Finite energy for a Gravitational Potential Falling Slower than 1/r

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Berezhiani-Comelli-Nesti-LP **JHEP 0807** 130 (2008)
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Comelli-Nesti-LP **PRD83** 084042 (2011)

Comelli-Nesti-LP **arXiv** 1105.3010

Potential falling slower than $1/r$?

Suppose we have a solution of Einstein equations with a static potential ϕ, $g_{tt} = -1 - 2\phi$
that, at large distances, falls off slower than $1/r$

- The total energy of the system would infinite. According Newton, source's total mass is $\sim$ flux of $\nabla\phi$

$$E = \frac{1}{4\pi G}\int_{\mathcal{S}_2} d^2x\, \vec{\nabla}\phi \cdot \vec{n}$$

  Finite E only if $\phi \sim 1/r$
- No such a solution in perturbative GR: Green function goes as $1/r$
- Modify gravity , Why do we need a non-Newtonian potential ?

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A 90 year-long successful theory
a single free parameter and it works great

- Equivalence principle 10^{-12} level
- Solar system tests (weak field) 10^{-4} level
- Binary pulsar (nonlinear) 10^{-3} level

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however .....

- there are some puzzling features at large distances
- CMB + Supernovae data require Dark energy
  $p = w\rho$, $w < 0$. Expanded acceleration
  Perhaps just a tiny (??) cosmological constant, $w = -1$,
  $\Lambda \sim (10^{-4}\,\text{eV})^4$ or a bizarre fluid?
- Is GR an isolated theory ? How rigid is GR ?

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Degrees of Freedom

- GR $\quad M_{pl}^2 E_{\mu\nu}^{(1)} = T_{\mu\nu}^{(1)}\,, \qquad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

  DOF $10 - 2 \times 4 = 2 \qquad$ 4 gauge modes $\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$
- Massive gravitons (Minkowski) have 5 DOF, more DOF needed
- Give up gauge symmetry. Fierz-Pauli theory (1939)

  $L_{FP} = M_{pl}^2 L_{\text{grav}}^{(2)} + M_{pl}^2 m^2 \left(a\, h_{\mu\nu} h^{\mu\nu} + b\, h^2\right)$

  $E_{\mu\nu}^{(1)} - \frac{1}{4} m^2 \left(a\, h_{\mu\nu} + b\, h\, \eta_{\mu\nu}\right) = M_{pl}^{-2} T_{\mu\nu}^{(1)} \qquad \partial^\nu E_{\mu\nu}^{(1)} = 0$

  4 constraints $\qquad$ DOF $10 - 4 = 6 = 5 + 1$
- The sixth mode is a ghost (Boulware-Deser).
  Absent in flat space when $a + b = 0$ (FP theory)
  present in curved space and at the nonlinear level
- When the ghost is projected out, light bending badly contradicts experiments (van Dam, Veltman, Zakharov) vdVZ discontinuity

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Beyond the Linear Level

Bottom line

Massive gravity around Minkowski background is problematic

- Linear FP theory suffers from vDVZ discontinuity
- Nonlinear effects are dominated by unknown UV physics
- Very low cutoff, solar system scale physics cannot be trusted if $m \sim H$

Giving up Minkowski background can help

- At linearized level there is no ghost, no vDVZ discontinuity
  FP tuning not needed
- Also propagation around curved background looks better

Rubakov, Dubowsky, Comelli-Nesti-Pilo

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Step 1: Recasting the mass term

- $g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu} + h^{\mu\alpha} h^{\nu}_{\alpha} + \cdots \quad \Rightarrow \quad g^{\mu\nu}\eta_{\mu\nu}$ represents a mass term
- To recover diff (gauge) invariance replace $\eta_{\mu\nu}$ by a dynamical (Stuckelberg) extra metric field $\tilde{q}_{\mu\nu}$
  $\eta_{\mu\nu} \to q_{\mu\nu}$
- New tensor from the two metric $X^{\mu}_{\nu} = g^{\mu\alpha} q_{\alpha\nu}$
- Typical mass terms are made out $\tau_n = \mathrm{Tr}(X^n)$

$$a\,(\tau_1 - 4)^2 + b\,(\tau_2 - 2\tau_1 + 4) = \left(a\,h_{\mu\nu}h^{\mu\nu} + b\,h^2\right) + \cdots$$

The Stuckelberg Trick in Massive GR: Bigravity I

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Step 2: Stuckelberg Dynamics

- The extra metrics is turned into a dynamical field

$$S_{MGR} = \int d^4x \left[\sqrt{g}\, M_{pl}^2\, R(g) + \kappa\, M_{pl}^2\, \sqrt{\tilde{g}}\, R(\tilde{g}) - 4(\tilde{g}g)^{1/4}\, V(X)\right]$$

- Matter couples only to $g_{\mu\nu}$
- Gauge symmetry: Diff
- When $\kappa \to \infty$, $\tilde{g}_{\mu\nu}$ gets non-dynamical and flat: $\tilde{g}_{\mu\nu} = e^a_\mu\, e^b_\nu \tilde{\eta}_{ab}$
  $e^a = d\phi^a$ and $\tilde{g}_{\mu\nu} = \partial_\mu \phi^a \partial_\nu \phi^b\, \tilde{\eta}_{ab}$
- When $\kappa \to \infty$ the stuckelberg fields are ϕ^a

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The Stuckelberg Trick in Massive GR: Bigravity III

- In the bigravity unitary gauge and $\kappa \to \infty$, the stuckelberg fields are ϕ^a
- Powerful formalism to treat in unified way both the Lorentz preserving and Lorentz breaking cases

  $X_{|bkg} = \text{Diag}(1, 1, 1, 1)$ Lorentz preserving (LI) background

  $X_{|bkg} = \text{Diag}(a, b, b, b)$ Lorentz breaking (LB) background only rotational symmetry is present
- For any V the LI background is always present
- Modified Einstein equations (Bigravity Unitary gauge)

$$M_{pl}^2 E^\mu_\nu + [Det(X)]^{1/4} \left[V \delta^\mu_\nu - 4(V'X)^\mu_\nu\right] = T^\mu_\nu$$
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- Powerful formalism to treat in unified way both the Lorentz preserving and Lorentz breaking cases

  $X_{|bkg} = \text{Diag}(1, 1, 1, 1)$ Lorentz preserving (LI) background

  $X_{|bkg} = \text{Diag}(a, b, b, b)$ Lorentz breaking (LB) background only rotational symmetry is present
- For any V the LI background is always present
- Modified Einstein equations (Bigravity Unitary gauge)

$$M_{pl}^2 \, E^\mu_\nu + [Det(X)]^{1/4} \left[V \, \delta^\mu_\nu - 4(V' X)^\mu_\nu \right] = T^\mu_\nu$$

$$\kappa \, M_{pl}^2 \, \tilde{E}^\mu_\nu + [Det(X)]^{-1/4} \left[V \, \delta^\mu_\nu + 4(V' X)^\mu_\nu \right] = 0$$

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Modifying Schwarzschild I

- Spherically symmetric ansatz

$$ds^2 = -J(r)\,dt^2 + K(r)\,dr^2 + r^2\,d\Omega^2$$

$$\tilde{d}s^2 = -C(r)\,dt^2 + A(r)\,dr^2 + 2D(r)\,dtdr + B(r)\,d\Omega^2$$

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Finding all solutions if very hard. Consider solutions with $D \neq 0$
Potential independent analysis: $g_{\mu\nu}$ is diagonal $\Rightarrow E^{\mu}_{\nu}$ diagonal $\Rightarrow (V'X)^{\mu}_{\nu}$ diagonal $\Rightarrow \tilde{E}^{\mu}_{\nu}$ diagonal $\Rightarrow \tilde{E}^1_1 = \tilde{E}^2_2 \Rightarrow K = J^{-1}$

- First result potential independent: $\psi = \phi$, leading PN physics same as in GR. Solar system tests are OK !

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Solution

$$J = \left(1 - \frac{2Gm_1}{r}\right) + 2G\,S\,r^{\gamma}\,, \qquad KJ = 1$$

$$C = c^2\omega^2\left(1 - \frac{2Gm_2}{\kappa\,r}\right) - \frac{2G}{c\,\omega^2\kappa}S\,r^{\gamma}\,, \qquad D^2 + AC = c^2\omega^4$$

$$B = \omega^2 r^2\,, \qquad A = \cdots$$

- Integration constants: m_1, m_2 and S. Determined by the parameters in V: c, ω
- When $\gamma < 2$, for $r \to \infty$
  $g \to \mathrm{diag}(-1, 1, 1, 1)$ and $g \to \omega^2\,\mathrm{diag}(-c^2, 1, 1, 1)$

  Lorentz Breaking asymptotics for $c \neq 1$
- When $S \neq 0$ nontrivial modification but still flat at infinity when $\gamma < 2$. When $-1 < \gamma < 2$ the large r behaviour is modified !
- In general the solution can by AdS or dS at infinity ($\gamma < 2$) not shown $\cdots$

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Total Energy

To be physical the solution must have finite total energy
Energy in GR is tricky

- Equivalence principle forbids localization of gravitational energy
  Hypothetical EMT of gravity: $T_{GR}(x_0) \sim \mathcal{F}(\partial g)_{|x_0}$. But at each x_0
  $g(x_0) \equiv \eta$ and $\partial g(x_0) = 0 \Rightarrow T_{GR}(x_0) = 0$
- Energy cannot be taken apart but must be considered as whole
  Locally there is no gravity !
- Energy in GR is the conserved charge associated with an arbitrary translation in time, diff generated by a timelike vector
- Equivalently, given a solution, its ADM energy is the value of the Hamiltonian
  Needed: a splitting of spacetime in space + time

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Energy as a Noether Charge

- Consider the Noether charge associated to timelike translations: $x^\mu \to x^\mu + \xi^\mu$, with $\xi^2 < 0$
- Choose a set boundary condition for dynamical variables, adjust boundary terms in the action so that the charge is a scalar (coordinate independent). NB a reference metric is needed. We use flat space
- Fixing the induced metric on the the 2-surface $t = const$, $r = \bar{r}$ with $\bar{r}$ large, we get the Nester expression for the energy

$$E = \frac{1}{32\pi G} \int_{S_t} d^2 z \, \epsilon_{\rho\sigma\mu\nu} \left(\xi^\tau \Pi^{\beta\lambda} \Delta\Gamma^\alpha_{\beta\gamma} \, \delta^{\mu\nu\gamma}_{\alpha\lambda\tau} + \bar{\nabla}_\beta \xi^\alpha \Delta\Pi^{\beta\lambda} \, \delta^{\mu\nu}_{\alpha\lambda} \right) \frac{\partial x^\rho}{dz^1} \frac{\partial x^\sigma}{dz^2} \, ,$$

- For Schwarzschild , $E = M$, even in Painleve coordinates. Actually does not depend on coordinates ! Ideal tool fo us

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Computation of the energy

- Boundary terms come only from the kinetic parts
  the potential has no role here
- Contribution of $R(g)$ $\qquad E = M - S\bar{r}^{\gamma+1}$
- Contribution of $R(\tilde{g})$ $\qquad \tilde{E} = \tilde{M}c^2 + S\bar{r}^{\gamma+1}$.
- Total energy, finite even when $\bar{r} \to \infty$!

$$E_{tot} = E + \tilde{E} = M + \tilde{M}c^2$$

- Beware ! Consider a the frozen $\tilde{g}$ theory, equivalent to $\kappa \to \infty$. The solution for g is similar, but there is no $\tilde{E}$ contribution. Energy is infinite !
- No decoupling effects of "heavy modes" of $\tilde{g}$, needed to account for all energy budget
- Effective field theories are tricky in gravity when energy is concerned, heavy modes warp spacetime and sometime cannot be neglected

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Conclusions and Outlook

- A non-standard Newton potential calls for modified gravity.
- Bigravity is great tool for studying massive deformation of GR
- No dDVZ discontinuity in bigravity massive deformation
- Spherically symmetric solution featuring:
    1. First nontrivial large distance modification of gravity
    2. Finite total energy

- Outlook
    - Full canonical analysis of bigravity. What does propagate ? Are all modes safe ?
    - Cosmological impact of massive deformation

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Massive Deformed GR

Not an easy task !

Challenge

Build a version of GR modified at large distances such that

- It is consistent with experiments in the solar system
- ideally valid up to the scale $\Lambda_2 = (M_{pl} m)^{1/2}$
  as for broken gauge theories, gauge boson mass $\sim m$
  $\Lambda_2 = m\, g^{-1} = m\, \left(\Lambda_2/M_{pl}\right)^{-1} \Rightarrow \Lambda_2^2 = m\, M_{pl}^2$

From GR valid up to distances $> 10^{-33}$ cm to

Massive GR valid up to distances $> \Lambda_2^{-1} \sim \left[10^{-33} m^{-1} (\text{cm})\right]^{1/2}$ cm

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From GR valid up to distances $> 10^{-33}$ cm to
Massive GR valid up to distances $> \Lambda_2^{-1} \sim \left[10^{-33} m^{-1} (\text{cm})\right]^{1/2}$ cm

Massive Deformed GR

Not an easy task !

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Build a version of GR modified at large distances such that

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Modifying Schwarzschild II

Class of exact solvable potentials

If $\{\lambda_i,\ i = 0, \cdots, 3\}$ are the eigenvalues of X, the potentials

$$V_n = \sum_{i_1 > i_2 \cdots > i_n} \lambda_{i_1} \lambda_{i_2} \cdots \lambda_{i_n}$$

lead to analytically solvable equations

Examples

$$V_1 = \frac{1}{6|\tilde{g}|}(\epsilon\,\epsilon\,\tilde{g}\,\tilde{g}\,\tilde{g}\,g) = \frac{1}{6\mathrm{Det}(X)}(\tau_1^3 - 3\,\tau_2\tau_1 + 2\,\tau_3)$$

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Energy I

- In adapted coordinates (t, x^i), ADM energy measured by an observer with a clock ticking t

$$H_{tot} = \int_{t=\text{const}} d^3x \left[\mathcal{H}\, N + \mathcal{H}_i\, N^i\right] + \int_{S^2,\, r\to\infty} d^2x \mathcal{B}$$

$$\overset{\text{on shell}}{=} \int_{S^2,\, r\to\infty} d^2x \mathcal{B}$$

S^2 is 2-sphere bounding space ($t = \text{const}$) at infinity

- The value of $\mathcal{B}$ and then the total energy depends on the detailed asymptotics of $g_{\mu\nu}$
- For asymptotically flat spacetime, $h_{ij} \sim \delta_{ij}/r$ at large r, and using asymptotics Cartesian coordinates x^i

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Energy II

- ADM limitations: derivatives of h_{ij} (extrinsic curvature) must fall-off at least as $1/r^2$ to be well defined
- Coordinates must be Cartesian at Infinity No good for our solution ! Large distances: $D \sim 1/\sqrt{r}$ (for $\gamma < -1$). Too slow
- Analogous to the Schwarzschild solution written in Painlevé coordinates: $dt = dT - f' dr$

$$ds^2 = -J\, dt^2 + J^{-1}\, dr^2 + r^2\, d\Omega^2$$
$$= -J\, dT^2 + 2f' J\, dTdr + dr^2 + 2f' J\, dTdr + r^2\, d\Omega^2$$
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ADM energy is zero in Painleve coordinates !! In reality is not defined in Painleve coordinates. Extrinsic curvature does not have the right fall-off

- We need a more general tool: Gravitational energy as a Noether charge

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