## New results in exclusive hard reactions <br> GPDs and TDAs

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B. Pire

CPhT, École Polytechnique, Palaiseau
based on work done with K Semenov-Tian-Shansky, L Szymanowski, J Wagner
Phys Rev D 2005 ; Phys Letters 2005; Phys Rev D 2010 ; Phys Rev D 2011

## Plan

$\Rightarrow$ QCD factorization for exclusive reactions

* Success in DVCS : See previous presentation, J. Bowles
$\rightarrow$ GPD properties
$\Rightarrow$ On Timelike Compton scattering
$\rightarrow$ NLO corrections
$\rightarrow$ access in UPC at LHC
BP, L Szymanowski, J Wagner , Phys Rev D. 2009 and 2011
$\Rightarrow$ Backward meson electroproduction
$\rightarrow$ from GPDs to TDAs
BP, K Semenov-Tian-Shansky, L Szymanowski, Phys Rev D. 2010 and 2011.


## QCD factorization in Exclusive processes

## DVCS



## Meson Production


$\Rightarrow$ Factorisation between a hard part (perturbatively calculable) and a soft part (non-perturbative) Generalized Parton Distribution demonstrated for

$$
\begin{aligned}
& Q^{2} \rightarrow \infty, x_{B}=\frac{Q^{2}}{Q^{2}+W^{2}} \text { fixed and } t \ll \text { fixed } \\
& \quad \text { D. Muller et al., Ji, Radyushkin, Collins et al. , '94, '96,'98 }
\end{aligned}
$$

## Generalised Parton Distributions

Non-Local operators (as in DIS) and non diagonal matrix elements $=$ soft part of the amplitude for exclusive reactions


GPD $=$ Fourier Transform of matrix elements

$$
\left.\left\langle N\left(p^{\prime}, \lambda^{\prime}\right)\right| \bar{\psi}(-z / 2)_{\alpha}[-z / 2 ; z / 2] \psi(z / 2)_{\beta}|N(p, \lambda)\rangle\right|_{z^{+}=0, z_{T}=0}
$$

ON THE LIGHT CONE $z^{2}=0$

$$
p^{\prime}-p=\Delta \quad \Delta^{2}=t \quad \Delta^{+}=-\xi\left(p+p^{\prime}\right)^{+} \quad x-x^{\prime}=2 \xi
$$

## Energy flow in GPDs

Three different regions

antiquark content
$\bar{q} q$ content
quark content

Two different evolution equations
as $\bar{q}\left(-x, Q^{2}\right)$
as $\Phi^{\pi}\left(z, Q^{2}\right)$
as $q\left(x, Q^{2}\right)$

DGLAP
$\rightarrow \delta(-x)$

ERBL
$\rightarrow \Phi_{a s}^{\pi}\left(z, Q^{2}\right)=6 z \bar{z}$

DGLAP

## Impact picture Representation

$t$ dependence of GPDs maps transverse position $b_{T}$ of quarks.
Fourier transform GPD at zero skewedness $q\left(x, b_{T}\right)=(2 \pi)^{-2} \int d^{2} \Delta_{T} e^{i \Delta_{T} \cdot b_{T}} H(x, \xi=0, t)$ probability

Generalize at $\xi \neq 0 \rightarrow$ Quantum femtophotography.
The $t$-dependence of dVCS localizes transversally the $q$ (DGLAP) or the $\bar{q} q$ pairs of size $\frac{1}{Q}$ (ERBL) in the proton

$$
\text { DGLAP region }(x>\xi)
$$

(a)


Femtophotography of quark in the proton
(b)


Femtophotography of quark-antiquark pair in the proton

## This is the reason I consider GPDs as a breakthrough in QCD physics

$\Leftrightarrow$ Beautiful progress in forward exclusive photon (DVCS) and meson (DVMP) experiments and analysis
$\rightleftharpoons$ Need to test universality of GPDs : TCS vs DVCS extractions
$\Rightarrow$ Need to better understand NLO and twist 3 contributions ( $\rightarrow \rho_{T}$ ) see A. Besse POSTER session
$\Rightarrow$ Extend forward case (= GPDs : $\bar{\psi} \psi$ operators ) to backward kinematics $\rightarrow$ TDAs : $\psi \psi \psi$ operators

## On spacelike vs timelike probe

$$
\gamma^{*}(q) N(p) \rightarrow \gamma^{*}\left(q^{\prime}\right) N^{\prime}\left(p^{\prime}\right) \quad \text { DVCS vs TCS }
$$


spacelike $q^{2}<0 ; q^{\prime 2}=0 \quad$ vs timelike $q^{2}=0 ; q^{\prime 2}>0$

$$
e N \rightarrow e^{\prime} N \gamma \quad \text { vs } \quad \gamma N \rightarrow N \mu^{+} \mu^{-}
$$

$\mathrm{LO}: \mathcal{A}_{D V C S}=\mathcal{A}_{T C S}^{*}$
$\mathrm{NLO}: \mathcal{A}_{D V C S} \neq \mathcal{A}_{T C S}^{*}$

$$
R_{T-S}^{q}=\frac{C_{1(\mathrm{TCS})}^{q}-C_{1(\mathrm{DVCS})}^{q^{*}}}{C_{0}^{q}} .
$$


$\rightleftharpoons$ Both timelike and spacelike data useful to check NLO analysis !

## GPDs at LHC (and RHIC)

$\stackrel{\text { Ultraperipheral Collisions : quasi real photons from proton beam }}{ }$
$\mu^{+} \mu^{-}$pair production


QED dominates over TCS but in specific kinematics
$\rightarrow$ cutting out QED with angular cuts :


GPDs are expected to be large at small $x \approx \xi$
$\xi \approx Q^{2} / s_{\gamma p}$
$\Rightarrow$ Probe of sea and gluon GPDs in small $x$ regime

## Observing TCS at LHC

$\Rightarrow$ Characteristic signal from interference (charge conj. odd)


## First data



CAUTION : TCS not in Monte Carlo!

# From Forward to Backward electroproduction 

From GPDs to TDAs

## Meson (or $\gamma$ ) deep electroproduction : 3 kinematics

$$
\gamma^{*}(q) N(p) \rightarrow M(k) N^{\prime}\left(p^{\prime}\right)
$$

define $t=(q-k)^{2}=\left(p^{\prime}-p\right)^{2}$

$$
u=\left(q-p^{\prime}\right)^{2}=(p-k)^{2}
$$

$\rightleftharpoons$ Forward region : $-t$ small $\rightarrow$ GPD domain
$\curvearrowleft$ Fixed angle region $-t \approx-u \rightarrow$ very small cross sections
$\leadsto$ Backward region : $-u$ small $\rightarrow$ TDA domain

Backward region may be analyzed similarly as Forward region with GPDs replaced by TDAs and many common features

$$
Q^{2}=-q^{2} \text { large }
$$

## from GPDs to TDAs

$\Rightarrow$ GPDs are not the adequate tool for describing backward hard electroproduction
$\Rightarrow$ Basic difference forward vs backward is the exchange of $\quad \bar{q} q \quad$ vs $q q q$
$\Rightarrow$ From $\bar{\psi}\left(z_{1}\right) \psi\left(z_{2}\right)$ to $\psi\left(z_{1}\right) \psi\left(z_{2}\right) \psi\left(z_{3}\right)$ operators

## TDAs : transition distribution amplitudes

In backward DVCS and backward meson electroproduction, one may factorize a non-perturbative part describing a baryon to photon or baryon to meson transition.

L.L.Frankfurt et al, PRD60(1999)

$$
\text { BP, L. Szymanowski, PRD } 71 \text {; PLB } 622 \text { (2005) }
$$

Kinematics (light-cone vectors $\mathbf{p}, \mathbf{n}$ ) $p_{1}=(1+\xi) p+\frac{M^{2}}{1+\xi} n$ $p_{\pi}=(1-\xi) p+\frac{m^{2}-\Delta_{T}^{2}}{1-\xi} n+\Delta_{T}$

$$
u=\left(p_{1}-p_{\pi}\right)^{2} \ll Q^{2} \sim O\left(W^{2}\right)
$$

skewness parameter : $\xi=\frac{Q^{2}}{2 W^{2}-Q^{2}}$

## Factorization

The perturbative part describes the $\gamma^{*} q q q \rightarrow q q q$ transition.


The non-perturbative part describes the proton-meson transition.


## Energy flow in TDAs

## Different regions

$\rightarrow$ Both for Baryon $\rightarrow$ Meson and Baryon $\rightarrow$ photon,
3 quarks are exchanged in the $t$-channel ; $x_{1}+x_{2}+x_{3}=2 \xi$


ERBL


DGLAP1


DGLAP2
$\Rightarrow$ ERBL region : $x_{i}>0 ; \quad$ (as for proton DA)
$\Rightarrow$ DGLAP1 regions : $x_{1}>0 ; x_{2}>0 ; x_{3}<0+$ permutations
$\Rightarrow$ DGLAP2 regions : $x_{1}>0 ; x_{2}<0 ; x_{3}<0+$ permutations
$\rightarrow$ Different physical picture and evolution equations

## Physical picture of TDAs

$\leadsto$ The TDAs provides information on the next to minimal Fock state in $P$

$P=\mid u u d \pi^{0}>$ or $\mid u d d \pi^{+}>$how one can find a meson in a proton
$\otimes$ Impact picture Representation Fourier transform $\vec{\Delta}_{T} \rightarrow \vec{b}_{T}$
As for GPDs, the $t$ dependence of TDAs maps the transverse quark position In the ERBL region: Transverse localization of $q q q$ core of size $\frac{1}{Q}$


Femtophoto of 3 quark core in the proton ERBL


Femtophoto of 2 quark in the proton

DGLAP1


Femtophoto of 2 antiquark in the meson

DGLAP2

## Evolution equations

$\leadsto$ Same operator as in DAs $\rightarrow$ Same renormalization group equations

$$
\begin{aligned}
& Q \frac{d}{d Q} F^{\uparrow \downarrow \uparrow}\left(x_{i}\right)=-\frac{\alpha_{s}}{2 \pi}\left[\frac{3}{2} C_{F} F^{\uparrow \downarrow \uparrow}\left(x_{i}\right)-\left(1+\frac{1}{N_{c}}\right) \mathcal{A}\right] \\
\mathcal{A}= & {\left[\left(\int_{-1+\xi}^{1+\xi} d x_{1}^{\prime}\left[\frac{x_{1} \rho\left(x_{1}^{\prime}, x_{1}\right)}{x_{1}^{\prime}\left(x_{1}^{\prime}-x_{1}\right)}\right]_{+}+\int_{-1+\xi}^{1+\xi} d x_{2}^{\prime}\left[\frac{x_{2} \rho\left(x_{2}^{\prime}, x_{2}\right)}{x_{2}^{\prime}\left(x_{2}^{\prime}-x_{2}\right)}\right]_{+}\right) F^{\uparrow \downarrow \uparrow}\left(x_{1}^{\prime}, x_{2}^{\prime}, x_{3}\right)\right.} \\
+ & \left(\int_{-1+\xi}^{1+\xi} d x_{1}^{\prime}\left[\frac{x_{1} \rho\left(x_{1}^{\prime}, x_{1}\right)}{x_{1}^{\prime}\left(x_{1}^{\prime}-x_{1}\right)}\right]_{+}+\int_{-1+\xi}^{1+\xi} d x_{3}^{\prime}\left[\frac{x_{3} \rho\left(x_{3}^{\prime}, x_{3}\right)}{x_{3}^{\prime}\left(x_{3}^{\prime}-x_{3}\right)}\right]_{+}\right) F^{\uparrow \downarrow \uparrow}\left(x_{1}^{\prime}, x_{2}, x_{3}^{\prime}\right) \\
+ & \left(\int_{1+\xi}^{1+\xi} d x_{2}^{\prime}\left[\frac{x_{2} \rho\left(x_{2}^{\prime}, x_{2}\right)}{x_{2}^{\prime}\left(x_{2}^{\prime}-x_{2}\right)}\right]_{+}^{1+\xi}+\int_{-1+\xi}^{1+\xi} d x_{3}^{\prime}\left[\frac{x_{3} \rho\left(x_{3}^{\prime}, x_{3}\right)}{x_{3}^{\prime}\left(x_{3}^{\prime}-x_{3}\right)}\right]_{+}^{1+\xi}\right) F^{\uparrow \downarrow \uparrow}\left(x_{1}, x_{2}^{\prime}, x_{3}^{\prime}\right) \\
+ & \frac{1}{2 \xi-x_{3}}\left(\int_{-1+\xi}^{1+\xi} d x_{1}^{\prime} \frac{x_{1}}{x_{1}^{\prime}} \rho\left(x_{1}^{\prime}, x_{1}\right)+\int_{-\xi}^{1+\xi} d x_{2}^{\prime} \frac{x_{2}}{x_{2}^{\prime}} \rho\left(x_{2}^{\prime}, x_{2}\right)\right) F^{\uparrow \downarrow \uparrow}\left(x_{1}^{\prime}, x_{2}^{\prime}, x_{3}\right) \\
+ & \left.\left.\frac{1}{2 \xi-x_{1}}\left(\int_{-1+\xi}^{1+\xi} d x_{2}^{\prime} \frac{x_{2}}{x_{2}^{\prime}} \rho\left(x_{2}^{\prime}, x_{2}\right)+\int_{-1+\xi}^{1+\xi} d x_{3}^{\prime} \frac{x_{3}^{\prime}}{x_{3}^{\prime}} \rho\left(x_{3}^{\prime}, x_{3}\right)\right) F^{\uparrow \downarrow \uparrow}\left(x_{1}, x_{2}^{\prime}, x_{3}^{\prime}\right)\right]\right\}
\end{aligned}
$$

with integration region restricted by : $\rho(x, y)=\theta(x \geq y \geq 0)-\theta(x \leq y \leq 0)$, and $x_{i}^{\prime} \in[-1+\xi, 1+\xi]$

No detailed study yet

## Dirac decomposition

decompose $\left.\left\langle\pi\left(p_{\pi}\right)\right| \epsilon^{i j k} u_{\alpha}^{i}\left(z_{1} n\right) u_{\beta}^{j}\left(z_{2} n\right) d_{\gamma}^{k}\left(z_{3} n\right)|p(p, s)\rangle\right|_{n^{2}=0}$ on independent Dirac structures ( $\Delta=p_{\pi}-p, 2 P=p_{\pi}+p$ )

$$
\begin{aligned}
&\left(u=\text { nucleon spinor, } \widehat{a}=a^{\mu} \gamma^{\mu}, \sigma_{a b}=[\widehat{a}, \widehat{b}] / 2\right) \\
&\left(v_{1}\right)_{\alpha \beta \gamma}=(\widehat{P} C)_{\alpha \beta}(\widehat{P} u)_{\gamma}\left(a_{1}\right)_{\alpha \beta \gamma}=\left(\widehat{P} \gamma^{5} C\right)_{\alpha \beta}\left(\gamma^{5} \widehat{P} u\right)_{\gamma} \\
&\left(v_{2}\right)_{\alpha \beta \gamma}=(\widehat{P} C)_{\alpha \beta}(\widehat{\Delta} u)_{\gamma}\left(a_{2}\right)_{\alpha \beta \gamma}=\left(\widehat{P} \gamma^{5} C\right)_{\alpha \beta}\left(\gamma^{5} \widehat{\Delta} u\right)_{\gamma} \\
&\left(t_{1}\right)_{\alpha \beta \gamma}=\left(\sigma_{P \mu} C\right)_{\alpha \beta}\left(\gamma^{\mu} \widehat{P} u\right)_{\gamma}\left(t_{2}\right)_{\alpha \beta \gamma}=\left(\sigma_{P \mu} C\right)_{\alpha \beta}\left(\gamma^{\mu} \widehat{\Delta} u\right)_{\gamma} \\
&\left(t_{3}\right)_{\alpha \beta \gamma}=\frac{1}{M}\left(\sigma_{P \Delta} C\right)_{\alpha \beta}(\widehat{P} u)_{\gamma}\left(t_{4}\right)_{\alpha \beta \gamma}=\frac{1}{M}\left(\sigma_{P \Delta} C\right)_{\alpha \beta}(\widehat{\Delta} u)_{\gamma}
\end{aligned}
$$

equivalent to helicity decomposition of $p \rightarrow \pi q q q$

$$
v_{1}-2 \xi v_{2}, a_{1}-2 \xi a_{2}, t_{1}-2 \xi t_{2} \text { survive at } \Delta_{T}=0
$$

and define scalar functions $V_{1}, A_{1}, T_{1}, V_{2}, A_{2}, T_{2}, T_{3}, T_{4}$

## Polynomiality property

As for GPDs Lorentz invariance constrains skewness dependence :
$\Rightarrow$ Define $x_{i}$ Mellin moments

$$
\left\langle x_{1}^{n_{1}} x_{2}^{n_{2}} x_{3}^{n_{3}} H_{s}^{\pi N}\right\rangle=\int d x_{1} \int d x_{2} \int d x_{3} \delta\left(x_{1}+x_{2}+x_{3}-2 \xi\right) x_{1}^{n_{1}} x_{2}^{n_{2}} x_{3}^{n_{3}} H_{s}^{\pi N}\left(x_{1}, x_{2}, x_{3}, \xi, \Delta\right)
$$

$\rightleftharpoons$ They are expressed through Form Factors of local operators
$\leadsto$ Get polynomials in $\xi$ up to $\xi^{n_{1}+n_{2}+n_{3}+1}$

$$
\begin{gathered}
\left\langle x_{1}^{n_{1}} x_{2}^{n_{2}} x_{3}^{\left.n_{3}\left\{V_{i}, A_{i}, T_{1,2}\right\}\right\rangle=} \sum_{n=1}^{N}(-1)^{N-n}(2 \xi)^{n} \sum_{i=0}^{n_{1}} \sum_{j=0}^{n_{2}} \sum_{k=0}^{n_{3}} \delta_{i+j+k, n} A_{i j k}^{\left\{V_{i}, A_{i}, T_{1,2}\right\}\left(n_{i}\right)}\left(\Delta^{2}\right)\right. \\
-(2 \xi)^{N+1} C_{N+1}^{\left\{V_{i}, A_{i}, T_{1,2}\right\}\left(n_{1}, n_{2}, n_{3}\right)}\left(\Delta^{2}\right) \quad\left(\xi^{N+1} \rightarrow\right. \text { D-term) } \\
\left\langle x_{1}^{n_{1}} x_{2}^{n_{2}} x_{3}^{n_{3}}\left\{T_{3,4}\right\}\right\rangle= \\
\text { TO BE SATISFIED BY ALL CONSISTENT MODELS }
\end{gathered}
$$

## Nucleon exchange in TDA framework

$\rightleftharpoons$ Write effective Lagrangian for $\pi \bar{N} N$ interaction :
$\mathcal{H}_{\mathrm{eff}}=-i g_{\pi N N} \bar{N}_{\alpha}\left(\sigma_{a}\right)_{\beta}^{\alpha} \gamma_{5} N^{\beta} \pi_{a}$

$\pi$
$\leadsto$ Get $\pi N$ matrix element

$$
\left\langle\pi_{a}\left(p_{\pi}\right)\right| \widehat{O}_{\rho \tau \chi}^{\alpha \beta \gamma}\left(\lambda_{i} n\right)\left|N_{\iota}\left(p_{1}\right)\right\rangle=\sum_{s_{p}}\langle 0| \widehat{O}_{\rho \tau \chi}^{\alpha \beta \gamma}\left(\lambda_{i} n\right)\left|N_{\kappa}\left(-\Delta, s_{p}\right)\right\rangle\left(\sigma_{a}\right)_{\iota}^{\kappa} \frac{\kappa g_{\pi N N} \bar{U}_{\varrho}\left(-\Delta, s_{p}\right)}{\Delta^{2}-M^{2}}\left(\gamma^{5} U\left(p_{1}, s_{1}\right)\right)_{\varrho} .
$$

$\leadsto$ Decompose on Dirac struct. and get contrib. to $I=\frac{1}{2} \pi N$ TDAs $\left\{V_{1}, A_{1}, T_{1}\right\}^{(\pi N)_{1 / 2}}\left(x_{1}, x_{2}, x_{3}\right)=\Theta_{\mathrm{ERBL}}\left(x_{1}, x_{2}, x_{3}\right) \times\left(g_{\pi N N}\right) \frac{M f_{\pi}}{\Delta^{2}-M^{2}} 2 \xi \frac{1}{(2 \xi)^{2}}\left\{V^{p}, A^{p}, T^{p}\right\}\left(\frac{x_{1}}{2 \xi}, \frac{x_{2}}{2 \xi}, \frac{x_{3}}{2 \xi}\right) ;$

$$
\begin{aligned}
\left\{V_{2}, A_{2}, T_{2}\right\}^{(\pi N)_{1 / 2}}\left(x_{1}, x_{2}, x_{3}\right) & =\frac{1}{2}\left\{V_{1}, A_{1}, T_{1}\right\}^{(\pi N)_{1 / 2}}\left(x_{1}, x_{2}, x_{3}\right) \quad ; \quad T_{3}^{(\pi N)}=T_{4}^{(\pi N)}=0 \\
& \text { with } \Theta_{\mathrm{ERBL}}\left(x_{1}, x_{2}, x_{3}\right) \equiv \prod_{k=1}^{3} \theta\left(0 \leq x_{k} \leq 2 \xi\right) .
\end{aligned}
$$

Nucleon exchange contrib. is a pure $D$ - term contribution.

## Models for TDAs

$\Rightarrow$ Closest object : Baryon Distribution Amplitude $\phi^{N}$ : known from RG analysis, Conf. Inv., Lattice , QCD sum rules
$\Rightarrow$ CHIRAL LIMIT of $p \rightarrow \pi$ TDA $(\xi \rightarrow 1)$

$$
\begin{aligned}
& <\pi^{a}(k)|O| P(p, s)>=\frac{-i}{f_{\pi}}<0\left|\left[Q_{5}^{a}, O\right]\right| P(p, s)> \\
& \left.\phi_{1}^{(\pi N)_{1 / 2}}\left(x_{1}, x_{2}, x_{3}, \xi=1, \Delta^{2}=M^{2}\right)\right|_{\substack{\text { soft } \\
\text { pion }}}=\frac{1}{24} \phi^{N}\left(x_{1}, x_{2}, x_{3}\right)+\frac{1}{6} \phi^{N}\left(x_{3}, x_{2}, x_{1}\right) ;
\end{aligned}
$$

$$
\begin{aligned}
& \left.\phi_{1}^{(\pi N)_{3 / 2}}\left(x_{1}, x_{2}, x_{3}, \xi=1, \Delta^{2}=M^{2}\right)\right|_{\substack{\text { soft } \\
\text { pion }}}=\frac{1}{4}\left(\phi^{N}\left(x_{1}, x_{2}, x_{3}\right)+\phi^{N}\left(x_{3}, x_{2}, x_{1}\right)\right) \text {; } \\
& \left.\left.\phi_{2}^{(\pi N)_{3 / 2}}\left(x_{1}, x_{2}, x_{3}, \xi=1, \Delta^{2}=M^{2}\right)\right|_{\substack{\text { soot } \\
\text { pion }}}=-\frac{1}{2} \phi_{1}^{(\pi N)_{3 / 2}}\left(x_{1}, x_{2}, x_{3}, \xi=1, \Delta^{2}=M^{2}\right) \right\rvert\,
\end{aligned}
$$

$\Rightarrow$ Skew the chiral limit away from $\xi=1$ limit
through quadruple distributions (cf Radyushkin's double dist. for GPDs) which populate the DGLAP regions.

## TDA modeling

from BP,LS + Kirill Semenov-Tian-Shansky, Phys Rev D82 (2010) 094030

"quark-diquark" $(\omega, v)$ coordinates

( $x_{1}, x_{2}, x_{3}$ ) barycentric coordinates

## Conclusions

GPDs and TDAs explore confinement dynamics of quarks in hadrons in a complementary way.
They are matrix elements of different non local light cone operators
GPDs extraction needs more understanding of NLO corrections
$\Rightarrow$ Timelike Compton Scattering $=$ a useful complement to dVCS
$\Rightarrow$ TCS data from ZEUS and LHC ... and from JLab12?
TDAs extraction is crucial to probe meson content of baryons
$\rightleftharpoons$ First signals at JLab at 6 GeV
$\Rightarrow$ CLAS12 proposals on pseudoscalar and vector meson production : backward $\varphi$ (strangeness in the nucleon)
$\Rightarrow$ PANDA at GSI-FAIR and COMPASS with $\pi$ beam : timelike channels for TDA extraction

## Please note

6th Int. Conf. on Quarks and Nuclear Physics


40 years of development of QCD
École Polytechnique, Palaiseau (France), APRIL 16-20, 2012 http ://qnp2012.sciencesconf.org/

