Max-Planck-Institut
für Gravitationsphysik
(Albert-Einstein-Institut)

Constructing black holes and black hole microstates

String theory and the fuzzball proposal

Clément Ruef, AEI

LAPTH, Annecy-Le-Vieux, March 8th 2011

Work done with I. Bena, N. Bobev, S. Giusto, N. Warner, G. Dall'Agata.
Work in Progress with G. Bossard

Many different groups


Interesting reviews

- *The fuzzball proposal for black holes : An elementary review* , Mathur, hep-th/0502050,
- *Black holes, black rings and their microstates*, Bena and Warner, hep-th/0701216,
- *The fuzzball proposal for black holes*, Skenderis and Taylor, 0804.0552,
- *Black Holes as Effective Geometries*, Balasubramanian, de Boer, El-Showk and Messamah, 0811.0263.

Motivation

Motivation : Quantum gravity

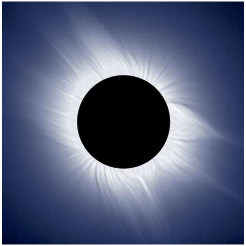
But the developed tools are quite general :

- Generation of gravity solutions
- Application to other string theoretical systems :
  Flux compactifications and Klebanov-Strassler type systems
- Possible applications to cosmology

Black hole issues

Fundamental black hole problems :

- Central singularity
- Microscopic understanding of the BH entropy
- Information paradox



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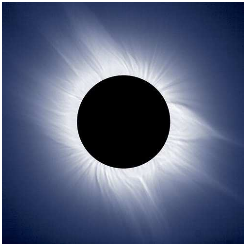

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What is a black hole ?

Black-hole entropy

Classically, a black hole has a macroscopic entropy :

$$S = \frac{A}{4G_N}$$

Uniqueness theorem $\longrightarrow$ only one single state !

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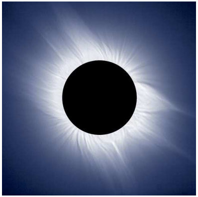
Statistically : e^S states.

Ex : $M = M_{\text{center galaxy}} \longrightarrow N = e^{10^{90}}$



Huge discrepancy !

Questions

- Where are the BH microstates ?
- What are the BH microstates ?
- How do the BH microstates behave ?
- What is the correct framework to understand the BH microstates ?

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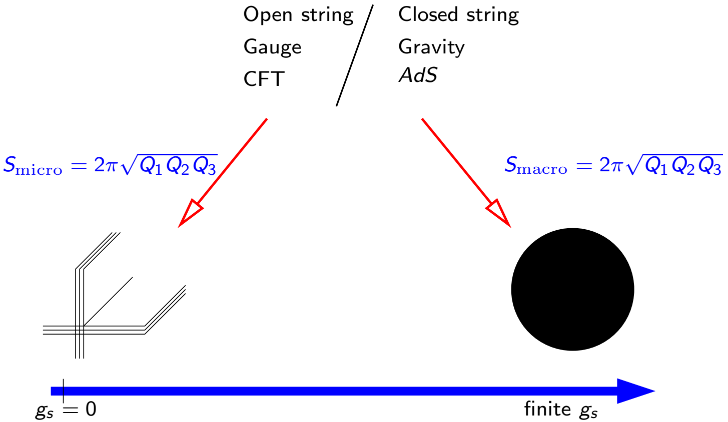
We need a theory of quantum gravity !

Strominger-Vafa counting

String theory provides partial answers :



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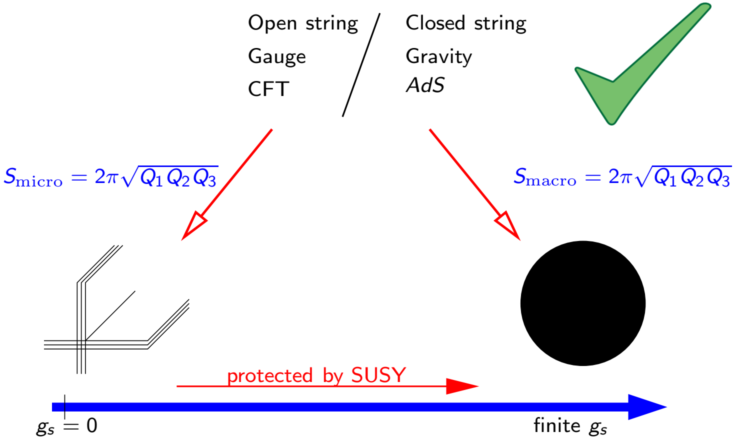


Remaining questions

$S_{\rm micro} = 2\pi\sqrt{Q_1 Q_2 Q_3}$ $\qquad$ $S_{\rm macro} = 2\pi\sqrt{Q_1 Q_2 Q_3}$

- How do the "microstates" transform while turning on g_s ?
- What about the singularity resolution and the information paradox ?

Thermodynamics

Two descriptions

A macroscopic one, continuous, in terms of thermodynamics and fluid mechanics. Pertinent for long scale effects.

The microscopic one, quantized, in terms of statistical/quantum mechanics. Pertinent for small scale effects.

Macroscopic state = statistical average of microscopic states

Black hole thermodynamics

Two descriptions ?

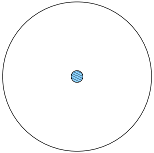
A macroscopic one, continuous, in terms of BH thermodynamics. Pertinent for long scale effects, like gravitational scattering, gravitational lensing...

General features

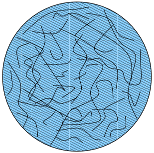
- Macroscopic state = statistical average of microscopic states
- Same long range behaviour as the BH $\longrightarrow$ same mass and charges
- Have to grow with g_s, as the BH. Non trivial statement !
- Horizon = Entropy $\longrightarrow$ no horizon

Modification at the horizon scale !

Key idea



QG effects : $l \sim l_P$

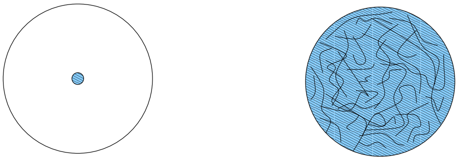


QG effects : $l \sim N^{\alpha} l_P \sim r_S$

$$l_P = \sqrt{\frac{\hbar G}{c^3}} \sim 10^{-35} m$$

Fuzzball proposal : Quantum gravity effects extend until the horizon size Mathur

The fuzzball proposal



BH microstates = a horizonless configuration with the same asymptotics as the BH

- Very fuzzy ? Fully stringy or only geometric ?
- Can the geometric solutions sample the space of microstates ?

Back to black hole issues

The fuzzball proposal could solve all the BH issues

- Central singularity resolved
- Microscopic understanding of the BH entropy

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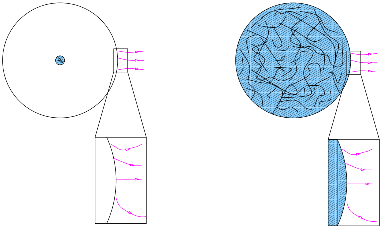
The fuzzball proposal could solve all the BH issues

- Central singularity resolved
- Microscopic understanding of the BH entropy
- Hypothesis leading to the information paradox do not hold anymore



Two charge story

A very large body of work for two-charge black holes

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Framework

- String theory : theory of quantum gravity (10D)
- Low energy limit : supergravity (10D or 11D)

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- String theory : theory of quantum gravity (10D)
- Low energy limit : supergravity (10D or 11D)
- We will physically describe 4D or 5D black holes. Other dimensions compactified
- Keep in mind stringy nature of the objects and interactions

I will switch between 11D/IIA/4D/5D supergravities.

11D Supergravity

The supergravity action :

$$2\kappa_{11}^2\, S = \int d^{11}x\sqrt{-G}\left(R - \frac{1}{2}|F^{(4)}|^2\right) - \frac{1}{6}\int A^{(3)} \wedge F^{(4)} \wedge F^{(4)}\,.$$

Field content :

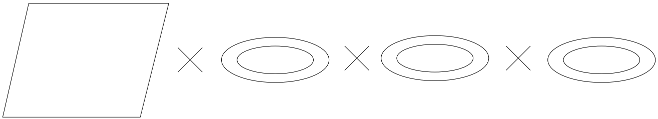
- $g_{\mu\nu}$ $\leftrightarrow$ spacetime
- $A^{(3)}$ $\leftrightarrow$ M2 and M5 branes

In 4D/5D, gravity coupled to Maxwell and scalar fields.

M Branes

From a 11D point of view, the charges come from M branes wrapping cycles along the compact T^6 :

$$\mathcal{M}_{11\mathrm{D}} = \mathbb{R}^{4,1} \times T^6 = \mathbb{R}^{4,1} \times T^2 \times T^2 \times T^2$$



M2 branes $\leftrightarrow$ electric charges   M5 branes $\leftrightarrow$ magnetic charges

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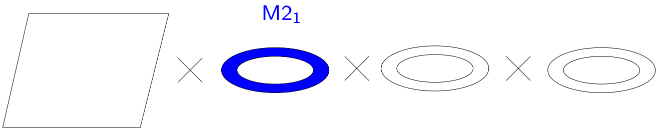

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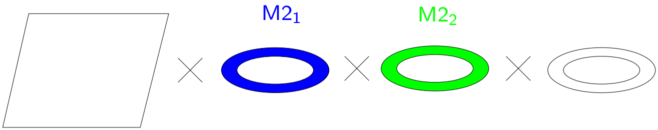

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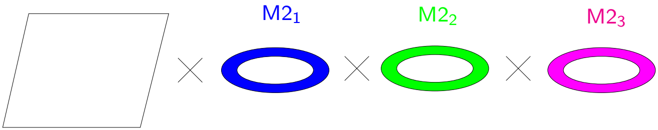

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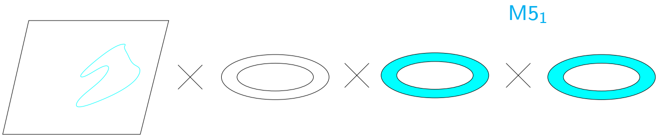

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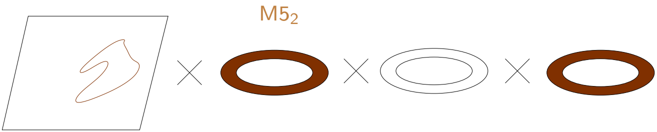

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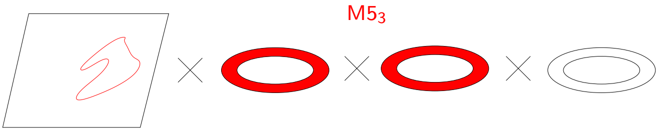

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Metric Ansatz

We want to describe five-dimensional solutions :

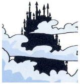
$$ds^2 = -Z^{-2}(dt+k)^2 + Z\, ds_4^2 + \sum_{I=1}^{3} X_I (dy_{I1}^2 + dy_{I2}^2)\,,$$

$$A^{(3)} = \sum_{I=1}^{3} \left(-Z_I^{-1}(dt+k) + B^{(I)}\right) \wedge dy_{I1} \wedge dy_{I2}$$

with $Z = (Z_1 Z_2 Z_3)^{1/3}$ and $X_I = Z/Z_I$.

This can describe either *black holes, black rings or regular, BPS or non-BPS* , solutions.

Fields and content

ds_2^4	base space	
$B^{(I)}$	magnetic charges	
Z_I	electric charges	
k	angular momentum	

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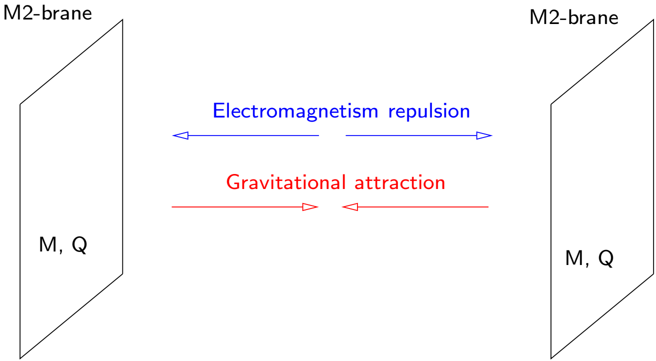
"Floating brane" Ansatz

Floating brane Ansatz



No global force, the branes are **mutually BPS**.

If SUSY, ansatz imposed

BPS equations

Supersymmetry reduces Einstein equations to a **first order system**.

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A four step procedure :

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Linear system of equations ! Bena, Warner

Gibbons-Hawking metrics

Assuming a triholomorphic $U(1)$ isometry, an hyperkähler space is Gibbons-Hawking :

$$ds_4^2 = V^{-1}(d\psi + A)^2 + V ds_3^2$$
$$V \text{ harmonic}, \quad dV = *_3 dA.$$

Ex : $V = \frac{1}{r}$, flat $\mathbb{R}^4$

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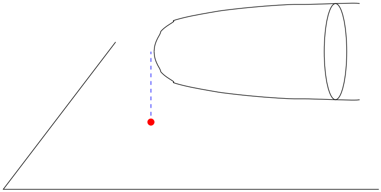
Ex : $V = 1 + \frac{1}{r}$, Taub-NUT space, interpolates between $\mathbb{R}^4$ and $\mathbb{R}^3 \times S^1$



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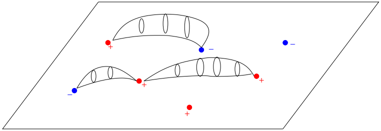
Ex : $V = 1 + \sum_i \frac{q_i}{|\vec{r} - \vec{r}_i|}$, Multi Taub-NUT space



BPS Solutions

Assuming this Ansatz, all BPS solutions have been found. They are given by 8 harmonic functions : Gauntlett, Gutowski, Hull, Pakis, Reall

- $V \leftrightarrow$ 
- $K_I \leftrightarrow$ 
- $L_I \leftrightarrow$ 
- $M \leftrightarrow$ 

BPS Solutions

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- black holes $S = 2\pi\sqrt{Q_1 Q_2 Q_3}$
- black rings, horizon $S^2 \times S^1$
- multicentered black holes
- smooth, regular solutions

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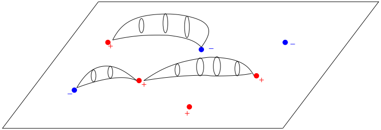
Smooth solutions

How to build smooth solutions ?

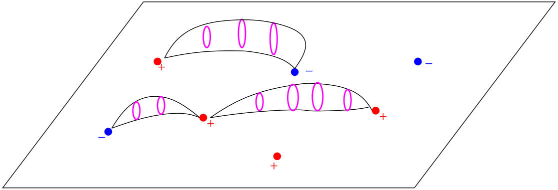
Start from a multi-centered Taub-NUT space

$$ds_4^2 = V^{-1}(d\psi + A)^2 + V ds_3^2$$
$$V = 1 + \sum_i \frac{q_i}{|\vec{r} - \vec{r}_i|}$$



The S^1 fiber shrinks at the each GH point $\longrightarrow$ **bubbles**

Smooth solutions

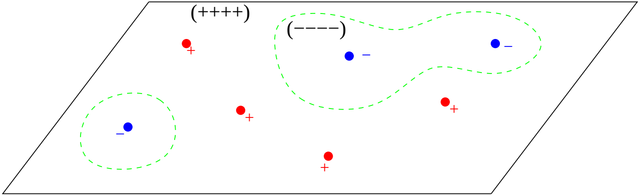


- Bubbles stabilized by magnetic fluxes
- No localized sources, no singularity
- **bf Integrability, or bubble equation** Denef ; Bena, Warner :

$$\sum_j \frac{< \Gamma_i, \Gamma_j >}{r_{ij}} = < \Gamma_i, h >$$

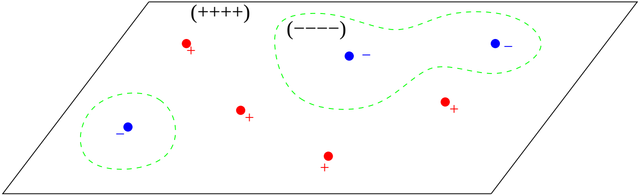
- **Fluxes create the charges seen at infinity**

Smooth solutions and ambipolar spaces



Need to start from an 4D ambipolar base. Signature switches from $(+,+,+,+)$ to $(-,-,-,-)$ $\longrightarrow$ seems to be highly singular !

Smooth solutions and ambipolar spaces



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Complete 11D (5D) solutions completely regular Giusto, Mathur, Saxena

Giving up the $U(1)$ isometry

One can count the entropy coming from the microstates $\longrightarrow$ **not enough**

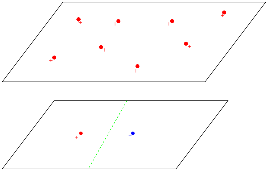
It was expected :
$U(1)$-isometry : cuts all the modes along the fiber !

Two-charge case : entropy comes from these modes

Giving up the $U(1)$ isometry

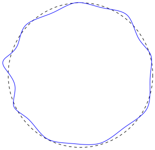
Look at a wiggling supertube dual to a smooth GH center
Problem : we need the Green function on an ambipolar GH space

- known for $(+, +, ..., +)$ centers Page
- can be found for ambipolar two centers $(+, -)$ from $AdS_3 \times S^2$
- Very hard problem in general



New solutions with a function $f(\theta)$ as parameter $\longrightarrow$ **Infinite dimensional moduli space** Bena, Bobev, Giusto, CR, Warner

Entropy enhancement mechanism



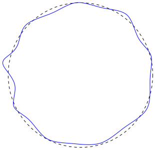
- Entropy of the supertube in flat space

$$S \sim \sqrt{Q_1 Q_2}$$

- Entropy of the supertube in dipole-charged background

$$S \sim \sqrt{Q_{1\mathrm{eff}} Q_{2\mathrm{eff}}}$$

Entropy enhancement mechanism



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Much more entropy than naively expected !

Bena, Bobev, CR, Warner

Other approaches to the fuzzball proposal

One can make use of the AdS/CFT correspondence in the context of the fuzzball proposal

- Identification of the microstates on the CFT side Skenderis, Taylor
- Computation of perturbative corrections of fuzzballs to the flat metric from a pure worldsheet point of view Giusto, Morales, Russo
- Precision counting on both sides of the correspondence, using indices and partition functions Sen

Gravity	Gauge
macroscopic	microscopic

All approaches, despite being very different, seem to confirm the conjecture.

Non-BPS black holes

What can we do without supersymmetry ?

Recent years : a lot of progress for extremal non-BPS black holes, through different approaches

- Fake superpotential and first order formalism Ceresole, Dall'Agata et al ; Andrianopoli, D'Auria, Trigiante et al ; Gimon, Larsen, Simon ; Perz, Galli, Jansen, Smyth, Van Riet, Vercnocke ;...
- Almost BPS equations Goldstein, Katmadas ; Bena, Giusto, CR, Warner
- Integrability conditions Andrianopoli, D'Auria, Orazi, Trigiante et al
- Reduction to three dimensions Clement, Galt'sov, Scherbluk et al ; Bossard et al ; Virmani et al ; Chemissany, Rosseel, Trigiante, Van Riet et al ; ...

Non-BPS black holes

What can we do without supersymmetry ?

Recent years : a lot of progress for extremal non-BPS black holes, through different approaches

- Fake superpotential and first order formalism Ceresole, Dall'Agata et al ; Andrianopoli, D'Auria, Trigiante et al ; Gimon, Larsen, Simon ; Perz, Galli, Jansen, Smyth, Van Riet, Vercnocke ;...
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Supersymmetry $\longrightarrow$ extremality

Almost BPS solutions

Fondamental idea : **SUSY broken by the relative orientation of the branes**

Solve almost the same system of equations

BPS system

$$\begin{aligned} dV &= *_3 dA \\ \Theta^{(I)} &= *_4 \Theta^{(I)} \\ \nabla^2 Z_I &= \frac{C_{IJK}}{2} *_4 [\Theta^{(J)} \wedge \Theta^{(K)}] \\ dk + *_4 dk &= Z_I \Theta^{(I)} \end{aligned}$$

Ex : BPS 4-charge black hole D6-D2-D2-D2

Almost BPS solutions

Fondamental idea : **SUSY broken by the relative orientation of the branes**

Solve almost the same system of equations

non-BPS system Goldstein, Katmadas

$$
\begin{aligned}
dV &= - *_3 dA \\
\Theta^{(I)} &= *_4 \Theta^{(I)} \\
\nabla^2 Z_I &= \frac{C_{IJK}}{2} *_4 [\Theta^{(J)} \wedge \Theta^{(K)}] \\
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\end{aligned}
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Ex : non-BPS 4-charge black hole $\overline{\mathrm{D6}}$-D2-D2-D2

Almost BPS solutions

Tools developed in the SUSY context can be used

Large class of new solutions : Bena, Dall'Agata, Giusto, CR, Warner

- **Black holes**
- **Black rings**
- **Multicentered black holes**
- **No microstates**

One recovers all solutions found with the fake superpotential approach, by solving linear systems.

Further generalization of the system of equations, and the solutions.

Floating brane vs extremality

First assumption : Floating brane ansatz $\sim$ extremality

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- Dualities : map solutions to solutions.
  BPS case : solution space is closed, all in (the closure of) the floating brane ansatz
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  Ex : New non-BPS doubly spinning black ring in Taub-NUT with dipole charges Dall'Agata, Giusto, CR ; Bena, Giusto, CR

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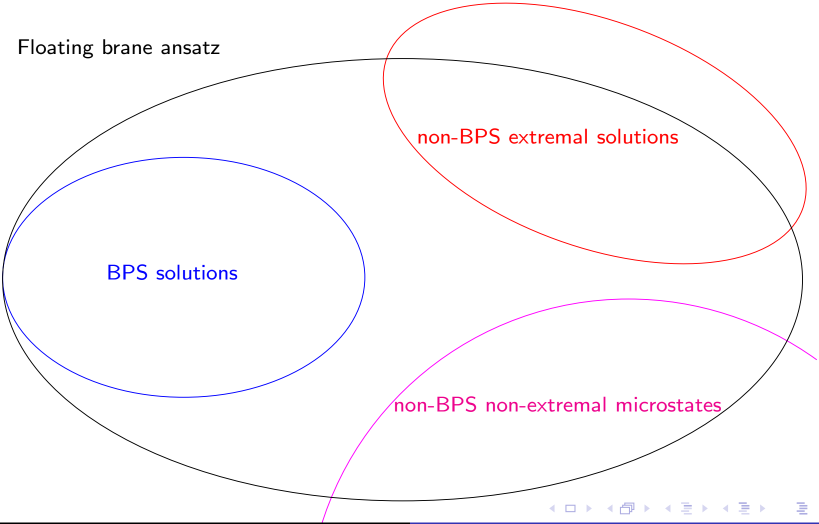
- Possible to obtain non-extremal microstates within the floating brane ansatz

Floating brane Ansatz



Linear systems

Key point : **the equations are solved in a linear way.**

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Underlying structure behind. **How can we make it explicit, and use it ?**

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Reduction to a three-dimensional problem

Three-dimensional approach

- In 3D, electric-magnetic duality $\longrightarrow$ gravity coupled to scalars

  Breitenlohner, Gibbons, Maison

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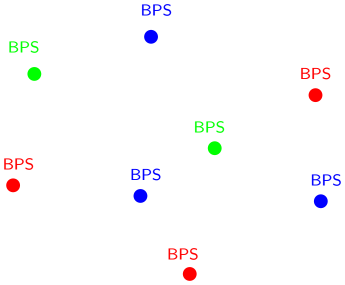
Extremal solutions $\longleftrightarrow$ Nilpotent orbits in $\mathcal{M}$
Graded system $\longleftrightarrow$ Lie algebra graded decomposition

Three-dimensional approach

Multicenter solutions

- Recover all BPS solutions

  Bossard, Nicolai, Stelle



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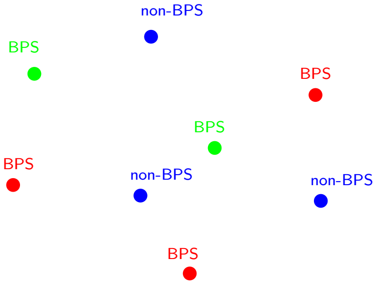
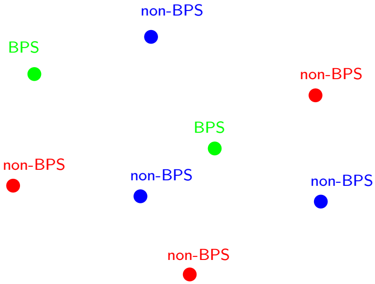


Three-dimensional approach

Multicenter solutions

- Recover all BPS solutions Bossard, Nicolai, Stelle
- Recover almost BPS solutions Perz, Galli ; Bossard ; Bossard, CR
- Find new solutions Bossard, CR



Floating brane and non-BPS microstates

Wider generalisation of the system of equations : Bena, Giusto, CR, Warner

$$
\begin{aligned}
R_{ab} &= 0 \\
\Theta^{(I)} &= *_4 \Theta^{(I)} \\
\nabla^2 Z_I &= \frac{C_{IJK}}{2} *_4 \left[\Theta^{(J)} \wedge \Theta^{(K)}\right] \\
dk + *_4 dk &= Z_I \Theta^{(I)}
\end{aligned}
$$

This system allows for microstates !

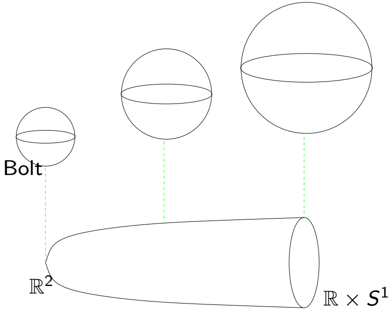
Bolt solutions

$R_{ab} = 0 \rightarrow$ Why not start with an Euclidean black hole ?

Bolt solutions

$R_{ab} = 0 \to$ Why not start with an Euclidean black hole ?

Lorentzian $\to$ Euclidean :
event horizon becomes a bolt, a non-trivial S^2
The space ends smoothly at $r = r_+$, and interpolates between $\mathbb{R}^2 \times S^2$ and $\mathbb{R}^3 \times S^1$



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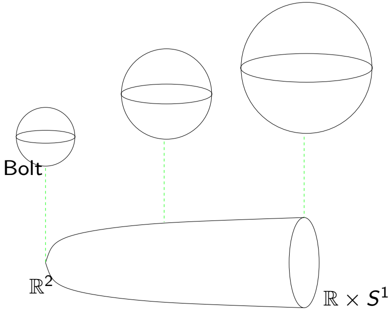
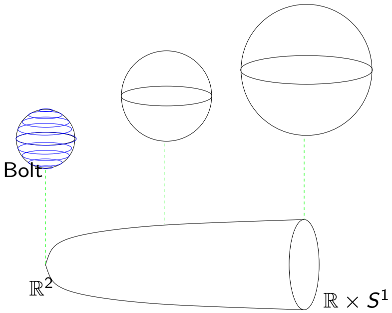
No singularity



Putting fluxes on the bolt

The bolt gives us an S^2 to put magnetic fluxes. As in the BPS case, this fluxes create the charges seen from infinity.
"Charges dissolved in fluxes"



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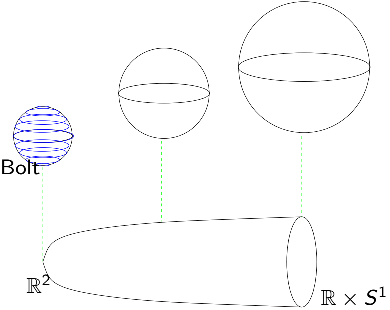
Regular solutions, no singularity, no horizon Bena, Giusto, CR, Warner; Bobev, CR



Have the same asymptotics as a **non-extremal** black hole

$$M = M_{sol} + \sum Q_I$$

Conclusion

- Black hole issues yet to be solved
- Fuzzball proposal :
  - physically intuitive motivated
  - rigorously defended, from various point of views
  - works in the two charge case
- Microstates built by putting magnetic fluxes on non-trivial two-cycles
- Entropy enhancement mechanism
- New non-BPS microstates

Perspectives

- Need to find more general solutions
- Extremal non-BPS microstates from 3D approach
- How much entropy can be obtained by the entropy enhancement mechanism ?
- Study of the (in-)stability of the non extremal microstates Mathur, Chowdhury
- Application to very early universe cosmology Mathur, Chowdhury

Thank you for your attention