sup>3</sup> & Steven Chu<sup>1,2,4</sup>

One of the central predictions of metric theories of gravity, such as general relativity, is that a clock in a gravitational potential  $U$  will run more slowly by a factor of  $1 + U/c^2$ , where  $c$  is the velocity of light, as compared to a similar clock outside the potential<sup>1</sup>. This effect, known as gravitational redshift, is important to the operation of the global positioning system<sup>2</sup>, timekeeping<sup>3,4</sup> and future experiments with ultra-precise, space-based clocks<sup>5</sup> (such as searches for variations in fundamental constants). The gravitational redshift has been measured using clocks on a tower<sup>6</sup>, an aircraft<sup>7</sup> and a rocket<sup>8</sup>, currently reaching an accuracy of  $7 \times 10^{-5}$ . Here we show that laboratory experiments based on quantum interference of atoms<sup>9,10</sup> enable a much more precise measurement, yielding an accuracy of  $7 \times 10^{-9}$ . Our result supports the view that gravity is a manifestation of space-time cur-

verify that there is zero variation between different clocks that move together through space-time. Still, the verification of local position invariance may be called the weakest link in the experimental underpinning of the EEP.

Our determination of the gravitational redshift is based on a re-interpretation of atom interferometry experiments that have been used to measure the acceleration of free fall<sup>9,10,14</sup>. As shown in Fig. 1a, a laser-cooled atom launched vertically upwards in a vacuum chamber is subjected to three pulses from a pair of anti-parallel, vertical laser beams having respective wavenumbers of  $k_1$  and  $k_2$ . Each laser pulse transfers the momentum  $\hbar(k_1 + k_2)$  (where  $\hbar$  is  $h/2\pi$ ,  $h$  being the Planck constant) of two photons to the atom (Fig. 1b). The recoil gives a combined momentum impulse of  $\hbar k$ , where  $k \equiv k_1 + k_2$ . The intensity and duration of the first laser pulse is

# Some important implications?

## General relativity tested on a tabletop

Atomic-clock experiment pins down accuracy of fundamental gravity measurement.

By measuring a spectacularly small difference in the ticks of two quantum clocks, physicists have proven a pillar of Albert Einstein's theory of gravity to be on firmer footing than ever before.

The experiment is the latest in a series of tests in which scientists have scrutinized one of Einstein's more profound predictions: that clocks in stronger gravitational fields run more slowly. For decades they have put clocks at higher elevations, where Earth's gravity is slightly weaker, and measured the ensuing changes. From a clock in a tower at Harvard University in Cambridge, Massachusetts, in the 1960s, to others flown on planes in the 1970s, to a clock that flew thousands of kilometres into space on a rocket in 1980, physicists have not been able to show that Einstein was wrong.

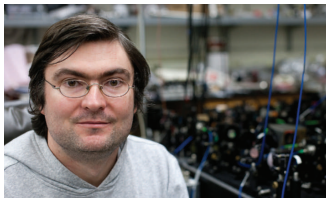
Now, a team led by Holger Müller of the University of California, Berkeley, has measured the time-shifting effects of gravity 10,000 times more accurately than ever before. They show that gravity's effect on time is predictable to 7 parts per billion (H. Müller, A. Peters and S. Chu *Nature* 463, 926–929, 2010). And they did it using two laboratory clocks with a height difference of just 0.11 metres — a set-up that seems quaintly small in this day of big physics. "Precision experiments on a tabletop are not something of the past," says Müller, whose research team consisted of Achim Peters of the Humboldt University of Berlin and Steven Chu, the US Secretary of Energy.

Many atomic clocks use the extremely regular pulsations of atoms shifting between excited energy states.

But "while the apparatus used in this experiment on the fundamental quantum frequency of a caesium atom associated with the atom's rest energy. This frequency was so high that physicists never thought to use it as a clock. But a special interferometer could measure the difference between two such clocks experiencing gravity's effect."

"What's fascinating about their work is that they were using the entire atom as a clock," says atomic-clock expert Jun Ye of the Joint Institute for Laboratory Astrophysics in Boulder, Colorado.

Müller and his team shot caesium atoms, cooled nearly to absolute zero, in an arc across a gap. Mid-stream, photons from a laser bumped the atoms into two, quantum-mechanical alternate realities. In one, an atom absorbed a photon and arced on a slightly higher path,



D. BRUGH

Holger Müller used laser-trap technology to test one of Einstein's predictions from general relativity.

experiencing a tiny weakening of gravity and speed-up of time. In the other, the atom stuck to the lower path, where gravity was stronger and time moved slightly more slowly. A difference in phase in the atom's fundamental frequency, measured by the interferometer, indicated a tiny difference in time.

### Laser traps

The experiment takes advantage of the laser atom trap, for which Chu won a Nobel prize in 1997. The data for the current study were obtained shortly after that,

when Chu was using the set-up to measure a different constant, the acceleration of gravity (A. Peters, K. Y. Chuang and S. Chu *Nature* 400, 849–852, 1999).

But Müller says that in October 2008, he had an epiphany that the same data could be used to show the constancy of gravity's effect on time. He e-mailed Chu, then the director of the Lawrence Berkeley National Laboratory in Berkeley, California, who responded three days later saying it was a good idea.

Chu says in an e-mail that he found time to work on the current study during nights, weekends and on planes — after putting in 70–80-hour weeks as energy secretary. "I like juggling a lot of balls," he says.

The result could one day have practical applications. If gravity's time-shifting effect were not constant, then researchers might have

had to worry about the accuracy of new atomic clocks as they are flown into orbit on Global Positioning System (GPS) satellites. But Müller has demonstrated the effect to be extraordinarily consistent. "Now we know that the physics is fine," he says.

The test also puts pressure on the Atomic Clock Ensemble in Space (ACES), an experiment being run by the European Space Agency that is due to be attached to the International Space Station in 2013. The current study already betters ACES's planned measurement of gravity's time-shifting effect by almost three orders of magnitude. ACES's principal investigator Christophe Salomon says that the mission will cost about €100 million (US\$136 million), plus the cost of a launch rocket. By comparison, Müller says that his tabletop apparatus cost much less than \$1 million. Salomon says that ACES is still justified because it will perform two other fundamental physics tests, as well as help researchers to improve the coordination of ground-based atomic clocks.

Physicist Clifford Will of Washington University in St. Louis, Missouri, says that Müller's result narrows the window for the alternative theories of gravity that some theorists are exploring. Will was also impressed that Chu found time to contribute to the study. "When was the last time that a sitting member of the president's cabinet had a paper in *Nature* on fundamental physics?" he asks.

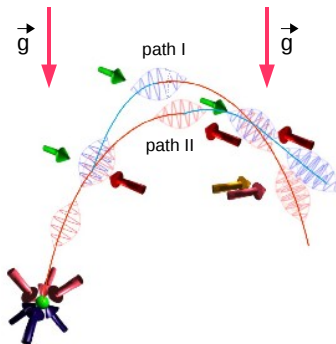
Eric Hand

- "A team led by Holger Müller of the University of California, Berkeley, has measured the time-shifting effects of gravity 10,000 times more accurately than ever before"
- "The test puts pressure on the Atomic Ensemble in Space, an experiment being run by the European Space Agency"
- "When was the last time that a sitting member of the president's cabinet had a paper in *Nature* on fundamental physics?" [Clifford Will]

# A raging controversy

- Holger Müller, Achim Peters & Steven Chu, *A precision measurement of the gravitational redshift by the interference of matter waves*, Nature **463**, 926 (2010)
- Holger Müller, Achim Peters & Steven Chu, *Atom gravimeters and gravitational redshift*, Nature **467**, E2 (2010)
- Mike Hohensee, Achim Peters, Steven Chu, Holger Müller *et al*, *Gravitational redshift, equivalence principle, and matter waves*, J. Phys. Conf. Ser. **264**, 012009 (2011)
- Mike Hohensee, Stephen Chu, Achim Peters & Holger Müller, *Equivalence principle and gravitational redshift*, Physical Review Letters **106**, 151102 (2011)
- Peter Wolf, Luc Blanchet, Christian Bordé, Serge Reynaud, Christophe Salomon & Claude Cohen-Tannoudji, *Atom gravimeters and gravitational redshift*, Nature **467**, E1 (2010)
- Peter Wolf, Luc Blanchet, Christian Bordé, Serge Reynaud, Christophe Salomon & Claude Cohen-Tannoudji, *Does an atom interferometer test the gravitational redshift at the Compton frequency?*, Classical and Quantum Gravity **28**, 145017 (2011)
- Supurna Sinha & Joseph Samuel, *Atom interferometers and the gravitational redshift*, arXiv:1102.2587 (2011)
- Domenico Giulini, *Equivalence principle, quantum mechanics, and atom-interferometric tests*, arXiv:1105.0749 (2011)

# Basic idea of this proposal



The atoms are viewed as “**clocks**” ticking at the de Broglie-Compton frequency

$$\omega_C = \frac{mc^2}{\hbar}$$

The “**atom-clocks**” propagate in the two paths I and II of the interferometer at different elevations in the gravitational field  $\vec{g}$ . They experience a measurable phase shift due to the gravitational redshift

# An analogy with clock experiments

- Two identical clocks are synchronized and moved at different elevations in a gravitational field. The total phase difference when the two clocks are brought back together is

$$\Delta\varphi_{\text{clock}} = \omega \left[ \int_{\text{I}} d\tau - \int_{\text{II}} d\tau \right] \equiv \omega \oint d\tau$$

where  $d\tau$  is the proper time and  $\omega$  the proper frequency

- The phase shift in an atom interferometer contains a contribution similar to the clock phase shift,

$$\Delta\varphi = \boxed{\omega_{\text{C}} \oint d\tau} + \Delta\varphi_{\text{el}}$$

with the role of the clock's proper frequency played by the atom's Compton frequency

# However the analogy looks suspect

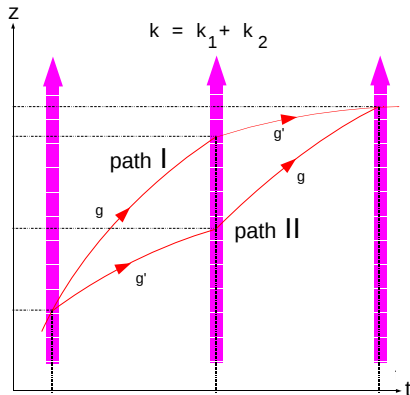
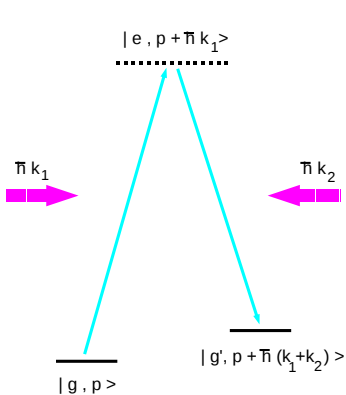
## Some differences with clock experiments

- 1 The phase shift in atom interferometry contains also a term  $\Delta\varphi_e$  coming from the **interaction with the lasers**
- 2 While the phase shift of clocks is valid in any gravitational field, in an atom interferometer it is known only for a **quadratic Lagrangian** and cannot be applied when there are important gravity gradients
- 3 In clock experiments the trajectories of the clocks are **continuously measured** (e.g. by exchange of electromagnetic signals) while in atom interferometry the trajectories of the atoms are not measured independently
- 4 In atom interferometry it is probably impossible to determine independently the trajectories of the wave packet without **destroying the interference pattern**
- 5 By contrast to ordinary clocks, the “**atom-clock**” is not a real clock since it does not deliver a physical signal at the Compton frequency



# Beam-slitting process in atom interferometry

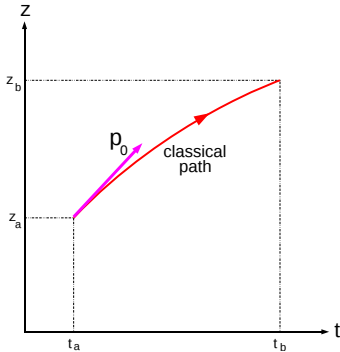
- The beam-splitter is realized through the interaction of atoms with laser beams resonant with an hyperfine atomic transition
- The atoms undergo a Raman transition  $g \rightarrow g'$ , resulting in a recoil velocity  $\hbar k$  where  $k = k_1 + k_2$  is the effective wave vector transferred to the atoms by the Raman lasers



# Free evolution of quantum matter wave

The calculation can be performed using Feynman's path integral formalism in the case of a Lagrangian quadratic in  $z$  and  $\dot{z}$  [Storey & Cohen-Tannoudji 1999]

$$L[z, \dot{z}] = a(t) \dot{z}^2 + b(t) \dot{z}z + c(t) z^2 + d(t) \dot{z} + e(t) z + f(t)$$



- Quantum propagator

$$K(z_b, t_b; z_a, t_a) = F(t_b, t_a) e^{\frac{i}{\hbar} S_{\text{cl}}(z_b, t_b; z_a, t_a)}$$

- Classical action

$$S_{\text{cl}}(z_b, t_b; z_a, t_a) = \int_{t_a}^{t_b} dt L[z_{\text{cl}}(t), \dot{z}_{\text{cl}}(t)]$$

- Final wavefunction

$$\psi(z_b, t_b) = G(t_b, t_a) \psi(z_a, t_a) e^{\frac{i}{\hbar} S_{\text{cl}}(z_b, t_b; z_a, t_a)}$$

(assuming that the initial state is a plane wave)

# Phase shift due to free propagation of atoms

Theorem [Bordé 1989, Kasevitch & Chu 1991, Storey & Cohen-Tannoudji 1999, Wolf & Tournenc 1999]

For any **quadratic** Lagrangian the phase difference due to the free propagation of atoms in the interferometer is zero,

$$\Delta\varphi_S = \frac{\Delta S_{\text{cl}}}{\hbar} = \frac{1}{\hbar} \oint L[z_{\text{cl}}(t), \dot{z}_{\text{cl}}(t)] dt = 0$$

- The proper time on the two paths I and II is the same
- The Compton frequency of the atom plays no role

# Total phase shift in the atom interferometer

$$\Delta\varphi = \Delta\varphi_\ell + \Delta\varphi_{gg'}$$

- 1 The main contribution comes from the interactions with lasers

$$\Delta\varphi_\ell = -\phi(z_a, 0) + \phi(z_b, T) + \phi(z_c, T) - \phi(z_d, T + T')$$

where  $\phi$  is the phase of the laser light as seen by the atom

- 2 The energy difference between the states  $g$  and  $g'$  must be taken into account

$$\Delta\varphi_{gg'} = \underbrace{\omega_{gg'}(T - T')}_{\text{clock term}}$$

- 3 For special configurations atom interferometers can be viewed as clocks beating at the frequency  $\omega_{gg'}$  [Bordé 2001, 2002]

# Prediction from general relativity

The Lagrangian (at Newtonian order) is

$$L_{\text{GR}}(z, \dot{z}) = -mc^2 \frac{d\tau}{dt} = -mc^2 + \frac{GMm}{r_{\oplus}} - mgz + \frac{1}{2}m\dot{z}^2 + \mathcal{O}\left(\frac{1}{c^2}\right)$$

$$\Delta\varphi = k g T^2$$

where  $g$  is the gravitational field,  $k$  is the effective wave vector of the lasers and  $T$  is the time interval between pulses

- The atom interferometer is a **gravimeter**. It measures the local acceleration of gravity  $g$ . The phase shift arises from the interactions with the lasers and the fact that the atoms are falling with respect to the experimental platform
- There is no dependence on the Compton frequency of the atom

# Modified Lagrangian formalism [Nordtvedt 1975, Haugan 1979, Will 1993]

We postulate that the mass of the atom depends on the position  $z$  through a violation of LPI. The Lagrangian is

$$L_{\text{modified}} = -m(z) c^2 + \frac{GMm(z)}{r_{\oplus}} - m(z) g z + \frac{1}{2} m(z) \dot{z}^2$$

This is modelled by assuming that a particular energy in the atom  $E_X$  behaves anomalously and we pose

$$E_X(z) = \overline{E}_X + \beta_X^{(a)} \overline{m} g z$$

where  $\overline{m}$  is the sum of rest masses of particles constituting the atom, and  $\beta_X^{(a)}$  is a LPI-violating coefficient depending on the type of energy  $X$  and the type of atom ( $a$ ). Denoting  $m_0$  the “normal” contribution to the mass of the atom the Lagrangian becomes

$$L_{\text{modified}} = -m_0 c^2 + \frac{GMm_0}{r_{\oplus}} - (1 + \beta_X^{(a)}) m_0 g z + \frac{1}{2} m_0 \dot{z}^2$$

# Testing the Einstein equivalence principle

- Violation of WEP or the universality of free fall (UFF)

$$\ddot{z} = -(1 + \beta_X^{(a)}) g$$

The trajectory of the atom is affected by the violation of LPI and the UFF-violating parameter is  $\beta_X^{(a)}$ .

- Violation of the redshift or universality of clock rates (UCR)

$$\frac{\Delta\nu}{\nu} = (1 + \alpha_X^{(a)}) \frac{g \Delta z}{c^2}$$

The UCR-violating parameter  $\alpha_X^{(a)}$  is given by

$$\alpha_X^{(a)} = \beta_X^{(a)} \left( \frac{\overline{E}_X}{\overline{m} c^2} \right)^{-1}$$

Tests of the redshift (or UCR) and UFF are not independent [Schiff 1960]. Since for typical energies involved we shall have  $\overline{E}_X \ll \overline{m} c^2$  this means that

$$\beta_X^{(a)} \ll \alpha_X^{(a)}$$

# Analysis of atom interferometry

The “atom-clock” that accumulates a phase is of identical composition to the falling object (the same atom). One has to consistently use the same value of  $\beta_X^{(a)}$  when calculating the modified trajectories and the phase difference, so

$$\Delta\varphi_S = 0$$

and the total phase shift is given by the light interactions only,

$$\Delta\varphi = \Delta\varphi_\ell = (1 + \beta_X^{(a)})k g T^2$$

- The fact that  $\beta_X^{(a)}$  appears in the final phase shift is entirely related to the light phase shift coming from the modified trajectories of the atoms
- The experiment measures the differential free fall acceleration of two test masses of different internal composition, with precision

$$\left| \beta_X^{(\text{Cs})} - \beta_X^{(\text{corner cube})} \right| \lesssim 7 \times 10^{-9}$$



# Atom interferometers test WEP i.e. universality of free fall

- Atom interferometers can be used for a test of the UFF between the atoms and macroscopic objects with current precision  $7 \times 10^{-9}$
- This is the most sensitive test comparing the free fall of **quantum** objects with classical test masses (a corner cube in practice)

This conclusion applies to the modified Lagrangian formalism which contains most alternative frameworks for analyzing violations of the EEP

- 1 All metric theories
- 2 Most non-metric theories
- 3 Models motivated by string theory [Damour & Polyakov 1994]
- 4 The  $TH_{\epsilon\mu}$  parametrized formalism and variants [Lightman & Lee 1973, Blanchet 1992]
- 5 Energy conservation frameworks [Nordtvedt 1975, Haugan 1979]
- 6 The standard model extension (SME) [Kostelecky 2006, Kostelecky & Tasson 2010]

# Multi Lagrangian formalism

We look for a violation of the LPI while supposing that the LLI are WEP are valid

- 1 With WEP we can consider local freely falling frames associated with test bodies falling with universal acceleration  $g$
- 2 With LLI clocks will measure in these frames a proper time  $d\tau$  proportional to the Minkowskian interval  $ds$
- 3 To violate LPI we allow for a proportionality factor  $f(\Phi)$  built from some anomalous field  $\Phi$  associated with gravity and depending on position
- 4 In an arbitrary frame this means that clocks measure

$$d\tau = f(\Phi) ds = f(\Phi) \sqrt{-g_{\mu\nu} dx^\mu dx^\nu / c^2}$$

# Testing the gravitational redshift

- ① In redshift experiments we obtain

$$\frac{\Delta\nu}{\nu} = (1 + \beta) \frac{g\Delta z}{c^2}$$

where the redshift-violating parameter  $\beta$  is given by  $\beta = c^2\zeta^{-1}f'_0/f_0$  where  $\zeta$  is such that  $g = \zeta d\Phi/dz$  [Will 1993]

- ② In atom interferometry experiments the phase difference accumulated by the “atom-clock” is no longer zero

$$\Delta\varphi_S = \omega_C \oint d\tau = \omega_C \oint \left[ 1 + \beta \frac{g\Delta z}{c^2} \right] ds = \beta k g T^2$$

- ③ The total phase shift is

$$\Delta\varphi = (1 + \beta) k g T^2$$

showing that the atom interferometer does test the redshift in that case

# Violation of Schiff's conjecture

## Schiff's conjecture [Schiff 1960]

Any complete and self-consistent theory of gravity that embodies WEP necessarily satisfies the full EEP, including LLI and LPI

The conjecture has been proved

- Within general formalisms such as the  $TH\varepsilon\mu$  formalism [Lightman & Lee 1973]
- Using general arguments based on energy conservation [Dicke 1964, Nordtvedt 1975, Haugan 1979]

Here the conjecture is violated because **we are assuming two Lagrangians for describing the same physical object**, and shall face severe problems associated with energy conservation

# Violation of Quantum Mechanics

The trajectories of massive bodies (classical particles or classical paths of wave packets) obey a Lagrangian which is different from the Lagrangian used to compute the proper time of clocks or the phase shift of matter waves

$$\begin{aligned} L_{\text{particle}} &= -m c^2 + \frac{GMm}{r_{\oplus}} - m g z + \frac{1}{2} m \dot{z}^2 \\ L_{\text{matter wave}} &= \left(1 + \beta \frac{g z}{c^2}\right) L_{\text{particle}} = -m c^2 + \frac{GMm}{r_{\oplus}} - (1 + \beta) m g z + \frac{1}{2} m \dot{z}^2 \end{aligned}$$

The formalism violates

- 1 The principle of least action for matter waves
- 2 The Feynman path integral formulation of Quantum Mechanics
- 3 More generally the particle-wave duality of Quantum Mechanics

Even worse, because the Feynman path integral formalism is violated, one is not allowed to use it for the derivation of phase shifts in an atom interferometer

Does atom interferometry test the gravitational redshift at Compton frequency?

- ① General relativity: **No**
  - No redshift effect
  - No dependence on the atom's Compton frequency
- ② Modified Lagrangian formalism: **No**
  - No redshift effect
  - No dependence on the atom's Compton frequency
  - Atom interferometers test the UFF but not the redshift
- ③ Multi Lagrangian formalism: **Yes but at the price of violating**
  - The Schiff conjecture
  - The principle of least action for matter waves
  - The Feynman path integral formulation of Quantum Mechanics
  - The particle-wave duality of Quantum Mechanics