Two-Loop Mixed QCD-EW Virtual Corrections to the Drell-Yan Production of Z and W bosons

Roberto BONCIANI

Laboratoire de Physique Subatomique et de Cosmologie,
Université Joseph Fourier/CNRS-IN2P3/INPG,
F-38026 Grenoble, France

In collaboration with: G. Degrassi and A. Vicini

Plan of the Talk

- Introduction
- Description of the Calculation

Introduction

- Drell-Yan production of Z and W bosons, $p(\bar{p}) \to Z \to l^+l^-$ and $p(\bar{p}) \to W \to l\nu$, is a fundamental process for an accurate check of the SM at hadron colliders. It has a big cross section and it is very sensitive to the properties of the gauge bosons
- DY production of W is important for the determination of the W mass (transverse mass and p_T distributions), that is suppose to be measured at Tevatron with $\Delta M_W \sim 15$MeV and at LHC even more precisely ($\Delta M_W \sim 7$MeV). This requires an accurate theoretical control on the distributions
- Background for processes of new physics as Z'/W' production (or $t\bar{t}$ ...)
- Possible determination of the luminosity (used in ratios of cross sections, as at Tevatron)
- Big impact on the distributions (and therefore on the determination of the W mass) comes from QCD ISR with QED final state radiation or real-virtual (FACTORIZED). At the level of $\Delta M_W \sim 10$ MeV also the mixed QCD-EW corrections may be important.
- Mixed QCD-EW corrections important also for the stabilization of the scale dipendence: NLO EW (partonic cross section) is leading order in α_S for what concerns the hadronic observable. The mixed corrections can reduce the scale variation

For these reasons we need a precise and reliable theoretical predicion.

Literature

QCD

- NLO/NNLO corrections to W/Z total production rate

  Altarelli, Ellis, Martinelli '79; Altarelli, Ellis, Greco, Martinelli '84; Hamberg, van Neerven, Matsuura '91; van Neerven, Zijstra '92

- Fully differential NLO to $l\bar{l}'$ (MCFM)

  Campbell, Ellis '99

- Fully differential NNLO to $l\bar{l}'$ (FEWZ)

  Anastasiou, Dixon, Melnikov, Petriello '04; Melnikov, Petriello '06

- Resummation LL/NLL in p_T^W/M_W (RESBOS)

  Balazs, Yuan '97

- NLO matched with resummation NLL in p_T^W/M_W

  Bozzi, Catani, De Florian, Ferrera, Grazzini '09

- NLO with PS (MC@NLO and POWHEG

  Frixione, Webber '02; Frixione, Nason, Oleari '07

Literature

EW

W production NLO

- Pole approximation

  Wackeroth, Hollik ’97; Baur et al. ’99

- Exact corrections

  Zykunov et al. ’01; Dittmaier, Krämer ’02; Baur, Wackeroth ’04; Arbuzov et al. ’06; Carloni Calame et al. ’06 (HORACE); Hollik, Kasprzik, Kniehl ’08

- Photon induced processes

  Dittmaier, Krämer ’05; Baur, Wackeroth ’04; Carloni Calame et al. ’06; Arbuzov et al. ’07 ...

Z production NLO

- Only QED

  Baur et al. ’98

- Exact corrections

  Baur et al. ’02; Zykunov et al. ’07; Carloni Calame et al. ’07 (HORACE)...

- Photon induced processes

  Carloni Calame et al. ’07 (HORACE)

Literature

Mixed (non factorizable) QCD-EW

Z production

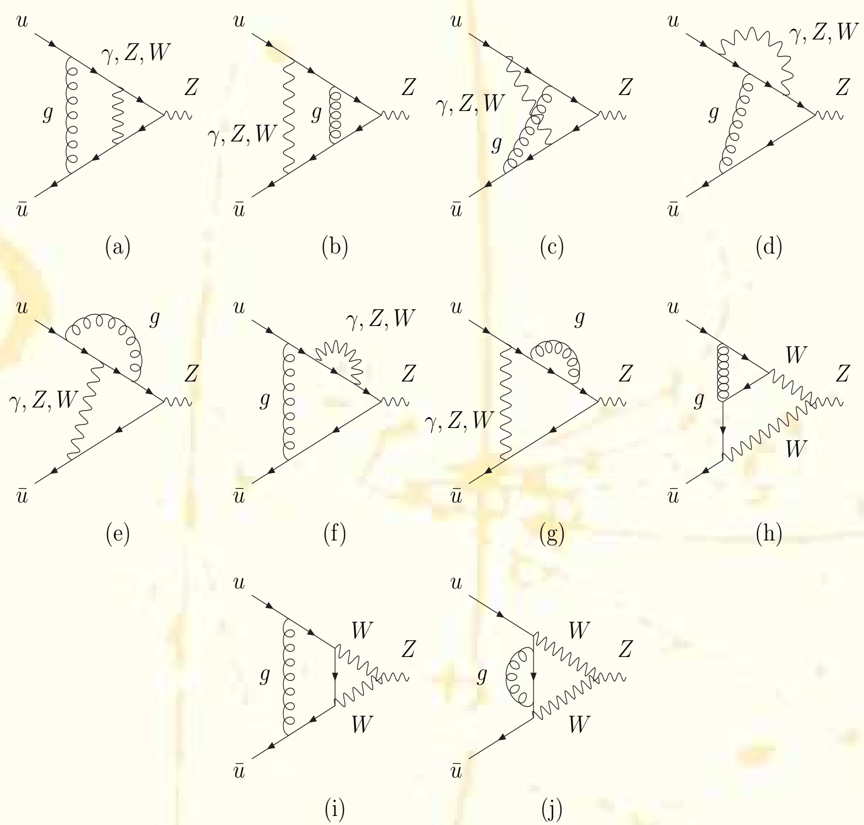
- In 2008 Kotikov, Kuhn and Veretin studied the mixed two-loop corrections to the form factors for a $U(1) \times U(1)$ and $SU(2) \times U(1)$ gauge theory with massive and massless gauge bosons.

  Kotikov, Kuhn, Veretin ’08

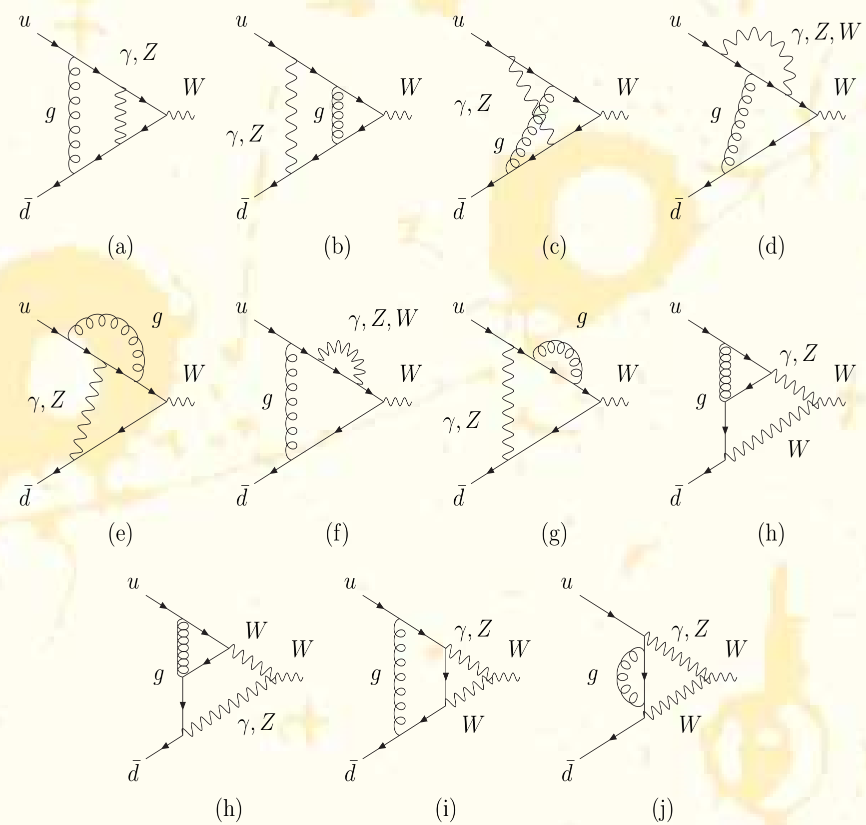
  - Analytic calculation in terms of Harmonic Polylogarithms
  - Peculiar structure of the corrections: factorization of the QCD and EW IR poles

Feynman Diagrams

- Since we concentrate on leptonic decay of the Z and W bosons $\Rightarrow$ QCD corrections are only initial-state corrections. At NNLO they involve vertex virtual corrections.
- The EW corrections, however, connect initial and final state. For the moment we consider resonant contributions:



40 diagrams contribute to the Z production



44 diagrams contribute to the W production

Form Factors

The vertex corrections to the two processes in the Standard Model can be described in terms of two form factors:

$$V^\mu(p_1, p_2) = G_L(q^2)\,\gamma^\mu \frac{(1+\gamma_5)}{2} + G_R(q^2)\,\gamma^\mu \frac{(1-\gamma_5)}{2}$$

For the W production, we have $G_R(q^2) = 0$.
The form factors $G_{L,R}(q^2)$ are expanded in power of the coupling constants α_w and α_S as follows:

$$\begin{aligned} G_{L,R} &= K_{Z,W}\Bigg[G_{L,R}^{(0l)} + \left(\frac{\alpha_S}{\pi}\right) G_{L,R}^{(1l,QCD)} + \left(\frac{\alpha_w}{\pi}\right) G_{L,R}^{(1l,EW)} + \left(\frac{\alpha_S}{\pi}\right)^2 G_{L,R}^{(2l,QCD)} \\ &\quad + \left(\frac{\alpha_w}{\pi}\right)\left(\frac{\alpha_S}{\pi}\right) G_{L,R}^{(2l,Mix)} + \ldots \Bigg] \\ K_Z &= \frac{i g_w}{c_w} \qquad K_W = \frac{i g_w}{\sqrt{2}} V_{ud}^* \end{aligned}$$

where, at the tree-level, we have:

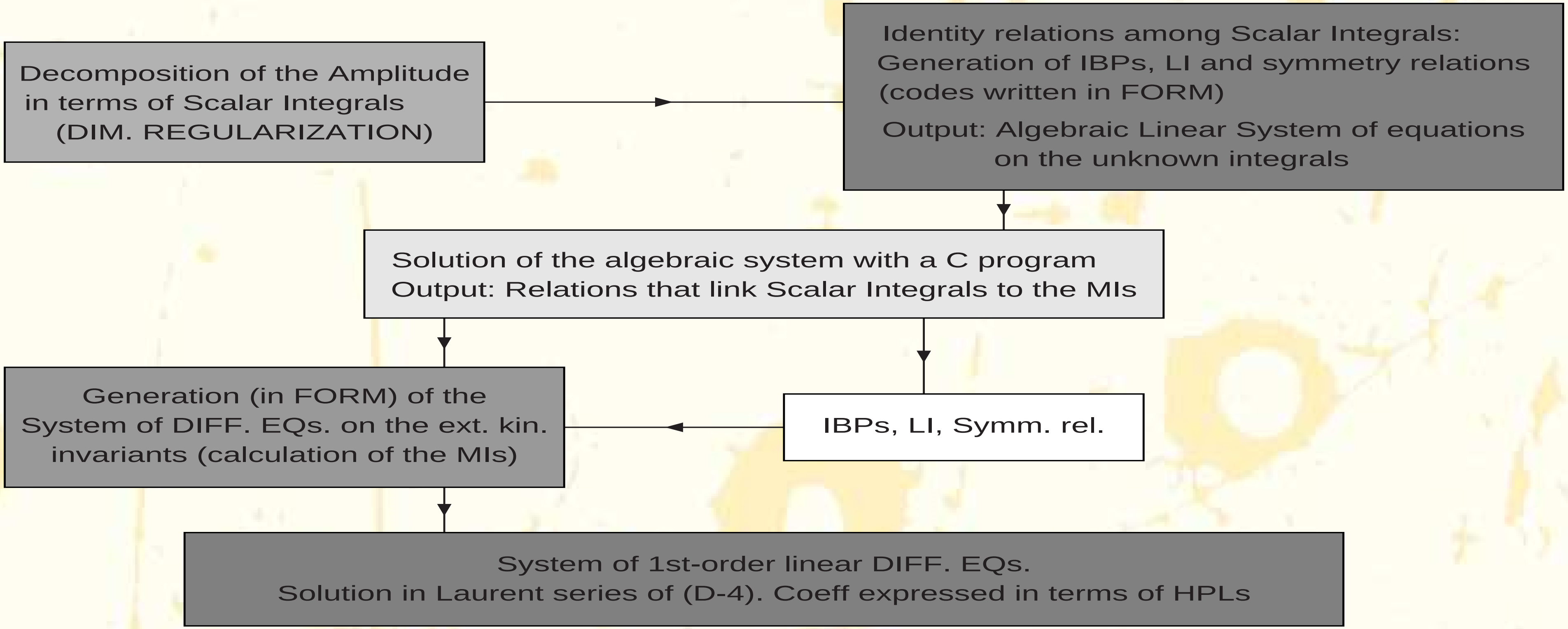
$$G_{Z,L}^{(0l)} = (v_u + a_u)\,, \qquad G_{Z,R}^{(0l)} = (v_u - a_u)\,, \qquad G_{W,L}^{(0l)} = 1$$

Structure of the Calculation

- We generate the two-loop diagrams with FeynArts
- We interface FeynArts with the reduction and we extract automatically the form factor
- We project out the form factors (or we interfere with the tree level) and we perform the reduction to the Master Integrals

- The diagrams containing both the Z and W masses are approximated expanding in $\Delta M^2 = M_Z^2 - M_W^2$ (the first order in ΔM^2 should be sufficient for phenomenological purposes). In this way we are reconducted to a problem with a single mass. The diagrams proportional to ΔM^2 have squared propagators and they are reconducted to the Masters via the reduction process

Laporta Algorithm and Diff. Equations



PUBLIC PROGRAMS

- AIR – Maple package
  (C. Anastasiou and A. Lazopoulos, JHEP **0407** (2004) 046)
- FIRE – Mathematica package (A. V. Smirnov, JHEP **0810** (2008) 107)
- REDUZE – REDUZE2 C++/GiNaC packages
  (C. Studerus, Comput. Phys. Commun. **181** (2010) 1293;
  A. von Manteuffel and C. Studerus, in preparation)

Differential Equations for the MIs

Differential Equations for the MIs

For a given topology, when the system of identities is not reducible, we have a small number of MI's. In the case of three-point functions:

$$F_i(Q^2, p_1^2, p_2^2) = \int d^D k_1 d^D k_2 \frac{S_1^{n_1} \cdots S_q^{n_q}}{D_1^{m_1} \cdots D_t^{m_t}}$$

Using all the identity-relations (IBP's, LI, Sym) we can construct the following system of first-order linear differential equations:

$$\frac{dF_i}{dQ^2} = \sum_j h_j(Q^2, m^2)\, F_j + \Omega_i$$

where $i, j = 1, ..., N_{MIs}$.

Ω_i | This term involves integrals of the class $I_{t-1,r,s}$ (sub-topologies) to be considered KNOWN

V. Kotikov, *Phys. Lett.* **B254** (1991) 158; **B259** (1991) 314; **B267** (1991) 123.
E. Remiddi, *Nuovo Cim.* **110A** (1997) 1435.

Form Factors: One-Loop Corrections

At the one-loop level we have for instance for the Z boson:

$$
\begin{aligned}
G_{Z;L,R} &= C(d)\left(\frac{\mu^2}{M_W^2}\right)^{\frac{4-d}{2}} \delta_{c_1c_2}\,\mathcal{G}_{Z;L,R} \\
\mathcal{G}^{(1l,QCD)}_{Z,L} &= C_F\,\frac{(v_u+a_u)}{2} f_0(d,x_W) \\
\mathcal{G}^{(1l,EW)}_{Z,L} &= \frac{(v_u+a_u)}{2}\left[Q_u^2 s_w^2 f_0(d,x_W) + \frac{(v_u+a_u)^2}{c_w^2} f_1(d,x_W,x_Z) + |V_{ud}|^2 f_2(d,x_W)\right] \\
&\quad + c_w^2 |V_{ud}|^2 f_3(d,xW) \\
\mathcal{G}^{(1l,QCD)}_{Z,R} &= C_F\,\frac{(v_u-a_u)}{2} f_0(d,x_W) \\
\mathcal{G}^{(1l,EW)}_{Z,R} &= \frac{(v_u-a_u)}{2}\left[Q_u^2 s_w^2 f_0(d,x_W) + \frac{(v_u-a_u)^2}{c_w^2} f_1(d,x_W,x_Z)\right]
\end{aligned}
$$

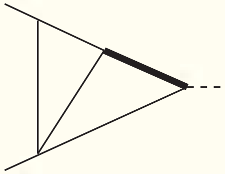
where :

$$
\begin{aligned}
f_0(d,x_W) &= \frac{1}{(d-4)^2} - \left[\frac{3}{4} - \frac{1}{2}H(0,x_W)\right]\frac{1}{(d-4)} + 1 - \frac{\zeta(2)}{4} - \frac{3}{8}H(0,x_W) + \frac{1}{4}H(0,0,x_W) \\
&\quad - \left[1 - \frac{3}{16}\zeta(2) - \frac{\zeta(3)}{4} - \left(\frac{1}{2} - \frac{\zeta(2)}{8}\right)H(0,x_W) + \frac{3}{16}H(0,0,x_W)\ldots\right](d-4) + \mathcal{O}(d-4)^2\,, \quad (1) \\
f_1(d,x_W,x_Z) &= \frac{1}{64(d-4)} + \frac{1}{128}H(0,x_W) - \frac{1}{64x_Z^2}\left[x_Z - 2x_Z^2 - x_Z(1-2x_Z)H(0,x_Z)\cdots\right.
\end{aligned}
$$

List of Master Integrals

van Neerven '86; Gonsalves '86; Fleischer, Kotikov, Veretin '99; Davydychev, Kalmykov '03; Aglietti, R. B. '03-'04 ....

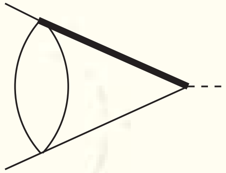
Solution for the new Masters



$$= \frac{1}{m^2} \sum_{i=-1}^{0} A_i \epsilon^i + \mathcal{O}(\epsilon)$$

$$A_{-1} = \frac{1}{16\,x}\Big[H(0,-1,0,x) - H(0,-1,-1,x)\Big]$$

$$A_0 = \frac{1}{16\,x}\Big[2\zeta(2)H(0,-1,x) + 5H(0,-1,-1,-1,x) - 2H(0,-1,-1,0,x) - H(0,-1,0,-1,x)$$
$$-2H(0,-1,0,0,x) - H(0,0,-1,-1,x) + H(0,0,-1,0,x)\Big]$$



$$= \sum_{i=-2}^{2} A_i \epsilon^i + \mathcal{O}(\epsilon^3)$$

$$A_{-2} = \frac{1}{32}$$

$$A_{-1} = \frac{1}{32}\Big[5 - 2H(-1,x) - \frac{2}{x}H(-1,x)\Big]$$

$$A_0 = \frac{1}{32}\Big\{19 + 2\zeta(2) - 2\Big(1+\frac{1}{x}\Big)[5H(-1,x) - 3H(-1,-1,x) + H(0,-1,x)]\Big\}$$

$$A_1 = \frac{1}{32}\Big\{65 + 10\zeta(2) - 2\zeta(3) + \Big(1+\frac{1}{x}\Big)[-2(19+2\zeta(2))H(-1,x) + 30H(-1,-1,x) - 18H(-1,-1,-1,x)$$
$$+6H(-1,0,-1,x) - 10H(0,-1,x) + 6H(0,-1,-1,x) - 2H(0,0,-1,x)]\Big\}$$

$$A_2 = \frac{211}{32} + \frac{9}{80}\zeta(2)^2 + \frac{1}{16}\Big\{19\zeta(2) - 5\zeta(3) - \Big(1+\frac{1}{x}\Big)\Big[(65+10\zeta(2)-2\zeta(3))H(-1,x) - (57+6\zeta(2))H(-1,-1,x)$$
$$+(19+2\zeta(2))H(0,-1,x) + 45H(-1,-1,-1,x) - 15H(-1,0,-1,x) - 15H(0,-1,-1,x)\cdots\Big]\Big\}$$

Harmonic Polylogarithms (HPLs)

- Weight $= 1$

$$H(0,x) = \ln x \quad H(-1,x) = \int_0^x \frac{dt}{1+t} = \ln(1+x) \quad H(1,x) = \int_0^x \frac{dt}{1-t} = -\ln(1-x)$$

- Weight > 1
  If $\vec{a} = \vec{0}$ we define $H(\vec{0},x) = \frac{1}{\omega!}\ln^\omega x$. If $\vec{a} \neq \vec{0}$:

$$H(\vec{a},x) = \int_0^x dt\, f(a_1,x)\, H(\vec{a}_{\omega-1},t) \qquad \frac{d}{dx} H(\vec{a},x) = f(a_1,x)\, H(\vec{a}_{\omega-1},x)$$

- The Algebra: $\omega_{\vec{a}} \times \omega_{\vec{b}} = \omega_{\vec{a}} \times \omega_{\vec{b}}$

$$H(\vec{a},x)\, H(\vec{b},x) = \sum_{\vec{c}=\vec{a}\uplus\vec{b}} H(\vec{c},x)$$

- Integration by Parts

$$H(m_1,...,m_q,x) = H(m_1,x)H(m_2,...,m_q,x) - ... + (-1)^{q+1} H(m_q,...,m_1,x)$$

- Connection with Nielsen's polylog and Spence functions:

$$S_{n,p}(x) = H(\vec{0}_n,\vec{1}_p,x) \quad Li_n(x) = H(\vec{0}_{n-1},1,x)$$

A.B.Goncharov, *Math. Res. Lett.* **5** (1998), 497-516.
E. Remiddi and J. A. M. Vermaseren, *Int. J. Mod. Phys.* **A15** (2000) 725.

Generalized Harmonic Polylogs (GHPLs)

- Weight $= 1$
  In addition to the usual basis functions $g(0,x) = 1/x$, $g(1,x) = 1/(1-x)$, $g(-1,x) = 1/(1+x)$, the following enlargement of the set is considered:

$$g(\pm 4, x) = \frac{1}{4 \mp x}, \quad g(c, x) = \frac{1}{x - e^{\frac{i\pi}{3}}}, \quad g(\bar{c}, x) = \frac{1}{x - e^{\frac{-i\pi}{3}}}, \quad g(\pm r, x) = \frac{1}{\sqrt{x(4 \mp r)}}$$

$$g(\mp 1 - r, x) = \frac{1}{(1 \pm x)\sqrt{x(4+r)}}, \quad g(\mp 1 + r, x) = \frac{1}{(1 \pm x)\sqrt{x(4-r)}}$$

  The weight 1 GHPLs are:

$$H(\pm 4; x) = \mp \log(4 \mp x) \pm 2\log 2, \quad H(c; x) = \log(x - 1/2 - i\sqrt{3}/2) - \log(-1/2 - i\sqrt{3}/2),$$

$$H(\bar{c}; x) = \log(x - 1/2 + i\sqrt{3}/2) - \log(-1/2 + i\sqrt{3}/2), \quad H(-r; x) = 2\log(\sqrt{x+4} + \sqrt{x}) - \log 2,$$

$$H(r; x) = 2\arcsin(\sqrt{x}/2), \quad H(-1-r; x) = 2/\sqrt{3}\,\arctan(\sqrt{3x/(4+x)}), \quad \dots$$

- Weight > 1
  $\vec{a} = \{0, \pm 1, \pm 4, c, \bar{c}, \pm r, \pm 1 \pm r\}$. If $\vec{a} = \vec{0}$ we define $H(\vec{0}, x) = \frac{1}{\omega!}\ln^{\omega} x$. If $\vec{a} \neq \vec{0}$:

$$H(\vec{a}, x) = \int_0^x dt\, g(a_1, x)\, H(\vec{a}_{\omega-1}, t) \qquad \frac{d}{dx} H(\vec{a}, x) = g(a_1, x)\, H(\vec{a}_{\omega-1}, x)$$

- The Algebra and the other properties of the HPLs are maintained

U. Aglietti and R. B., *Nucl. Phys.* **B698** (2004) 277.

Checks

- Numerical checks
  - Numerical checks on the Master Integrals using FIESTA
- Analytical checks
  - Many Master were already used in other calculations
  - We checked the QCD part against the two-loop calculation by van Neerven et al.
  - The self energies match with Djouadi-Gambino
  - We are checking the Z form factors against Kotikov-Kuhn-Veretin.

Conclusions

- Drell-Yan is one of the best studied processes in hadronic physics
- The theoretical description involved the effort of many groups and is done at the moment at NNLO + resummation for what concerns QCD. EW NLO corrections are available. At the moment the complete set of mixed QCD-EW corrections is not known
- We calculated analytically the virtual QCD-EW corrections for Z and W production in the resonant region
- The complete evaluation of the cross section needs the evaluation of the real emission