Automation of One-Loop Calculations with Golem/Samurai

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Outline

NLO WISHLIST . . . DONE!

Recent Progress on $2 \to 4(5)$

- $pp \to W+$ 3 jets Rocket (2009), Blackhat (2009)
- $pp \to t\bar{t}b\bar{b}$ Denner-Dittmaier (2009), Helac-NLO (2009)
- $pp \to Z(\gamma)+$ 3 jets Blackhat (2010)
- $pp \to t\bar{t}jj$ Helac-NLO (2010)
- $pp \to W^+W^-b\bar{b}$ Denner-Dittmaier (2010), Helac-NLO (2010)
- $pp \to W^+W^+jj$ Rocket (2010)
- $pp \to W+$ 4 jets Blackhat (2011)
- $pp \to b\bar{b}b\bar{b}$ Golem/Samurai (2011)
- $pp \to W^+W^-jj$ Rocket (2011)

Several methods/codes "available on the market"

Automation → the use of methods for controlling processes automatically, often reducing manpower

Virtues of Automation (for Calculations of Scattering Amplitudes):

- **Optimization** / **Self-organization**
- **Avoid human mistakes**
- **Process-independent** techniques

This is not a new idea → fully exploited at the tree-level

Automation at One-Loop

At the One-Loop level, amazing work has been done towards Automation:

- "Proofs of concept": **HELAC-NLO**, **MadLoop**

  Helac-nlo : Bevilacqua, Czakon, van Hameren, Papadopoulos, Pittau, Worek

  MadLoop: Hirschi, Frederix, Frixione, Garzelli, Maltoni, Pittau

  - Upgrade of tree-level tools combined with integrand level reduction
  - Fully integrated with real radiation and subtraction terms to produce finite results

- There is also an **"algebraic way"** to Automation

  FeynArts/FormCalc/LoopTools: T. Hahn

  - Generate unintegrated amplitudes with Feynman diagrams
  - Manipulate and simplify them
  - Perform the reduction

Automation at One-Loop: "Algebraic Way"

Main features of the "Algebraic Way":

- Amplitudes generated with Feynman diagrams
- Algebraic manipulations are allowed before starting the numerical integration
- The generation of numerators is executed separately from the numerical reduction
- Optimization: grouping of diagrams, smart caching
- Control over sub-parts of the computation (move in/out subsets of diagrams)
- Algebra in dimension d, different schemes

Great flexibility in the reduction
Choice between different algorithms at runtime

Automation with GoSam

Golem/Samurai

Algebraic generation of **d-dimensional integrands** via **Feynman diagrams**

Reduction at the Integrand Level: **d-dimensional extension of OPP reduction**

Target: provide an **automated tool** for stable evaluation of one-loop matrix elements

- **be general/model independent** (QCD, EW, BSM)
- **interface with existing tools** like MadEvent, Sherpa, PowHEG, ...
- build upon **open source tools only** (i.e. Samurai, Golem95, QGraf, Form, Spinney, Haggies, QCDLoop, OneLOop)
- **support open standards**

Diagram Generation

Cullen, **Greiner**, **Heinrich**, **Luisoni**, **Mastrolia**, **G.O.**, **Reiter**, **Tramontano**

An automated amplitude generation based on Feynman diagrams
(distributed as a python package)

- FORM
  J.A.M. Vermaseren, (1991)
- QGRAF
  P. Nogueira, (1993)
- Haggies
  T. Reiter, (2009)
- Spinney
  Cullen, Koch-Janusz, Reiter, (2010)

To be released soon!

Diagram Reduction

Default Option: **Samurai**
Mastrolia, G.O., Reiter, Tramontano (2010)

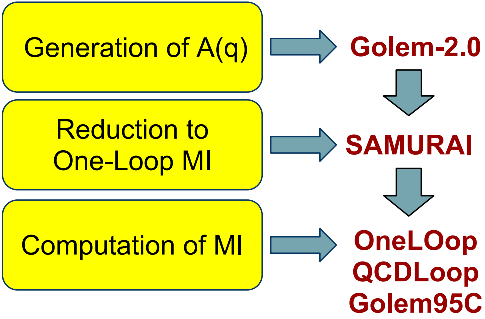
- OPP Reduction Algorithm G.O., Papadopoulos, Pittau (2007)
- d-dimensional extension Ellis, Giele, Kunszt, Melnikov (2008)
- Coefficients of Polynomials via DFT Mastrolia et al. (2008)
- Model-independent Computation of the full Rational Term

Other options available (at runtime):

Golem95
Binoth, Cullen, Guillet, Heinrich, Kleinschmidt, Pilon, Reiter, Rodgers (2008)

Tensorial Integrand-level Reduction
Heinrich, G.O., Reiter, Tramontano (2010)

Standard GoSam reduction:



This process is **fully automated**

A walk through GoSam

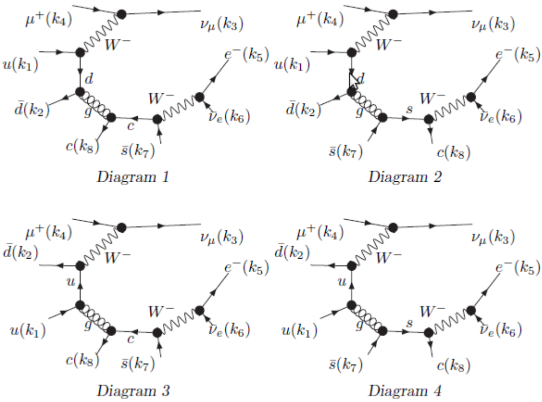
Preparation of the "card": we use as example $u\,\bar{d} \to \bar{s}\,c\,e^-\,\bar{\nu_e}\,\mu^+\,\nu_\mu$

```
in= u,d~
out= nmu, mu+, e-, ne~, s~, c
model=smdiag
```

models can be added via **FeynRules** (Duhr) or **LanHEP** (Semenov)

```
order=gw,4,4; order=gs,2,4
zero=mB,mC,mS,mU,mD,me,mmu
one=gs,e
helicities=-+-+-++-
extensions=samurai, dred
```

Building the code : check the details before the run



Building the code : check the details before the run

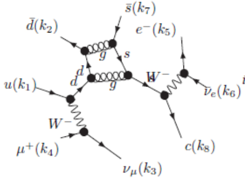


Diagram 36

$S' = S^{\{2\}}_{Q \to q+(-k3+k2+k1-k4)}$, rk = 2

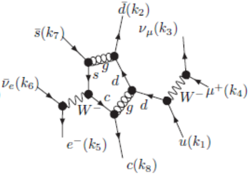


Diagram 3

$S' = S_{Q \to -q-(-k3+k2+k1-k4)}$,
rk = 3

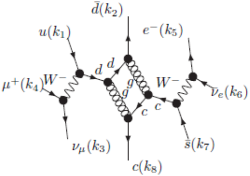
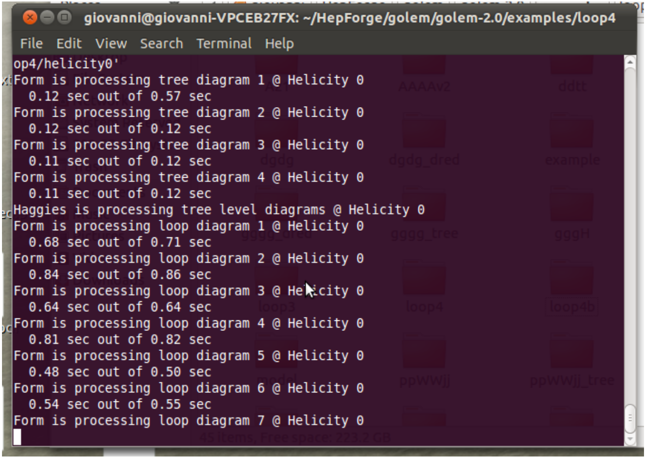


Diagram 31

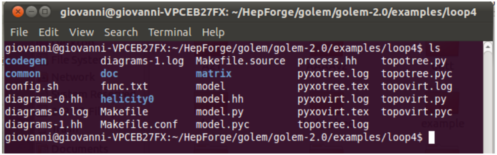
$S' = S^{\{3\}}_{Q \to -q-(-k3+k1-k4)}$, rk = 2

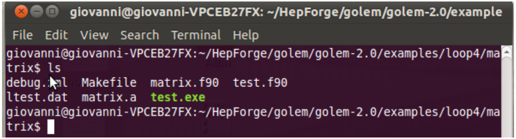
A walk through GoSam

Building the code : Spinney+ Haggies



Execution: all the code is ready






```
NLO/LO, finite part -15.91575134226371
NLO/LO, single pole 7.587050691447690
NLO/LO, double pole -5.333333333333456
```

Table 8 of arXiv:1104.2327: **Melia, Melnikov, Rontsch, Zanderighi**

Precision Tests

Use the decomposition of the numerator function $N(\bar{q})$ after determining all coefficients

$$
\begin{aligned}
N(\bar{q}) &= \sum_{i<<m}^{n-1} \Delta_{ijk\ell m}(\bar{q}) \prod_{h\neq i,j,k,\ell,m}^{n-1} \bar{D}_h + \sum_{i<<\ell}^{n-1} \Delta_{ijk\ell}(\bar{q}) \prod_{h\neq i,j,k,\ell}^{n-1} \bar{D}_h + \\
&+ \sum_{i<<k}^{n-1} \Delta_{ijk}(\bar{q}) \prod_{h\neq i,j,k}^{n-1} \bar{D}_h + \sum_{i<j}^{n-1} \Delta_{ij}(\bar{q}) \prod_{h\neq i,j}^{n-1} \bar{D}_h + \sum_{i}^{n-1} \Delta_{i}(\bar{q}) \prod_{h\neq i}^{n-1} \bar{D}_h
\end{aligned}
$$

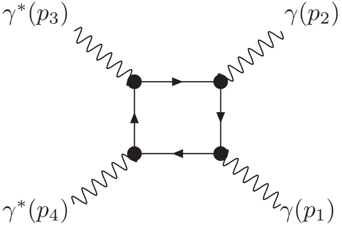
1. **Global** $(N = N)$**-test**
2. **Local** $(N = N)$**-test**
3. **Power-test**

Are those methods **reliable** in detecting **unstable phase space points**?

Approaching the Gram - I

- We approach a kinematic configuration which can lead to large cancellations

- Fermion loop with two massless and two massive vector particles

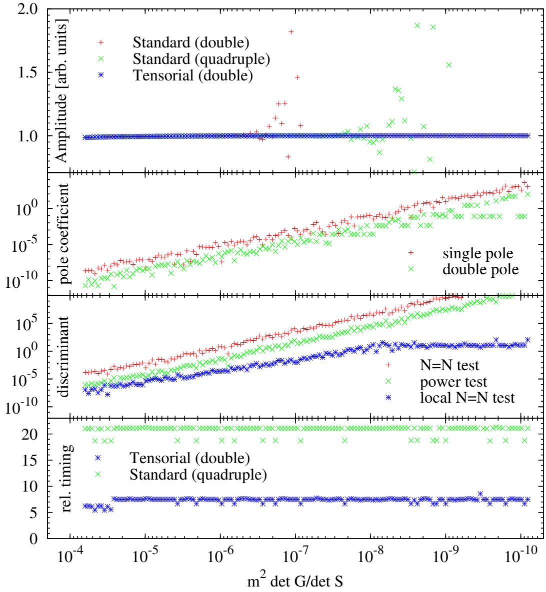


$$p_{1,2} = (E, 0, 0, \pm E) \quad p_{1,2}^2 = 0$$
$$p_{3,4} = (E, 0, \pm Q \sin\theta, \pm Q \cos\theta)$$
$$p_{3,4}^2 = m^2$$
$$E = \sqrt{m^2 + Q^2}$$

- The Gram-det vanishes when $Q \to 0$ (m and θ are fixed)

$$\det G = 32\, E^4 Q^2 \sin^2\theta$$

Approaching the Gram - II



Example: Alternative reduction paths

Samurai/Tensorial Reduction/Golem95

$u\bar{u} \to d\bar{d}$

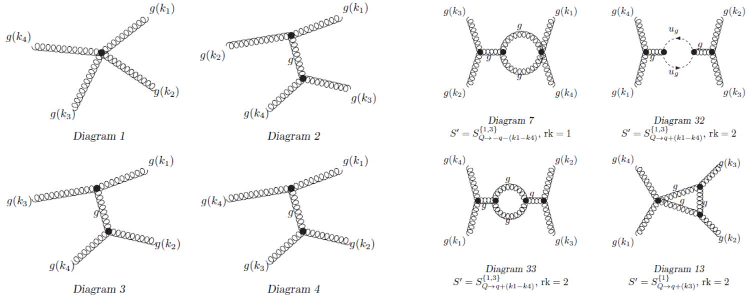
1. Evaluation with Samurai, sampling of diagram groups
2. Evaluation with Samurai, sampling of individual diagrams
3. Tensorial Reconstruction + Reduction of numetens with Samurai
4. Evaluation with Golem95

Method	finite part	single pole	double pole
1	-3.433053565229151	-14.62937842683104	-5.333333333333338
2	-3.433053565229129	-14.62937842683102	-5.333333333333342
3	-3.433053565229163	-14.62937842683104	-5.333333333333342
4	-3.433053565229146	-14.62937842683102	-5.333333333333332

Calculations tested with Golem/Samurai

1. $\gamma + \gamma \rightarrow \gamma + \gamma$
2. $\bar{u} + d \rightarrow e^- + \bar{\nu}_e$
3. $\bar{u} + u \rightarrow d + \bar{d}$
4. $d + g \rightarrow d + g$
5. $u + \bar{d} \rightarrow e^+ + \nu_e + g$
6. $u + \bar{d} \rightarrow e^+ + \nu_e + s + \bar{s}$
7. $u + \bar{d} \rightarrow e^+ + \nu_e + g + g$
8. $d + \bar{d} \rightarrow e^+ + e^- + g$
9. $d + \bar{d} \rightarrow W^+ + W^-$
   (with/without leptonic decays)
10. $d + \bar{d} \rightarrow t + \bar{t}$
11. $b + g \rightarrow H + b$
12. $u + \bar{u} \rightarrow g + \gamma$
13. $u + g \rightarrow u + \gamma$
14. $g + g \rightarrow g + \gamma$
15. $g + g \rightarrow g + g$
16. $g + g \rightarrow Z + g$
17. $g + g \rightarrow Z + Z$
    (with/without leptonic decays)
18. $g + g \rightarrow W^+ + W^-$
    (with/without leptonic decays)
19. $e^+e^-(\rightarrow Z) \rightarrow \bar{d}dgg$
20. $u\bar{d} \rightarrow \bar{c}se^+\nu_e\mu^+\nu_\mu$
21. $u\bar{d} \rightarrow \bar{s}ce^-\bar{\nu}_e\mu^+\nu_\mu$

EXAMPLE: $gg \to gg$

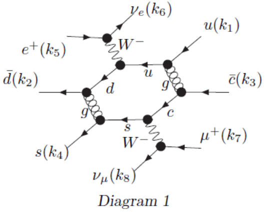


	Golem/Samurai	hep-ph/0609054
LO	14.120983050796795	14.120983050796804
NLO/LO finite	-124.02475579423496	-124.02475579423495
NLO/LO $1/\epsilon$	44.003597347101028	44.003597347101035
NLO/LO $1/\epsilon^2$	-12.000000000000002	-12.000000000000000

Comparison with: hep-ph/0609054 Binoth, Guillet, Heinrich

EXAMPLE: $pp \to W^+W^+jj$

$$u\bar{d} \to \bar{c}se^+\nu_e\mu^+\nu_\mu$$



Helicities

Index	1	2	3	4	5	6	7	8
0	−	+	+	−	+	−	+	−

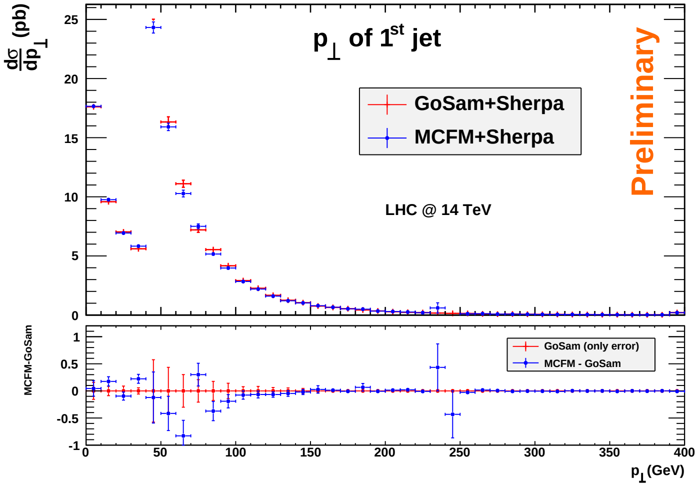
Golem/Samurai (NLO/LO):

```
finite part 23.3596455167118
single pole 13.6255429251954
double pole -5.33333333333343
```

Comparison with Melia, Melnikov, Rontsch, Zanderighi

Example: GoSam + Sherpa

$W + jet$ at LHC (14 TeV)



(Thanks to **Jennifer Archibald** and **Gionata Luisoni**)

GoSam and Complex Masses

Complex masses **are supported**!

Golem95C

(Cullen, Guillet, Heinrich, Kleinschmidt, Pilon, Reiter, Rodgers)

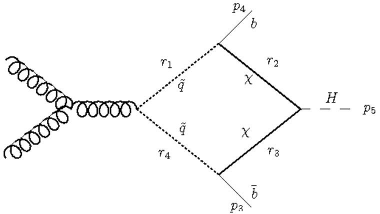
- Upgrade of previous *Golem95* library
- Real and complex masses are supported
- Interface for tensorial reconstruction

`arXiv:1101.5595`

Samurai v2.1

(to be released soon!)

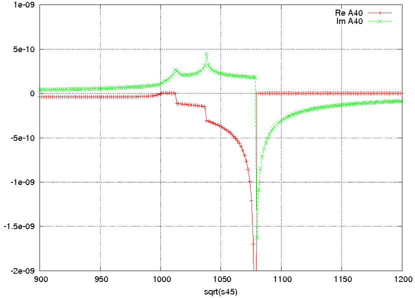
Example: MSSM Higgs and Complex Masses



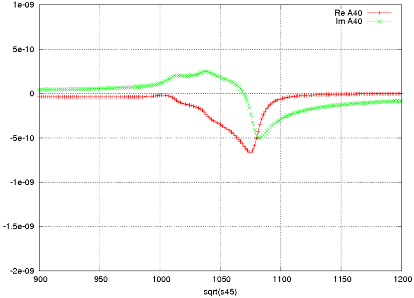
Production of a heavy neutral MSSM Higgs boson and a $\bar{b}b$ pair in gluon fusion.

The loop contains two squarks and two neutralinos

Real Masses



Complex Masses



Golem95C – arXiv:1101.5595

Conclusions: Golem/Samurai

There are many valuable approaches/codes to One-Loop Calculations
(too much to fit in a 17+3 minutes)

GoSam is a flexible and broadly applicable tool

- it is based on Feynman diagrams
- it uses a d-dimensional reduction (no additional techniques required for rational terms)
- it will be publicly available, as soon as we complete the testing
- it uses some of the best techniques on the market

We look forward to **interacting/interfacing** with other tools

More results soon!

EXTRA SLIDES

Alternative Path: the "Tensorial Way"

Tensorial Reconstruction at the Integrand Level
Heinrich, G.O., Reiter, Tramontano JHEP 1010:105,2010

In this work:

- We **tested** the methods for the **detection of instabilities**
- We proposed a **"rescue-system"** alternative to higher precision routines
- We proposed an **optimized reconstruction method**

Idea: tensorial reconstruction performed at the integrand level by means of a sampling in the integration momentum.

$$\mathcal{N}(q) = \sum_{r=0}^{R} C_{\mu_1 \dots \mu_r} q_{\mu_1} \dots q_{\mu_r} \Longrightarrow \hat{N}(q)$$

$\hat{N}(q)$ is the "reconstructed numerator" written as a tensor
– numerically identical to the initial $\mathcal{N}(q)$ –

ORIGINAL "MASTER" FORMULA FOR THE INTEGRAND

4-dim identity at the integrand level for $N(q)$ in terms of **4-dim** D_i

$$
\begin{aligned}
N(q) &= \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i \\
&+ \sum_{i_0<i_1<i_2}^{m-1} \left[c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2) \right] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\
&+ \sum_{i_0<i_1}^{m-1} \left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1) \right] \prod_{i \neq i_0, i_1}^{m-1} D_i + \sum_{i_0}^{m-1} \left[a(i_0) + \tilde{a}(q; i_0) \right] \prod_{i \neq i_0}^{m-1} D_i
\end{aligned}
$$

$\tilde{d}(q)$, $\tilde{c}(q)$, $\tilde{b}(q)$, $\tilde{a}(q)$ are "spurious" terms that vanish upon integration

ORIGINAL "MASTER" FORMULA FOR THE INTEGRAND

4-dim identity at the integrand level for $N(q)$ in terms of **4-dim** D_i

$$
\begin{aligned}
N(q) &= \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3)\right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i \\
&+ \sum_{i_0<i_1<i_2}^{m-1} \left[c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2)\right] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\
&+ \sum_{i_0<i_1}^{m-1} \left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1)\right] \prod_{i \neq i_0, i_1}^{m-1} D_i + \sum_{i_0}^{m-1} \left[a(i_0) + \tilde{a}(q; i_0)\right] \prod_{i \neq i_0}^{m-1} D_i
\end{aligned}
$$

Rational Terms

In this approach, Rational Terms require a separate computation

G.O., Papadopoulos, Pittau (2007)

Draggiotis, Garzelli, Malamos, Pittau (2009-2011)

Important points

1. The **functional form** of the **OPP master formula** is **universal** (**process independent**)

2. The **only information required in order to extract the coefficients** of the master integrals is the knowledge of **the numerical value of the numerator function for a finite set of values $\{q_i\}$ of the integration momentum**

3. The process becomes particularly simple if we **choose** $\{q_i\}$ such that sets of **denominators D_i vanish** ("cuts")

G.O., Papadopoulos, Pittau (2007)

IDENTITY IN D-DIMENSIONS: $q \to \bar{q}$

Ellis, Giele, Kunszt, Melnikov (2008), Melnikov, Schulze (2010)
Mastrolia, G.O., Reiter, Tramontano (2010)

Reconstruct directly d-dimensional denominators

$$\begin{aligned} N(\bar{q}) \;=\; & \sum_{i<<m}^{n-1} \Delta_{ijk\ell m}(\bar{q}) \prod_{h\neq i,j,k,\ell,m}^{n-1} \bar{D}_h + \sum_{i<<\ell}^{n-1} \Delta_{ijk\ell}(\bar{q}) \prod_{h\neq i,j,k,\ell}^{n-1} \bar{D}_h + \\ & + \sum_{i<<k}^{n-1} \Delta_{ijk}(\bar{q}) \prod_{h\neq i,j,k}^{n-1} \bar{D}_h + \sum_{i<j}^{n-1} \Delta_{ij}(\bar{q}) \prod_{h\neq i,j}^{n-1} \bar{D}_h + \sum_{i}^{n-1} \Delta_{i}(\bar{q}) \prod_{h\neq i}^{n-1} \bar{D}_h \end{aligned}$$

1. Denominators in d-dimensions: $D_h \to \bar{D}_h$
2. $N(\bar{q})$ is d-dimensions: $N(q) \to N(q, \mu^2)$
3. The polynominals in the coefficients have a **more complicated structure**

Identity in d-dimensions: Coefficients

- Add a spurious pentagon term in μ^2

$$\Delta_{ijk\ell m}(\bar{q}) = c_{5,0}^{(ijk\ell m)}\,\mu^2$$

- The coefficients have a more complicated structure

Box in 4-dimensions

$$\Delta_{ijk\ell}(q) = c_{4,0} + c_{4,1}\,\tilde{F}(q)$$

Box in d-dimensions

$$\Delta_{ijk\ell}(\bar{q}) = c_{4,0} + c_{4,2}\,\mu^2 + c_{4,4}\,\mu^4 + \left(c_{4,1} + c_{4,3}\,\mu^2\right)\tilde{F}(q)$$