Analysis of the anomalous-dimension matrix of *n*-jet operators to four loops in SCET

Leonardo Vernazza

In collaboration with

Valentin Ahrens and Matthias Neubert

THEP

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

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INTRODUCTION

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- Understanding the structure of *infrared singularities* in gauge theory amplitude has been a long standing issue.

- Recently, it has been shown that they can be mapped onto UV divergences of n-jet operators in SCET. (Becher,Neubert, 2009)

- This means they can be described by means of an anomalous dimension, whose structure is constrained by: (Becher,Neubert, 2009; Gardi, Magnea 2009)

  - *soft-collinear factorization,*
  - *color conservation,*
  - *non-abelian exponentiation,*
  - *collinear limit.*

- A conjecture has been formulated, which has an extremely simple form and it should hold to all order in perturbation theory.

MOTIVATION

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- Important phenomenological applications in higher order log resummation for *n*-jet processes.

- Interesting for the understanding of the deeper structure of QCD: the anomalous dimension predicts only pairwise interactions among different partons.

- It implies Casimir scaling of the cusp anomalous dimension, in contrast with results obtained using the AdS/CFT correspondence in the strong-coupling behavior.

- This does not tell if and at which order a violation of the Casimir scaling could arise in perturbation theory. A diagrammatic analysis excluded it up to 3 loop, and at 4 loops in terms with higher Casimir invariants.
  (Becher, Neubert 2009; Gardi, Magnea 2009)

- Our aim is to complete the diagrammatic analysis at four loop.

INFRARED DIVERGENCES OF GAUGE THEORY AMPLITUDES

- Given a UV renormalized, on-shell *n*-parton scattering amplitude with IR divergences regularized in $d = 4 - 2\epsilon$ dimensions, one obtains the finite remainder free from IR divergences from

$$|\mathcal{M}_n(\{\underline{p}\},\mu)\rangle = \lim_{\epsilon\to 0} \mathbf{Z}^{-1}(\epsilon,\{\underline{p}\},\mu)|\mathcal{M}_n(\epsilon,\{\underline{p}\})\rangle.$$

- The multiplicative renormalization factor **Z** derives from an anomalous dimension $\mathbf{\Gamma}$:

$$\mathbf{Z}(\epsilon,\{\underline{p}\},\mu) = \mathbf{P}\exp\left[\int_\mu^\infty \mathbf{\Gamma}(\{\underline{p}\},\mu)\right].$$

- The anomalous dimension is conjectured to be very simple:

$$\mathbf{\Gamma}(\{\underline{p}\},\mu) = \sum_{(i,j)} \frac{\mathbf{T}_i\cdot\mathbf{T}_j}{2}\gamma_{\text{cusp}}(\alpha_s)\ln\frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s).$$

(Becher, Neubert 2009; Gardi, Magnea 2009)

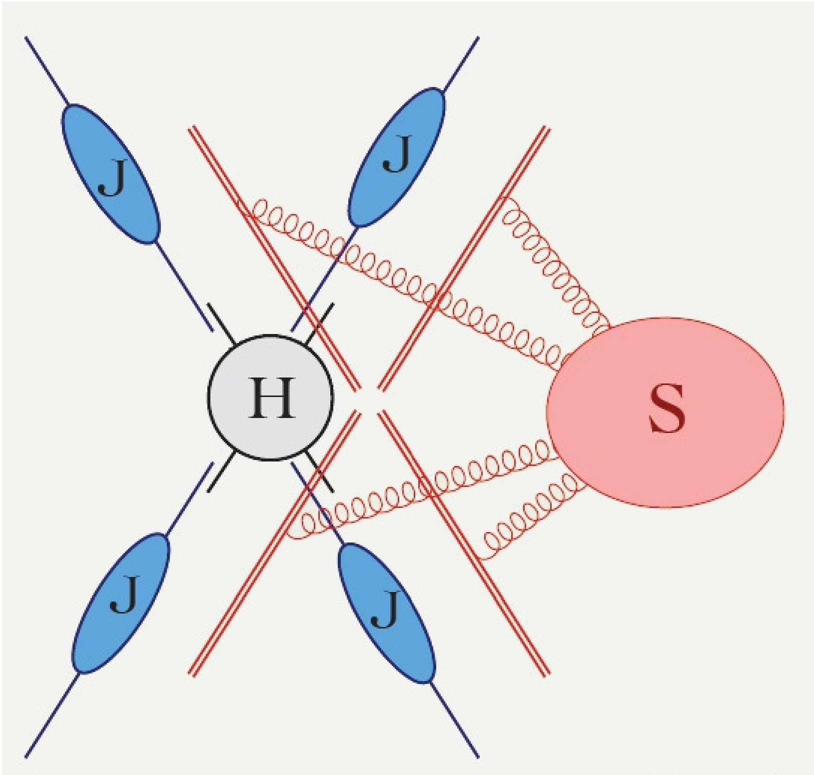
- Semiclassical origin of IR singularities: completely determined by color charges and momenta of external partons; only color dipole correlations.

Constraint on Γ: Soft-Collinear Factorization

- The conjecture is driven by the identification of on-shell amplitudes with Wilson coefficients of *n*-jet operators in SCET: (Becher, Neubert 2009)

$$|\mathcal{M}_n(\{\underline{p}\},\mu)\rangle = |\mathcal{C}_n(\{\underline{p}\},\mu)\rangle \times [\text{on-shell spinors and polarization vectors}]$$

- Amplitudes of *n*-jet operators in SCET factorizes into
  - A hard function $H = |\mathcal{C}_n\rangle$ which depends on large momentum transfer $s_{ij} = (p_i \pm p_j)^2$
  - *n*-jets depending on the collinear momenta of each collinear sector p_i with $p_i^2 \ll |s_{ij}|$
  - A soft function $\mathcal{S}$ depending on the soft scales $\Lambda_{ij} = \dfrac{p_i^2 p_j^2}{s_{ij}}$



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CONSTRAINT ON Γ: SOFT-COLLINEAR FACTORIZATION

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- The identification $|\mathcal{M}_n\rangle = |\mathcal{C}_n\rangle$ allows to use properties of the soft-collinear factorization to constrain $\mathbf{\Gamma}$. First $\mathbf{\Gamma} = \mathbf{\Gamma_h}$
- Then, invariance under the renormalization group assure that

$$\mathbf{\Gamma_h} = \mathbf{\Gamma_{c+s}}$$

- Soft-collinear factorization gives then

$$\mathbf{\Gamma_{c+s}}(s_{ij}) = \mathbf{\Gamma_s}(\Lambda_{ij}) + \underbrace{\sum_{\mathbf{i}} \Gamma_c^i(p_i^2)\mathbf{1}}_{p_i^2 \text{ dependence must cancel}}$$

- Given that $\Gamma_c^i(L_i) = -\Gamma_{\text{cusp}}^i(\alpha_s) L_i + \gamma_c^i(\alpha_s)$ with $L_i = \ln\frac{\mu^2}{-p_i^2}$, one obtains

$$\frac{\partial \mathbf{\Gamma}_s(\{\mathbf{L}\})}{\partial L_i} = \Gamma_{\text{cusp}}^i(\alpha_s) \quad \Rightarrow \quad \mathbf{\Gamma}_s(\{\underline{\beta}\},\mu) \stackrel{?}{=} -\sum_{(i,j)} \frac{\mathbf{T}_i \cdot \mathbf{T}_j}{2} \gamma_{\text{cusp}}(\alpha_s)\beta_{ij} + \sum_i \gamma_s^i(\alpha_s).$$

(Becher, Neubert 2009; Gardi,Magnea 2009)

$$\ln\frac{-s_{ij}\mu^2}{(-p_i^2)(-p_j^2)} = \beta_{ij} = L_i + L_j - \ln\frac{\mu^2}{-s_{ij}}$$

soft log

collinear log

hard log

Constraint on Γ: non-abelian exponentiation

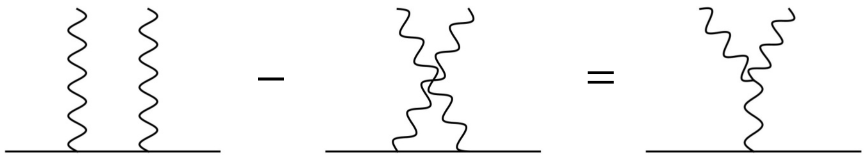
- The soft function is a matrix element of Wilson lines:

$$\mathcal{S}(\{\underline{n}\},\mu) = \langle 0 | \mathbf{S}_1(0)\ldots\mathbf{S}_n(0) | 0 \rangle = \exp(\widetilde{\mathcal{S}}(\{\underline{n}\},\mu))$$

- The exponent $\widetilde{\mathcal{S}}$ receives contributions only from Feynman diagrams whose color weights are color-connected ("maximally non-abelian")

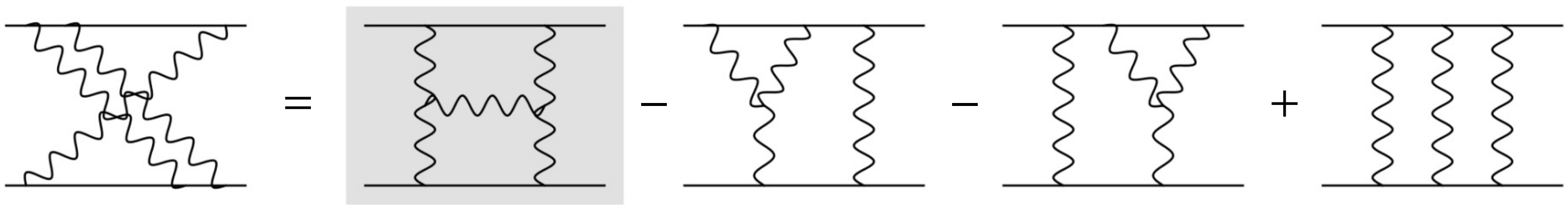
(Gatheral 1983; Frenkel and Taylor 1984)

- Color structures can be simplified using the Lie commutation relation:



$$\mathbf{T}^a\mathbf{T}^b - \mathbf{T}^b\mathbf{T}^a = if^{abc}\mathbf{T}^c$$

- Use this to decompose color structures into a sum over products of connected webs



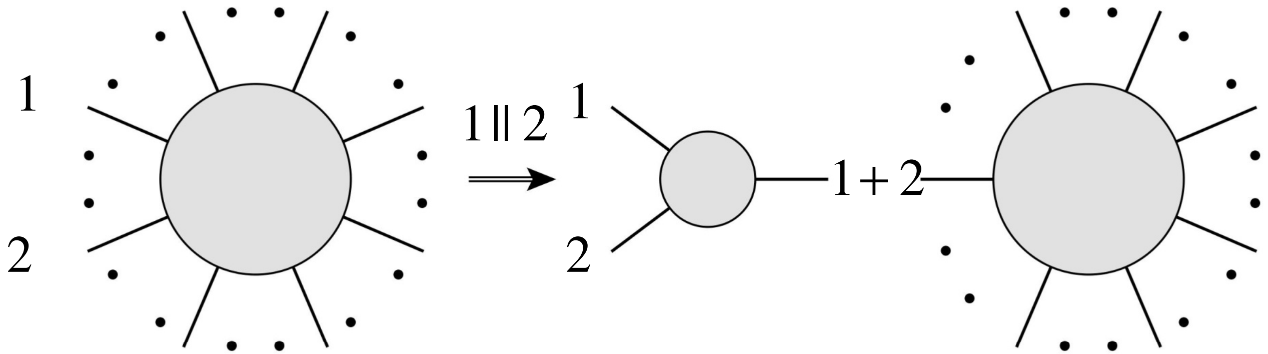
- Only single connected webs contribute to the exponent $\widetilde{\mathcal{S}}$.

CONSTRAINT ON Γ: CONSISTENCY WITH THE COLLINEAR LIMIT

- When two partons become collinear, an *n*-point amplitudes reduces to a (*n*-1)-parton amplitude times a splitting function:

(Berends, Giele 1989; Mangano, Parke 1991; Kosower 1999; Catani,De Florian, Rodrigo 2003)

$$|\mathcal{M}_n(\{p_1, p_2, p_3, \ldots, p_n\})\rangle = \mathbf{Sp}(\{p_1, p_2\})\,|\mathcal{M}_{n-1}(\{P, p_3, \ldots, p_n\})\rangle + \ldots$$

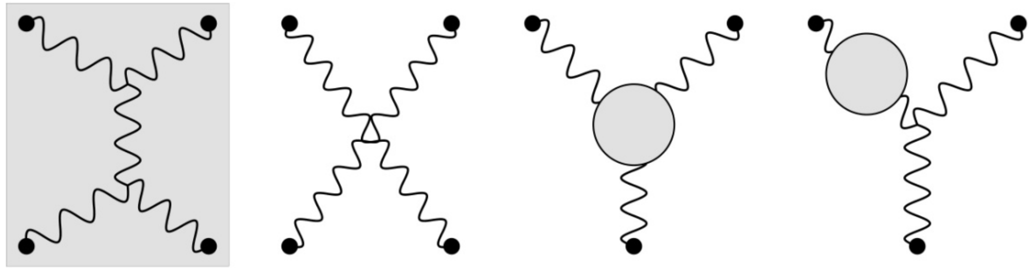


$$\boldsymbol{\Gamma}_{\mathrm{Sp}}(\{p_1, p_2\}, \mu) = \boldsymbol{\Gamma}(\{p_1, \ldots, p_n\}, \mu) - \boldsymbol{\Gamma}(\{P, p_3, \ldots, p_n\}, \mu)\big|_{\mathbf{T}_P \to \mathbf{T}_1 + \mathbf{T}_2}$$

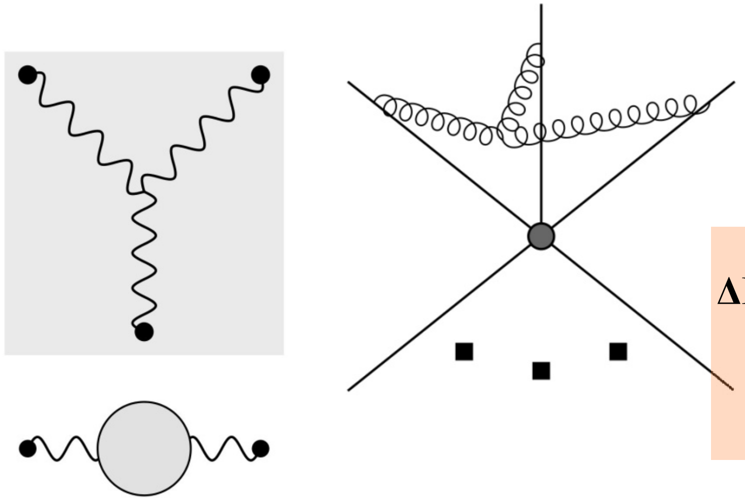
(Becher, Neubert 2009)

- $\boldsymbol{\Gamma}_{\mathrm{Sp}}$ must be independent of momenta and colors of partons 3,…*n.*

DIAGRAMMATIC ANALYSIS: ONE, TWO AND THREE LOOPS



- One loop
  - one leg: $\mathbf{T}_i^2 = C_i$
  - two legs: $\mathbf{T}_i \cdot \mathbf{T}_j$



- Two loops
  - one leg: $-if^{abc}\mathbf{T}_i^a\mathbf{T}_i^b\mathbf{T}_i^c = \frac{C_A C_i}{2}$
  - two legs: $-if^{abc}\mathbf{T}_i^a\mathbf{T}_i^b\mathbf{T}_j^c = \frac{C_A}{2}\mathbf{T}_i \cdot \mathbf{T}_j$
  - three legs: $-if^{abc}\mathbf{T}_i^a\mathbf{T}_j^b\mathbf{T}_k^c$
- Incompatible with soft-collinear factorization.

- Three loops
  - Three new structures compatible with soft-collinear factorization:

$$\Delta\Gamma_3 = \sum_{(i,j,k,l)} f^{adx} f^{bcx}\mathbf{T}_i^a\mathbf{T}_j^b\mathbf{T}_k^c\mathbf{T}_l^d \left(\frac{\bar{f}_1(\alpha_s)}{4} \ln\frac{(-s_{ij})(-s_{kl})}{(-s_{ik})(-s_{jl})} + F(\beta_{ijkl}, \beta_{iklj} - \beta_{iljk}) \right) - \bar{f}_2(\alpha_s) \sum_{(i,j,k)} f^{adx} f^{bcx} (\mathbf{T}_i^a\mathbf{T}_i^b)_+ \mathbf{T}_j^c\mathbf{T}_k^d$$

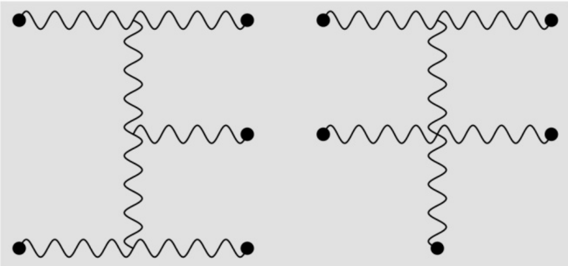
conformal cross ratio

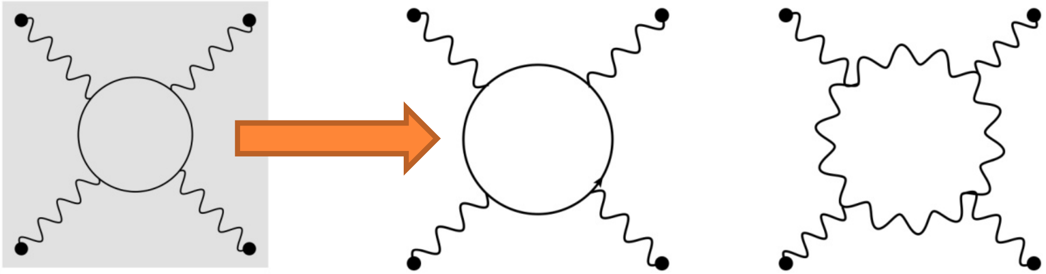
$$\beta_{ijkl} = \beta_{ij} + \beta_{kl} - \beta_{ik} - \beta_{jl}$$

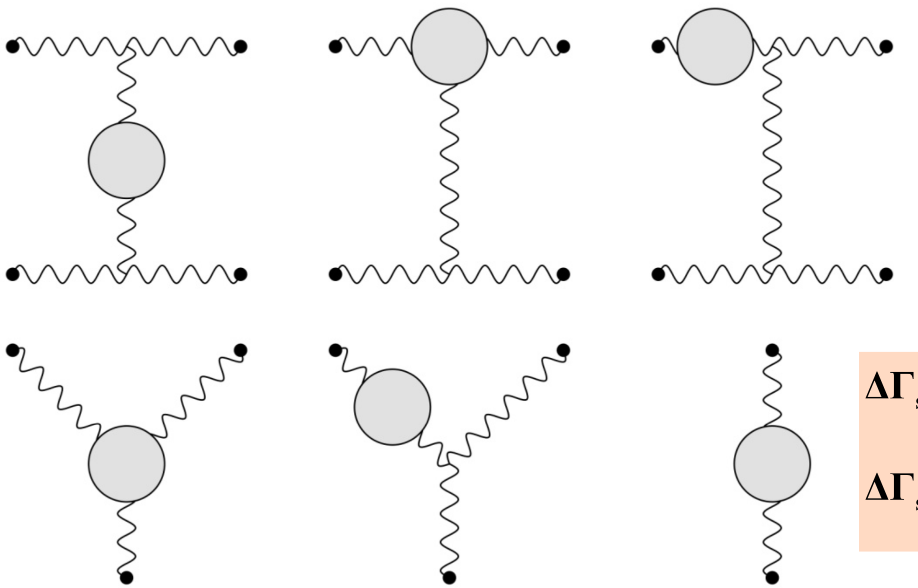
  - $\bar{f}_1$ and $\bar{f}_2$ are not compatible with collinear limit: the splitting function depends on colors and momenta of additional partons.
  - An exception is $F(\beta_{ijkl}, \beta_{iklj} - \beta_{iljk})$, if it vanishes in all collinear limits. It is possible that such a function exists.

(Becher, Neubert 2009; Dixon,Gardi,Magnea,2009)

DIAGRAMMATIC ANALYSIS: FOUR LOOPS I







- At four loops structures involving higher Casimir invariants appears:

$$\mathcal{D}_{ijkl} = d_F^{abcd}\left(\mathbf{T}_i^a\mathbf{T}_j^b\mathbf{T}_k^c\mathbf{T}_l^d\right)_+ , \quad d_R^{a_1\ldots a_n} = \mathrm{tr}[(\mathbf{T}_R^{a_1}\ldots\mathbf{T}_R^{a_n})_+]$$

- There are possible new structures compatible with soft-collinear factorization:

$$\Delta\Gamma_{s4,1} \propto \sum_{(i,j)} \beta_{ij}\left[D_{iijj}\, g_1(\alpha_s) + D_{iiij}\, g_2(\alpha_s)\right] + \sum_{(i,j,k)} \beta_{ij}\, D_{ijkk}\, g_3(\alpha_s),$$

$$\Delta\Gamma_{s4,2} = \sum_{(i,j)}\left[D_{iijj}\, g_4(\alpha_s) + D_{iiii}\, g_5(\alpha_s)\right] + \sum_{(i,j,k,l)} D_{ijkl}\, G_1(\beta_{ijkl}, \beta_{iklj} - \beta_{iljk}).$$

- Again, they are not compatible with the collinear limit, except $G_1(\beta_{ijkl}, \beta_{iklj} - \beta_{iljk})$, if it vanishes in all collinear limits.

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DIAGRAMMATIC ANALYSIS: FOUR LOOPS II

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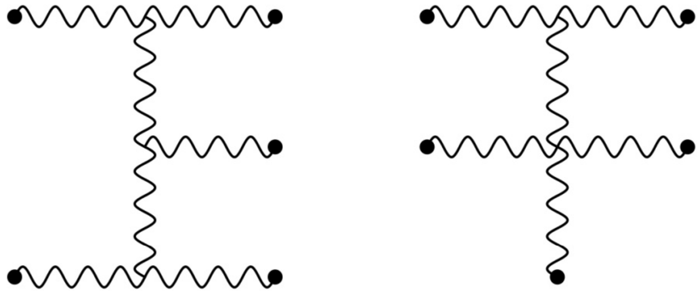

- The two webs have color structure

$$\mathcal{T}_{ijklm} \equiv f^{adx} f^{bcy} f^{exy} (\mathbf{T}_i^a \mathbf{T}_j^b \mathbf{T}_k^c \mathbf{T}_l^d \mathbf{T}_m^e)_+ .$$

- There are two structures compatible with soft-collinear factorization:

$$\Delta\Gamma_{s,4} = \sum_{(i,j,k)} \mathcal{T}_{iijkk}\, \bar{g}(\alpha_s)\beta_{ij} + \sum_{(i,j,k,l,m)} \mathcal{T}_{ijklm}\, G_2(\beta_{ijkm}, \beta_{ikmj} - \beta_{imjk}, \beta_{ijml}, \beta_{imlj} - \beta_{iljm}),$$

- The first function is incompatible with the collinear limit, the second function cannot be excluded, if it vanishes in all collinear limits.

- Applied to the two-jet case, it means that the Casimir scaling of the cusp anomalous dimension is still preserved:

$$\frac{\Gamma^q_{\text{cusp}}(\alpha_s)}{C_F} = \frac{\Gamma^g_{\text{cusp}}(\alpha_s)}{C_A} = \gamma_{\text{cusp}}(\alpha_s)$$

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CONCLUSION

- *Infrared singularities* in gauge theory amplitude can be mapped onto UV divergences of n-jet operators in SCET.
- They can be described by means of an anomalous dimension, whose structure is constrained by soft-collinear factorization, non-abelian exponentiation, and two-parton collinear limit.
- The anomalous dimension is expected to have a very simple structure. It should hold to all order in perturbation theory.
- We have completed a diagrammatic analysis up to four loop, showing that only new structures proportional to functions vanishing in all collinear limits can appear.
- No violation of Casimir scaling of the cusp anomalous dimension arise.

BACKUP

- The formal solution for **Z** up to four loops in perturbation theory reads

$$
\begin{aligned}
\ln \mathbf{Z} = {} & \frac{\alpha_s}{4\pi}\left(\frac{\Gamma_0'}{4\epsilon^2}+\frac{\boldsymbol{\Gamma}_0}{2\epsilon}\right)+\left(\frac{\alpha_s}{4\pi}\right)^2\left(-\frac{3\beta_0\Gamma_0'}{16\epsilon^3}+\frac{\Gamma_1'-4\beta_0\boldsymbol{\Gamma}_0}{16\epsilon^2}+\frac{\boldsymbol{\Gamma}_1}{4\epsilon}\right) \\
& +\left(\frac{\alpha_s}{4\pi}\right)^3\left(\frac{11\beta_0^2\Gamma_0'}{72\epsilon^4}-\frac{5\beta_0\Gamma_1'+8\beta_1\Gamma_0'-12\beta_0^2\boldsymbol{\Gamma}_0}{72\epsilon^3}+\frac{\Gamma_2'-6\beta_0\boldsymbol{\Gamma}_1-6\beta_1\boldsymbol{\Gamma}_0}{36\epsilon^2}+\frac{\boldsymbol{\Gamma}_2}{6\epsilon}\right) \\
& +\left(\frac{\alpha_s}{4\pi}\right)^4\left(-\frac{25\beta_0^3\Gamma_0'}{192\epsilon^5}+\frac{13\beta_0^2\Gamma_1'+40\beta_1\beta_0\Gamma_0'-24\beta_0^3\boldsymbol{\Gamma}_0}{192\epsilon^4}\right. \\
& \qquad -\frac{7\beta_0\Gamma_2'+9\beta_1\Gamma_1'+15\beta_2\Gamma_0'-24\beta_0^2\boldsymbol{\Gamma}_1-48\beta_0\beta_1\boldsymbol{\Gamma}_0}{192\epsilon^3} \\
& \qquad \left.+\frac{\Gamma_3'-8\beta_0\boldsymbol{\Gamma}_2-8\beta_1\boldsymbol{\Gamma}_1-8\beta_2\boldsymbol{\Gamma}_0}{64\epsilon^2}+\frac{\boldsymbol{\Gamma}_3}{8\epsilon}\right)+O(\alpha_s^5)
\end{aligned}
$$

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