Tribimaximal Mixing From Small Groups

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Based on arXiv:1012.2842

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Neutrino Mixing Matrix

➢ Neutrinos have mass and the different flavors can mix

Super-Kamiokande Collaboration, Y. Fukuda *et al.*, "Evidence for oscillation of atmospheric neutrinos," *Phys. Rev. Lett.* **81** (1998) 1562–1567, hep-ex/9807003

SNO Collaboration, Q. R. Ahmad *et al.*, "Direct evidence for neutrino flavor transformation from neutral-current interactions in the Sudbury Neutrino Observatory," *Phys. Rev. Lett.* **89** (2002) 011301, nucl-ex/0204008.

➢ Pontecorvo-Maki-Nakagawa-Sakata matrix

B. Pontecorvo, "Mesonium and antimesonium," *Sov. Phys. JETP* **6** (1957) 429.

Z. Maki, M. Nakagawa, and S. Sakata, "Remarks on the unified model of elementary particles," *Prog. Theor. Phys.* **28** (1962) 870–880.

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \underbrace{\begin{pmatrix} U_{e1} & U_{e2} & U_{e3}\\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3}\\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3}\end{pmatrix}}_{U_{\mathrm{PMNS}}} \begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

➢ Completely analogous to the CKM matrix in the quark sector (except that for CKM, the down-type quarks are rotated)

Neutrino Mixing Matrix

What we know about the mixing angles ...

$$U_{\mathrm{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12}-s_{13}s_{23}c_{12}e^{i\delta} & c_{23}c_{12}-s_{13}s_{23}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12}-s_{13}c_{23}c_{12}e^{i\delta} & -s_{23}c_{12}-s_{13}c_{23}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & e^{-i\delta}s_{13} \\ 0 & 1 & 0 \\ -e^{i\delta}s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

T. Schwetz, M. A. Tortola, and J. W. F. Valle, "Three-flavour neutrino oscillation update," *New J. Phys.* **10** (2008) 113011, 0808.2016.

Angle	1σ	2σ	3σ
θ_{12}	$32.46^\circ - 34.82^\circ$	$31.31^\circ - 36.27^\circ$	$30.00^\circ - 37.47^\circ$
θ_{23}	$41.55^\circ - 49.02^\circ$	$38.65^\circ - 52.54^\circ$	$36.87^\circ - 54.94^\circ$
θ_{13}	$0.00^\circ - 9.28^\circ$	$0.00^\circ - 11.54^\circ$	$0.00^\circ - 13.69^\circ$

Harrison-Perkins-Scott Matrix

Presently our best guess ...

P. F. Harrison, D. H. Perkins, and W. G. Scott, "Tri-bimaximal mixing and the neutrino oscillation data," *Phys. Lett.* **B530** (2002) 167, hep-ph/0202074.

$$U_{\mathrm{HPS}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

Suggestive of an underlying symmetry ...

Some groups that have been considered in the literature:

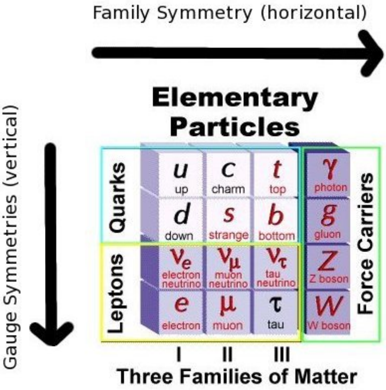
Review: G. Altarelli and F. Feruglio, "Discrete Flavor Symmetries and Models of Neutrino Mixing," 1002.0211.

$S_3,\ D_4,\ D_7,\ A_4,\ A_5,\ \widetilde{T},\ S_4,\ (C_3 \times C_3) \rtimes_\varphi C_3,\ C_7 \rtimes_\varphi C_3,\ \mathrm{PSL}_2(7)$

$\rightsquigarrow$ As a paradigm, we will consider a model with $A_4 \times C_3$ symmetry and then generalize it to other symmetry groups

Horizontal Symmetries

➢ Introduce relations between families of quarks and leptons



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Some groups that have been considered in the literature:

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$$S_3, \quad D_4, \quad D_7, \quad A_4, \quad A_5, \quad \widetilde{T}, \quad S_4, \quad (C_3 \times C_3) \rtimes_\varphi C_3, \quad C_7 \rtimes_\varphi C_3, \quad \mathrm{PSL}_2(7)$$

$\rightsquigarrow$ As a paradigm, we will consider a model with $A_4 \times C_3$ symmetry and then generalize it to other symmetry groups

Altarelli-Feruglio Model Revisited

G. Altarelli and F. Feruglio, "Tri-Bimaximal Neutrino Mixing, A4 and the Modular Symmetry," *Nucl. Phys.* **B741** (2006) 215–235, hep-ph/0512103.

❶ Symmetries of the model

$$\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times A_4 \times \mathbb{Z}_3$$

❷ Particle content and charges

Field	$\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$	A_4	$\mathbb{Z}_3$	$A_4 \times \mathbb{Z}_3$
L	$(\mathbf{2},\text{-}1)$	$\mathbf{3}$	ω	$\mathbf{3'}$
e	$(\mathbf{1},\ 2)$	$\mathbf{1}$	ω^2	$\mathbf{1'}$
μ	$(\mathbf{1},\ 2)$	$\mathbf{1''}$	ω^2	$\mathbf{1^{(8)}}$
τ	$(\mathbf{1},\ 2)$	$\mathbf{1'}$	ω^2	$\mathbf{1^{(5)}}$
h_u	$(\mathbf{2},\ 1)$	$\mathbf{1}$	1	$\mathbf{1}$
h_d	$(\mathbf{2},\text{-}1)$	$\mathbf{1}$	1	$\mathbf{1}$
φ_T	$(\mathbf{1},\ 0)$	$\mathbf{3}$	1	$\mathbf{3}$
φ_S	$(\mathbf{1},\ 0)$	$\mathbf{3}$	ω	$\mathbf{3'}$
ξ	$(\mathbf{1},\ 0)$	$\mathbf{1}$	ω	$\mathbf{1''}$

❸ Breaking the family symmetry

$$\varphi_T = (v_T, v_T, v_T), \ \ \varphi_S = (v_S, 0, 0), \ \ \xi = v_\xi$$

What Information Do We Need for the Analysis?

➢ Which terms in the Lagrangian are invariant?
   → Tensor product of irreps
   → Character table
   → Dimension of conjugacy classes

➢ How do we contract the flavor indices?
   → Clebsch-Gordan-Coefficients
   → Explicit form of representation matrices

Group Information from GAP

The GAP Group, "GAP – Groups, Algorithms, and Programming, Version 4.4.12.", http://www.gap-system.org

"GAP is a system for computational discrete algebra, with particular emphasis on Computational Group Theory."

```
group := SmallGroup(36,11);;
Display(StructureDescription(group));
chartab := Irr(group);;
Display(chartab);
SizesConjugacyClasses(CharacterTable(group));
LoadPackage("repsn");;
for i in [1..Size(chartab)] do
  rep := IrreducibleAffordingRepresentation(chartab[i]);
  for el in Elements(group) do
    Display(el^rep);
  od;
od;
```

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➢ Specify the group that we will work with

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➢ The "human readable" name of the group

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➢ The character table

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  od;
od;
```

➢ Dimensions of the conjugacy classes

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  for el in Elements(group) do
    Display(el^rep);
  od;
od;
```

➢ The matrices for the representations

The Character Table of $\mathbf{A_4} \times \mathbf{C_3}$

	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8	K_9	K_{10}	K_{11}	K_{12}
$\mathbf{1}$	1	1	1	1	1	1	1	1	1	1	1	1
$\mathbf{1'}$	1	1	ω^2	1	1	ω^2	ω	ω^2	ω^2	ω	ω	ω
$\mathbf{1''}$	1	1	ω	1	1	ω	ω^2	ω	ω	ω^2	ω^2	ω^2
$\mathbf{1'''}$	1	ω^2	1	1	ω	ω^2	1	1	ω	ω^2	1	ω
$\mathbf{1^{(4)}}$	1	ω	1	1	ω^2	ω	1	1	ω^2	ω	1	ω^2
$\mathbf{1^{(5)}}$	1	ω^2	ω^2	1	ω	ω	ω	ω^2	1	1	ω	ω^2
$\mathbf{1^{(6)}}$	1	ω	ω	1	ω^2	ω^2	ω^2	ω	1	1	ω^2	ω
$\mathbf{1^{(7)}}$	1	ω^2	ω	1	ω	1	ω^2	ω	ω^2	ω	ω^2	1
$\mathbf{1^{(8)}}$	1	ω	ω^2	1	ω^2	1	ω	ω^2	ω	ω^2	ω	1
$\mathbf{3}$	3	0	3	-1	0	0	3	-1	0	0	-1	0
$\mathbf{3'}$	3	0	3ω	-1	0	0	$3\omega^2$	ω	0	0	$1+\omega$	0
$\mathbf{3''}$	3	0	$3\omega^2$	-1	0	0	3ω	$1+\omega$	0	0	ω	0

$\omega = e^{2\pi i/3}$ is the primitive third root of unity

Decomposition of Tensor Products

From the character table and the dimensions of the conjugacy classes:

$$\begin{aligned}
&\mathbf{1}\otimes\mathbf{1}=\mathbf{1}\\
&\mathbf{1}\otimes\mathbf{1}^{(5)}=\mathbf{1}^{(5)}\\
&\mathbf{1}\otimes\mathbf{3}'=\mathbf{3}'\\
&\mathbf{1}'\otimes\mathbf{1}^{(4)}=\mathbf{1}^{(8)}\\
&\mathbf{1}'\otimes\mathbf{3}=\mathbf{3}''\\
&\mathbf{1}''\otimes\mathbf{1}^{(4)}=\mathbf{1}^{(6)}\\
&\mathbf{1}''\otimes\mathbf{3}=\mathbf{3}'\\
&\mathbf{1}'''\otimes\mathbf{1}^{(5)}=\mathbf{1}^{(8)}\\
&\mathbf{1}'''\otimes\mathbf{3}'=\mathbf{3}'\\
&\mathbf{1}^{(4)}\otimes\mathbf{1}^{(7)}=\mathbf{1}''\\
&\mathbf{1}^{(5)}\otimes\mathbf{1}^{(5)}=\mathbf{1}^{(6)}\\
&\mathbf{1}^{(5)}\otimes\mathbf{3}'=\mathbf{3}\\
&\mathbf{1}^{(6)}\otimes\mathbf{3}=\mathbf{3}'\\
&\mathbf{1}^{(7)}\otimes\mathbf{3}=\mathbf{3}'\\
&\mathbf{1}^{(8)}\otimes\mathbf{3}'=\mathbf{3}
\end{aligned}$$

$$\begin{aligned}
&\mathbf{1}\otimes\mathbf{1}'=\mathbf{1}'\\
&\mathbf{1}\otimes\mathbf{1}^{(6)}=\mathbf{1}^{(6)}\\
&\mathbf{1}\otimes\mathbf{3}''=\mathbf{3}''\\
&\mathbf{1}'\otimes\mathbf{1}^{(5)}=\mathbf{1}^{(7)}\\
&\mathbf{1}'\otimes\mathbf{3}'=\mathbf{3}\\
&\mathbf{1}''\otimes\mathbf{1}^{(5)}=\mathbf{1}'''\\
&\mathbf{1}''\otimes\mathbf{3}'=\mathbf{3}''\\
&\mathbf{1}'''\otimes\mathbf{1}^{(6)}=\mathbf{1}''\\
&\mathbf{1}'''\otimes\mathbf{3}''=\mathbf{3}''\\
&\mathbf{1}^{(4)}\otimes\mathbf{1}^{(8)}=\mathbf{1}^{(5)}\\
&\mathbf{1}^{(5)}\otimes\mathbf{1}^{(6)}=\mathbf{1}\\
&\mathbf{1}^{(5)}\otimes\mathbf{3}''=\mathbf{3}'\\
&\mathbf{1}^{(6)}\otimes\mathbf{3}'=\mathbf{3}''\\
&\mathbf{1}^{(7)}\otimes\mathbf{3}'=\mathbf{3}''\\
&\mathbf{1}^{(8)}\otimes\mathbf{3}''=\mathbf{3}'
\end{aligned}$$

$$\begin{aligned}
&\mathbf{1}\otimes\mathbf{1}''=\mathbf{1}''\\
&\mathbf{1}\otimes\mathbf{1}^{(7)}=\mathbf{1}^{(7)}\\
&\mathbf{1}'\otimes\mathbf{1}'=\mathbf{1}''\\
&\mathbf{1}'\otimes\mathbf{1}^{(6)}=\mathbf{1}^{(4)}\\
&\mathbf{1}'\otimes\mathbf{3}''=\mathbf{3}'\\
&\mathbf{1}''\otimes\mathbf{1}^{(6)}=\mathbf{1}^{(8)}\\
&\mathbf{1}''\otimes\mathbf{3}''=\mathbf{3}\\
&\mathbf{1}'''\otimes\mathbf{1}^{(7)}=\mathbf{1}^{(6)}\\
&\mathbf{1}^{(4)}\otimes\mathbf{1}^{(4)}=\mathbf{1}'''\\
&\mathbf{1}^{(4)}\otimes\mathbf{3}=\mathbf{3}\\
&\mathbf{1}^{(5)}\otimes\mathbf{1}^{(7)}=\mathbf{1}^{(4)}\\
&\mathbf{1}^{(6)}\otimes\mathbf{1}^{(6)}=\mathbf{1}^{(5)}\\
&\mathbf{1}^{(6)}\otimes\mathbf{3}''=\mathbf{3}\\
&\mathbf{1}^{(7)}\otimes\mathbf{3}''=\mathbf{3}
\end{aligned}$$

$$\begin{aligned}
&\mathbf{1}\otimes\mathbf{1}'''=\mathbf{1}'''\\
&\mathbf{1}\otimes\mathbf{1}^{(8)}=\mathbf{1}^{(8)}\\
&\mathbf{1}'\otimes\mathbf{1}''=\mathbf{1}\\
&\mathbf{1}'\otimes\mathbf{1}^{(7)}=\mathbf{1}'''\\
&\mathbf{1}''\otimes\mathbf{1}''=\mathbf{1}'\\
&\mathbf{1}''\otimes\mathbf{1}^{(7)}=\mathbf{1}^{(5)}\\
&\mathbf{1}'''\otimes\mathbf{1}'''=\mathbf{1}^{(4)}\\
&\mathbf{1}'''\otimes\mathbf{1}^{(8)}=\mathbf{1}'\\
&\mathbf{1}^{(4)}\otimes\mathbf{1}^{(5)}=\mathbf{1}'\\
&\mathbf{1}^{(4)}\otimes\mathbf{3}'=\mathbf{3}'\\
&\mathbf{1}^{(5)}\otimes\mathbf{1}^{(8)}=\mathbf{1}''\\
&\mathbf{1}^{(6)}\otimes\mathbf{1}^{(7)}=\mathbf{1}'\\
&\mathbf{1}^{(7)}\otimes\mathbf{1}^{(7)}=\mathbf{1}^{(8)}\\
&\mathbf{1}^{(8)}\otimes\mathbf{1}^{(8)}=\mathbf{1}^{(7)}
\end{aligned}$$

$$\begin{aligned}
&\mathbf{1}\otimes\mathbf{1}^{(4)}=\mathbf{1}^{(4)}\\
&\mathbf{1}\otimes\mathbf{3}=\mathbf{3}\\
&\mathbf{1}'\otimes\mathbf{1}'''=\mathbf{1}^{(5)}\\
&\mathbf{1}'\otimes\mathbf{1}^{(8)}=\mathbf{1}^{(6)}\\
&\mathbf{1}''\otimes\mathbf{1}'''=\mathbf{1}^{(7)}\\
&\mathbf{1}''\otimes\mathbf{1}^{(8)}=\mathbf{1}^{(4)}\\
&\mathbf{1}'''\otimes\mathbf{1}^{(4)}=\mathbf{1}\\
&\mathbf{1}'''\otimes\mathbf{3}=\mathbf{3}\\
&\mathbf{1}^{(4)}\otimes\mathbf{1}^{(6)}=\mathbf{1}^{(7)}\\
&\mathbf{1}^{(4)}\otimes\mathbf{3}''=\mathbf{3}''\\
&\mathbf{1}^{(5)}\otimes\mathbf{3}=\mathbf{3}''\\
&\mathbf{1}^{(6)}\otimes\mathbf{1}^{(8)}=\mathbf{1}'''\\
&\mathbf{1}^{(7)}\otimes\mathbf{1}^{(8)}=\mathbf{1}\\
&\mathbf{1}^{(8)}\otimes\mathbf{3}=\mathbf{3}''
\end{aligned}$$

$$\begin{aligned}
&\mathbf{3}\otimes\mathbf{3}=\mathbf{1}+\mathbf{1}'''+\mathbf{1}^{(4)}+2\otimes\mathbf{3}\\
&\mathbf{3}\otimes\mathbf{3}''=\mathbf{1}'+\mathbf{1}^{(5)}+\mathbf{1}^{(8)}+2\otimes\mathbf{3}''\\
&\mathbf{3}'\otimes\mathbf{3}''=\mathbf{1}+\mathbf{1}'''+\mathbf{1}^{(4)}+2\otimes\mathbf{3}
\end{aligned}$$

$$\begin{aligned}
&\mathbf{3}\otimes\mathbf{3}'=\mathbf{1}''+\mathbf{1}^{(6)}+\mathbf{1}^{(7)}+2\otimes\mathbf{3}'\\
&\mathbf{3}'\otimes\mathbf{3}'=\mathbf{1}'+\mathbf{1}^{(5)}+\mathbf{1}^{(8)}+2\otimes\mathbf{3}''\\
&\mathbf{3}''\otimes\mathbf{3}''=\mathbf{1}''+\mathbf{1}^{(6)}+\mathbf{1}^{(7)}+2\otimes\mathbf{3}'
\end{aligned}$$

Decomposition of Tensor Products

From the character table and the dimensions of the conjugacy classes:

$$\begin{aligned}
&1 \otimes 1 = 1\\
&1 \otimes 1^{(5)} = 1^{(5)}\\
&1 \otimes 3' = 3'\\
&1' \otimes 1^{(4)} = 1^{(8)}\\
&1' \otimes 3 = 3''\\
&1'' \otimes 1^{(4)} = 1^{(6)}\\
&1'' \otimes 3 = 3'\\
&1''' \otimes 1^{(5)} = 1^{(8)}\\
&1''' \otimes 3' = 3'\\
&1^{(4)} \otimes 1^{(7)} = 1''\\
&1^{(5)} \otimes 1^{(5)} = 1^{(6)}\\
&1^{(5)} \otimes 3' = 3\\
&1^{(6)} \otimes 3 = 3'\\
&1^{(7)} \otimes 3 = 3'\\
&1^{(8)} \otimes 3' = 3
\end{aligned}$$

$$\begin{aligned}
&1 \otimes 1' = 1'\\
&1 \otimes 1^{(6)} = 1^{(6)}\\
&1 \otimes 3'' = 3''\\
&1' \otimes 1^{(5)} = 1^{(7)}\\
&1' \otimes 3' = 3\\
&1'' \otimes 1^{(5)} = 1'''\\
&1'' \otimes 3' = 3''\\
&1''' \otimes 1^{(6)} = 1''\\
&1''' \otimes 3'' = 3''\\
&1^{(4)} \otimes 1^{(8)} = 1^{(5)}\\
&1^{(5)} \otimes 1^{(6)} = 1\\
&1^{(5)} \otimes 3'' = 3'\\
&1^{(6)} \otimes 3' = 3''\\
&1^{(7)} \otimes 3' = 3''\\
&1^{(8)} \otimes 3'' = 3'
\end{aligned}$$

$$\begin{aligned}
&1 \otimes 1'' = 1''\\
&1 \otimes 1^{(7)} = 1^{(7)}\\
&1' \otimes 1' = 1''\\
&1' \otimes 1^{(6)} = 1^{(4)}\\
&1' \otimes 3'' = 3'\\
&1'' \otimes 1^{(6)} = 1^{(8)}\\
&1'' \otimes 3'' = 3\\
&1''' \otimes 1^{(7)} = 1^{(6)}\\
&1^{(4)} \otimes 1^{(4)} = 1'''\\
&1^{(4)} \otimes 3 = 3\\
&1^{(5)} \otimes 1^{(7)} = 1^{(4)}\\
&1^{(6)} \otimes 1^{(6)} = 1^{(5)}\\
&1^{(6)} \otimes 3'' = 3\\
&1^{(7)} \otimes 3'' = 3
\end{aligned}$$

$$\begin{aligned}
&1 \otimes 1''' = 1'''\\
&1 \otimes 1^{(8)} = 1^{(8)}\\
&1' \otimes 1'' = 1\\
&1' \otimes 1^{(7)} = 1'''\\
&1'' \otimes 1'' = 1'\\
&1'' \otimes 1^{(7)} = 1^{(5)}\\
&1''' \otimes 1''' = 1^{(4)}\\
&1''' \otimes 1^{(8)} = 1'\\
&1^{(4)} \otimes 1^{(5)} = 1'\\
&1^{(4)} \otimes 3' = 3'\\
&1^{(5)} \otimes 1^{(8)} = 1''\\
&1^{(6)} \otimes 1^{(7)} = 1'\\
&1^{(7)} \otimes 1^{(7)} = 1^{(8)}\\
&1^{(8)} \otimes 1^{(8)} = 1^{(7)}
\end{aligned}$$

$$\begin{aligned}
&1 \otimes 1^{(4)} = 1^{(4)}\\
&1 \otimes 3 = 3\\
&1' \otimes 1''' = 1^{(5)}\\
&1' \otimes 1^{(8)} = 1^{(6)}\\
&1'' \otimes 1''' = 1^{(7)}\\
&1'' \otimes 1^{(8)} = 1^{(4)}\\
&1''' \otimes 1^{(4)} = 1\\
&1''' \otimes 3 = 3\\
&1^{(4)} \otimes 1^{(6)} = 1^{(7)}\\
&1^{(4)} \otimes 3'' = 3''\\
&1^{(5)} \otimes 3 = 3''\\
&1^{(6)} \otimes 1^{(8)} = 1'''\\
&1^{(7)} \otimes 1^{(8)} = 1\\
&1^{(8)} \otimes 3 = 3''
\end{aligned}$$

$$\begin{aligned}
&3 \otimes 3 = 1 + 1''' + 1^{(4)} + 2 \otimes 3\\
&3 \otimes 3'' = 1' + 1^{(5)} + 1^{(8)} + 2 \otimes 3''\\
&3' \otimes 3'' = 1 + 1''' + 1^{(4)} + 2 \otimes 3
\end{aligned}$$

$$\begin{aligned}
&3 \otimes 3' = 1'' + 1^{(6)} + 1^{(7)} + 2 \otimes 3'\\
&3' \otimes 3' = 1' + 1^{(5)} + 1^{(8)} + 2 \otimes 3''\\
&3'' \otimes 3'' = 1'' + 1^{(6)} + 1^{(7)} + 2 \otimes 3'
\end{aligned}$$

Invariant Lagrangian

➢ Terms that are invariant, have 2 leptons and mass dimension ≤ 6:

$$L\,L\,h_u\,h_u\,\varphi_S, \quad L\,L\,h_u\,h_u\,\xi, \quad L\,e\,h_d\,\varphi_T, \quad L\,\mu\,h_d\,\varphi_T, \quad L\,\tau\,h_d\,\varphi_T$$

Invariant Lagrangian

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$$\mathbf{3'}\otimes\mathbf{3'}\otimes\mathbf{1}\otimes\mathbf{1}\otimes\mathbf{3'}=\left(\mathbf{1'}+\mathbf{1^{(5)}}+\mathbf{1^{(8)}}+2\times\mathbf{3''}\right)\otimes\mathbf{3'}=2\times\mathbf{1}+2\times\mathbf{1'''}+2\times\mathbf{1^{(4)}}+7\times\mathbf{3}$$

Invariant Lagrangian

- Terms that are invariant, have 2 leptons and mass dimension ≤ 6:

$$L\,L\,h_u\,h_u\,\varphi_S,\quad L\,L\,h_u\,h_u\,\xi,\quad L\,e\,h_d\,\varphi_T,\quad L\,\mu\,h_d\,\varphi_T,\quad L\,\tau\,h_d\,\varphi_T$$

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- Contract family indices:

$$\frac{1}{\sqrt{3}}L_2\,L_3\,h_u\,h_u\,\varphi_{S,1}+\frac{1}{\sqrt{3}}L_3\,L_1\,h_u\,h_u\,\varphi_{S,2}+\frac{1}{\sqrt{3}}L_1\,L_2\,h_u\,h_u\,\varphi_{S,3}+\frac{1}{\sqrt{3}}L_1\,L_1\,h_u\,h_u\,\xi$$
$$+\frac{1}{\sqrt{3}}L_2\,L_2\,h_u\,h_u\,\xi+\frac{1}{\sqrt{3}}L_3\,L_3\,h_u\,h_u\,\xi$$

Invariant Lagrangian

- Terms that are invariant, have 2 leptons and mass dimension ≤ 6:

$$L\,L\,h_u\,h_u\,\varphi_S, \quad L\,L\,h_u\,h_u\,\xi, \quad L\,e\,h_d\,\varphi_T, \quad L\,\mu\,h_d\,\varphi_T, \quad L\,\tau\,h_d\,\varphi_T$$

$$\mathbf{3'}\otimes\mathbf{3'}\otimes\mathbf{1}\otimes\mathbf{1}\otimes\mathbf{3'}=\left(\mathbf{1'}+\mathbf{1^{(5)}}+\mathbf{1^{(8)}}+2\times\mathbf{3''}\right)\otimes\mathbf{3'}=2\times\mathbf{1}+2\times\mathbf{1'''}+2\times\mathbf{1^{(4)}}+7\times\mathbf{3}$$

- Contract family indices:

$$\tfrac{1}{\sqrt{3}}L_2\,L_3\,h_u\,h_u\,\varphi_{S,1}+\tfrac{1}{\sqrt{3}}L_3\,L_1\,h_u\,h_u\,\varphi_{S,2}+\tfrac{1}{\sqrt{3}}L_1\,L_2\,h_u\,h_u\,\varphi_{S,3}+\tfrac{1}{\sqrt{3}}L_1\,L_1\,h_u\,h_u\,\xi$$
$$+\tfrac{1}{\sqrt{3}}L_2\,L_2\,h_u\,h_u\,\xi+\tfrac{1}{\sqrt{3}}L_3\,L_3\,h_u\,h_u\,\xi$$

- Contract $\mathrm{SU}(2)$ indices and substitute *vevs* $\langle\varphi_S\rangle=(v_S,0,0)$, etc:

$$\tfrac{1}{\sqrt{3}}L_2^{(1)}\,L_3^{(1)}\,v_u\,v_u\,v_S+\tfrac{1}{\sqrt{3}}L_1^{(1)}\,L_1^{(1)}\,v_u\,v_u\,v_\xi+\tfrac{1}{\sqrt{3}}L_2^{(1)}\,L_2^{(1)}\,v_u\,v_u\,v_\xi+\tfrac{1}{\sqrt{3}}L_3^{(1)}\,L_3^{(1)}\,v_u\,v_u\,v_\xi$$

Finally, the Mixing Angles

- Mass matrices

$$M_{\ell^+} = \frac{v_d v_T}{M} \cdot \begin{array}{c} \\ L_1^{(2)} \\ L_2^{(2)} \\ L_3^{(2)} \end{array} \begin{array}{c} \begin{array}{ccc} e & \mu & \tau \end{array} \\ \begin{pmatrix} -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} & \frac{1}{2\sqrt{3}} & \frac{1}{2\sqrt{3}} \\ -\frac{1}{\sqrt{3}} & \frac{1}{2\sqrt{3}} & \frac{1}{2\sqrt{3}} \end{pmatrix} \end{array} \qquad M_\nu = \frac{v_u v_u}{M^2} \cdot \begin{array}{c} \\ L_1^{(1)} \\ L_2^{(1)} \\ L_3^{(1)} \end{array} \begin{array}{c} \begin{array}{ccc} L_1^{(1)} & L_2^{(1)} & L_3^{(1)} \end{array} \\ \begin{pmatrix} \frac{v_\xi}{\sqrt{3}} & 0 & 0 \\ 0 & \frac{v_\xi}{\sqrt{3}} & \frac{v_S}{2\sqrt{3}} \\ 0 & \frac{v_S}{2\sqrt{3}} & \frac{v_\xi}{\sqrt{3}} \end{pmatrix} \end{array}$$

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➢ Singular value decomposition: $\hat{M}_{\ell^+} = D_L M_{\ell^+} D_R^\dagger, \quad \hat{M}_\nu = U_L M_\nu U_R^\dagger$

$$D_{\mathrm{L}} = \begin{pmatrix} -0.5774+i\,0.0000 & -0.5774+i\,0.0000 & -0.5774+i\,0.0000 \\ 0.5738-i\,0.0636 & -0.2319+i\,0.5287 & -0.3420-i\,0.4652 \\ 0.5731-i\,0.0702 & -0.3474-i\,0.4612 & -0.2257+i\,0.5314 \end{pmatrix}, \quad U_{\mathrm{L}} = \begin{pmatrix} 0.0000 & -0.7071 & -0.7071 \\ -1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.7071 & -0.7071 \end{pmatrix}$$

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- Neutrino mixing matrix: $U_{\mathrm{PMNS}} = D_{\mathrm{L}} U_{\mathrm{L}}^\dagger$ (needs rephasing)

$$U_{\mathrm{PMNS}} = \begin{pmatrix} 0.8165+i\,0.0000 & 0.5774+i\,0.0000 & 0.0000+i\,0.0000 \\ 0.4058-i\,0.0449 & -0.5738+i\,0.0636 & 0.0778+i\,0.7028 \\ 0.4052-i\,0.0497 & -0.5731+i\,0.0702 & -0.0860-i\,0.7019 \end{pmatrix}$$

- Mixing angles: $\theta_{12} = 35.26, \quad \theta_{23} = 45.00, \quad \theta_{13} = 0.00$ Tribimaximal ✔

Finally, the Mixing Angles

➢ Mass matrices

$$M_{\ell^+} = \frac{v_d v_T}{M} \cdot \begin{array}{c} \\ L_1^{(2)} \\ L_2^{(2)} \\ L_3^{(2)} \end{array} \begin{array}{c} \begin{array}{ccc} e & \mu & \tau \end{array} \\ \begin{pmatrix} -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} & \frac{1}{2\sqrt{3}} & \frac{1}{2\sqrt{3}} \\ -\frac{1}{\sqrt{3}} & \frac{1}{2\sqrt{3}} & \frac{1}{2\sqrt{3}} \end{pmatrix} \end{array} \qquad M_\nu = \frac{v_u v_u}{M^2} \cdot \begin{array}{c} \\ L_1^{(1)} \\ L_2^{(1)} \\ L_3^{(1)} \end{array} \begin{array}{c} \begin{array}{ccc} L_1^{(1)} & L_2^{(1)} & L_3^{(1)} \end{array} \\ \begin{pmatrix} \frac{v_\xi}{\sqrt{3}} & 0 & 0 \\ 0 & \frac{v_\xi}{\sqrt{3}} & \frac{v_S}{2\sqrt{3}} \\ 0 & \frac{v_S}{2\sqrt{3}} & \frac{v_\xi}{\sqrt{3}} \end{pmatrix} \end{array}$$

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$$|U_{\mathrm{PMNS}}| = \begin{pmatrix} 0.8165 & 0.5774 & 0.0000 \\ 0.4082 & 0.5774 & 0.7071 \\ 0.4082 & 0.5774 & 0.7071 \end{pmatrix}$$

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Finally, the Mixing Angles

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$$M_{\ell^+} = \frac{v_d v_T}{M} \cdot \begin{matrix} & e & \mu & \tau \\ L_1^{(2)} & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \\ L_2^{(2)} & -\frac{1}{\sqrt{3}} & \frac{1}{2\sqrt{3}} & \frac{1}{2\sqrt{3}} \\ L_3^{(2)} & -\frac{1}{\sqrt{3}} & \frac{1}{2\sqrt{3}} & \frac{1}{2\sqrt{3}} \end{matrix} \qquad M_\nu = \frac{v_u v_u}{M^2} \cdot \begin{matrix} & L_1^{(1)} & L_2^{(1)} & L_3^{(1)} \\ L_1^{(1)} & \frac{v_\xi}{\sqrt{3}} & 0 & 0 \\ L_2^{(1)} & 0 & \frac{v_\xi}{\sqrt{3}} & \frac{v_S}{2\sqrt{3}} \\ L_3^{(1)} & 0 & \frac{v_S}{2\sqrt{3}} & \frac{v_\xi}{\sqrt{3}} \end{matrix}$$

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➢ Neutrino mixing matrix: $U_\mathrm{PMNS} = D_\mathrm{L} U_\mathrm{L}^\dagger$ (needs rephasing)

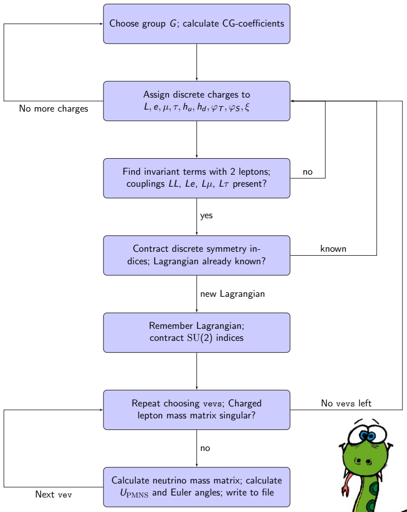
$$|U_\mathrm{PMNS}| = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ 1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ 1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

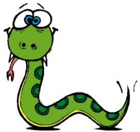
➢ Mixing angles: $\theta_{12} = 35.26, \ \theta_{23} = 45.00, \ \theta_{13} = 0.00$ Tribimaximal ✔

What is the Bottom Line?

- We used the Altarelli-Feruglio model only as a paradigm
- The analysis is completely independent of the family symmetry
- `GAP` gives us all the relevant information about the group
- Complexity of group is hereby irrelevant
- We use `Python` to interact w/`GAP` and do symbolic manipulations

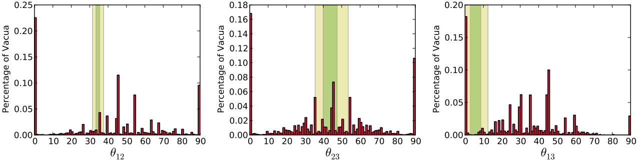
Scanning for the Models



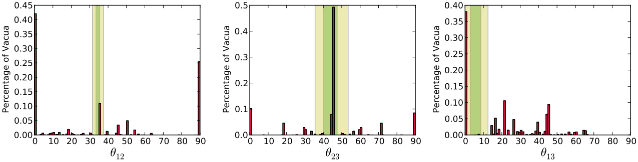


- Consider $A_4 \times \mathbb{Z}_3$
- 39,900 inequivalent models/Lagrangians
- 22,932 models w/non-singular mass matrices
- 4,481 consistent w/experiment at 3σ level (19.5%)
- 4,233 are tribimaximal (18.5%)
- Probably largest set of viable neutrino models ever constructed!

Most Likely Mixing Angles

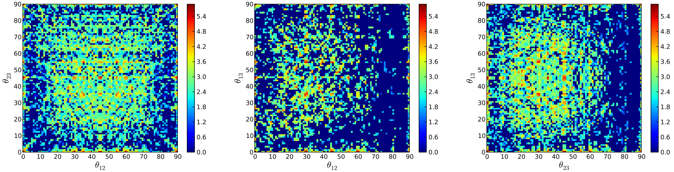


(a) Number of models that give θ_{ij} with no constraints on the other 2 angles. Each histogram has 15,992,118 entries.

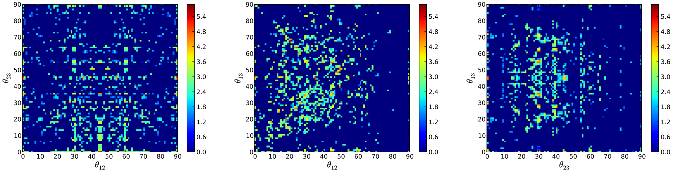


(b) Number of models that give θ_{ij} with the other 2 angles restricted to their 3σ interval. The histograms have 838,289, 148,886 and 225,844 entries, respectively.

Correlation Between Pairs of Mixing Angles

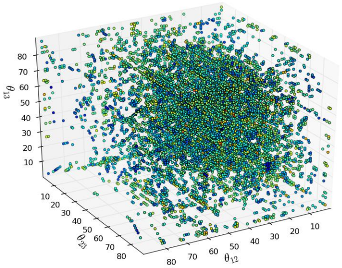


(c) Number of models that give θ_{ij} and θ_{mn} with no constraint on the remaining angle. Each histogram has 15,768,810 entries.

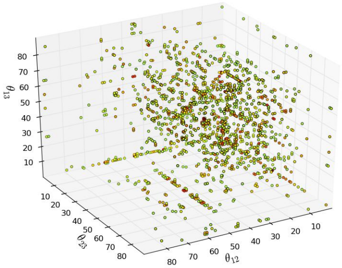


(d) Number of models that give θ_{ij} and θ_{mn} with the remaining angle restricted to its 3σ interval. The histograms have 2,591,752, 4,060,640 and 1,214,874 entries, respectively.

Correlation Between All Mixing Angles



(e) The 12,230 bins that are ≥ 1.

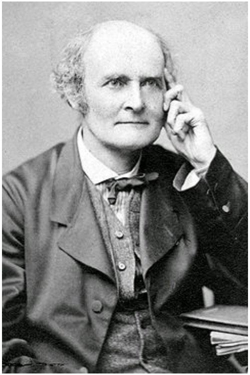


(f) The 1,586 bins that are ≥ 1000.

Are We Looking Under the Lamppost?

Small Groups

Arthur Cayley (1821-1895) is the first to systematically construct groups; in 1854, he determined all groups of order 4 and 6 . . .



Small Groups

The first few of the 1048 groups of order ≤ 100

✔ $= \mathrm{U}(n)$ and ✔ $= \mathrm{SU}(n)$ for $n = 2, 3$

GAP ID	Group	**3**	U(3)	U(2)	U(2)×U(1)	A_4
[1, 1]	1	✗	✗	✗	✗	✗
[2, 1]	C_2	✗	✗	✗	✗	✗
[3, 1]	C_3	✗	✗	✗	✗	✗
[4, 1]	C_4	✗	✗	✗	✗	✗
[4, 2]	$C_2 \times C_2$	✗	✗	✗	✗	✗
[5, 1]	C_5	✗	✗	✗	✗	✗
[6, 1]	S_3	✗	✔	✔	✔	✗
[6, 2]	C_6	✗	✗	✗	✗	✗
[7, 1]	C_7	✗	✗	✗	✗	✗
[8, 1]	C_8	✗	✗	✗	✗	✗

Small Groups

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[8, 2]	$C_4 \times C_2$	✗	✗	✗	✗	✗
[8, 3]	D_4	✗	✔ (SU)	✔ (U)	✔ (U)	✗
[8, 4]	Q_8	✗	✔ (SU)	✔ (SU)	✔ (SU)	✗
[8, 5]	$C_2 \times C_2 \times C_2$	✗	✗	✗	✗	✗
[9, 1]	C_9	✗	✗	✗	✗	✗
[9, 2]	$C_3 \times C_3$	✗	✗	✗	✗	✗
[10, 1]	D_5	✗	✔ (SU)	✔ (U)	✔ (U)	✗
[10, 2]	C_{10}	✗	✗	✗	✗	✗
[11, 1]	C_{11}	✗	✗	✗	✗	✗
[12, 1]	$C_3 \rtimes_{\varphi} C_4$	✗	✔ (SU)	✔ (SU)	✔ (SU)	✗

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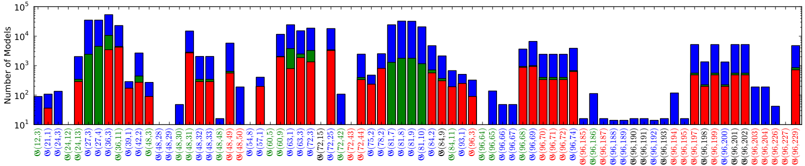
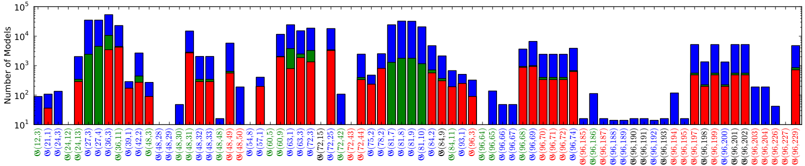
GAP ID	Group	**3**	U(3)	U(2)	U(2)×U(1)	A_4
[12, 2]	C_{12}	✗	✗	✗	✗	✗
[12, 3]	A_4	✔	✔	✗	✗	✔
[12, 4]	D_6	✗	✔	✔	✔	✗
[12, 5]	$C_6 \times C_2$	✗	✗	✗	✗	✗
[13, 1]	C_{13}	✗	✗	✗	✗	✗
[14, 1]	D_7	✗	✔	✔	✔	✗
[14, 2]	C_{14}	✗	✗	✗	✗	✗
[15, 1]	C_{15}	✗	✗	✗	✗	✗
[16, 1]	C_{16}	✗	✗	✗	✗	✗
[16, 2]	$C_4 \times C_4$	✗	✗	✗	✗	✗

All Models



➢ There are 1048 groups of order ≤ 100

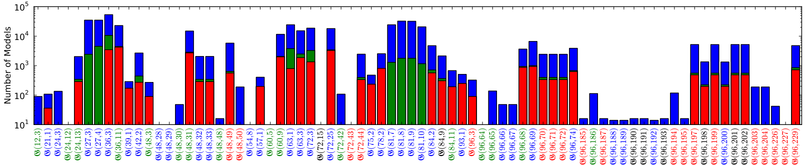
All Models



➢ There are 1048 groups of order ≤ 100

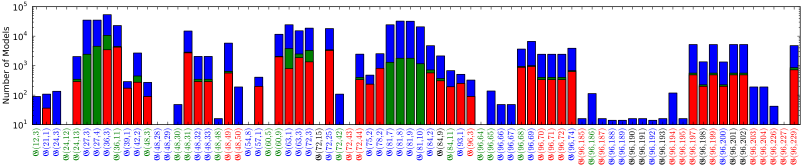
➢ 90 out of these 1048 groups have a 3-dimensional irrep

All Models



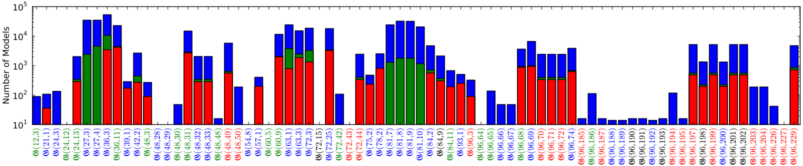
- ➢ There are 1048 groups of order ≤ 100
- ➢ 90 out of these 1048 groups have a 3-dimensional irrep
- ➢ 76 out of these 90 groups can be scanned in less than 60 days

All Models



- ➢ There are 1048 groups of order ≤ 100
- ➢ 90 out of these 1048 groups have a 3-dimensional irrep
- ➢ 76 out of these 90 groups can be scanned in less than 60 days
- ➢ 9 groups (12%) only have singular mass matrices

All Models



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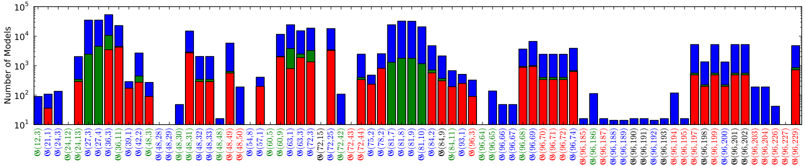

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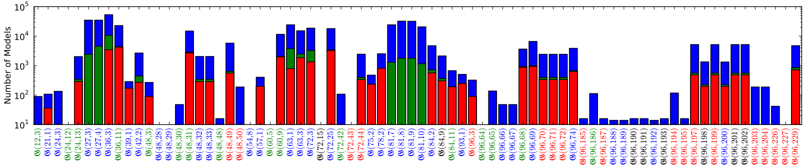
All Models



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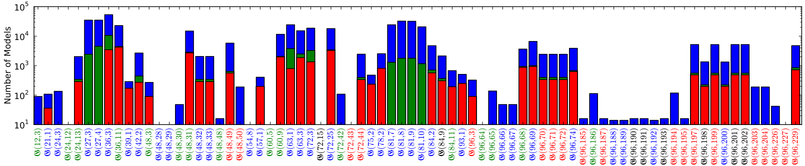
All Models



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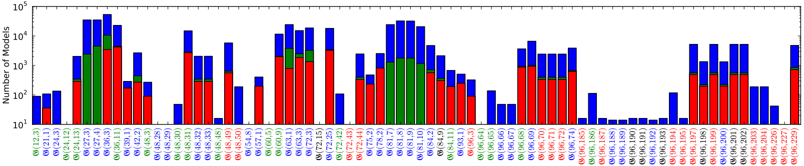
- 38 groups (50%) have tribimaximal models
- Smallest group that can have TBM: $\mathfrak{G}(21,1) = T_7$. Special?
- Largest fraction of TBM models: $\mathfrak{G}(39,1) = T_{13}$. Special?

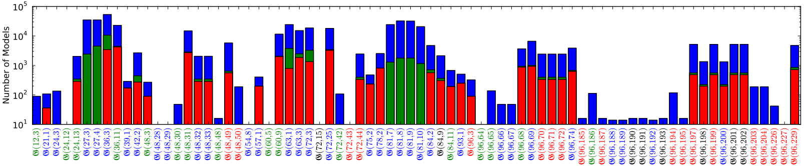
All Models



➢ Is There a Connection Between A_4 and Tribimaximal Mixing?

- Blue x-axis label: $\mathfrak{g} \subset \mathrm{U}(3)$
- Red x-axis label: $\mathfrak{g} \supset A_4$
- Green x-axis label: $A_4 \subset \mathfrak{g} \subset \mathrm{U}(3)$

All Models



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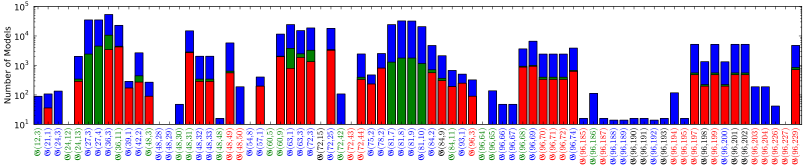
➢ 35 groups have A_4 subgroup, but only 16 realize TBM

All Models



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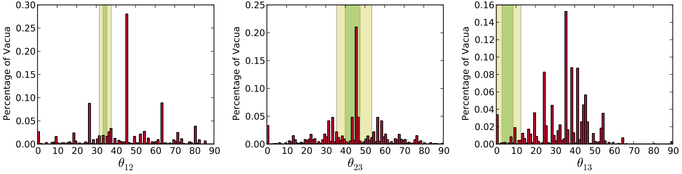
➢ 35 groups have A_4 subgroup, but only 16 realize TBM

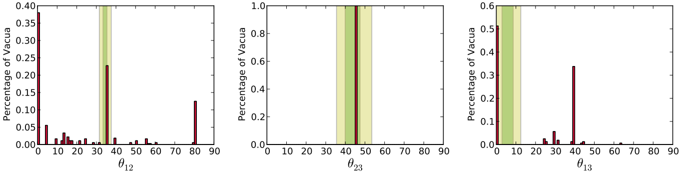
➢ $\mathfrak{G}(84,9)$, $\mathfrak{G}(96,198)$, $\mathfrak{G}(96,201)$, $\mathfrak{G}(96,202)$ neither subsets of $\mathrm{U}(3)$ nor contain A_4 subgroup but realize TBM

The Metacyclic Group T_7

➢ Special? Every model in 3σ band is TBM!



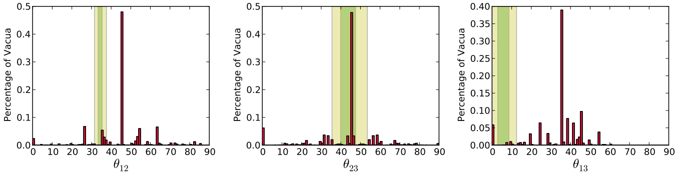
(g) Vacua that give θ_{ij} with no constraints on the other 2 angles.



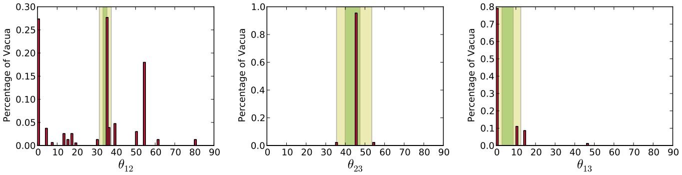
(h) Vacua that give θ_{ij} with the other 2 angles restricted to their 3σ interval.

The Metacyclic Group T_{13}

➢ Largest fraction of TBM vacua! Striking correlation $3\sigma \leftrightarrow$ TBM



(i) Vacua that give θ_{ij} with no constraints on the other 2 angles.



(j) Vacua that give θ_{ij} with the other 2 angles restricted to their 3σ interval.

Conclusions

- Constructed thousands of new models of tribimaximal mixing
  - 18.5% of all $A_4 \times \mathbb{Z}_3$ models are TBM. Encouraging!
  - Prediction for θ_{13}: If A_4 and $\theta_{13} \lesssim 12^\circ \rightsquigarrow \theta_{13} = 0^\circ$
  - Prediction for δ: 0°
  - Correlations between mixing angles: Fix two, predict the third
- Constructed specific models
  - $\theta_{13} \neq 0$ possible: $\theta_{12} \simeq 34^\circ$, $\theta_{23} \simeq 41^\circ$ and $\theta_{13} \simeq 5^\circ$
  - Altarelli-Feruglio model works with $\mathbb{Z}_2$: $A_4 \times \mathbb{Z}_2 \simeq \Sigma(24)$
  - TBM possible for T_7, $\Sigma(24)$, T_{13}, T_{14}, $\Delta(48)$, T_{19}, ...
- Is A_4 special? Are TBM and A_4 connected?
  - 50% of the 76 groups we scanned can accomodate TBM
  - Metacyclic group T_{13} has larger fraction of TBM models
  - Smallest group w/TBM is T_7