NNLO QCD corrections to $b \to uW^*$

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Zhao-ting Pan

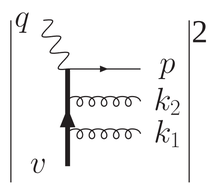
Introduction

The measurements of inclusive semileptonic B meson decays,such as $\overline{\mathrm{B}} \to X_u l \bar{\nu}_l$ and $\overline{\mathrm{B}} \to X_c l \bar{\nu}_l$, allow a precise determination of the CKM matrix element $|V_{ub}|$ and $|V_{cb}|$.Total decay rates of the B meson are described by a local Operator Product Expension(OPE) in inverse powers of the b-quark mass m_b. To the leading order in $1/m_b$, the total B meson decay rate is equivalent to the decay rate of the on-shell b quark, which can be calculated in perturbation theory. However,experimental collaborations need to impose cuts on the kinematic variables. It is therefore of great interest to consider differential decay distributions, from which it is possible to derive predictions for partial decay rates with arbitrary cuts. Different frameworks were developed in order to accont for the effects due to cuts on the kinematic space. At the NLO, the triple differential distribution of the inclusive semileptonic decay $\overline{\mathrm{B}} \to X_u l \bar{\nu}_l$ has been calculated. At the NNLO, the virtual corrections are known (as well as more complete studies made using SCET). In this work, we take into account the calculation of the real emission.

IBP's Method for cut diagram

Using the Cutkosky rules

First we consider the following double-real contribution at NNLO:

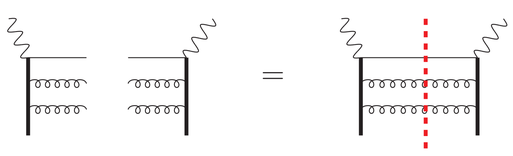


$$\sim \int \frac{d^d k_1 d^d k_2 \delta(p^2)\delta(k_1^2)\delta(k_2^2)\,[\ldots]}{k_1^4 k_2^4 p^4}$$

Using the Cutkosky rules, we can replace the delta-functions in the above integral by the difference of two propagators, with different $i0$ prescription:

$$2\,i\,\pi\delta\left(p^2-m^2\right) \to \frac{1}{p^2-m^2+i0} - \frac{1}{p^2-m^2-i0}.$$

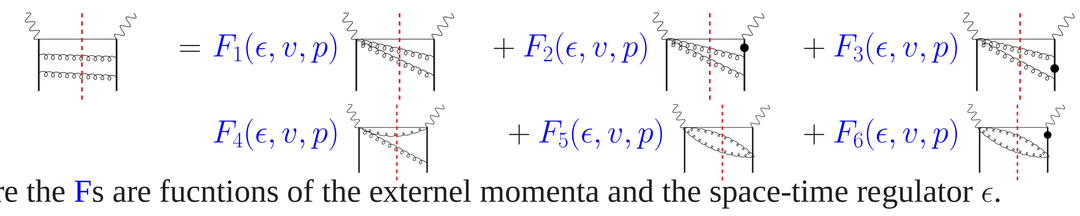
the r.h.s of last equation is equal to a forward scattering diagram:



After using Cutkosky rules, the phase-space integrals can then be evaluated in the same algorithmic way as in the multi-loop integral case: using IBP's in the "Laporta algorithm"

Reduction of cut diagrams

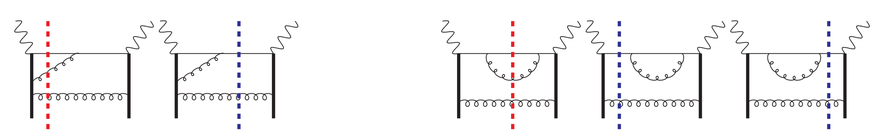
The Laporta Algorithm is implemented in the C++ program REDUZE. We use it to perform the reduction of the cut diagrams to a small set of independent scalar integrals called "Master Integrals". The above diagram can be, for instance, expressed as follows:



where the Fs are fucntions of the externel momenta and the space-time regulator ϵ.

Different cuts

The contribution coming from the real emission at NNLO can be grouped in two different categories: two-cut or three-cut diagrams:

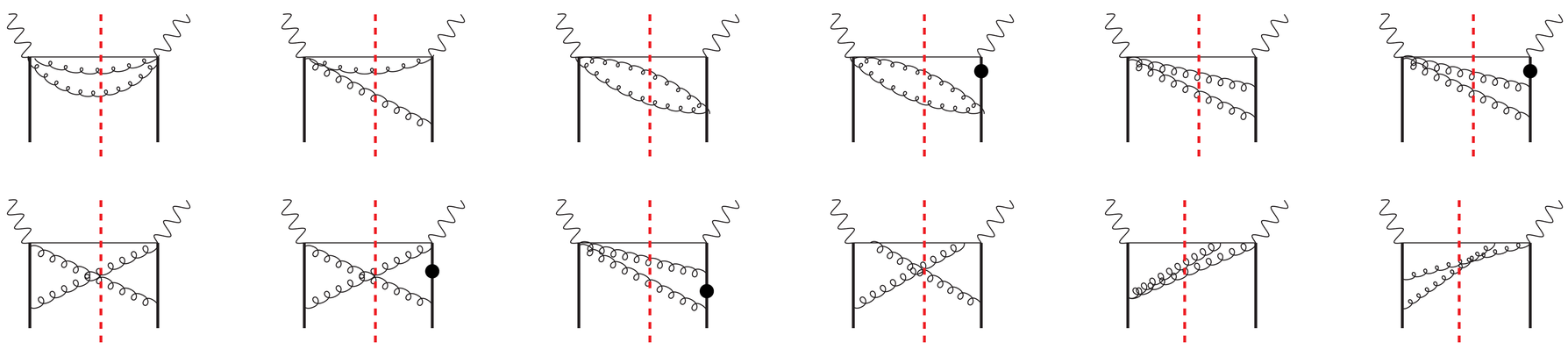


So far, we considered the three-cut diagrams and found out the Masters Integrals.

The Master Integrals for the three-cut diagrams and the Differential Equations

Master Integrals for the three-cut digrams

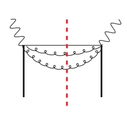
We list all the Master Integrals for the three-cut diagrams that we found in the reduction process:



A dot on the propagator line means that the propagator is raised to power 2. Bold lines represent a massive propagator. Thin and curly lines denote massless propagators.

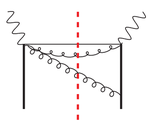
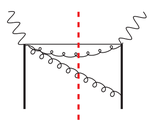
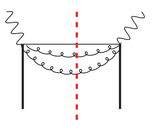
Master integrals of three-cut digrams

We normalize the three-body phase space integral to 1:

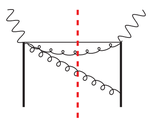


$$= 1$$

The MIs satisfy a system of first-order linear differential equations in the dimensionless variables $t = \sqrt{1-\frac{4p^2}{m_b^2 z^2}}$ and $z = -\frac{2\,v\cdot p}{m_b^2}$. A simple example is the following:

$$\frac{\partial}{\partial t}$$  $$= \left[\frac{1}{t}+\frac{1}{1-t}-\frac{1}{t+1}+(D-4)\left(\frac{3}{2}\frac{1}{1-t}-\frac{3}{2}\frac{1}{t+1}+\frac{1}{t}\right)\right]$$  $$+\frac{1}{z m_b^2}\left[\frac{2}{1-t}-\frac{3}{2}\frac{1}{1-t}+\frac{2}{1+t}+(D-4)\frac{3}{2}\frac{1}{1+t}-\frac{4}{t}-(D-4)\frac{3}{t}\right]$$ 

The initial condition is found imposing the regularity of the function in t = 0. In general, we look for the solution as a Laurent series in $(D-4)$. In this case we find:



$$= \frac{2}{m_b^2 z}\left[\frac{1}{t+1}+\frac{1}{1-t}\right] + \mathcal{O}\left((D-4)\right),$$

Conclusions

- We used the Laporta algorithm (implemented in the computer program REDUZE) to perform the reduction to the Master Integrals for the three-cut diagrams of the NNLO QCD corrections to the decay process $b \to uW^*$. Some MIs are already solved. Others have a more complicated structure and need a further study. We found at most three MIs per topology.
- The two-cut diagrams still need to be investigated.