F-theory and Model Building

Andrés Collinucci

ASC, LMU, Munich

Grenoble, July 23, 2011

A promising tool

- F-theory: An intuitive tool for embedding GUT models into string theory
- Goes beyond perturbative D-brane situations
- Algebraic geometry $\rightarrow$ spectacular control over F-term aspects of scenarios

BHV/DW Revival

$\sim$ 220 hits on "find t F-theory" on spires:

- Pre-KKLT: $\sim$ 90 for $2003 > t \geq 1996$
- Post-KKLT, Pre-BHV/DW: $\sim$ 20 for $2008 > t \geq 2003$
- Post-BHV/DW: $\sim$ 110 for $t \geq 2008$

Leitmotiv of talk: Euphoric phases vs. Contemplative phases

F-theory

A promising tool

- F-theory: An intuitive tool for embedding GUT models into string theory
- Goes beyond perturbative D-brane situations
- Algebraic geometry $\rightarrow$ spectacular control over F-term aspects of scenarios

BHV/DW Revival

$\sim$ 220 hits on "find t F-theory" on spires:

- Pre-KKLT: $\sim$ 90 for $2003 > t \geq 1996$
- Post-KKLT, Pre-BHV/DW: $\sim$ 20 for $2008 > t \geq 2003$
- Post-BHV/DW: $\sim$ 110 for $t \geq 2008$

Leitmotiv of talk: Euphoric phases vs. Contemplative phases

A promising tool

- F-theory: An intuitive tool for embedding GUT models into string theory
- Goes beyond perturbative D-brane situations
- Algebraic geometry $\rightarrow$ spectacular control over F-term aspects of scenarios

BHV/DW Revival

$\sim$ 220 hits on "find t F-theory" on spires:

- Pre-KKLT: $\sim$ 90 for $2003 > t \geq 1996$
- Post-KKLT, Pre-BHV/DW: $\sim$ 20 for $2008 > t \geq 2003$
- Post-BHV/DW: $\sim$ 110 for $t \geq 2008$

Leitmotiv of talk: Euphoric phases vs. Contemplative phases

Outline

1. Introduction: F-theory from M-theory

2. Modern developments

3. Pressing issue: Fluxes

Outline

D6-branes in IIA

Supergravity solution:

$$
\begin{aligned}
ds^2 &= (1 + M_1/r)^{-1/2} \left(-dt^2 + \sum_{i=1}^{6} dx_i^2 \right) \\
&+ (1 + M_1/r)^{1/2} \left(dr^2 + r^2 \, d\Omega_2^2 \right) \\
e^{\phi} &= e^{\phi_0} (1 + M_1/r)^{-3/4} \qquad C_\mu = (0, \vec{A}) \, .
\end{aligned}
$$

String coupling varies in space. It decreases as $r \to 0$.

Main feature: The D6-brane "backreacts" on $(g_{\mu\nu}, \phi, C_\mu)$.

Strong coupling $\to 11d$

Kaluza-Klein uplift to 11 dimensions:

$$g^{(11)}_{mn} = \begin{pmatrix} g^{(11)}_{\theta\theta} & g^{(11)}_{\theta\mu} \\ g^{(11)}_{\theta\mu} & g^{(11)}_{\mu\nu} \end{pmatrix} = \begin{pmatrix} e^{\phi} & C_\mu \\ C_\mu & g_{\mu\nu} \end{pmatrix},$$

string coupling measured by radius of S^1.

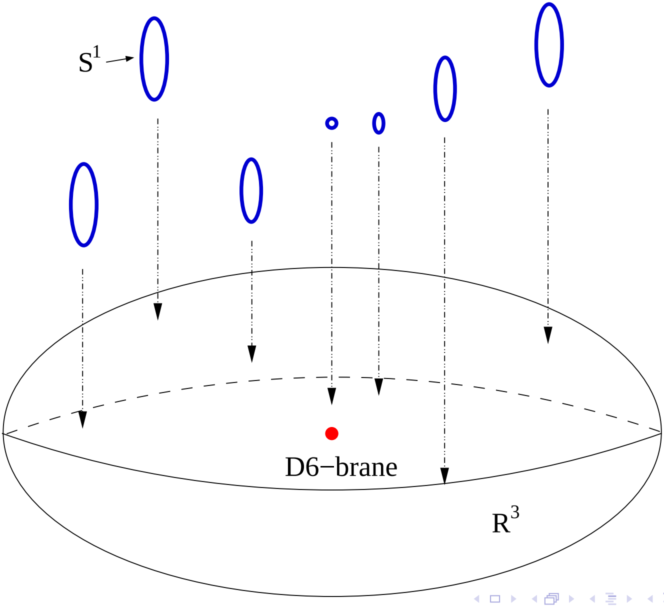
D6 lifts to

$$\begin{aligned} ds^2_{11} &= -dt^2 + \sum_{i=1}^{6} dx_i^2 + (1 + M_1/r)\,(dr^2 + r^2\, d\,\Omega_2^2) \\ &+ (1 + M_1/r)^{-1}\,(d\theta + A_\phi \cdot d\,\phi)^2\,. \end{aligned}$$

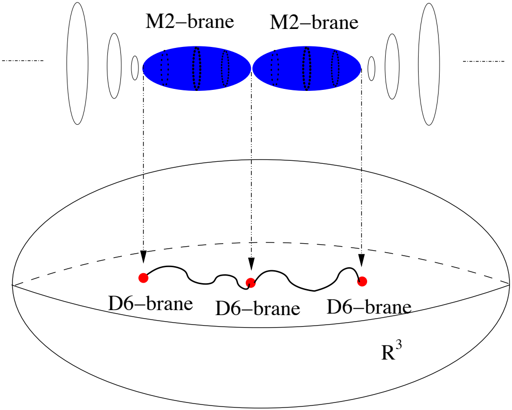
Completely geometric

Taub-NUT

The D6-brane lifts to a pure geometry: The Taub-NUT space.



Non-Abelian singularities



Taub-NUT centers approach → ADE singularity
→ new light states ∼ enhanced gauge group

The axio-dilaton

Define the complex scalar $\tau \equiv C_{(0)} + i\, e^{\phi}$.

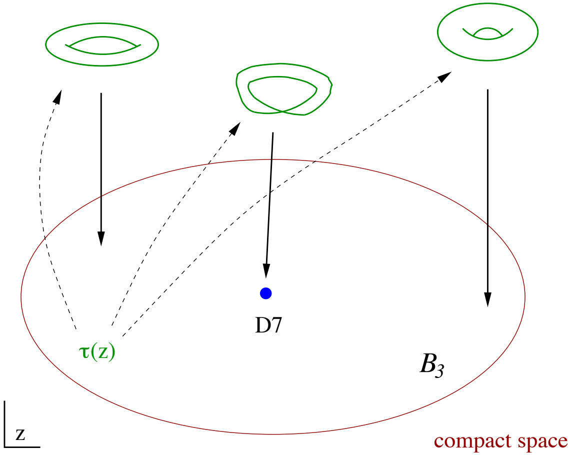
Conjectured *exact* symmetry of IIB:

$$\tau \to \frac{a\,\tau + b}{c\,\tau + d}\,,$$

for $a, b, c, d \in \mathbb{Z}$ and $a\,d - c\,d = 1$.

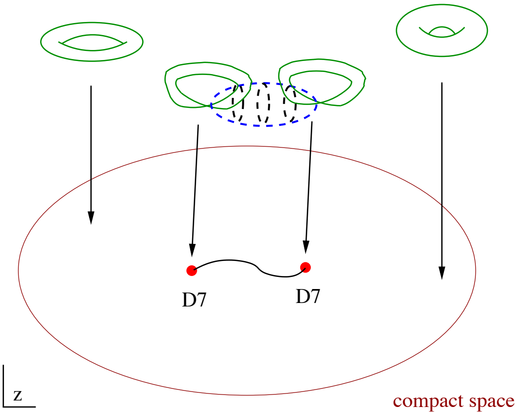
$SL(2\,,\mathbb{Z})$, *S-duality* group. Same as modular group of the torus.

Torus-fibration



F-theory encodes compactification + axio-dilaton data into one 4 + 8-dimensional geometric object: $\mathbb{R}^4 \times CY_4 = \mathbb{R}^4 \times B_3 \tilde{\times} T^2$

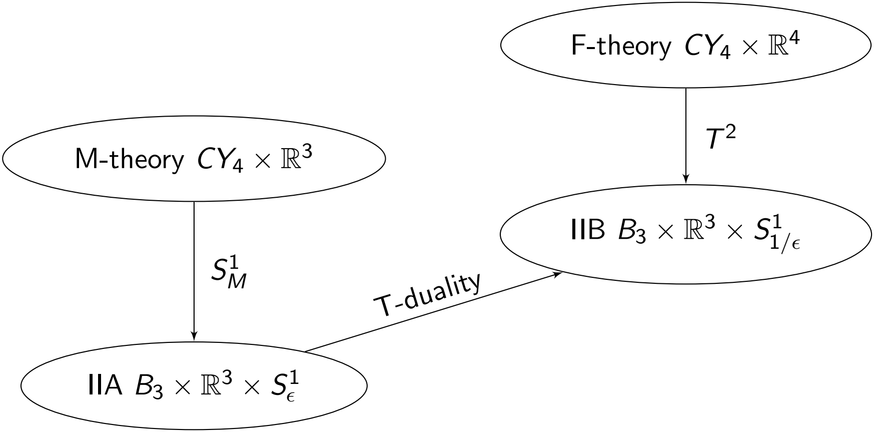
Non-Abelian singularities



Singular fibers approach → ADE singularity of the CY_4

M/F-theory duality

Unlike M-theory, extra two dimensions are not physical. However, by T-duality, one can make sense of it through M-theory.



Outline

Modern developments

[Beasley, Heckman, Vafa], [Donagi, Wijnholt] '08

- E8 structure: Can package GUT reps., generate perturbatively forbidden Yukawa's.
- New way of breaking GUT group via fluxes without making $U(1)_Y$ massive.
- Decoupling: Can send $M_{pl} \to \infty$ while keeping g_{YM} constant. Need: GUT 7-brane wraps shrinkable 4-cycle

Expectation of finiteness

- Internal 3-fold should be positively curved (Fano) $\rightsquigarrow$ only about 100 exist.
- Decoupling $\iff$ GUT brane on Del Pezzo $\rightsquigarrow$ only 8 exist.

$\Longrightarrow$ Can make genericity predictions.
Can do "local model building"

Modern developments

[Beasley, Heckman, Vafa], [Donagi, Wijnholt] '08

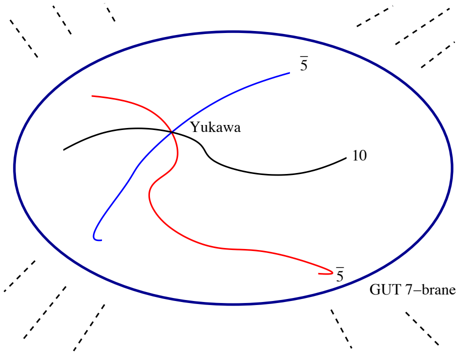
- E8 structure: Can package GUT reps., generate perturbatively forbidden Yukawa's.
- New way of breaking GUT group via fluxes without making $U(1)_Y$ massive.
- Decoupling: Can send $M_{pl} \to \infty$ while keeping g_{YM} constant. Need: GUT 7-brane wraps shrinkable 4-cycle

Expectation of finiteness

- Internal 3-fold should be positively curved (Fano) $\rightsquigarrow$ only about 100 exist.
- Decoupling $\iff$ GUT brane on Del Pezzo $\rightsquigarrow$ only 8 exist.

$\implies$ Can make genericity predictions.
Can do "local model building"

Local model building



'Zoom in' on GUT 7-brane

...'zoom in' on a patch *within* GUT 7-brane

Finiteness debunked

[Córdova '09]

- Shrinkability to a point → ~~Fano 3-fold~~.
  Otherwise, only shrinkable to a curve.
- Once we give up Fano → Del Pezzo's no longer priviledged: Infinite series of possible surfaces.
- Matter curves are tightly interrelated:

$$3\,\Sigma_{10} - \Sigma_5 + 5\,c_1(S) = 0$$

- Yuwaka points are tightly interrelated:

$$p(SU(7)) + 15\,p(E_6) - 22\,p(SO(12)) = 30\,c_1(S) \cdot c_1(S)$$

So finiteness is a myth. On the other hand, things get constrained already at the local level.

Finiteness debunked

[Córdova '09]

- Shrinkability to a point $\longrightarrow$ ~~Fano 3-fold~~. Otherwise, only shrinkable to a curve.
- Once we give up Fano $\longrightarrow$ Del Pezzo's no longer priviledged: Infinite series of possible surfaces.
- Matter curves are tightly interrelated:

$$3\,\Sigma_{10} - \Sigma_5 + 5\,c_1(S) = 0$$

- Yuwaka points are tightly interrelated:

$$p(SU(7)) + 15\,p(E_6) - 22\,p(SO(12)) = 30\,c_1(S)\cdot c_1(S)$$

So finiteness is a myth. On the other hand, things get constrained already at the local level.

Outline

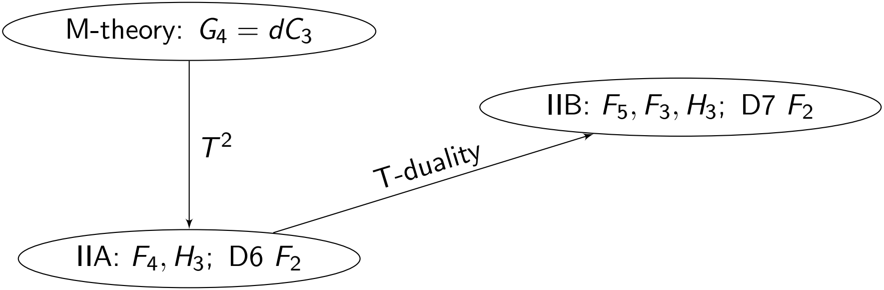
1. Introduction: F-theory from M-theory

2. Modern developments

3. Pressing issue: Fluxes

Fluxes

Axio-dilaton $\tau = C_0 + ie^{-\phi}$ is geometrized, the rest is not.

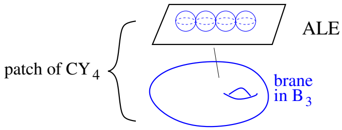


G_4 controls important data:

- D3-tadpole $\sim \int_{X_4} G \wedge G$
- Chiral spectrum in 4d
- F-term $\rightsquigarrow$ constraints on moduli: E.g. $W \sim \int G_4 \wedge \Omega^{0,4}$

Spectral cover

Clever trick: Treat CY_4 locally as an ALE fibration over GUT-brane



Procedure:

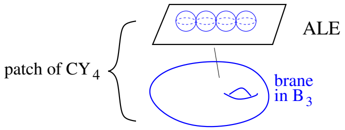
1. Define 5-fold cover $\pi : \tilde{S} \to S$.
2. Define appropriate line bundle $\mathcal{L}$ over $\tilde{S}$
3. Push $\mathcal{L}$ onto S: $\pi_*(\mathcal{L}) \sim \mathcal{V}$. V is vector bundle that encodes G_4-flux indirectly.

Drawbacks:

- Only locally defined. See works by [Dolan, Marsano, Saulina, Schäfer-Nameki] and [Grimm, Kerstan, Palti, Weigand] for global completions.
- Works only for gauge groups G that are commutants in E_8 of structure group of V. Only good for $SU(5)$ models.

Spectral cover

Clever trick: Treat CY_4 locally as an ALE fibration over GUT-brane



Procedure:

1. Define 5-fold cover $\pi : \tilde{S} \longrightarrow S$.
2. Define appropriate line bundle $\mathcal{L}$ over $\tilde{S}$
3. Push $\mathcal{L}$ onto S: $\pi_*(\mathcal{L}) \sim \mathcal{V}$. V is vector bundle that encodes G_4-flux indirectly.

Drawbacks:

- Only locally defined. See works by [Dolan, Marsano, Saulina, Schäfer-Nameki] and [Grimm, Kerstan, Palti, Weigand] for global completions.
- Works only for gauge groups G that are commutants in E_8 of structure group of V. Only good for $SU(5)$ models.

Poincaré invariance

$$G_4 = \omega^{(2)}_{B_3} \wedge d\theta_M \wedge d\theta_T$$

$$\Big\downarrow \text{IIA}$$

$$H_3 = \omega^{(2)}_{B_3} \wedge d\theta_T \xrightarrow{\text{T}} S^1\text{-fibration over } \mathbb{R}^3$$

$\rightsquigarrow$ ~~Poincaré invariance~~

$$G_4 = \omega^{(4)}_{B_3}$$

$$\Big\downarrow \text{IIA}$$

$$F_4 = \omega^{(4)}_{B_3} \xrightarrow{\text{T}} F_5 = \omega^{(4)}_{B_3} \times d\theta_T$$

where spacetime $= \mathbb{R}^3 \times S^1_T \rightsquigarrow$

~~Poincaré invariance~~

G_4 must have exactly 1 leg along fiber $\rightsquigarrow$ cannot be $\alpha^{(1,1)} \wedge \omega^{(1,1)}$

HARD!

Poincaré invariance

$$G_4 = \omega^{(2)}_{B_3} \wedge d\theta_M \wedge d\theta_T$$

$$\downarrow \text{IIA}$$

$$H_3 = \omega^{(2)}_{B_3} \wedge d\theta_T \xrightarrow{\text{T}} S^1\text{-fibration over } \mathbb{R}^3$$

$\rightsquigarrow$ ~~Poincaré invariance~~

$$G_4 = \omega^{(4)}_{B_3}$$

$$\downarrow \text{IIA}$$

$$F_4 = \omega^{(4)}_{B_3} \xrightarrow{\text{T}} F_5 = \omega^{(4)}_{B_3} \times d\theta_T$$

where spacetime $= \mathbb{R}^3 \times S^1_T \rightsquigarrow$

~~Poincaré invariance~~

G_4 must have exactly 1 leg along fiber $\rightsquigarrow$ cannot be $\alpha^{(1,1)} \wedge \omega^{(1,1)}$

HARD!

Poincaré invariance

$$G_4 = \omega^{(2)}_{B_3} \wedge d\theta_M \wedge d\theta_T$$

$$\big\downarrow \text{IIA}$$

$$H_3 = \omega^{(2)}_{B_3} \wedge d\theta_T \xrightarrow{\text{T}} S^1\text{-fibration over } \mathbb{R}^3$$

$\rightsquigarrow$ ~~Poincaré invariance~~

$$G_4 = \omega^{(4)}_{B_3}$$

$$\big\downarrow \text{IIA}$$

$$F_4 = \omega^{(4)}_{B_3} \xrightarrow{\text{T}} F_5 = \omega^{(4)}_{B_3} \times d\theta_T$$

where spacetime $= \mathbb{R}^3 \times S^1_T \rightsquigarrow$

~~Poincaré invariance~~

G_4 must have exactly 1 leg along fiber $\rightsquigarrow$ cannot be $\alpha^{(1,1)} \wedge \omega^{(1,1)}$

HARD!

G_4-fluxes

[A. Braun, A. C., R. Valandro]

Rewrite Weierstrass equation for elliptic fibration:

$$y^2 = x^3 + f\,x + g \quad \longrightarrow Y_+\,Y_- + a_6 = X\,Q$$

Impose constraint $a_6 = \rho\,\tau$

$\longrightarrow$ New algebraic 4-cycles:

$$\Sigma_4 : Y_\pm = 0 \quad \cap \quad \rho = 0 \quad \cap \quad X = 0\,.$$

New elements of $H^{2,2} \cap H^2(\mathbb{Z})$. **Not** intersections of 2 divisors in CY_4

$$G_4 \text{ flux} \sim \text{Poincaré dual to } \Sigma_4$$

G_4-fluxes

[A. Braun, A. C., R. Valandro]

Rewrite Weierstrass equation for elliptic fibration:

$$y^2 = x^3 + f\,x + g \quad \longrightarrow Y_+\,Y_- + a_6 = X\,Q$$

Impose constraint $a_6 = \rho\,\tau$

$\longrightarrow$ New algebraic 4-cycles:

$$\Sigma_4 : Y_\pm = 0 \quad \cap \quad \rho = 0 \quad \cap \quad X = 0\,.$$

New elements of $H^{2,2} \cap H^2(\mathbb{Z})$. **Not** intersections of 2 divisors in CY_4

G_4 flux $\sim$ Poincaré dual to Σ_4

Comparison with IIB

Claim: In weak coupling limit, our new $G_4 \longleftrightarrow F_2$ on D7-branes.

Checks for generic, D7/O7 configuration:

- Induced D3-charges match:

$$\frac{1}{2} \int_{D7} F_2^2 = -\frac{1}{2} \int_{CY_4} G_4^2$$

- Cplex str. mod. of CY_4 match 7-brane moduli

$$CY_4 : Y_+ Y_- + \rho\,\tau = X\,Q \quad \longleftrightarrow \quad D7 : \eta^2 + \xi^2 \rho\,\tau = 0$$

Checks for, generic $\mathbf{D7}_{O(1)}$ with $\mathbf{D7}_{Sp(1)}$ and $SU(2)$ stacks:

- Induced D3-charges match for perturbative case
- Can compute chiral index at $\Sigma = D7_{O(1)} \cap D7_{Sp(1)}$ intersection:

$$\int_{\Sigma} F_{D7_{O(1)}} - F_{D7_{Sp(1)}} = \int_{\Sigma \times \mathbb{P}^1} G_4$$

Comparison with IIB

Claim: In weak coupling limit, our new $G_4 \longleftrightarrow F_2$ on D7-branes.

Checks for generic, D7/O7 configuration:

- Induced D3-charges match:

$$\frac{1}{2}\int_{D7} F_2^2 = -\frac{1}{2}\int_{CY_4} G_4^2$$

- Cplex str. mod. of CY_4 match 7-brane moduli

$$CY_4 : Y_+ Y_- + \rho\,\tau = X\,Q \quad \longleftrightarrow \quad D7 : \eta^2 + \xi^2\,\rho\,\tau = 0$$

Checks for, generic D7$_{O(1)}$ with D7$_{Sp(1)}$ and $SU(2)$ stacks:

- Induced D3-charges match for perturbative case
- Can compute chiral index at $\Sigma = D7_{O(1)} \cap D7_{Sp(1)}$ intersection:

$$\int_{\Sigma} F_{D7_{O(1)}} - F_{D7_{Sp(1)}} = \int_{\Sigma\times\mathbb{P}^1} G_4$$

Conclusion

- Via its application to GUT model building, F-theory has gotten a facelift.
- Some aspects about genericity may have been oversold.
- However, the ideas are bright $\longrightarrow$ subject even more fascinating than expected.
- For the past year or so, more focus on fundamental aspects.