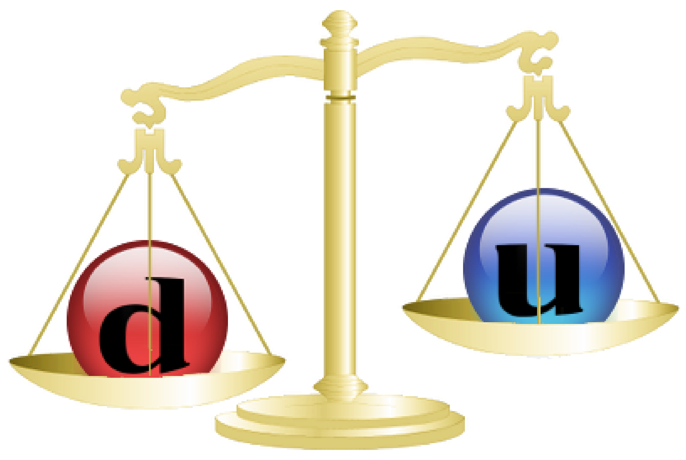

Determination of the m_u and m_d quark masses from $\eta \to 3\pi$ decay

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Work presented in [1].

Quark masses ..

.. are fundamental parameters of SM. However, color confinement prohibits their direct determination. Instead, we use some observable depending on m_q in some theory and compare its theoretical prediction with the corresponding exp. value. $\Rightarrow$ They are theory dependent and depend on renormalization scheme and scale. We use *current quark masses* (= as they occur in QCD Lagrangian) in $\overline{MS}$ scheme.

Light quark masses

For the determination of the light quark masses there are used the following types of the methods

- **QCD sum rules (SR)**
- **Lattice QCD (LQCD)**
- **Effective field theories for QCD (EFT)**

Whereas m_s and the isospin averaged $\hat{m} = \frac{m_u+m_d}{2}$ are determined independently from both SR and LQCD with a reasonable precision with compatible results. For determination of individual m_u and m_d they both need additional input (mainly from ChPT). [Because of elmag. corrections to isospin breaking observables.]

Chiral perturbation theory

Among these EFT, ChPT plays a prominent role. Its use is affected by the following disadvantages:

- non-renormalizable theory $\Rightarrow$ Lagrangian contains ∞ number of LECs: $F_0, B_0, L_i, C_i, \ldots$
- m_q scaling – in all physical quantities $\ni B_0 m_q$
- Kaplan-Manohar ambiguity of changing $m_u \to m_u + \beta m_d m_s$ (cycl.) with $L_{6,7,8}$ and C_is

$\Rightarrow$ ChPT can determine only quark mass ratios and needs some input fixing the physical definition of the masses (large N_C, lattice, …). For m_q determination we combine isosymmetric results from SR and LQCD with an isospin-breaking study from ChPT.

$\eta \to 3\pi$

$\eta \to 3\pi$ decay possible only in the isospin breaking world. Moreover, elmag. effects are there very small $\Rightarrow$ its amplitude proportional directly to $m_d - m_u$,

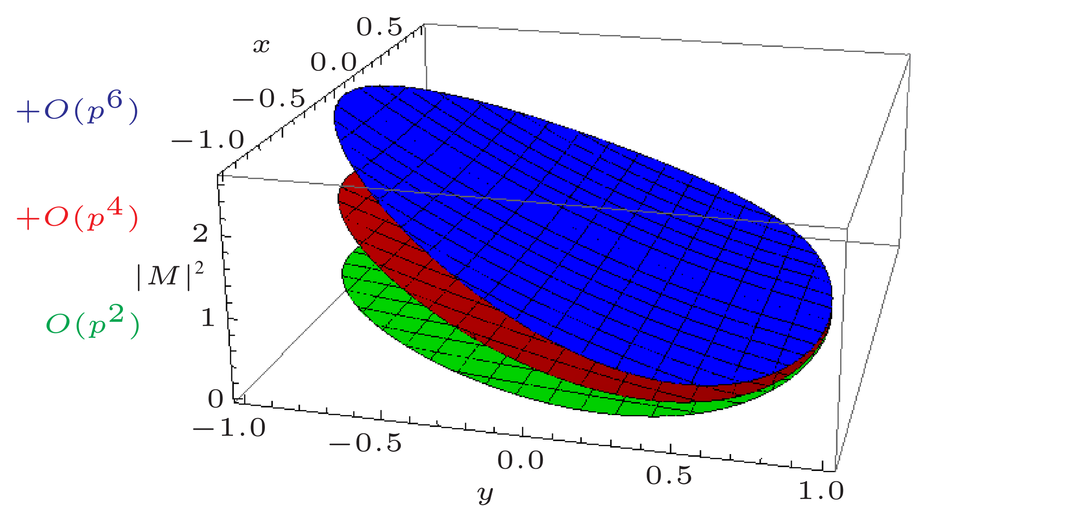
$$\mathcal{A}(s,t,u) = \frac{\sqrt{3}}{4R} M(s,t,u) \text{ with } R = \frac{m_s - \hat{m}}{m_d - m_u}.$$

$\Rightarrow$ Measuring its decay rate Γ one can determine R. In a good approximation $m_{\pi^\pm} = m_{\pi^0}$, then the $0: \eta \to 3\pi^0$ decay is related to $x: \eta \to \pi^+\pi^-\pi^0$.

$\eta \to 3\pi$ in ChPT

However, computation of M in ChPT problematic:

- Slow chiral convergence – from exp. Γ we have $R_{LO} = 19.1$, $R_{NLO} = 31.8$, $R_{NNLO} = 41.3$.



- Dalitz parameters discrepancy

	KLOE [2]	ChPT [3]
a	-1.09 ± 0.02	-1.271 ± 0.075
b	0.124 ± 0.012	0.394 ± 0.102
d	0.057 ± 0.017	0.055 ± 0.057
f	0.14 ± 0.02	0.025 ± 0.160
g	~ 0	0
α	-0.030 ± 0.005	0.013 ± 0.016
β	?	-0.002 ± 0.025

Origin of the discrepancy?

Possible explanations of this discrepancy

- incorrect determination of NNLO LECs C_i
- higher-order final state rescatterings
- influence of slow convergence of $\pi\pi$ scatt., …

This has inspired alternative approaches taking different assumptions than ChPT [4, 5, 6, 7]. However, they do not fix normalization $\Rightarrow$ unavoidable to match to ChPT $\Rightarrow$ need to find a region where both these approaches compatible.

Influence of the C_is

NNLO M depend on subsets of 102 C_is, whose determination is needed before any reliable prediction. But many C_is just estimated (resonance saturation, …).

C_i-independent relations of Dalitz parameters:

1. $\left(4(b+d) - a^2 - 16\alpha\right)\big|_C = 0$
2. $\left(a^3 - 4ab + 4ad + 8f - 8g\right)\big|_C = 0$ (CIR)
3. $\beta|_C = 0$

	KLOE [2]	ChPT [3]	NREFT [4]
rel$_1$	0.02 ± 0.12	-0.03 ± 0.72	0.35 ± 0.13
rel$_2$	0.12 ± 0.21	-0.13 ± 1.4	0.44 ± 0.20
$10^3\beta$	?	-2 ± 25	-4.2 ± 0.7

ChPT and KLOE values seems to be in a good correspondence $\Rightarrow$ Exp. re-measurement of these combinations desirable in order to test this explanation.

First analysis

Assume that all the discrepancy can be included into a small real polynomial ΔP (as e.g. a change of C_i values). $\Rightarrow$ We add ΔP to NNLO ChPT M and fit it from KLOE $\to$ "corrected M". We find that the corrections are indeed small (on the phys. region) – it corresponds to the change of "our parameters" in P:

	A	B	C	D	E	F
$P^{(4)}$	0.46(1)	1.95(10)	−0.6(2)	1.04(2)		
$P^{(6)}$	0.58(1)	2.4(2)	0.3(34)	1.6(24)	5(150)	−4(84)
$P^{(6)}_{\text{corr.}}$	0.575(6)	1.99(4)	−6.8(3)	0.94(3)	−31(3)	20(1)

Even without changing the unitarity part, $M_{\text{corr.}}$ reproduces well KLOE. The value of R is shifted

$$R = 37.7 \pm 2.8 \quad \text{[ChPT+Disp.+KLOE]}.$$

(The error estimated from ChPT convergence of R.) Here we have imposed no definite explanation of the discrepancy, just parametrized it by the polynomial. However, if the discrepancy could be included into the change of the C_is, relations (CIR) have to be fulfilled. Interestingly rel$_2$ $\Rightarrow$ $(\Delta E + 2\Delta F = 0)$ and we observe $\Delta E \sim -36$ and $\Delta F \sim 24$.

Our parametrization ..

.. is based on basic assumptions of QFT together with some hierarchy of various contributions to M (inspired by a very basic chiral counting and/or numerical studies). It takes into account two final-state rescatterings and can include full isospin violation $m_{\pi^\pm} \neq m_{\pi^0}$. By setting its parameters to specific values it can reproduce the NNLO ChPT M exactly.

$$M_{+-0}(s,t,u) = P(s,t,u) + U(s,t,u),$$

$$P = A_x M_\eta^2 + B_x(s-s_0) + C_x(s-s_0)^2 + E_x(s-s_0)^3 + D_x[(t-s_0)^2 + (u-s_0)^2] + F_x[(t-s_0)^3 + (u-s_0)^3]$$

U = unitary contribution.

Second analysis

A different analysis can be performed assuming:

1. physical M can be reproduced by our parametr.
2. ChPT determination of M reliable at least in the region specified below (fixing normalization)

Fixing normalization

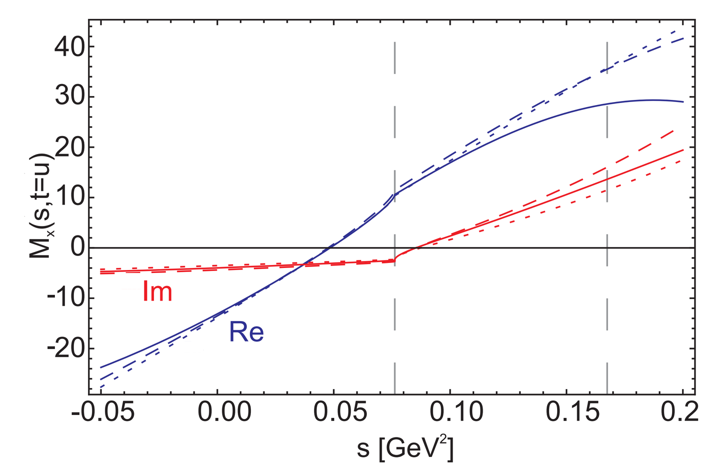
In order to fix the normalization of M we have to find the region, where its chiral convergence is fast. At NLO, 3 points on cut $s = u$ accidentally coincide

a. M is of order $O(m_\pi^2)$ (Adler zero),
b. $\text{Re}\, M = 0$,
c. higher order corrections to the slope are small,

It is usually employed for normalization. However, this proves not to be the case at NNLO. Using our parametr. and NNLO ChPT $M \Rightarrow$ the prescription

- fit on $t = u$ cut
- match only Im of ChPT under phys. threshold
- use the interpolation between order-by-order and "resummed" fits of our parametr. to ChPT

Second analysis: results



Dotted lines – order-by-order fit
Dashed lines – "resummed" fit
Solid line – final fit

$\Rightarrow$ $R = 37.8 \pm 3.3$ [Disp.+KLOE].

The sources of error:

- of the exp. distribution – for now dominant
- of the analytic continuation of the distribution
- from fixing the normalization

Combined results

Since the dominant sources of errors in both analysis were different (and the values are compatible), we can combine them into

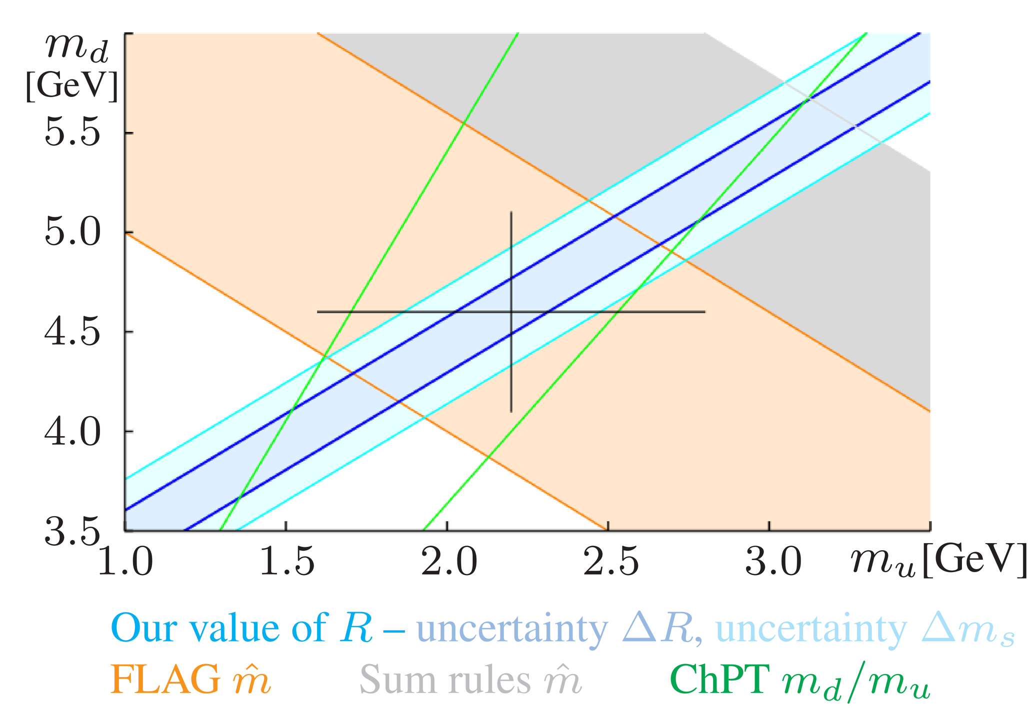
$$R = 37.7 \pm 2.2.$$

Another isospin breaking parameter $Q^2 = \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}$. Using $\frac{m_s}{\hat{m}} \sim 28$ from lattice, $Q = 23.3 \pm 0.8$.

Combining various constraints (in $\overline{MS}$, at $\mu = 2$ GeV)

$\Rightarrow$ The currently most precise values of $m_{u,d}^{\mu=2\,\text{GeV}}$:

$m_u = (2.2 \pm 0.6)$ MeV, $m_d = (4.6 \pm 0.5)$ MeV.



Dalitz parametrization

M (normalized to the center of Dalitz plot) usually parametrized

$$\frac{|M_x(x,y)|^2}{|M_x(0)|^2} = 1 + ay + by^2 + dx^2 + fy^3 + gx^2y + \ldots,$$

$$\frac{|M_0(x,y)|^2}{|M_0(0)|^2} = 1 + 2\alpha z + 2\beta y(3z - 4y^2) + \ldots,$$

$$x = \frac{\sqrt{3}(u-t)}{2m_\eta(m_\eta - 3m_\pi)},\quad y = \frac{3(s_0 - s)}{2m_\eta(m_\eta - 3m_\pi)},\quad z = x^2 + y^2.$$

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