PHENOMENOLOGY OF IRREVERSIBLE PROCESSES FROM GRAVITY

Ayan Mukhopadhyay

LPTHE, Université Pierre et Marie Curie, Paris, France

Collaborator: Ramakrishnan Iyer (USC, Los Angeles, USA)

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Gauge/Gravity duality in a nutshell

4D gauge theory at large λ and N	$\leftrightarrow$	5D classical theory of gravity
A *pure* state	$\leftrightarrow$	A *horizonless regular* solution
An *equilibrium* state	$\leftrightarrow$	A *black hole* solution
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eg.

$< t_{\mu\nu} >$ in a state $\leftrightarrow$ Asymptotic deviation of *metric* from pure AdS giving Balasubramanian-Krauss stress tensor

Universality and Einstein's equations

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- Einstein's equations give us fluid mechanics, so can they also give us phenomenology of general irreversible phenomena?

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- We consider states which have equidistribution of quasiparticles in a CFT in internal space which includes color, spin, flavor charge, etc. at any point in phase space so we have one effective quasiparticle distibution $f(\mathbf{x}, \mathbf{v})$.

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- Are there special solutions which can be determined by energy-momentum tensor alone? Do they give derivation of phenomenological equations of irreversible processes?

The quasiparticle distribution

- The *local* velocity moments of the quasiparticle distribution in a local inertial frame :

$$f_{i_1 \dots i_n}(\mathbf{x}, t) = \int d^3v \, \left(v_{i_1} - u_{i_1}(\mathbf{x}, t) \right) \dots \left(v_{i_n} - u_{i_n}(\mathbf{x}, t) \right) \, f(\mathbf{x}, \mathbf{v}, t)$$

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- The energy-momentum tensor is parametrised by 10 variables
(i) the average density $n(\mathbf{x}, t)$ given by $< 1 >$,
(ii) the average velocity $u_i(\mathbf{x}, t)$,
(iii) the root mean square velocity giving $T(\mathbf{x}, t)$, and
(iv) the five independent traceless second velocity moment giving $p_{ij}(\mathbf{x}, t)$, the shear stress tensor.
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- Boltzmann equation is equivalent to coupled local equations for infinite velocity moments. Exceptionally, the variables $n(\mathbf{x}, t)$, $\mathbf{u}(\mathbf{x}, t)$, $T(\mathbf{x}, t)$ follow usual hydrodynamic equations coupling only to the shear-stress tensor $p_{ij}(\mathbf{x}, t)$ and the partially traced third velocity moment $S_i(\mathbf{x}, t)$, the heat current.

Conservative solutions of the Boltzmann equation (1/2)

- In the conservative solutions, all higher moments have special solutions where they are just *algebraic functions* of the 10 variables of the energy-momentum tensor $\left(n, \mathbf{u}, p_{ij}\right)(\mathbf{x}, t)$ and their *spatial* derivatives in the local inertial frame where the mean velocity vanishes [AM, Iyer]. Thus they have no independent dynamical parts. For example, in a monoatomic gas

$$S_i = \frac{15pR}{2B^{(2)}}\frac{\partial T}{\partial x_i} + \frac{3}{2B^{(2)}}\left(2RT\frac{\partial p_{ir}}{\partial x_r} + 7Rp_{ir}\frac{\partial T}{\partial x_r} - \frac{2p_{ir}}{\rho}\frac{\partial p}{\partial x_r}\right) + ... \quad ,$$

where p is the pressure and $B^{(2)}$ is given by the collision kernel.

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  where p is the pressure and $B^{(2)}$ is given by the collision kernel.
- The ten variables $\left(n, \mathbf{u}, p_{ij}\right)(\mathbf{x}, t)$ obey a closed set of dynamical equations where all the phenomenological coefficients are determined by the heat kernel of the Boltzmann equation.
- Further, there are special *purely hydrodynamic* solutions where $p_{ij}(\mathbf{x}, t)$ is an algebraic solution of the hydrodynamic variables,

$$p_{ij}(\mathbf{x}, t) = \frac{p}{B^{(2)}}\left(\frac{\partial u_m}{\partial x_n} + \frac{\partial u_n}{\partial x_m} - \frac{2}{3}\delta_{mn}\frac{\partial u_r}{\partial x_r}\right) + \ldots,$$

  giving the known *normal* solutions discovered earlier and leading to determination of all transport coefficients [Enskog, Chapman, Burnett, Stewart].

Conservative solutions of the Boltzmann equation (2/2)

- Covariantization :

$$
\begin{aligned}
u_i &\rightarrow u_\mu, \text{ with } u_\mu \eta^{\mu\nu} u_\nu = -1, \text{ and } u^0 > 0. \\
\pi_{ij} &\rightarrow \pi_{\mu\nu}, \text{ with } \pi_{\mu\nu}\eta^{\mu\nu} = 0 \text{ and } \pi_{\mu\nu} u^\nu = 0, \\
\text{time derivative} &\rightarrow u^\mu \partial_\mu, \\
\text{spatial derivative} &\rightarrow P_\mu{}^\nu \partial_\nu, \text{ with the spatial projector} \\
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- The phenomenological equations of u^μ, T and $\pi_{\mu\nu}$ can be determined on general grounds. We expect that the conservative solutions will form the universal sector at large N and λ, where the semiclassical Boltzmann equation loses its validity. All the universal phenomenological coefficients will be determined by gravity. Caution : Many coefficients do not vanish at strong coupling.

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- One can also argue that any arbitrary nonequilibrium state that thermalizes approximates an appropriate conservative solution at sufficiently late time. *Thus these phenomenological equations hold for general irreversible processes which are independent of initial state dynamics. Caution : Useful only at strong coupling*
- Conjecture : The phenomenological equations of the energy-momentum tensor give all solutions in pure gravity which equilibrates thermally with a regular future horizon for appropriate values of the phenomenological coefficients. [AM, Iyer]

Principles of general Weyl-covariant phenomenology (1/3)

- In gravity, it is natural to use Landau definitions :

u^μ : velocity of energy transport,

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- Derivatives ∂_μ can be promoted to Weyl-covariant derivatives with a Weyl connection made from the hydrodynamic variables only [Loganayagam]. As a result,$u \cdot \partial$ is promoted to a *Weyl-covariant time-derivative* $\mathcal{D}$.

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- Without loss of generality,
$\pi_{\mu\nu} = \pi^{(h)}_{\mu\nu} + \pi^{(nh)}_{\mu\nu}$, where
$\pi^{h}_{\mu\nu}$ is purely hydrodynamic built out of u^μ, T and their derivatives,
$\pi^{nh}_{\mu\nu}$ is independent dynamically from the hydrodynamic variables.

Principles of general Weyl-covariant phenomenology (2/3)

- The purely hydrodynamic part $\pi_{\mu\nu}^{(h)}$ is slowly varying spatio-temporally and can be systematically obtained using Weyl-covariance only in the derivative expansion (T times the length scale of variation). Up to second order in derivative expansion in gravity

$$\begin{aligned}\pi_{\mu\nu}^{(h)} &= -2(\pi T)^3\sigma_{\mu\nu} + (2-\ln 2)(\pi T)^2\mathcal{D}\sigma_{\mu\nu}\\ &+2(\pi T)^2\left(\sigma_\mu{}^\alpha\sigma_{\alpha\nu} - \frac{1}{3}P_{\mu\nu}\sigma_{\alpha\beta}\sigma^{\alpha\beta}\right)\\ &+\ln 2(\pi T)^2(\sigma_\mu{}^\alpha\omega_{\alpha\nu} + \sigma_\nu{}^\alpha\omega_{\alpha\mu}) + O(\epsilon^3),\end{aligned}$$

where

$$\begin{aligned}\sigma_{\mu\nu} &= \frac{1}{2}P_\mu{}^\alpha P_\nu{}^\beta(\partial_\alpha u_\beta + \partial_\beta u_\alpha) - \frac{1}{3}P_{\mu\nu}(\partial\cdot u),\\ \omega_{\mu\nu} &= \frac{1}{2}P_\mu{}^\alpha P_\nu{}^\beta(\partial_\alpha u_\beta - \partial_\beta u_\alpha).\end{aligned}$$

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- $\pi^{(nh)}_{\mu\nu}$ is slowly varying spatially but not temporally near equilibrium as quasinormal modes of gravity indicate. Its equation of motion can be constructed phenomenologically using both derivative and amplitude expansion (the typical value/equilibrium pressure), where we sum up time-derivatives at each order. Also $\pi^{(nh)}_{\mu\nu} = 0$, because gravity also admits purely hydrodynamic limit. Its equation is Weyl-covariant too.

Principles of general Weyl-covariant phenomenology (3/3)

- Up to some orders in derivative and amplitude expansion, the most general equation for $\pi^{(nh)}_{\mu\nu}$ is

$$\begin{aligned}\left(\sum_{n=0}^{\infty} D_R^{(1,n)}(\pi T)^n\mathcal{D}^n\right)\pi^{(nh)}_{\mu\nu} \quad = \quad & \frac{(\pi T)\lambda_1}{2}\left(\pi^{(nh)\alpha}_{\mu}\sigma_{\alpha\nu}+\pi^{(nh)\alpha}_{\nu}\sigma_{\alpha\mu}-\frac{2}{3}P_{\mu\nu}\pi^{(nh)}_{\alpha\beta}\sigma^{\alpha\beta}\right)\\ & +\frac{(\pi T)\lambda_2}{2}\left(\pi^{(nh)\alpha}_{\mu}\omega_{\alpha\nu}+\pi^{(nh)\alpha}_{\nu}\omega_{\alpha\mu}\right)\\ & -(\pi T)^4\sum_{n=0}^{\infty}\sum_{\substack{m=0\\ n+m\text{ is even}}}^{n} D_R^{(2,n,m)}(\pi T)^n\\ & \sum_{\substack{a,b=0\\ a+b=n\\ |a-b|=m}}^{n}\left[\mathcal{D}^a\pi_{ik}\mathcal{D}^b\pi_{kj}-\frac{1}{3}\delta_{ij}\mathcal{D}^a\pi_{pq}\mathcal{D}^b\pi_{pq}\right]\\ & +O(\epsilon^2\delta,\epsilon\delta^2,\delta^3).\end{aligned}$$

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& + \frac{(\pi T)\lambda_2}{2}\left(\pi_\mu^{(nh)\alpha}\omega_{\alpha\nu} + \pi_\nu^{(nh)\alpha}\omega_{\alpha\mu}\right) \\
& -(\pi T)^4 \sum_{n=0}^{\infty} \sum_{\substack{m=0 \\ n+m \text{ is even}}}^{n} D_R^{(2,n,m)}(\pi T)^n \\
& \qquad \sum_{\substack{a,b=0 \\ a+b=n \\ |a-b|=m}}^{n} \left[\mathcal{D}^a\pi_{ik}\mathcal{D}^b\pi_{kj} - \frac{1}{3}\delta_{ij}\mathcal{D}^a\pi_{pq}\mathcal{D}^b\pi_{pq}\right] \\
& + O(\epsilon^2\delta, \epsilon\delta^2, \delta^3).
\end{aligned}
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- New non-hydrodynamic phenomenological coefficients : $D_R^{(1,n)}$, $D_R^{(2,n,m)}$, λ_1, λ_2, ... (by definition these are dimensionless).

Special case : Homogeneous relaxation from gravity

- There are special *purely non-hydrodynamic solutions* which are spatially homogeneous. Here u^μ and T are constants, so $\pi^{(h)}_{\mu\nu} = 0$. In a global inertial frame $u^\mu = (1, 0, 0, 0)$. Also in this frame $\pi^{nh)}_{00} = \pi^{(nh)}_{0i} = \pi^{(nh)}_{i0} = 0$ and $\pi^{(nh)}_{ij}(t)$, so the conservation of energy and momentum are trivially satisfied.

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- The dual solutions of gravity in these special case can be argued to be manifestly regular at the future horizon in ingoing Eddington-Finkelstein coordinates [AM, Iyer]. We can show that the metric is regular provided the equation for $\pi^{(nh)}_{ij}$ is satisfied with $\mathcal{D} = d/dt$. We also find $D^{(1,0)}_R = -1$, $D^{(1,1)}_R = (\pi/2) + (1/4)\ \ln 2$, etc.; $D^{(2,0,0)}_R = 1/2$, etc. A complicated explicit recursion relation can be derived for $D^{(1,n)}_R$'s and $D^{(2,n,m)}_R$'s.

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- Connection to quasinomal modes : Consider the series

$$D_R(\omega) = \sum_{n=0}^{\infty} D_R^{(1,n)} (-i\pi T\omega)^n,$$

and analytically continue it to the LHP of ω. If this series has simple zeroes at discrete points in the LHP, then the analytic continuation of $\pi_{ij}(\omega)$ in LHP also has simple poles at those points. This should reproduce all the quasinormal modes of black branes when $\mathbf{k} = 0$. We hope to verify this numerically.

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- What are the field-theoretic definitions of the general phenomenological coefficients?
- How to check the conjecture in gravity for general spatially inhomogeneous non-hydrodynamic configurations? We are trying to do this numerically.