The 4D physics of 5D GUTs

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DESY

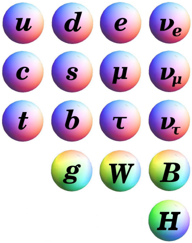
based on

arXiv:110x.xxxx with Sylvain Fichet, Sabine Kraml
arXiv:1007.0321 with Sylvain Fichet, Sabine Kraml, Ritesh Singh
arXiv:0906.2957 with Sylvain Fichet, Arthur Hebecker, Sabine Kraml

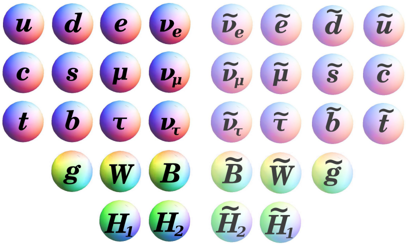
Outline

Introduction and motivation

The Standard Model



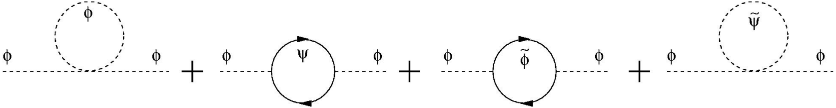
Beyond the Standard Model: Supersymmetry



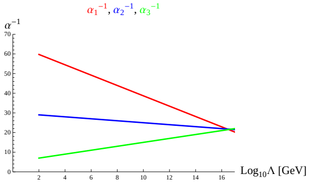
- MSSM = minimal SUSY extension of Standard Model
- each SM boson gets a fermionic superpartner
- each SM fermion gets a bosonic superpartner
- two Higgs doublets (+ “Higgsino” superpartners)

Why SUSY?

- solves the hierarchy problem if broken at $\lesssim$ TeV:
  Higgs mass corrections cancel between superpartners



- SUSY = unique non-trivial extension of Poincaré symmetry in 3+1 dimensions
- MSSM with R-parity contains dark matter candidate:
  LSP, "lightest supersymmetric particle"
- MSSM: gauge couplings unify at $\sim 10^{16}$ GeV



$\rightarrow$ pointing to Grand Unification of all three SM forces?

A flaw of SUSY: too many parameters

SUSY breaking parameterised by

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2}\left(M_3\,\tilde{g}\tilde{g} + M_2\,\widetilde{W}\widetilde{W} + M_1\,\widetilde{B}\widetilde{B} + \text{h.c.}\right)\\ &-\left(a_u^{IJ}\,\tilde{u}_I^c\tilde{q}_J H_2 - a_d^{IJ}\,\tilde{d}_I^c\tilde{q}_J H_1 - a_e^{IJ}\,\tilde{e}_I^c\tilde{\ell}_J H_1 + \text{h.c.}\right)\\ &- m^2_{q\,IJ}\,\widetilde{q}_I^*\widetilde{q}_J - m^2_{\ell\,IJ}\,\tilde{\ell}_I^*\tilde{\ell}_J - m^2_{u\,IJ}\,\tilde{u}_I^{c*}\tilde{u}_J^c - m^2_{d\,IJ}\,\tilde{d}_I^{c*}\tilde{d}_J^c - m^2_{e\,IJ}\,\tilde{e}_I^{c*}\tilde{e}_J^c\\ &- m^2_{H_1}\,|H_1|^2 - m^2_{H_2}|H_2|^2 - \left(m_3^2\,H_2H_1 + \text{h.c.}\right)\end{aligned}$$

In addition, gauge couplings + Yukawas + Higgsino mass μ: $\sim$ 120 free parameters for the minimal model alone

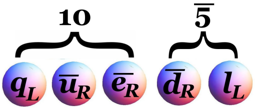
More fundamental framework needed to constrain parameters

Grand Unification

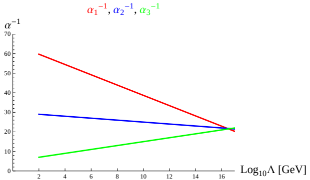
(MS)SM: gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$,
broken to $SU(3)_C \times U(1)_{EM}$ at $\langle H \rangle \approx 10^2$ GeV

Observations:

- (MS)SM matter in complete representations of SU(5) or SO(10)



- MSSM: gauge couplings unify at $M_{\rm GUT} \approx 10^{16}$ GeV



GUT Hypothesis: gauge group SU(5) (or larger),
broken to $SU(3)_C \times SU(2)_L \times U(1)_Y$ at $M_{\rm GUT}$

Grand Unification: Problems

In minimal models:

- (MS)SM Higgs fields not in SU(5) representations: doublet-triplet splitting problem
- Prediction: proton decay, no evidence yet
- Prediction: mass relations for (MS)SM fermions can be OK for 3rd generation, violated for light generations

Problems can be solved in non-minimal models

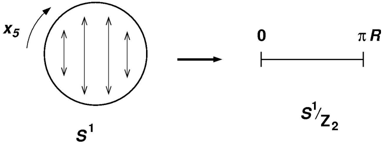
Particularly elegant solutions involve extra dimensions

Extra dimensions

Idea: 3 + 1 large dimensions, d small ones
"small" = compact, characteristic length $< (\text{TeV})^{-1} \approx 10^{-18}$ m or smaller

Simplest version: $d = 1$, circle S^1 → Kaluza, Klein 1920s

More realistic: interval $S^1/\mathbb{Z}_2$ ("orbifold")



- 5d "bulk" with 4d boundaries = "branes"
- $\mathbb{Z}_2$ projection allows for chiral fermions; only $\mathcal{N} = 1 SUSY$
- new mechanisms for symmetry breaking by boundary conditions
  convenient for orbifold GUTs: → Kawamura '00...
  choose radius $1/M_{\text{GUT}}$ ⇒ 4d effective field theory below 10^{16} GeV

Generalisation: Toroidal orbifolds $T^d/\mathbb{Z}_n$

5D SUSY GUTs

Compactification provides natural mechanisms for solving these problems

D.o.f. of 4D effective theory = bulk zero modes and brane-localised fields

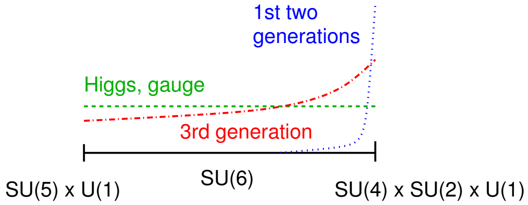
- Can use orbifold projection to project out part of gauge boson zero modes
  ($\Leftrightarrow$ assign specific boundary conditions to bulk fields)
  $\Rightarrow$ gauge symmetry partially broken below compactification scale
- Similar: Can project out e.g. colour triplet component of Higgs zero mode
  $\Rightarrow$ natural doublet-triplet splitting
- Or can put Higgs on a brane where GUT group is broken
  $\Rightarrow$ natural doublet-triplet splitting
- Operators from such branes
  $\Rightarrow$ small violation of GUT fermion mass relations
- Operators leading to fast proton decay: can be forbidden

Models

An example

→ Burdman/Nomura '03

4D gauge group = SU(3)×SU(2)×U(1)[×U(1)]



5d gauge supermultiplet → 4d gauge supermultiplet $-\theta\bar{\theta}A_\mu\sigma^\mu + \ldots$

$\oplus$ 4d chiral adjoint $\Phi = \Sigma + iA_5 + \ldots$

- Φ zero modes $\supset$ MSSM Higgs doublets: Gauge-Higgs unification at M_{GUT}
- MSSM matter from bulk hypermultiplets. Must assign boundary conditions and introduce extra brane fields to decouple exotics.
- bulk mass terms ⇒ zero modes distorted ⇒ Yukawa hierarchies

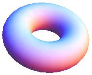
5D GUTs from heterotic string theory

→ Dixon et al. ’85-’86, Hamidi/Vafa ’86, Narain et al. ’86, Ibáñez/Nilles/Quevedo ’87,. . .

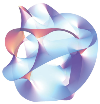
UV completion: orbifold compactifications of $E_8 \times E_8$ heterotic string theory

Strings live in 10 dimensions → 6 compact (+ small)

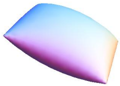
- torus compactification on T^6: 4D theory calculable, but too much SUSY



- To get $\mathcal{N} = 1$ SUSY in 4D must compactify on “Calabi-Yau” space: complicated, 4D theory unknown



- Calculable, singular limits of Calabi-Yau spaces: Toroidal orbifolds $T^6/\mathbb{Z}_N$
  Effective field theory has all essential features of field theoretic orbifolds



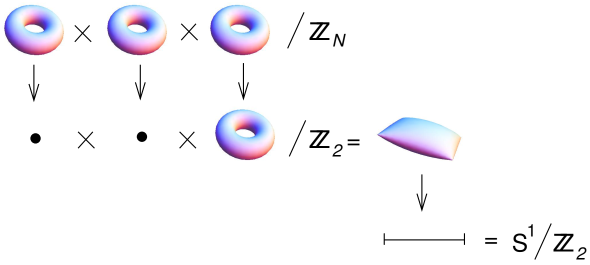
Recent realistic examples (4D theory = MSSM):

→ Buchmüller/Hamaguchi/Lebedev/Ratz ’05/’06, Lebedev et al. ’06-’08

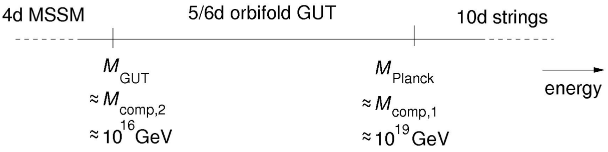
5D GUTs from heterotic string theory

Anisotropic heterotic orbifold compactification:

Compactify 5 radii at $\sim 1/M_{\text{Planck}}$, one at $\sim 1/M_{\text{GUT}}$



$\Rightarrow$ effective 5D model between M_{GUT} and M_{Planck}:



5D GUTs from holography

AdS/CFT: Duality between type IIB strings on $AdS_5 \times S^5$ and 4D $\mathcal{N} = 4$ SYM

$\Downarrow$ (much handwaving)

"duality" between weakly coupled field theory on warped $S^1/\mathbb{Z}_2$
and strongly coupled gauge theory on 4D Minkowski

Heuristic picture: $\rightarrow$ e.g. Arkani-Hamed/Porrati/Randall '00

- 5D Randall-Sundrum-like model with UV brane and IR brane
- UV brane fields $\sim$ external states to 4D dual
- IR brane fields $\sim$ composites of 4D dual
- bulk fields $\sim$ partly elementary, partly composite

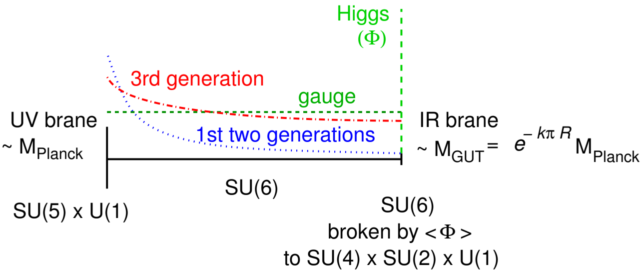
Can argue to get 5D calculable dual e.g. for technicolour-like theories
or for GUT breaking by strong dynamics if IR brane scale $\approx M_{\text{GUT}}$

However: all calculations done within weakly coupled 5D framework. 4D compositeness picture is nice but not too helpful. 4D dual always unknown.

An example

→ Nomura/Poland/Tweedie '06

Warped extra dimension, "slice of AdS_5"
UV scale $\sim$ Planck scale, IR scale $\sim$ GUT scale



- Higgs doublets from brane field Φ:
  pseudo-Goldstone bosons of SU(6) broken to SU(4)$\times$ SU(2)$\times$ U(1)
- matter fields from bulk hypermultiplets
- bulk mass terms ⇒ Yukawa hierarchies

Soft term patterns

1. Gaugino mass unification

Assume SUSY breaking is separated from GUT breaking:
Goldstino is gauge singlet

Leads to gaugino mass unification:

$$M_1 = M_2 = M_3 \qquad \text{at } M_{\text{GUT}}$$

(Common assumption in most scenarios)

2. Degenerate Higgs mass matrix

Characteristic for our examples:
Degenerate Higgs mass matrix at the GUT scale,

$m_{H_1}^2 + |\mu|^2 = m_{H_2}^2 + |\mu|^2 = |B\mu|$, i.e.

$$V_{\text{Higgs}} = \frac{1}{2} \left(\overline{H}_1 \; H_2\right) \begin{pmatrix} m^2 & m^2 \\ m^2 & m^2 \end{pmatrix} \begin{pmatrix} H_1 \\ \overline{H}_2 \end{pmatrix} + \text{ quartic},$$

where $m^2 \equiv m_{H_i}^2 + |\mu|^2 = |B\mu|$

Reason: special structure of the Higgs sector

Origin of degenerate Higgs mass matrix

SUSY GUT with chiral adjoint Φ

Adjoint of GUT group G decomposes under SM gauge group as

$$\begin{aligned} \mathbf{Ad}(G) &\to (\mathbf{1},\mathbf{2})_{-1/2} \oplus (\mathbf{1},\mathbf{2})_{1/2} \oplus \ldots \\ \Phi &\to \quad H_1 \quad \oplus \quad H_2 \quad \oplus \ldots \end{aligned}$$

If Higgs part of $\Phi - \Phi^\dagger$ massless at tree-level — e.g. being a

- pseudo-Nambu-Goldstone Boson
- gauge boson in higher dimensions
- ...

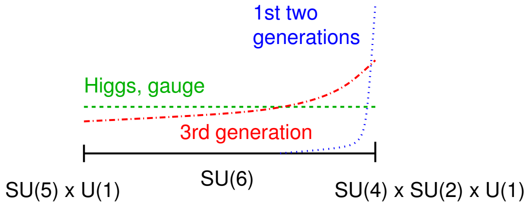
then

$$\begin{aligned} V \supset m^2 \,\mathrm{tr}\left(\Phi + \Phi^\dagger\right)^2 &\supset m^2(H_1 + \overline{H}_2)(\overline{H}_1 + H_2) \\ &= m^2|H_1|^2 + m^2|H_2|^2 + m^2\left(H_1 H_2 + \mathrm{h.c.}\right) \end{aligned}$$

$$\Rightarrow \; m_{H_1}^2 + |\mu|^2 = m_{H_2}^2 + |\mu|^2 = |B\mu|$$

DHMM in gauge-Higgs unification

Recall Burdman-Nomura model:



5d gauge supermultiplet $\rightarrow$ 4d gauge supermultiplet $-\theta\bar{\theta}A_\mu\sigma^\mu + \ldots$

$\oplus$ 4d chiral adjoint $\Phi = \Sigma + iA_5 + \ldots$

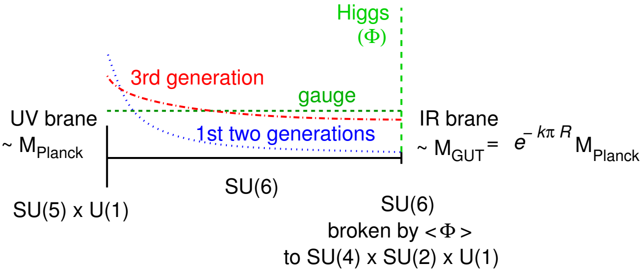
5d gauge invariance: mass term only for $\Sigma \sim \Phi + \Phi^\dagger$, not for $A_5 \sim \Phi - \Phi^\dagger$
Boundary conditions: only $H_1, H_2 \subset \Phi$ have zero modes

$$\Rightarrow \;\; V \supset m^2(H_1 + \overline{H}_2)(\overline{H}_1 + H_2) + \ldots$$

$\Leftrightarrow$ degenerate Higgs mass matrix

DHMM in holographic GUTs

Recall holographic GUT model:



- In gaugeless limit: $\Phi - \Phi^\dagger$ contains pseudo-Goldstones of broken SU(6)
- With gauge couplings: not all pseudo-Goldstone bosons eaten, no $H_1 - \overline{H}_2$ mass at tree-level
  $\Rightarrow$ degenerate Higgs mass matrix

3. Sfermion masses and trilinears

Strongly dependent on details of SUSY breaking
Generic possibilities in 5D models:

- **Radion mediation**: Radius R extends to chiral multiplet
  $T = R + iB_5 + \theta^2 F^T$ → Chacko/Luty '00
  Very constrained: couplings mostly dictated by geometry
- **Brane fields**: 4D brane multiplet F-terms F_Z
  Less constrained: couplings roughly $\sim$ wave-function profiles at brane
- **Generically also**: SUSY breaking in 4D gravitational multiplet
  Parametrise by chiral compensator $1 + \theta^2 F^\varphi$

Phenomenology

Analysis of gauge-Higgs unified model

Higgs mass degeneracy at GUT scale constrains Higgs potential at electroweak scale.
Can we get realistic phenomenology?

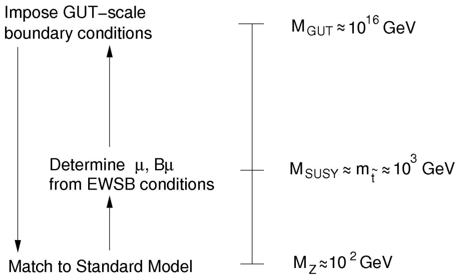
Example model: $\rightarrow$ Burdman/Nomura '03, Hebecker/March-Russell/Ziegler '08

- 5d gauge-Higgs unified model
- 3rd generation in bulk
- first two generations on brane (nearly)
- Radion-mediated SUSY breaking: $F^T \neq 0$ and $F^\varphi \neq 0$
  No brane sources (more predictivity)
- 5D Chern–Simons term crucial for gaugino masses
  $\rightarrow$ extra parameter: CS coefficient c
- 3rd generation matter soft terms $\leftarrow$ 2 bulk-brane mixing angles ϕ_Q, ϕ_L
- 1st two generation matter soft terms $\sim$ Yukawas: neglect
- fundamental model parameters thus $\{F^T, F^\varphi, c, \phi_Q, \phi_L\}$
  $\leftrightarrow \{M_{1/2}, \tan\beta, M_Z, \phi_Q, \phi_L\}$

How a SUSY spectrum generator works

- important dimensionful MSSM parameters:
  - Higgs sector: $m^2_{H_1}$, $m^2_{H_2}$, $B\mu$, μ
  - gaugino masses
  - sfermion soft masses
  - trilinear terms

  Evolve to electroweak scale $\rightarrow$ generically predicts wrong M_Z, cannot match to Standard Model
- usual procedure: exchange μ and $B\mu$ at high scale
  for M_Z and $\tan\beta = \langle H_2 \rangle / \langle H_1 \rangle$ at low scale
- public codes (SuSpect $\rightarrow$ Djouadi et al. '02 / SOFTSUSY $\rightarrow$ Allanach '01) determine spectrum iteratively:

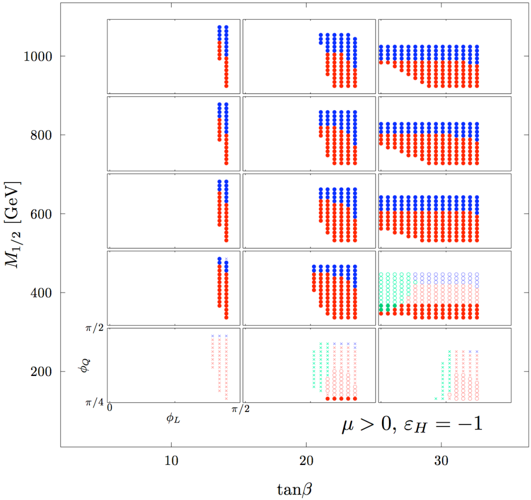


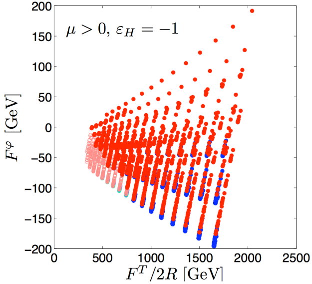
RG analysis of DHMM models

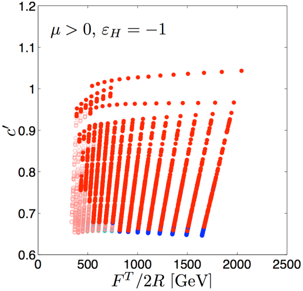
Challenge for implementing DHMM relations:

- DHMM relation holds between μ, $B\mu$, $m^2_{H_i}$ at GUT scale, but μ and $B\mu$ are outputs in standard spectrum generators
  $\rightarrow$ how to restrict scans to points with DHMM?
- Ideally would like μ and $B\mu$ to be inputs and predict $\tan\beta$
  $\rightarrow$ difficult to implement, numerical problems especially at large $\tan\beta$
- Next best thing: still use $\tan\beta$ and M_Z as inputs,
  adjust $m^2_{H_i}$ iteratively to satisfy DHMM relations
  This turns out to be numerically stable

Grid scan finds points with realistic EWSB





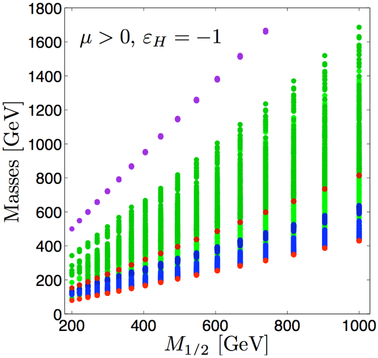


Neutralino, stau, selectron LSP.

Small points excluded by LEP or B-physics.

Note CS parameter necessary: $c = 0$ excluded.

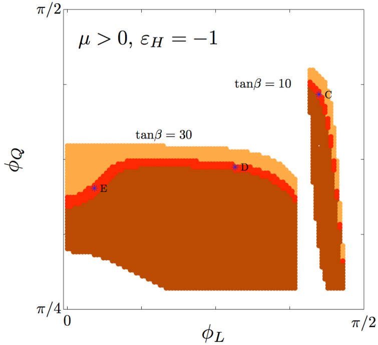
Sparticle masses in neutralino LSP region



Neutralinos, staus, selectrons, gluino

Note small NLSP-LSP mass difference
Also $\tilde{\chi}_2^0$ heavier than selectrons (sometimes also heavier than stau):
decay $\tilde{\chi}_2^0 \to \ell^\pm \tilde{\ell}^\mp \to \ell^\pm \ell^\mp \tilde{\chi}_1^0$ kinematically allowed, large BR
"Same-flavour-opposite-sign" dilepton signature at LHC

Neutralino relic density



Red band = relic density lies within 3σ of WMAP5 observation.
Orange region: Ωh^2 too low (other DM components besides $\tilde{\chi}_0$ required)
Brown region: Ωh^2 too high (with standard cosmology)

DHMM with MCMC

Now go beyond this example and do a more general MCMC scan

Two representative choices for sfermion soft terms:

- universal sfermion soft terms
  (representing generic models)
- vanishing 1st and 2nd generation sfermion soft terms
  (for models where soft terms $\sim$ Yukawa hierarchy, e.g. GHU)

Simplified picture: no flavour violation yet ($\rightarrow$ later!)

Two kinds of prior probability distribution:

- flat prior in dimensionful parameters and in $\tan\beta$
- "naturalness prior", disfavouring fine-tuned points:
  fine-tuning defined by

  $$c = \max_i \left| \frac{d \log M_Z}{d \log a_i} \right|$$

  where $\{a_i\}$ = parameters. Weight all points with $1/c$.

Universal sfermions

Setup: 10 chains with 10^6 points each

Tools: SOFTSUSY, microMEGAs → Allanach '01, Bélanger/Boudjema/Pukhov/Semenov '01

Parameter ranges:

parameters	from	to
$\tan\beta$	2	60
$M_{1/2}$	0	2 TeV
$m_{\text{sfermions}}$	0	5 TeV
A_0	-10 TeV	10 TeV

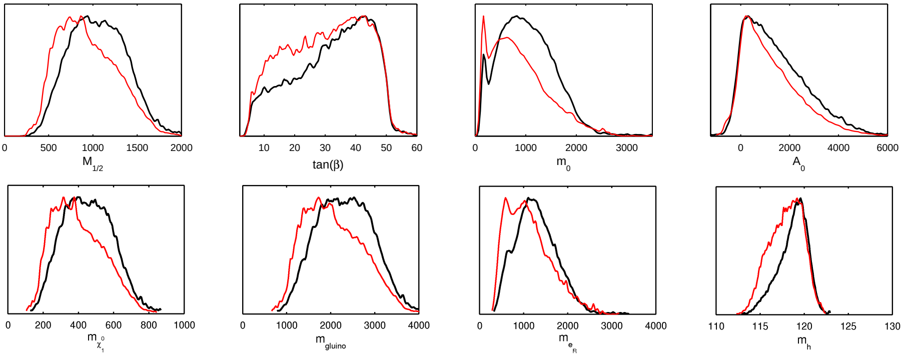
Experimental constraints:

observable	value
m_h	> 114.4 GeV
m_t	173.1 ± 1.4 GeV
m_W	80.398 ± 0.025 GeV
$b \to s\gamma$	$(3.52 \pm 0.34) \times 10^{-4}$

observable	value
$B_s \to \mu^+\mu^-$	$\leq 5.8 \times 10^{-8}$
$\Delta a_\mu^{\text{SUSY}}$	$\leq 4.48 \times 10^{-9}$
Ωh^2	0.113 ± 0.0105
SUSY masses	LEP + Tevatron limits

Results for universal sfermions

Marginalized posterior probability distributions:



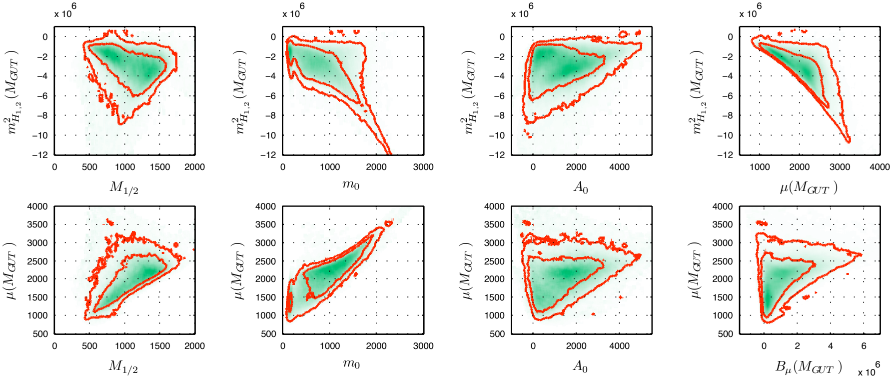
Black and red lines = flat and naturalness prior

Lower peak in m_0 and m_{e_R} distributions:

DM relic density reduced by coannihilation with sleptons (→ later)

Results for flat prior, universal sfermions

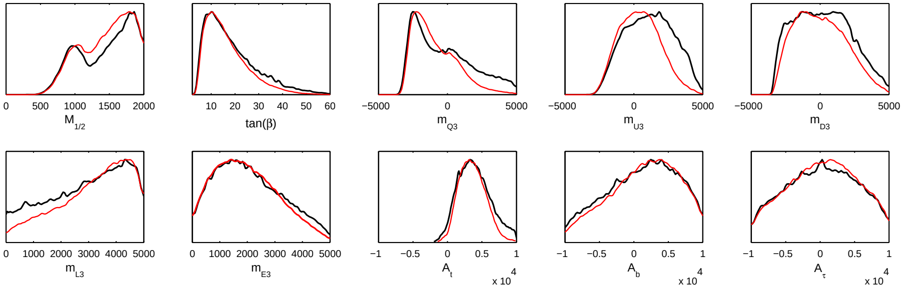
Correlations:



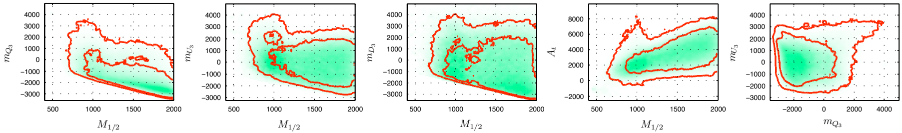
Green shades = likelihood;
red contours = 68% and 95% probability

Results for vanishing 1st and 2nd gen. soft terms

Marginalized posterior probability distributions:



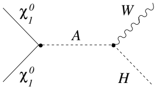
Correlations (naturalness prior only):



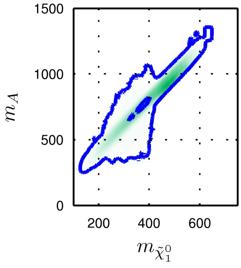
The dark matter constraint

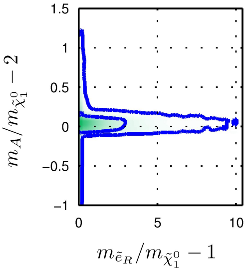
Assuming all dark matter is neutralino LSP:
Stringent constraint from χ_1^0 relic density, $\Omega_{\rm DM} h^2 = 0.113 \pm 0.011$ (WMAP)

- In generic regions of parameter space: relic density too large
- Need to enhance annihilation cross section. Most important mechanism: near-resonant pseudoscalar Higgs exchange, requires $m_A \approx 2 m_{\chi_1^0}$



- potentially also important: coannihilation with sleptons if $m_{\tilde{e},\tilde{\tau}} \approx m_{\chi_1^0}$





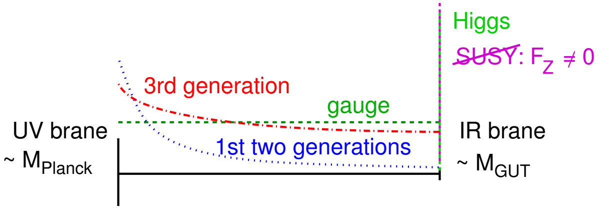
In progress: The SUSY flavour problem in 5D

Unconstrained MSSM: expect large FCNCs and $\not{CP}$
Reason: squark and slepton mass matrices contain phases,
are generally not diagonal in CKM basis

Ignore $\not{CP}$, focus on FCNCs for now

5D MSSM: Flavour problem can be alleviated
Reason: Yukawa hierarchy from localisation ⇒ similar hierarchy for soft terms

E.g. in holographic GUT with SUSY breaking on IR brane:



In progress: The SUSY flavour problem in 5D

Yukawa matrices from wave function localisation: e.g. up-type quarks

$$y_u = \begin{pmatrix} \lambda_{11}\,\epsilon^4 & \lambda_{12}\,\epsilon^3 & \lambda_{13}\,\epsilon^2 \\ \lambda_{21}\,\epsilon^3 & \lambda_{22}\,\epsilon^2 & \lambda_{23}\,\epsilon \\ \lambda_{31}\,\epsilon^2 & \lambda_{32}\,\epsilon & \lambda_{33} \end{pmatrix}$$

$\lambda_{ij} = \mathcal{O}(1)$ numbers, $\epsilon \approx 0.1$ from wave functions evaluated on IR brane

Can reproduce observed masses and mixings

Soft scalar masses:

$$m_u^2 = \frac{|F_Z|^2}{M^2} \begin{pmatrix} \kappa_{11}\,\epsilon^4 & \kappa_{12}\,\epsilon^3 & \kappa_{13}\,\epsilon^2 \\ \kappa_{12}^*\,\epsilon^3 & \kappa_{22}\,\epsilon^2 & \kappa_{23}\,\epsilon \\ \kappa_{13}^*\,\epsilon^2 & \kappa_{23}^*\,\epsilon & \kappa_{33} \end{pmatrix}$$

In general not diagonal in CKM basis $\Rightarrow$ FCNCs
but approximately diagonal since right hierarchy structure

How much tuning of the κ_{ij} is still needed to evade FCNC constraints?

Conclusions

- Can embed the MSSM into more fundamental frame: 5D GUTs
- Fifth dimension solves many problems of conventional 4D GUTs
- Interesting classes of models predictive on gaugino and Higgs soft terms: Degenerate Higgs mass matrix $m_{H_1}^2 + |\mu|^2 = m_{H_2}^2 + |\mu|^2 = |B\mu|$
- Squark and slepton soft terms depend more on SUSY breaking details Fifth dimension can alleviate flavour problem
- Realistic examples can be found: e.g. gauge-Higgs unified model
- MCMC studies reveal allowed regions of general parameter space Most stringent constraint (as of 2010): Dark matter relic density
- Early LHC (2011/12) will start probing parameter space
- In progress: To what extent is tuning still needed to solve flavour problem?

Thank you!

Backup

How to scan a large-dimensional parameter space

The problem:
Given

- a model (e.g. the MSSM) with some parameters (e.g. soft ~~SUSY~~ terms and SM couplings at the GUT scale)
- some experimental data (e.g. measurements of SM couplings, lower bounds on superpartner masses) with associated uncertainties

we can ask:

- What values for the parameters best describe the data?
- What regions of parameter space are experimentally ruled out?
- Which measurements provide the strongest constraints?

Note we do not (yet) compare different models (e.g. “does the MSSM fit the data better than the SM”?), just different parameter sets for the same model

How to scan a large-dimensional parameter space

Grid scans:

- inefficient
- need to know beforehand which parameter regions are promising
- strong correlations $\Rightarrow$ resources wasted on less interesting regions

Better: Markov Chain Monte Carlo (MCMC)

- “Random walk in parameter space”
- computationally less expensive
- more focus on interesting parameter regions, where data is well described by model
- allows for interpretation with methods of Bayesian statistics
- Application to MSSM:
  $\rightarrow$ Baltz/Gondolo ’04, Allanach/Lester ’05, de Austri et al. ’06...

MCMC explained

Assuming a model and given some measurement data D with errors, we define for some parameter point P

- the likelihood $L(D|P)$ of P
  $\equiv$ probability for P to reproduce D within errors, $L(D|P) \sim e^{-\chi^2}$
- the prior (probability) $\pi(P)$ of P
  $\equiv$ probability assigned to P before consideration of data; subjective!
- the (posterior) probability $p(P|D)$ of P

$$p(P|D) \equiv \frac{L(D|P) \cdot \pi(P)}{\text{normalization}}$$

A few remarks on priors:

The prior reflects theoretical bias.

E.g. for MSSM: soft masses should all be in TeV range (otherwise main motivations for SUSY are spoiled); disfavour points relying on large cancellations between parameters (fine-tuning is unnatural).

Once experimental constraints are strong enough, probabilities become practically independent of prior.

MCMC explained

The Metropolis algorithm

- Start with a parameter point P and calculate its probability $p(P|D)$
- Pick a nearby parameter point P' at random and calculate $p(P'|D)$
- If $p(P'|D) > p(P|D)$, append P' to the chain
- If $p(P'|D) \leq p(P|D)$, either append P' to the chain (with probability $p(P'|D)/p(P|D)$), or otherwise append P to the chain again
- Iterate, starting from the newly appended point

MCMC in practice

Ideal outcome:

- Collection of points clustering in regions of higher probability
- more precisely: point distribution samples posterior probability (i.e. reproduces it in the limit of large number of points)
- “interesting” regions covered better
- well suited for Bayesian interpretation, “LHC weather forecasts”. . . (but that’s an inexact science: prior dependence!)

Possible pitfalls:

- allowed region may be disconnected: run several chains
- “burn-in” phase required: discard initial points
- step size tricky. Adaptive algorithms no longer sample target distribution