Two-Loop Mixed QCD-EW Virtual Corrections to the Drell-Yan Production of $Z$ and $W$ bosons

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Plan of the Talk

- Introduction
- Description of the Calculation
Introduction

- Drell-Yan production of $Z$ and $W$ bosons, $p(\bar{p}) \rightarrow Z \rightarrow l^+ l^-$ and $p(\bar{p}) \rightarrow W \rightarrow l \nu$, is a fundamental process for an accurate check of the SM at hadron colliders. It has a big cross section and it is very sensitive to the properties of the gauge bosons.

- DY production of $W$ is important for the determination of the $W$ mass (transverse mass and $p_T$ distributions), that is suppose to be measured at Tevatron with $\Delta M_W \sim 15$ MeV and at LHC even more precisely ($\Delta M_W \sim 7$ MeV). This requires an accurate theoretical control on the distributions.

- Background for processes of new physics as $Z'/W'$ production (or $t\bar{t}$ ...)

- Possible determination of the luminosity (used in ratios of cross sections, as at Tevatron)

- Big impact on the distributions (and therefore on the determination of the $W$ mass) comes from QCD ISR with QED final state radiation or real-virtual (FACTORIZED). At the level of $\Delta M_W \sim 10$ MeV also the mixed QCD-EW corrections may be important.

- Mixed QCD-EW corrections important also for the stabilization of the scale dipendence: NLO EW (partonic cross section) is leading order in $\alpha_S$ for what concerns the hadronic observable. The mixed corrections can reduce the scale variation.

For these reasons we need a precise and reliable theoretical prediction.
QCD

- NLO/NNLO corrections to $W/Z$ total production rate
  - Altarelli, Ellis, Martinelli ’79; Altarelli, Ellis, Greco, Martinelli ’84; Hamberg, van Neerven, Matsuura ’91; van Neerven, Zijstra ’92

- Fully differential NLO to $l\bar{l}'$ (MCFM)
  - Campbell, Ellis ’99

- Fully differential NNLO to $l\bar{l}'$ (FEWZ)
  - Anastasiou, Dixon, Melnikov, Petriello ’04; Melnikov, Petriello ’06

- Resummation LL/NLL in $p_T^W/M_W$ (RESPSOS)
  - Balazs, Yuan ’97

- NLO matched with resummation NLL in $p_T^W/M_W$
  - Bozzi, Catani, De Florian, Ferrera, Grazzini ’09

- NLO with PS (MC@NLO and POWHEG)
  - Frixione, Webber ’02; Frixione, Nason, Oleari ’07
Literature

**EW**

**W production NLO**

- Pole approximation
  - Wackeroth, Hollik '97; Baur et al. '99
- Exact corrections
  - Zykunov et al. '01; Dittmaier, Krämer '02; Baur, Wackeroth '04; Arbuzov et al. '06; Carloni Calame et al. '06 (HORACE); Hollik, Kasprzik, Kniehl '08
- Photon induced processes

**Z production NLO**

- Only QED
  - Baur et al. '98
- Exact corrections
  - Baur et al. '02; Zykunov et al. '07; Carloni Calame et al. '07 (HORACE)...
- Photon induced processes
  - Carloni Calame et al. '07 (HORACE)
Mixed (non factorizable) QCD-EW

Z production

In 2008 Kotikov, Kuhn and Veretin studied the mixed two-loop corrections to the form factors for a $U(1) \times U(1)$ and $SU(2) \times U(1)$ gauge theory with massive and massless gauge bosons.

Kotikov, Kuhn, Veretin '08

- Analytic calculation in terms of Harmonic Polylogarithms
- Peculiar structure of the corrections: factorization of the QCD and EW IR poles
Since we concentrate on leptonic decay of the $Z$ and $W$ bosons $\Rightarrow$ QCD corrections are only initial-state corrections. At NNLO they involve vertex virtual corrections.

The EW corrections, however, connect initial and final state. For the moment we consider resonant contributions:

40 diagrams contribute to the $Z$ production

44 diagrams contribute to the $W$ production
Form Factors

The vertex corrections to the two processes in the Standard Model can be described in terms of two form factors:

\[ V^\mu(p_1,p_2) = G_L(q^2) \gamma^\mu \frac{(1 + \gamma_5)}{2} + G_R(q^2) \gamma^\mu \frac{(1 - \gamma_5)}{2} \]

For the $W$ production, we have $G_R(q^2) = 0$.

The form factors $G_{L,R}(q^2)$ are expanded in power of the coupling constants $\alpha_w$ and $\alpha_S$ as follows:

\[
G_{L,R} = K_{Z,W} \left[ G_{L,R}^{(0l)} + \left( \frac{\alpha_S}{\pi} \right) G_{L,R}^{(1l,QCD)} + \left( \frac{\alpha_w}{\pi} \right) G_{L,R}^{(1l,EW)} + \left( \frac{\alpha_S}{\pi} \right)^2 G_{L,R}^{(2l,QCD)} \right. \\
+ \left( \frac{\alpha_w}{\pi} \right) \left( \frac{\alpha_S}{\pi} \right) G_{L,R}^{(2l,Mix)} + \ldots \right]
\]

\[
K_Z = \frac{ig_w}{c_w}, \quad K_W = \frac{ig_w}{\sqrt{2}} V_{ud}^*
\]

where, at the tree-level, we have:

\[
G_{Z,L}^{(0l)} = (v_u + a_u), \quad G_{Z,R}^{(0l)} = (v_u - a_u), \quad G_{W,L}^{(0l)} = 1
\]
Structure of the Calculation

- We generate the two-loop diagrams with FeynArts
- We interface FeynArts with the reduction and we extract automatically the form factor
- We project out the form factors (or we interfere with the tree level) and we perform the reduction to the Master Integrals

The diagrams containing both the $Z$ and $W$ masses are approximated expanding in $\Delta M^2 = M_Z^2 - M_W^2$ (the first order in $\Delta M^2$ should be sufficient for phenomenological purposes). In this way we are reconducted to a problem with a single mass. The diagrams proportional to $\Delta M^2$ have squared propagators and they are reconducted to the Masters via the reduction process.
Decomposition of the Amplitude in terms of Scalar Integrals (DIM. REGULARIZATION)

Identity relations among Scalar Integrals:
Generation of IBPs, LI and symmetry relations (codes written in FORM)
Output: Algebraic Linear System of equations on the unknown integrals

Solution of the algebraic system with a C program
Output: Relations that link Scalar Integrals to the MIs

Generation (in FORM) of the System of DIFF. EQs. on the ext. kin. invariants (calculation of the MIs)

IBPs, LI, Symm. rel.

System of 1st-order linear DIFF. EQs.
Solution in Laurent series of (D-4). Coeff expressed in terms of HPLs

PUBLIC PROGRAMS

AIR – Maple package
(C. Anastasiou and A. Lazopoulos, JHEP 0407 (2004) 046)


REDUZE – REDUZE2 C++/GiNaC packages
Differential Equations for the MIs

For a given topology, when the system of identities is not reducible, we have a small number of MI’s. In the case of three-point functions:

\[ F_i(Q^2, p_1^2, p_2^2) = \int d^D k_1 d^D k_2 \frac{S_{i_1}^{m_1} \cdots S_{i_q}^{m_q}}{D_{i_1}^{m_1} \cdots D_{i_t}^{m_t}} \]

Using all the identity-relations (IBP’s, LI, Sym) we can construct the following system of first-order linear differential equations:

\[ \frac{dF_i}{dQ^2} = \sum_j h_j(Q^2, m^2) F_j + \Omega_i \]

where \( i, j = 1, ..., N_{MIs} \).

\[ \Omega_i \quad \text{This term involves integrals of the class } I_{t-1,r,s} \text{ (sub-topologies) to be considered KNOWN} \]

At the one-loop level we have for instance for the $Z$ boson:

\[
G_{Z,L,R} = C(d) \left( \frac{\mu^2}{M_W^2} \right)^{\frac{4-d}{2}} \delta_{c_1 c_2} G_{Z,L,R}
\]

\[
G_{Z,L}^{(\text{11,QCD})} = C_F \frac{(v_u + a_u)}{2} f_0(d, x_W)
\]

\[
G_{Z,L}^{(\text{11,EW})} = \frac{(v_u + a_u)}{2} \left[ Q_u^2 s_w^2 f_0(d, x_W) + \frac{(v_u + a_u)^2}{c_w^2} f_1(d, x_W, x_Z) + |V_{ud}|^2 f_2(d, x_W) \right] + c_w^2 |V_{ud}|^2 f_3(d, x_W)
\]

\[
G_{Z,R}^{(\text{11,QCD})} = C_F \frac{(v_u - a_u)}{2} f_0(d, x_W)
\]

\[
G_{Z,R}^{(\text{11,EW})} = \frac{(v_u - a_u)}{2} \left[ Q_u^2 s_w^2 f_0(d, x_W) + \frac{(v_u - a_u)^2}{c_w^2} f_1(d, x_W, x_Z) \right]
\]

where:

\[
f_0(d, x_W) = \frac{1}{(d-4)^2} - \left[ \frac{3}{4} - \frac{1}{2} H(0, x_W) \right] \frac{1}{(d-4)} + 1 - \frac{\zeta(2)}{4} - \frac{3}{8} H(0, x_W) + \frac{1}{4} H(0, 0, x_W) + \frac{1}{4} H(0, 0, x_W) \\
- \left[ 1 - \frac{3}{16} \zeta(2) - \frac{\zeta(3)}{4} - \left( \frac{1}{2} - \frac{\zeta(2)}{8} \right) H(0, x_W) + \frac{3}{16} H(0, 0, x_W) \ldots \right] (d-4) + \mathcal{O}(d-4)^2,
\]

\[
f_1(d, x_W, x_Z) = \frac{1}{64(d-4)} + \frac{1}{128} H(0, x_W) - \frac{1}{64 x_Z^2} \left[ x_Z - 2 x_Z^2 - x_Z (1 - 2 x_Z) H(0, x_Z) \ldots \right]
\]
List of Master Integrals

van Neerven '86; Gonsalves '86; Fleischer, Kotikov, Veretin '99; Davydychev, Kalmykov '03; Aglietti, R. B. '03-'04 ....
Solution for the new Masters

\[ \frac{1}{m^2} \sum_{i=0}^{0} A_i \epsilon^i + O(\epsilon) \]

\[ A_{-1} = \frac{1}{16} \left[ H(0, -1, 0, x) - H(0, -1, -1, x) \right] \]

\[ A_0 = \frac{1}{16} \left[ 2 \zeta(2) H(0, -1, x) + 5H(0, -1, -1, -1, x) - 2H(0, -1, -1, 0, x) - H(0, -1, 0, -1, x) \right] \]

\[ A_{-2} = \frac{1}{32} \left( 5 - 2H(-1, x) - \frac{2}{x} H(-1, x) \right) \]

\[ A_{-1} = \frac{1}{32} \left( 19 + 2\zeta(2) - 2 \left( 1 + \frac{1}{x} \right) \right) \left( 5H(-1, x) - 3H(-1, -1, x) + H(0, -1, x) \right) \]

\[ A_0 = \frac{1}{32} \left( 65 + 10\zeta(2) - 2\zeta(3) + \left( 1 + \frac{1}{x} \right) \right) \left( -2(19 + 2\zeta(2)) H(-1, x) + 30H(-1, -1, x) - 18H(-1, -1, -1, x) + 6H(-1, 0, -1, x) - 10H(0, -1, -1, x) + 6H(0, -1, -1, -1, x) - 2H(0, 0, -1, x) \right) \]

\[ A_1 = \frac{211}{32} + \frac{9}{80} \zeta(2)^2 + \frac{1}{16} \left( 19\zeta(2) - 5\zeta(3) - \left( 1 + \frac{1}{x} \right) \right) \left( (65 + 10\zeta(2) - 2\zeta(3)) H(-1, x) - (57 + 6\zeta(2)) H(-1, -1, x) + (19 + 2\zeta(2)) H(0, -1, x) + 45H(-1, -1, -1, x) - 15H(-1, 0, -1, x) - 15H(0, -1, -1, x) \right) \]

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Harmonic Polylogarithms (HPLs)

- Weight $= 1$

\[ H(0, x) = \ln x \quad H(-1, x) = \int_0^x \frac{dt}{1 + t} = \ln (1 + x) \quad H(1, x) = \int_0^x \frac{dt}{1 - t} = -\ln (1 - x) \]

- Weight $> 1$

If $\vec{a} = \vec{0}$ we define $H(\vec{0}, x) = \frac{1}{\omega!} \ln \omega x$. If $\vec{a} \neq \vec{0}$:

\[ H(\vec{a}, x) = \int_0^x dt \ f(a_1, x) \ H(\vec{a}_{\omega-1}, t) \quad \frac{d}{dx} H(\vec{a}, x) = f(a_1, x) H(\vec{a}_{\omega-1}, x) \]

- The Algebra: $\omega \vec{a} \times \omega \vec{b} = \omega \vec{a} \times \omega \vec{b}$

\[ H(\vec{a}, x) \ H(\vec{b}, x) = \sum_{\vec{c}=\vec{a} \oplus \vec{b}} \ H(\vec{c}, x) \]

- Integration by Parts

\[ H(m_1, ..., m_q, x) = H(m_1, x)H(m_2, ..., m_q, x) - ... + (-1)^{q+1} H(m_q, ..., m_1, x) \]

- Connection with Nielsen's polylog and Spence functions:

\[ S_{n,p}(x) = H(\vec{0}_n, \vec{1}_p, x) \quad Li_n(x)=H(\vec{0}_{n-1}, 1, x) \]


Generalized Harmonic Polylogs (GHPLs)

**Weight = 1**

In addition to the usual basis functions $g(0, x) = 1/x$, $g(1, x) = 1/(1 - x)$, $g(-1, x) = 1/(1 + x)$, the following enlargement of the set is considered:

\[
\begin{align*}
g(\pm 4, x) &= \frac{1}{4 \mp x}, \quad g(c, x) = \frac{1}{x - e^{\frac{i\pi}{3}}}, \quad g(\bar{c}, x) = \frac{1}{x - e^{-\frac{i\pi}{3}}}, \quad g(\pm r, x) = \frac{1}{\sqrt{x(4 \mp r)}} \\
g(\mp 1 - r, x) &= \frac{1}{(1 \pm x)\sqrt{x(4 + r)}}, \quad g(\mp 1 + r, x) = \frac{1}{(1 \pm x)\sqrt{x(4 - r)}}
\end{align*}
\]

The weight 1 GHPLs are:

\[
\begin{align*}
H(\pm 4; x) &= \mp \log (4 \mp x) \pm 2 \log 2, \quad H(c; x) = \log (x - 1/2 - i\sqrt{3}/2) - \log (-1/2 - i\sqrt{3}/2), \\
H(\bar{c}; x) &= \log (x - 1/2 + i\sqrt{3}/2) - \log (-1/2 + i\sqrt{3}/2), \quad H(-r; x) = 2 \log (\sqrt{x + 4} + \sqrt{x}) - \log 2, \\
H(r; x) &= 2 \arcsin (\sqrt{x}/2), \quad H(-1 - r; x) = 2/\sqrt{3} \arctan (\sqrt{3x/(4 + x)}), \ldots
\end{align*}
\]

**Weight > 1**

\[\vec{a} = \{0, \pm 1, \pm 4, c, \bar{c}, \pm r, \pm 1 \pm r\}.\] If $\vec{a} = \vec{0}$ we define $H(\vec{0}, x) = \frac{1}{\omega!} \ln^\omega x$. If $\vec{a} \neq \vec{0}$:

\[
\frac{d}{dx} H(\vec{a}, x) = g(a_1, x) \frac{d}{dx} H(\vec{a}_{\omega-1}, x) = \int_0^x dt \ g(a_1, x) H(\vec{a}_{\omega-1}, t)
\]

The Algebra and the other properties of the HPLs are maintained

Checks

- Numerical checks
  - Numerical checks on the Master Integrals using FIESTA

- Analytical checks
  - Many Master were already used in other calculations
  - We checked the QCD part against the two-loop calculation by van Neerven et al.
  - The self energies match with Djouadi-Gambino
  - We are checking the $Z$ form factors against Kotikov-Kuhn-Veretin.
Conclusions

- Drell-Yan is one of the best studied processes in hadronic physics

- The theoretical description involved the effort of many groups and is done at the moment at NNLO + resummation for what concerns QCD. EW NLO corrections are available. At the moment the complete set of mixed QCD-EW corrections is not known.

- We calculated analytically the virtual QCD-EW corrections for $Z$ and $W$ production in the resonant region.

- The complete evaluation of the cross section needs the evaluation of the real emission.