The NNLO Charm Contribution to ε_K and ΔM_K

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Outline

- Introduction to ε_K
- η_{cc} at NNLO
- Numerics

Tension in the Data?

Experiment:

ε_K is measured precisely:

$|\varepsilon_K| = 2.228(11)\times10^{-3}$.

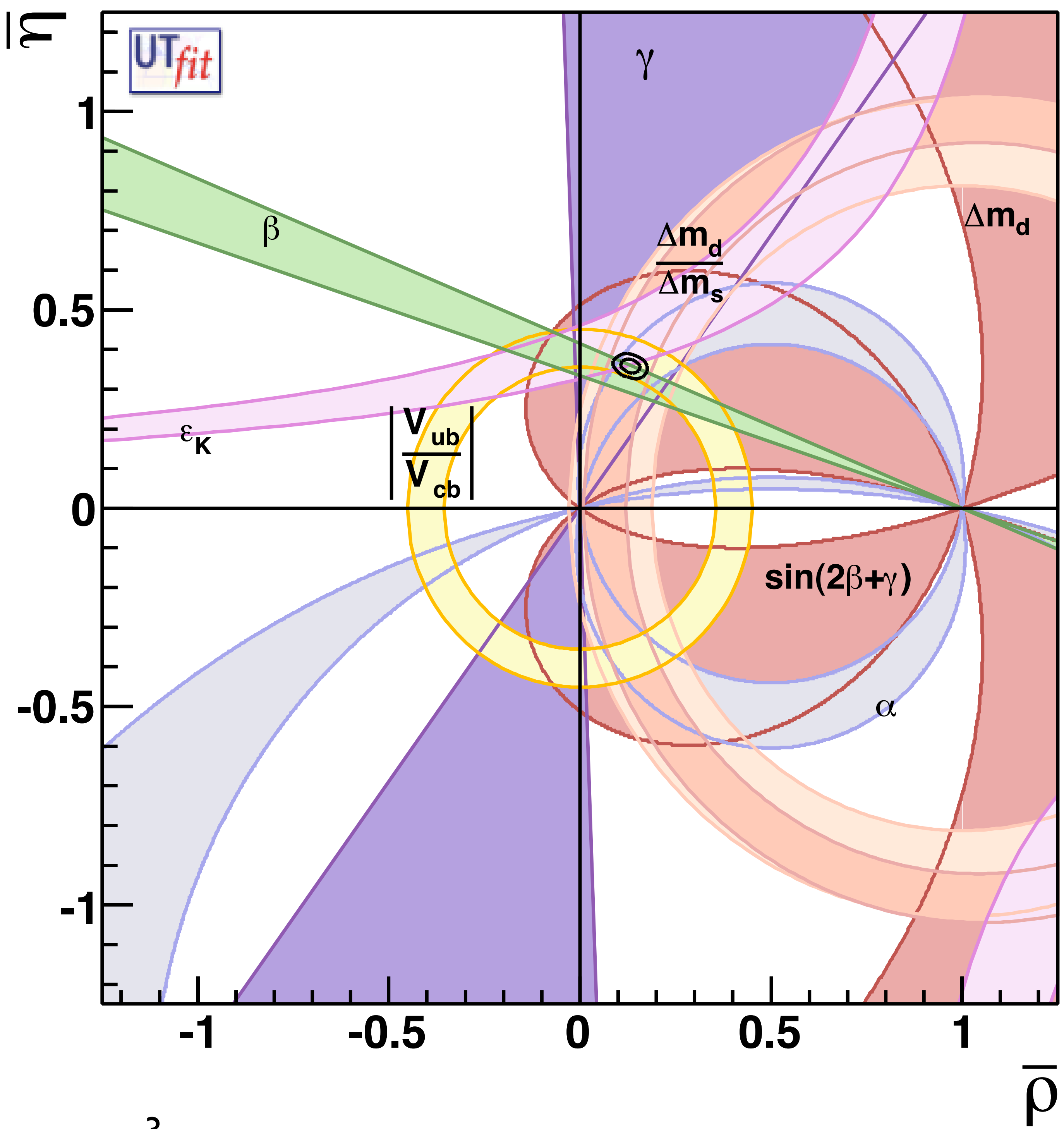
[PDG2010]

Theory:

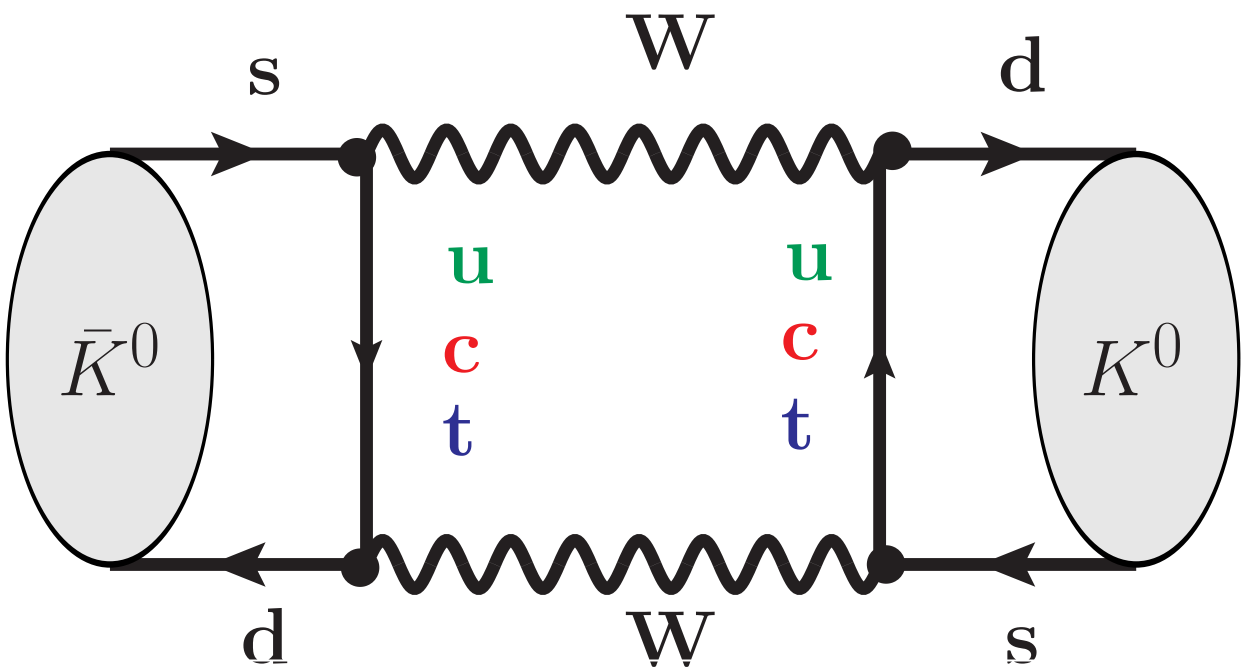
ε_K can be calculated reliably:

$|\varepsilon_K| = 1.90(27)\times10^{-3}$.

[Brod, Gorbahn '10; current SM prediction]



εK in the Standard Model

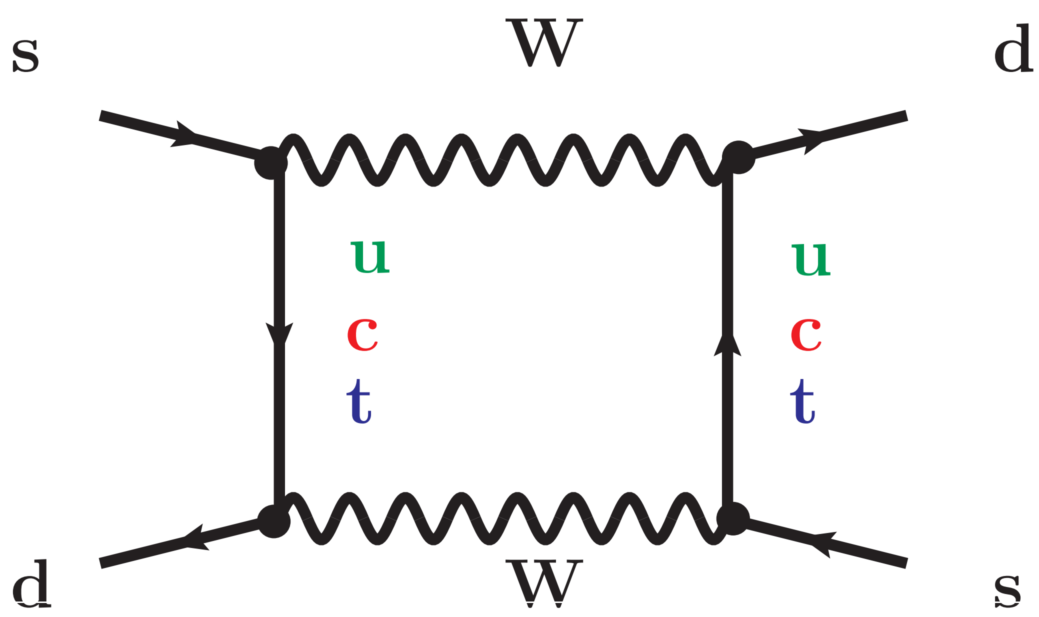


$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- Top-quark contribution CKM suppressed
  ⇒ sensitivity to deviations from MFV
- Chiral Enhancement of non-SM operators

$|\Delta S|=2$ Hamiltonian

$$H^{|\Delta S|=2} \propto \left[\lambda_c^2\eta_{cc}S\left(\frac{m_c^2}{M_W^2}\right)+\lambda_t^2\eta_{tt}S\left(\frac{m_t^2}{M_W^2}\right)+\lambda_c\lambda_t\eta_{ct}S\left(\frac{m_c^2}{M_W^2},\frac{m_t^2}{M_W^2}\right)\right]\tilde{Q}^{|\Delta S|=2}$$



CKM parameters:

$$\lambda_i = V_{is}^* V_{id}$$

$$\lambda_u = -\lambda_c - \lambda_t$$

Re $\langle K^0|H^{|\Delta S|=2}|\bar{K}^0\rangle \Rightarrow \Delta m_K$ **Im** $\langle K^0|H^{|\Delta S|=2}|\bar{K}^0\rangle \Rightarrow \varepsilon_K$

|ΔS|=2 Hamiltonian

$$H^{|\Delta S|=2} \propto \left[\lambda_c^2 \eta_{cc} S\left(\frac{m_c^2}{M_W^2}\right) + \lambda_t^2 \eta_{tt} S\left(\frac{m_t^2}{M_W^2}\right) + \lambda_c \lambda_t \eta_{ct} S\left(\frac{m_c^2}{M_W^2}, \frac{m_t^2}{M_W^2}\right)\right] \tilde{Q}^{|\Delta S|=2}$$

$$\approx \lambda_{\text{Cabibbo}}^{10} \times \eta_{tt} \times \frac{m_t^2}{M_W^2}$$

$\lambda_{\text{Cabibbo}} \approx 0.2$

Dominant contribution (≈+75%)

η_{tt} = 0.5765(65) at NLO QCD [Buras et al. '90]

|ΔS|=2 Hamiltonian

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$$\approx \lambda_{\text{Cabibbo}}^6 \times \eta_{ct} \times \frac{m_c^2}{M_W^2} \log \frac{m_c^2}{M_W^2}$$

Contributes ≈+45%

$\eta_{ct} = 0.496(47)$ at NNLO QCD [Brod, Gorbahn '10]

New three-loop calculation: +7% shift w.r.t. NLO!

|ΔS|=2 Hamiltonian

$$H^{|\Delta S|=2} \propto \left[\lambda_c^2 \eta_{cc} S\left(\frac{m_c^2}{M_W^2}\right) + \lambda_t^2 \eta_{tt} S\left(\frac{m_t^2}{M_W^2}\right) + \lambda_c \lambda_t \eta_{ct} S\left(\frac{m_c^2}{M_W^2}, \frac{m_t^2}{M_W^2}\right)\right] \tilde{Q}^{|\Delta S|=2}$$

$$\approx \lambda^6_{\text{Cabibbo}} \times \eta_{cc} \times \frac{m_c^2}{M_W^2}$$

Smallest contribution (≈-18%)

η_{cc} = 1.40(35) at NLO QCD [Herrlich, Nierste '94, '03]

How large are the NNLO corrections?

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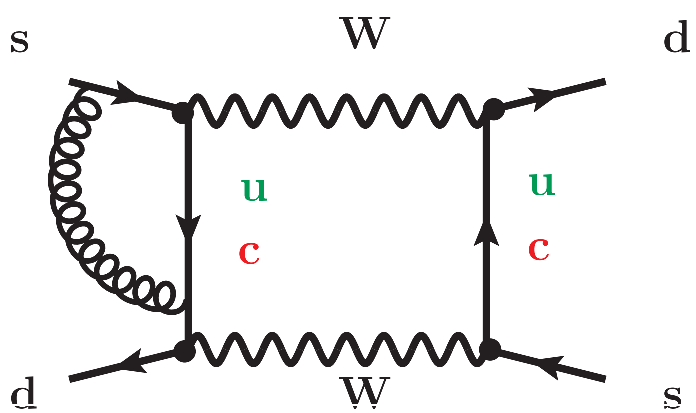
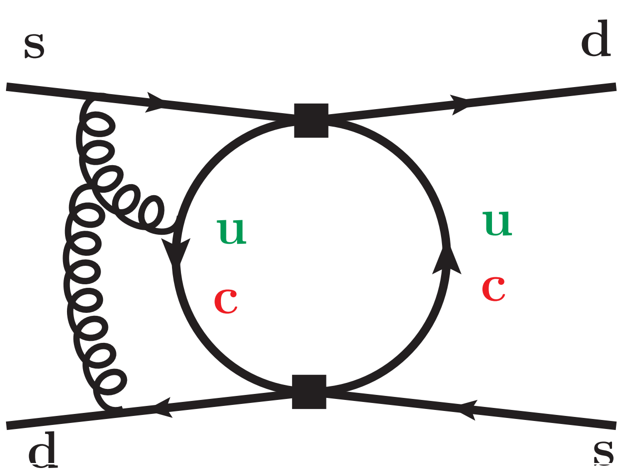
Too deep we delved there, and woke the nameless fear.

[J. R. R. Tolkien, The Lord of the Rings]

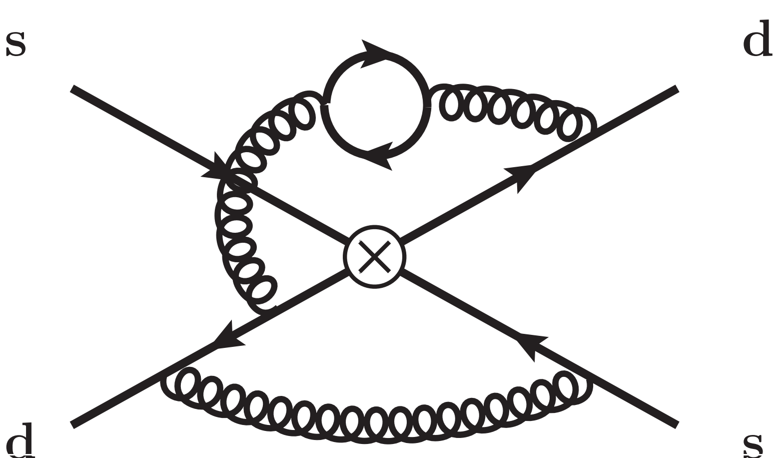
η_{cc} at NNLO: Calculation



Matching at M_W: vanishes by GIM [Witten '76]



Running to m_c:
only $\Delta S=1$ operators contribute



Matching at m_c:

- Match to effective 3-flavour theory
- O(100 000) 3-loop Feynman diagrams

μ_{NP}

μ_{Wt}

μ_b

μ_c

Λ_{QCD}

η_{cc} at NNLO: Result...

$$\langle Q_2 Q_2 \rangle = \frac{69738523}{113400} + \frac{47407}{8505}\pi^2 - \frac{1733}{810}\pi^4 + \frac{1872}{35}\sqrt{3}\,\mathrm{ImLi}_2((-1)^{1/3})$$
$$+24(\mathrm{ImLi}_2((-1)^{1/3}))^2 - \frac{32}{27}\pi^2\log(2)^2 + \frac{32}{27}\log(2)^4 + \frac{563}{18}\log\frac{\mu_c^2}{m_c^2}$$
$$+\frac{32}{3}\pi^2\log\frac{\mu_c^2}{m_c^2} + \frac{193}{3}\log^2\frac{\mu_c^2}{m_c^2} + \frac{256}{9}\mathrm{Li}_4(1/2) - \frac{15145}{54}\zeta_3$$

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$\Rightarrow \eta_{cc} = 1.87$

Corresponds to +36% shift!

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$\Rightarrow \eta_{cc} = 1.87 \pm ???$

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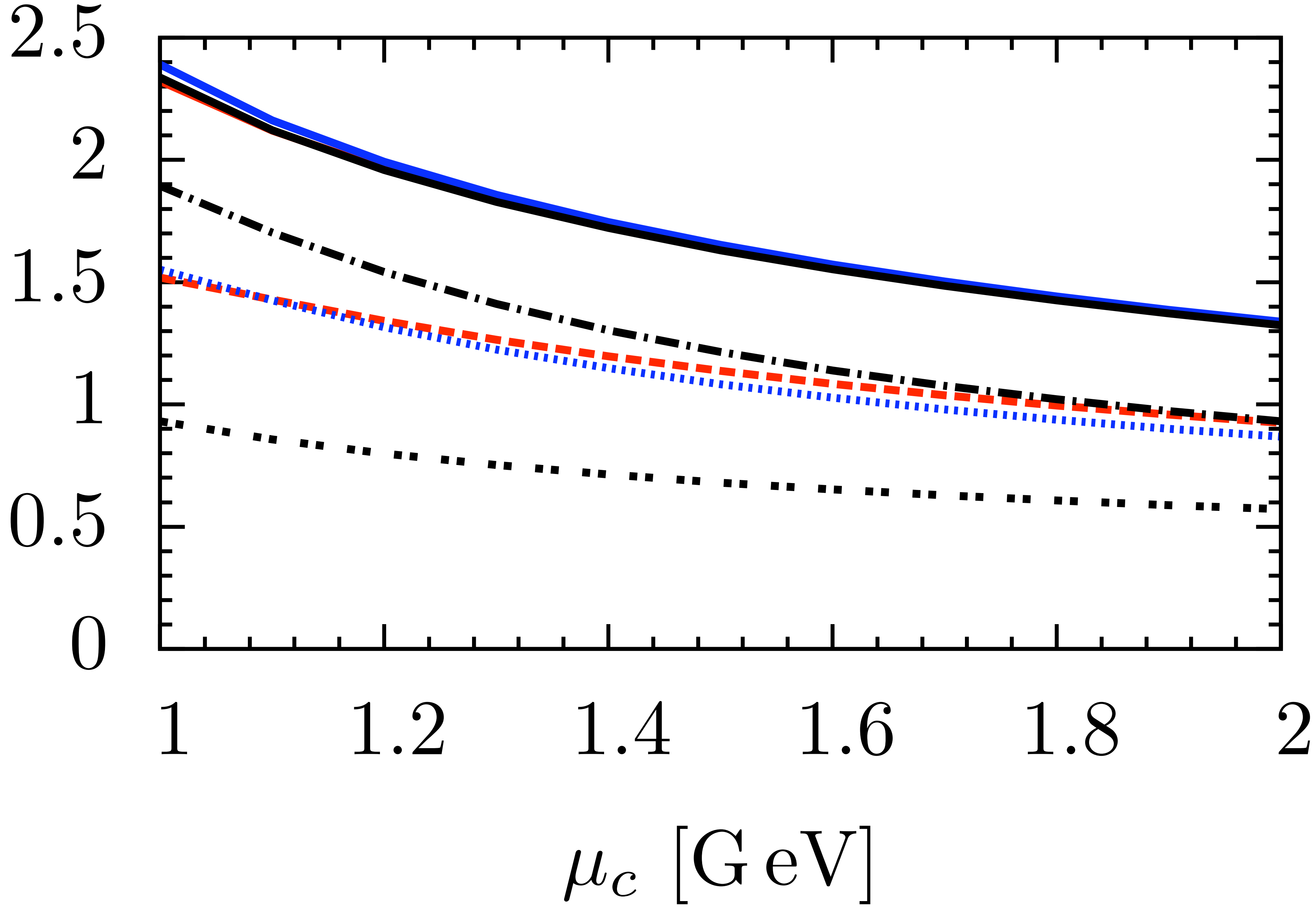
η_{cc} at NNLO: Scale Dependence

Matching @ mc(mc)



η_{cc} at NNLO: Uncertainty

- Divergence of perturbation series, renormalons?
- Nonperturbative → perturbative matching at NNLO?
- Compute on the lattice?

Preliminary prescription:
Add NNLO shift and scale uncertainty in quadrature

$$\eta_{cc} = 1.87 \pm 0.76$$

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$|\varepsilon_K|$ - Numerics

using only experimental and lattice input:

$$|\varepsilon_K| \propto \kappa_\varepsilon B_K |V_{cb}|^2 \xi_s \sin\beta$$

$$\times(|V_{cb}|^2 \xi_s \cos\beta \ \eta_{tt} S(x_t) + \eta_{ct} S(x_c,x_t) - \eta_{cc} S(x_c))$$

$\vert V_{cb}\vert$	0.0406(13)
$\sin(2\beta)$	0.671(23)
ξ_s	1.243(28)
B_K	0.725(26)
κ_ε	0.94(2)

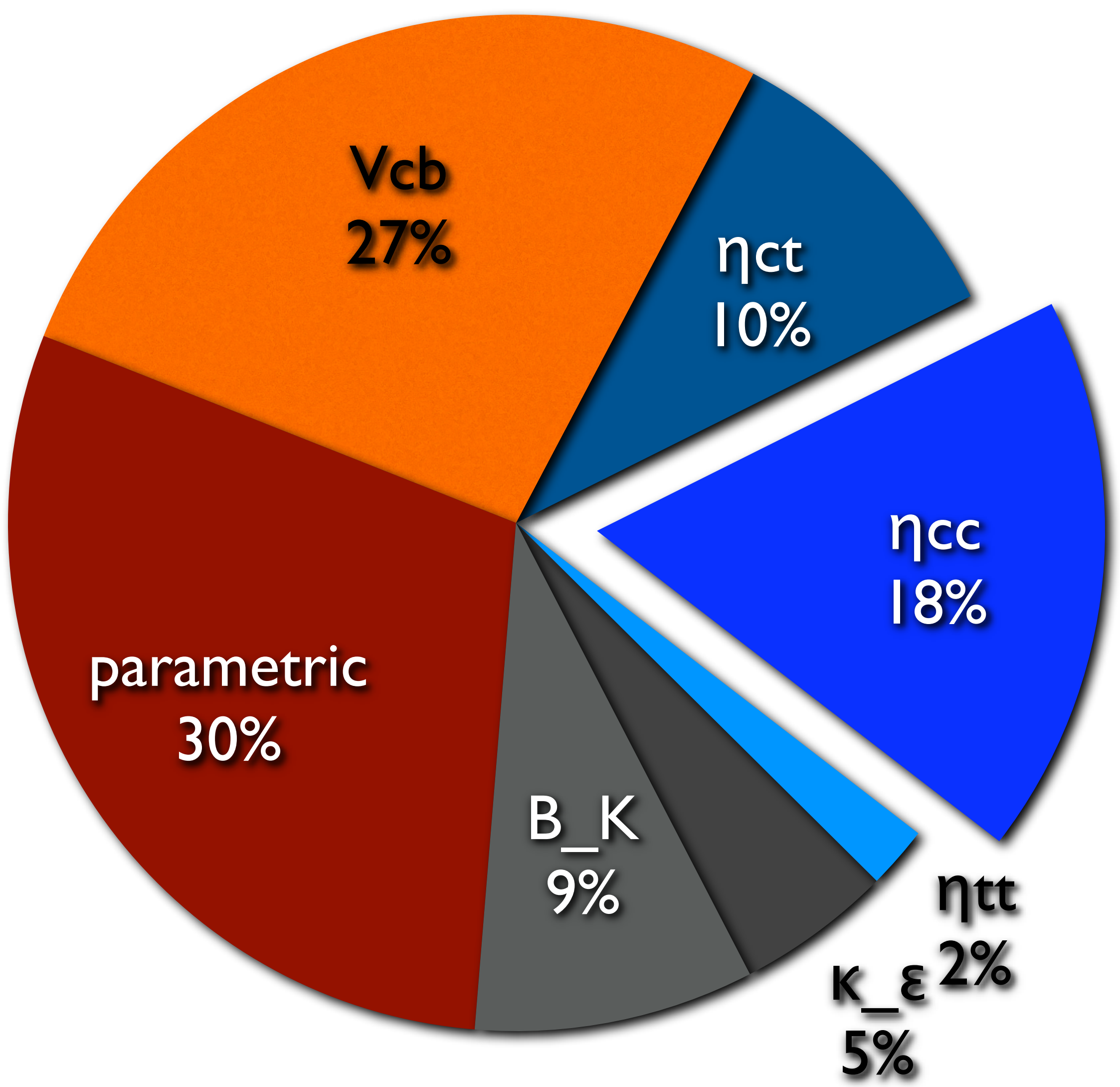
η_{ct}	0.496(47)
η_{cc}	1.87(76)
η_{tt}	0.5765(65)

$$\xi_s = \frac{F_{B_s}\sqrt{\hat{B}_s}}{F_{B_d}\sqrt{\hat{B}_d}} \qquad x_q = \frac{m_q^2}{M_W^2}$$

$|\varepsilon_K|$ - Result & Error Budget

$|\varepsilon_K| = 1.81(28) \times 10^{-3}$ using $\eta_{cc} = 1.87(76)$

cf. $|\varepsilon_K| = 1.90(27) \times 10^{-3}$ using $\eta_{cc} = 1.38(53)$ (NLO!)



cf. $|\varepsilon_K| = 1.53(23) \times 10^{-3}$
using $|V_{cb}| = 0.0387(11)$ (exclusive)

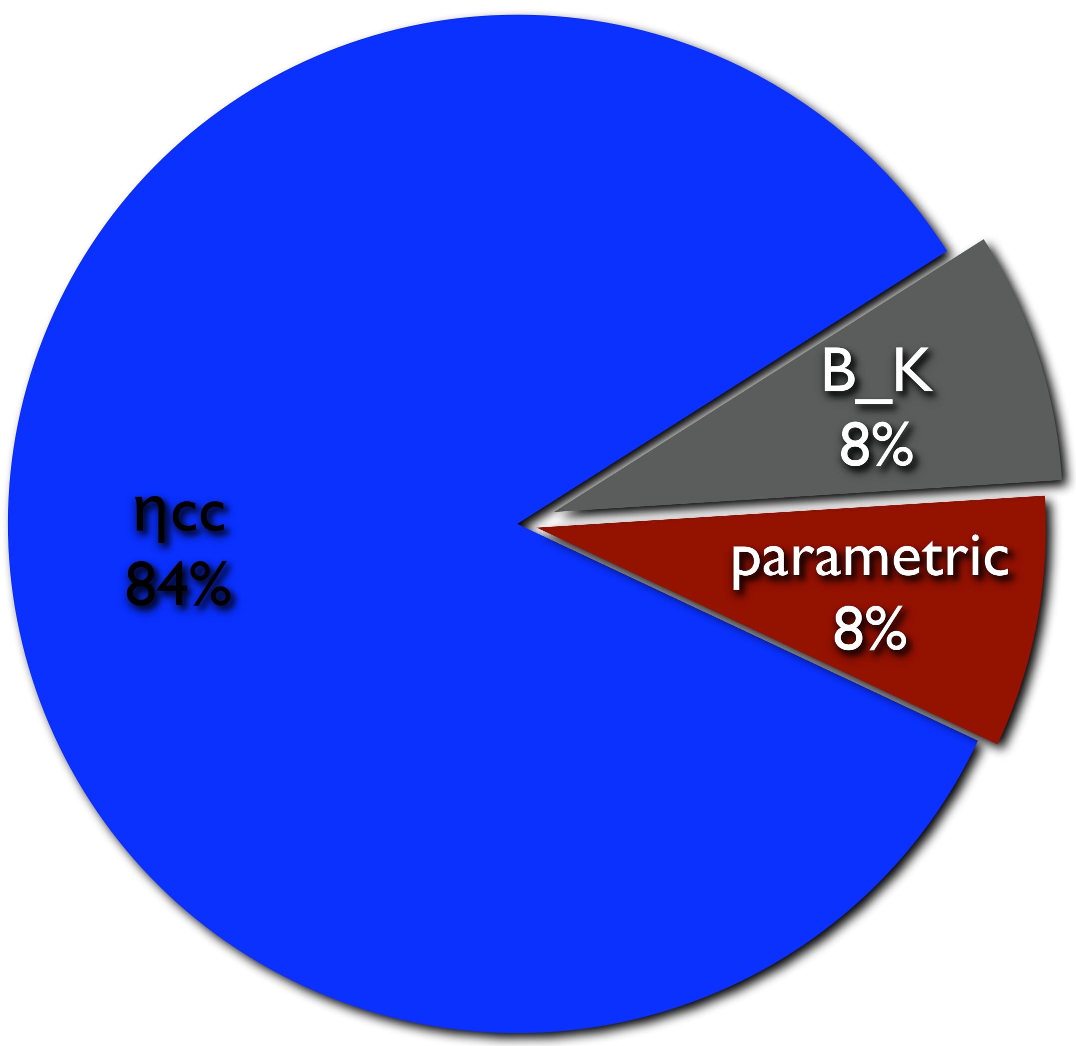
Experiment [PDG '10]:

$$|\epsilon_K| \stackrel{\text{exp.}}{=} 2.228(11) \times 10^{-3}$$

$\Delta M_K(SD)$ -- Numerics

$\Delta M_K(SD) = (3.0 \pm 1.2) \times 10^{-15}$ GeV using $\eta_{cc} = 1.87(76)$

cf. $\Delta M_K(SD) = (2.2 \pm 0.9) \times 10^{-15}$ GeV using $\eta_{cc} = 1.38(53)$ (NLO!)



$\Delta M_K(SD)$
$= 3.483(6) \times 10^{-15}$ GeV
[PDG '10]

(86% short distance)

Conclusion

- ε_K is sensitive to scales of new physics.
- NNLO calculation yields +36% shift of charm-quark contribution to ε_K, leading to $\eta_{cc} = 1.87(76)$.
- SM prediction for ε_K is shifted by ≈-5% to $\varepsilon_K = 1.81(28)\times 10^{-3}$.

We might already see new physics --
but first we need to understand the Standard Model!

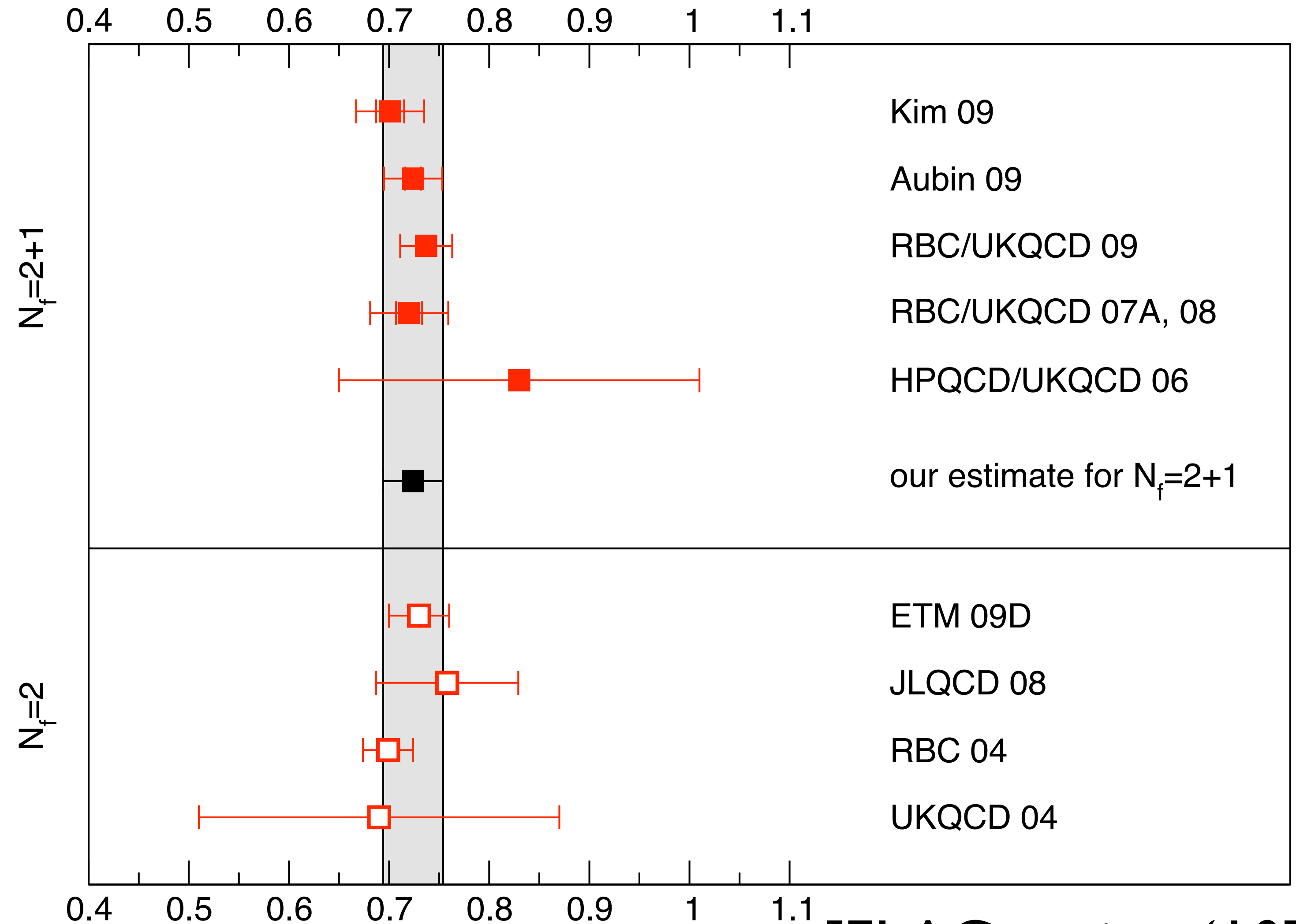
Backup

Determination of B_K

$$\langle K^0|\tilde{Q}^{|\Delta S|=2}|\bar{K}^0\rangle \propto \widehat{B}_K = 0.725(26)$$

Huge progress:

- Unquenched calculations
- Small pion masses
- Different lattice spacings
- Main error: matching to perturbative renormalisation



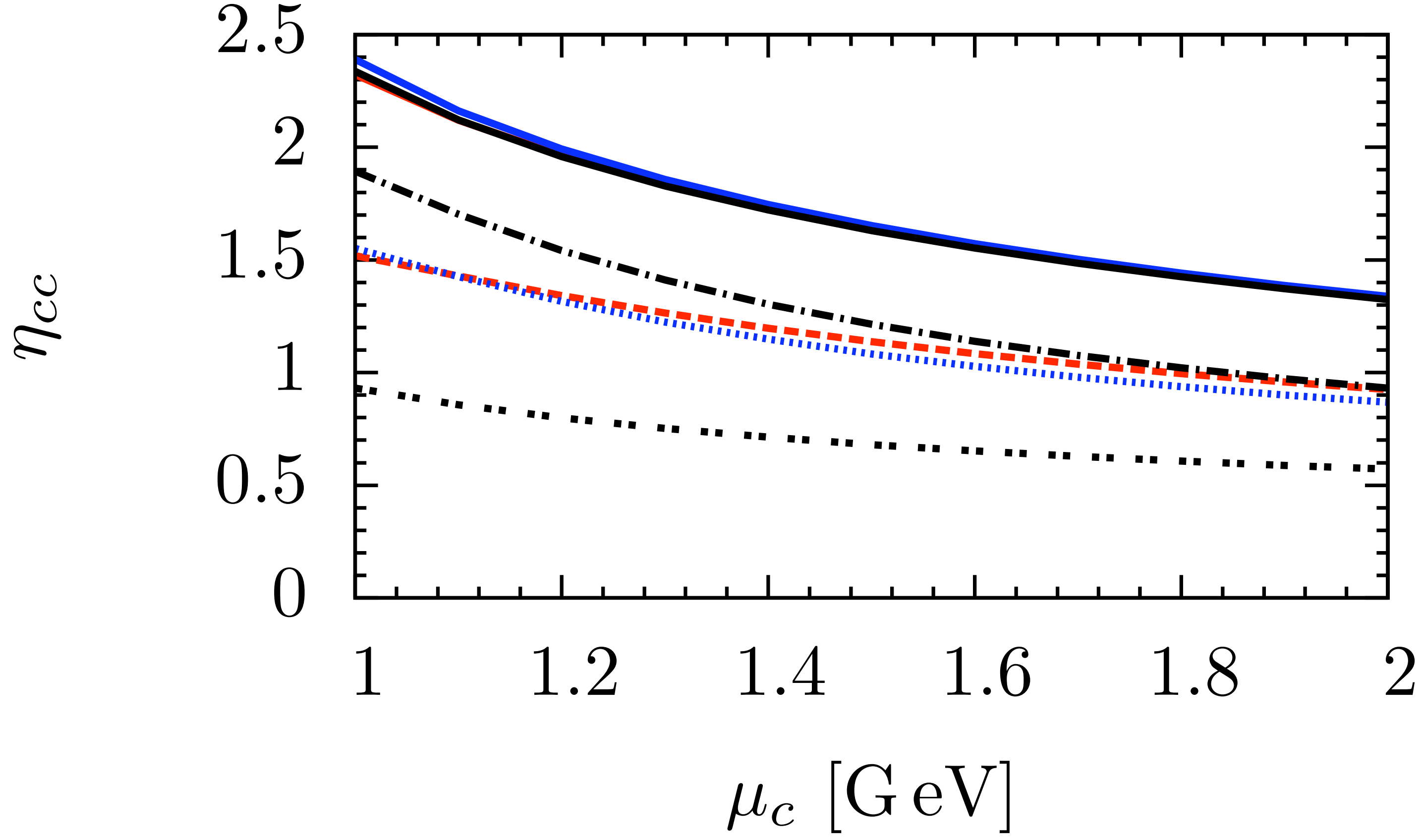
[FLAG review '10]

η_{cc} at NNLO: Convergence

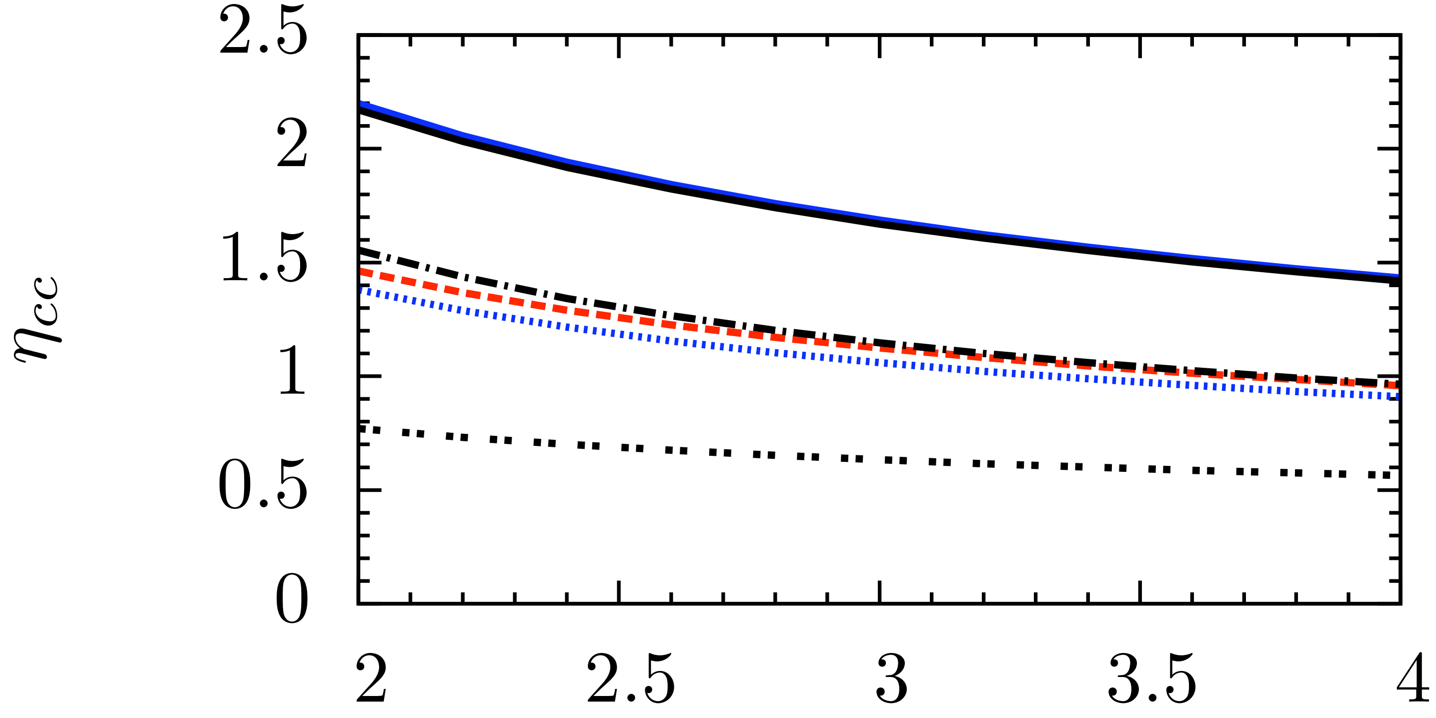
$$\eta_{cc}/\alpha_s\text{^}(2/9) = 1$$
$$+ \alpha_s \quad (0.25 + 0.64\, L_c)$$
$$+ \alpha_s\text{^}2\ (1.20 + 0.44\, L_c + 1.08\, L_c\text{^}2)$$

$L_c = \log(m_c/M_W) = -\ 4.14$ $\qquad$ $\alpha_s = \alpha_s(m_c) = 0.35$

η_{cc} at NNLO: Scale Dependence



Matching @ mc(mc)



Matching @ 3 GeV