Pseudoscalar-photon mixing in an expanding Universe

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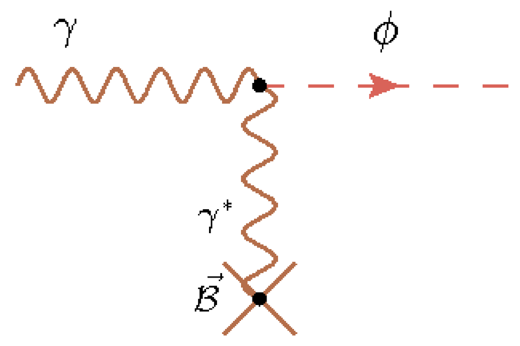
Abstract

We establish the equation of motion of pseudoscalar particles coupled to an electromagnetic field in a classical gravitational background through the use of conformal time and flat geometry. We show that in general the expansion of the universe leads to larger mixing than in a stationary universe. We also show that for a broad range of parameters, one can obtain a resonance mixing, i.e. a region in which the mixing becomes maximum!

Mixing Diagram

This is how they mix [1]



1. Dichroism
2. Birefringence

Features Of Expanding Universe

- The universe is expanding and flat
- The metric is $diag(1, -a^2(t), -a^2(t), -a^2(t))$ and not the Minkowski one.

  'Switch to conformal time parameter!'
- This facilitates the following
  1. The scaled metric that resembles the Minkowski one. The scale factor depends only on time
  2. Using of normal prescription of writing action with background gravity
  3. A few change of variables keeps the equation look less formidable!
  4. Approximate solution to the coupled system!

Methodology

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = dt^2 - a^2(t)\delta_{ij}dx^i dx^j \quad (1)$$

where,

$$g_{\mu\nu} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & -a^2 & 0 & 0 \\ 0 & 0 & -a^2 & 0 \\ 0 & 0 & 0 & -a^2 \end{vmatrix}$$

Next, we go to the conformal time by rescaling the physical time as,

$$a(\eta)d\eta = dt$$

Hence, in the conformal time the metric looks like -

$$g_{\mu\nu} = \begin{vmatrix} a^2 & 0 & 0 & 0 \\ 0 & -a^2 & 0 & 0 \\ 0 & 0 & -a^2 & 0 \\ 0 & 0 & 0 & -a^2 \end{vmatrix} = a^2(\eta)\eta_{\mu\nu} \quad (2)$$

We note, $\sqrt{-g} = a^4(\eta)$ & $\chi = a(\eta)\phi$. The first term in the action, namely, S_p

$$\frac{1}{2}\int \sqrt{-g}\, d^4x \left[g^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta} - m^2\phi^2\right] \quad (3)$$

will then become,

$$\int d\eta\, d^3x \left[\frac{1}{2}\partial^2\chi - \chi^2\left\{\frac{m^2a^2(\eta)}{2} - \frac{a''(\eta)}{2a(\eta)}\right\}\right] \quad (4)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = f_{\mu\nu}$$
$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu = \frac{f^{\mu\nu}}{a^4} \quad (5)$$

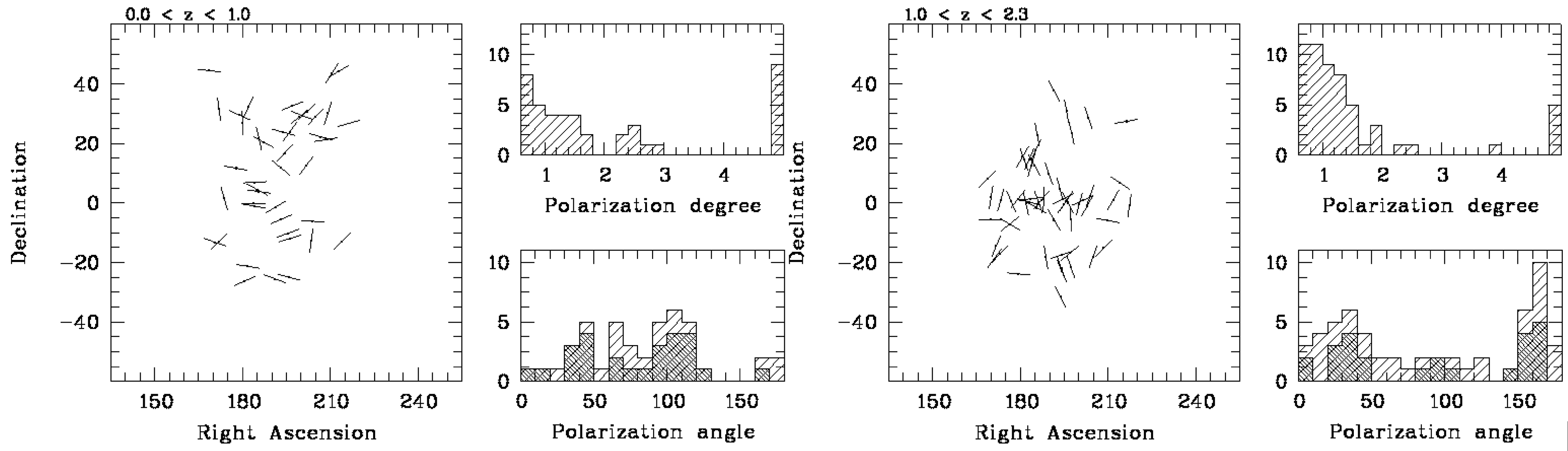
Since,

$$f^{\mu\nu} = \eta^{\mu\alpha}\eta^{\nu\beta}f_{\alpha\beta}$$

Hence, the term S_g remains unchanged, and we put this in terms of new variables as,

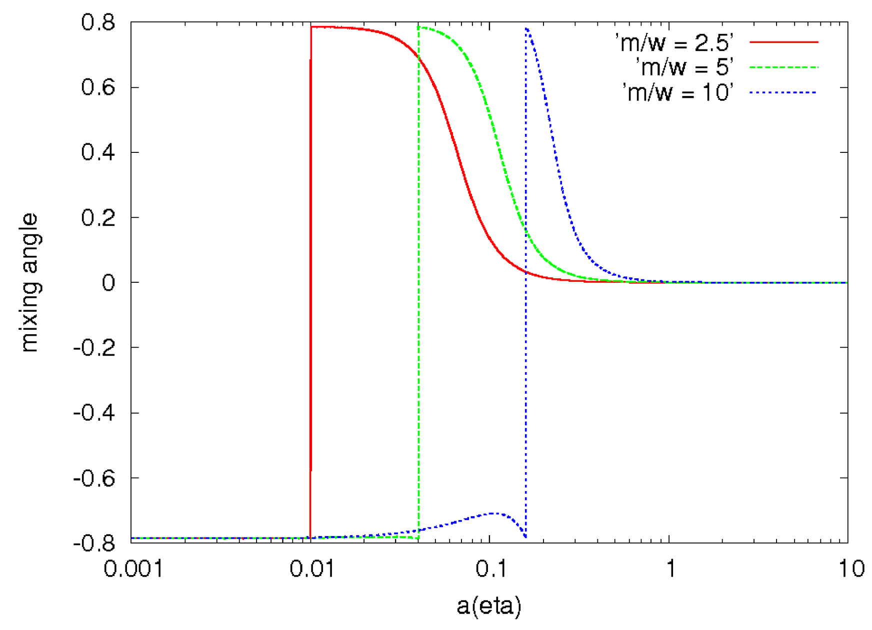
$$\frac{1}{4}\int d\eta\, d^3x \left[f^{\mu\nu}f_{\mu\nu}\right] = 0 \quad (6)$$

Observations

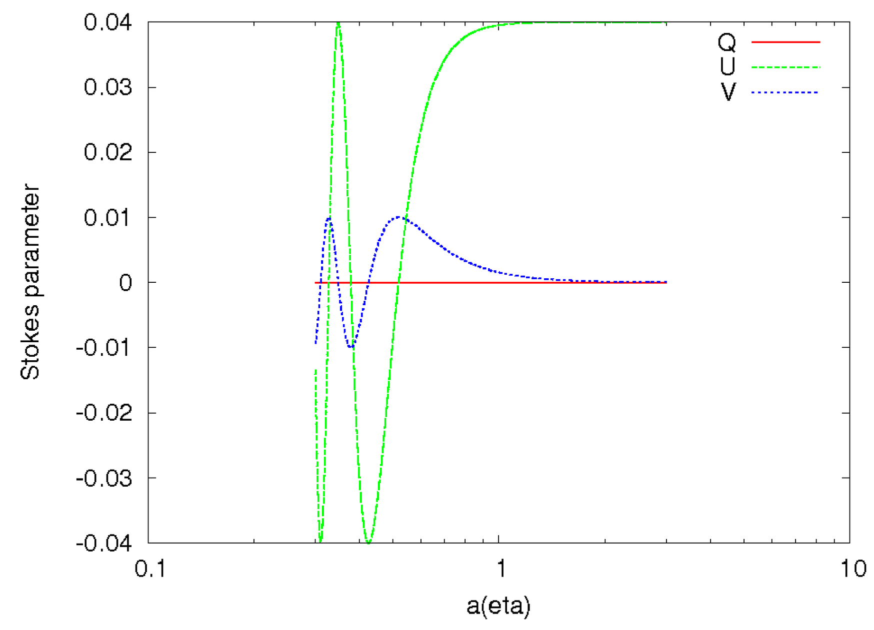


[2]

Mixing Angle



Stokes Parameter



Methodology cntd. I

$$\frac{1}{4}k\int \sqrt{-g}\, d^4x \left[-\tilde{F}^{\mu\nu}F_{\mu\nu}\phi\right] = 0 \quad (7)$$

We note that,

$$\tilde{F}^{\mu\nu} = \varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$$

. This Levi-Civita symbol, despite being isotropic, scales due to the expansion of the universe and can be written like,

$$\varepsilon^{\mu\nu\rho\sigma} = -\sqrt{-g}\epsilon^{\mu\nu\rho\sigma}$$

where the normal epsilon in fully covariant form, has a relation with its fully contravariant part, which is such to be,

$$\epsilon^{\mu\nu\rho\sigma} = -\epsilon_{\mu\nu\rho\sigma}$$

and further, it is normallized as -

$$\epsilon^{\mu\nu\rho\sigma} = \begin{cases} +1 & \text{if indices are evenly permuted,} \\ -1 & \text{if indices are not evenly permuted,} \\ 0 & \text{if indices are not all different.} \end{cases}$$

Extracting Lagrangian

The action of the system with gravitational background is as follows

$$\begin{aligned} S &= S_p + S_g + S_i \\ &= \frac{1}{2}\int \sqrt{-g}\, d^4x \left[g^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta} - m^2\phi^2\right] \\ &- \frac{1}{4}\int \sqrt{-g}\, d^4x \left[F^{\mu\nu}F_{\mu\nu}\right] \\ &- \frac{1}{4}k\int \sqrt{-g}\, d^4x \left[\tilde{F}^{\mu\nu}F_{\mu\nu}\phi\right] \end{aligned} \quad (8)$$

The resulting Lagrangians, which follow from this, are extracted below term by term.

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_p + \mathcal{L}_g + \mathcal{L}_i \\ &= \frac{1}{2}\left[\chi'^2 - \nabla\chi^2 - \left(m^2a^2(\eta) - \frac{a''(\eta)}{a(\eta)}\right)\chi^2\right] \\ &- \frac{1}{4}\left[f^{\mu\nu}f_{\mu\nu}\right] \\ &- \frac{1}{4}a^{-1}(\eta)\left[k\tilde{f}^{\mu\nu}f_{\mu\nu}\chi\right] \end{aligned} \quad (9)$$

Plasma Effects

The pair of equations (13,14) which looks like these, after the approximations -

$$\left[\frac{d^2}{d\eta^2} + (m^2a^2(\eta) + p_\chi^2)\right]\chi = -\frac{k}{a(\eta)}\vec{B}\cdot\vec{E}$$
$$\left[\frac{d^2}{d\eta^2} + (p_{\vec{E}}^2)\right]\vec{E} = \frac{k}{a(\eta)}\frac{\partial^2\chi}{\partial\eta^2}\vec{B} \quad (10)$$

Corrections arising due to medium

$$S_{\gamma_m} = \int \sqrt{-g}\omega_p^2(\eta)A_\mu A^\mu\, d^4x \quad (11)$$

which, when simplified looks like this -

$$S_{\gamma_m} = \int \underbrace{\omega_p^2 a(\eta)}\, A_\mu A^\mu\, d^4x \quad (12)$$

The underbrace shows the variation of photon mass with the scale factor (in conformal time).

EL Equation

The two in-homogenous gauge equation can be joined together -

$$\Box\vec{E} = k\frac{\partial^2}{\partial\eta^2}\left[\frac{\chi}{a(\eta)}\right]\vec{B} \quad (13)$$

Approximations made -

1. smallness of longitudinal component
2. static external magnetic field
   - However the electric field in outer space is absent and it can be dropped as is

Similarly, the pseudoscalar equation can be simplified and written (without any approximation) as -

$$\left[\Box\chi + \left\{m^2a^2(\eta) - \frac{a''(\eta)}{a(\eta)}\right\}\chi\right] = -\frac{k}{a(\eta)}\vec{E}\cdot\vec{B} \quad (14)$$

without approximation.

Conclusion

- Primary Objective is fullfilled - qualitatively
- The Result is independent of photon frequency - The effect is not seen in radio - has not been tested in X-ray/Gamma ray!

  'We need everything further away from these quasars to be polarised!'
- Else we have to rule out -
  1. Either the very low mass ALP's
  2. or the faraway magnetic field with cosmological lengthscale.
  3. Recent Studies On GRB spectra shows departure from synchrotron radiation and signature of photon-ALP mixing [3]
  4. Similar remarks are made by studying their empirically established Luminosity relations [4]

Thank You!

References

[1] S. Das et. al. J. Cosmol. Astropart. Phys., 0506, (2005), 002

[2] D. Hutsemékers, R. Cabanac, H. Lamy and D. Sluse, Astronomy and Astrophysics, 441, 915, (2005)

[3] O. Mena, S. Razzaque & F. Villaecusa-Navarro, arXiv:1101.1903

[4] C. Burrage, *Nuc. Phys B*, **194**, 190-195, *(2009)*