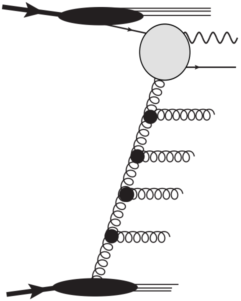
Unintegrated parton densities at small x

Martin Hentschinski

IFT Autonoma University
Madrid/CSIC

Based on results obtained with F. Hautmann & H. Jung and
C. Salas Hernandez & A. Sabio Vera



MOTIVATION & OUTLINE

definition of unintegrated gluon @ small x:

- high energy factorization of partonic QCD amplitudes ➡ BFKL evolution
- k_T-factoriation [Catani, Hautmann '94]: extension to hadronic scattering with $Q^2 \gg \Lambda^2_{\text{QCD}}$ ➡ resummation of collinear (DGLAP) and small x (BFKL) logarithms at a time in a consistent way

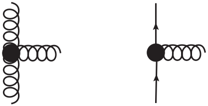
(I) definition of unintegrated seaquark density for CASCADE
[with F. Hautmann and H. Jung]

- MonteCarlo based on CCFM evolution: interpolation between DGLAP and BFKL
- LO MC realization of k_T-factorization at small x

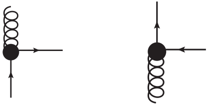
(II) NLO BFKL gluon density: [with C. Salas Hernandez and A. Sabio Vera]

- NLO BFKL: systematic determination of higher order corrections (running coupling!)
- requires resummation of (anti-)collinear logarithms on top
- fit to combined HERA data

CCFM evolution based on principle of color coherence ➡ **gauge bosons** emissions



unintegrated gluon and valence quark



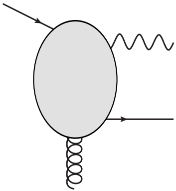
not present

Consequences for CASCADE parton shower (exclusive radiation!):

- only gluonic emissions, no quark ➡ jets purely gluonic
- naturally for DGLAP and also for NLO BFKL

Consequences for the hard process:

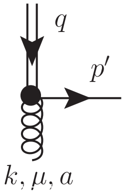
- quark $\equiv$ valence quark, seaquark only through α_s correction ➡ deal with collinear divergence
- example process: $qg^* \to qZ$ (forward DY)
- here: restrict to gluon-quark splitting as last evolution step



Gauge invariant off-shell factorization for quarks @ high c.o.m. energies

→ **reggeized quarks** – analogy to reggeized gluons for BFKL

- here: use to factorize $\sigma_{qg^* \to Zq} \to \hat{\sigma}_{*qq^* \to Z} \otimes P_{q^* g^*}$ at tree-level
- yields re-organization of QCD diagrams through **effective vertices**



$$= igt^a \left(\gamma^\mu - \not{q} \frac{(n^+)^\mu}{k^+} \right) \qquad \text{etc.}$$

- gauge invariant definition of off-shell Matrix Elements

$$\hat{\sigma}_{q\bar{q}^* \to Z}(\nu, \boldsymbol{q}^2) = \underbrace{\sqrt{2} G_F M_Z^2 (V_q^2 + A_q^2)}_{\text{Z-coupling}} \times \frac{\pi}{N_c} \delta(\nu - M_Z^2 - \boldsymbol{q}^2)$$

- gluon-quark splitting $P_{q^* g^*}(z) = T_R$: poor approximation
  [compare : DGLAP $P_{qg}(z) = T_R(z^2 + (1-z)^2)$]

- reason: energy not conserved for high energy factorization $z = 0$
- here: possible to include while keeping off-shell gauge invariance
  ➡ re-obtain k_T-dependent splitting function [Catani, Hautmann '94]

$$P_{qg}^{\text{CH}}\left(z, \boldsymbol{k}^2, \boldsymbol{q}^2\right) = T_R \left(\frac{\boldsymbol{q}^2}{\boldsymbol{q}^2 + z(1-z)\boldsymbol{k}^2}\right)^2 \left[P_{qg}(z) + 4z^2(1-z)^2 \frac{\boldsymbol{k}^2}{\boldsymbol{q}^2}\right]$$

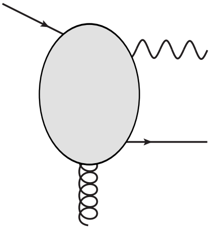
Note: ensures correct all order high energy collinear resummation

Definition of q_T-dependent sea-quark density:

$$\mathcal{Q}^{\text{sea}}(x, \boldsymbol{q}^2, \mu^2) := \frac{1}{\boldsymbol{q}^2} \int_x^1 dz \int_0^{\mu^2/z} d\boldsymbol{k}^2 P_{qg}^{\text{CH}}\left(z, \boldsymbol{k}^2, \boldsymbol{q}^2\right) \mathcal{G}_{\text{CCFM}}^{\text{gluon}}\left(\frac{x}{z}, \boldsymbol{k}^2, \bar{\mu}^2\right)$$

* correct collinear limit & small x resummation (CH-splitting + gluon density) & gauge invariance verified

* two choices for the hard scale $\bar{\mu}^2$: factorization scale $\bar{\mu}^2 = \mu^2$ (inclusive) or angular ordering scale $\bar{\mu}^2 = \frac{\boldsymbol{q}^2 + (1-z)\boldsymbol{k}^2}{(1-z)^2}$ (CCFM)



confront with full $\hat{\sigma}_{qg^* \to Zq}$ (in k_T-fact.) [Ball, Marzani, '09]:
define 'renormalized' cross-section $\bar{\sigma}_{qg^* \to Zq}$

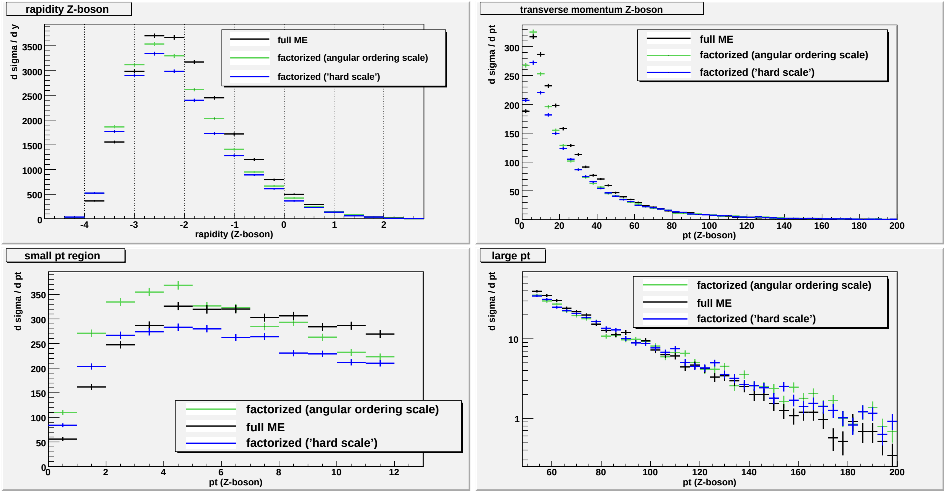
$$\bar{\sigma}(\nu, \boldsymbol{k}^2) \equiv \hat{\sigma}(\nu, \boldsymbol{k}^2) - \int_x^1 \frac{dz}{z} \int \frac{d\boldsymbol{q}^2}{\boldsymbol{q}^2} \hat{\sigma}_{q\bar{q}^* \to Z} P_{qg}^{\mathsf{CH}}$$

- 'renormalized' $\bar{\sigma}$ subleading in high energy ($\hat{s}_{qg^*} \gg Q^2, \boldsymbol{q}^2, \boldsymbol{k}^2$) and collinear ($Q^2 \gg \boldsymbol{q}^2 \gg \boldsymbol{k}^2$) limit ➡ 'higher order' correction
- factorized expression misses s-channel corrections + approximate kinematics ($\hat{\sigma}_{q\bar{q}^* \to Z}$)

$$\delta(z\nu - M_Z^2 - \boldsymbol{q}^2) \leftrightarrow \delta(z\nu - M_Z^2 - \frac{\boldsymbol{q}^2}{1-z} - z\boldsymbol{k}^2)$$

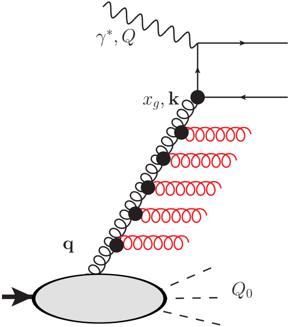
- k_T-factorization increases accuracy in kinematics, but does not capture finite z-correction: factorization is always approximation!

Numerical comparision full ME versus factorized



- ∃ already several NLO BFKL fits to HERA data [Altarelli, Ball, Forte], [White, Thorne], [Ellis, Kowalski, Lipatov, Ross, Watt], [Peschanski, Royon, Schoeffel]....
- Saddle point approximation, leading twist, complicated to use → need for a new NLO unintegrated gluon density → use for LHC analysis
- Use analytical study as check for construction of an NLO BFKL MC

Gluon density: convolution of BFKL Green's function f_{BFKL} & proton impact factor $\phi_P(\boldsymbol{k})$



$$\mathcal{G}(x, \boldsymbol{q}^2; \mu^2) = \int \frac{d^2\boldsymbol{k}}{2\pi} f_{\text{BFKL}}(x, \boldsymbol{q}, \boldsymbol{k}) \frac{\phi_P(\boldsymbol{k})}{\boldsymbol{k}^2}$$

Structure function
$F_2(x, Q^2) =:$ ugd ⊗ photon impact factor

Green's function as double Mellin transform

$$f_{\text{BFKL}}(x, \boldsymbol{k}, \boldsymbol{q}) = \frac{1}{\boldsymbol{k}^2} \int \frac{d\gamma d\omega}{(2\pi i)^2} \frac{\left(\frac{1-x}{x}\right)^{\omega}}{\omega - \bar{\alpha}_s \hat{\chi}_{\text{BFKL}}(\gamma)} \left(\frac{\boldsymbol{k}^2}{\boldsymbol{q}^2}\right)^{\gamma}$$

core: NLO BFKL kernel for DIS [Fadin, Lipatov, 1997]

$$\bar{\alpha}_s \hat{\chi}_{\mathrm{BFKL}}(\gamma) = \bar{\alpha}_s \chi_0(\gamma) + \bar{\alpha}_s^2 \left[\chi_1(\gamma) + \chi_{\mathrm{DIS}}(\gamma) + \hat{\chi}_{\mathrm{R.C.}}(\gamma)\right] + \bar{\alpha}_s^3 \chi_{\mathrm{RG}}(\gamma)$$

- χ_0, χ_1: conformal LO, NLO kernel
- $\chi_{\mathrm{DIS}}(\gamma) = -\chi_0(\gamma)\chi_0'(\gamma)$: change of reggeization scale: $Q^2 \gg Q_0^2$
- $\chi_{\mathrm{R.C.}}$: running coupling corrections

NLO kernel leads to numerical instability – $\exists$ (anti-)collinear γ-poles in χ_{NLO} ➡ resummation ➡ RG improved NLO BFKL

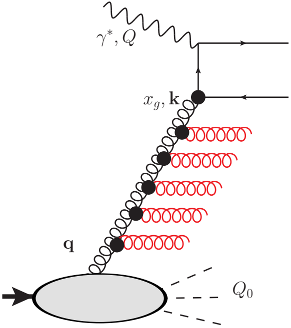
Often: introduces additional ω-dep. $\chi(\gamma) \to \chi(\omega; \gamma)$

Here: extend 'all-pole'-approximation [Sabio Vera, 05] to DIS scenario

$$\bar{\alpha}_s^3 \chi_{RG}^R(\gamma) = \sum_{m=0}^{\infty} \left\{ \left(\frac{1-\gamma+m-\bar{\alpha}_s \bar{b}}{2} \right) \cdot \left(\sqrt{1 + \frac{4\bar{\alpha}_s(1+\bar{\alpha}_s \bar{a})}{(1-\gamma+m-\bar{\alpha}_s)^2}} - 1 \right) - \frac{\bar{\alpha}_s(1+\bar{\alpha}_s\bar{a})}{1-\gamma+m} - \frac{\bar{\alpha}_s^2 \bar{b}}{(1-\gamma+m)^2} + \frac{\bar{\alpha}_s^2}{(1-\gamma+m)^3} \right\} - \bar{\alpha}_s \left[(1+\bar{\alpha}_s a)\Psi(\gamma - \bar{\alpha}_s b) - \Psi(\gamma)\right] - \bar{\alpha}_s^2 \left[b\Psi'(\gamma) - a\Psi(\gamma)\right]$$

Advantage:

- can solve ω-integral analytically ➡ BFKL kernel exponentiates
- possible to translate back to momentum/coordinate-space ➡ BFKL MC, dipole model ...



Structure function given as convolution in γ-space

$$F_2(x, Q^2) = \int_{1/2-i\infty}^{1/2+i\infty} \frac{d\gamma}{2\pi i} \left(\frac{Q^2}{Q_0^2}\right)^{\gamma} \left(\frac{1-x}{x}\right)^{\bar{\alpha}_s \chi_{\text{BFKL}}(\gamma)}$$
$$h_{\gamma^* g^* \to q\bar{q}}(\gamma, \bar{\alpha}_s \chi_{\text{BFKL}}(\gamma)) \cdot \Phi_P(\gamma; \delta, A)$$

$\Phi_P(\gamma; \delta)$: Mellin transform of model for proton impact factor

$$\phi_P\left(\frac{\boldsymbol{q}^2}{Q_0^2}, \delta, A\right) = A \left(\frac{\boldsymbol{q}^2}{Q_0^2}\right)^{\delta} e^{-\boldsymbol{q}^2/Q_0^2}$$

$h_{\gamma^* g^* \to q\bar{q}}(\gamma, \omega)$: kinematical improved LO photon impact factor [Bialas, Navelet, Peschanski, 01] ➡ energy conservation ➡ exptected to capture important part of full NLO corrections running coupling corrections

- $\hat{\chi}_{\text{R.C.}}$: operator in γ-space ➡ for symmetric treatment reduces to action on photon and proton, complicated commutator terms cancel at NLO, $\hat{\chi}_{\text{R.C.}} \to \chi_{\text{R.C.}}$

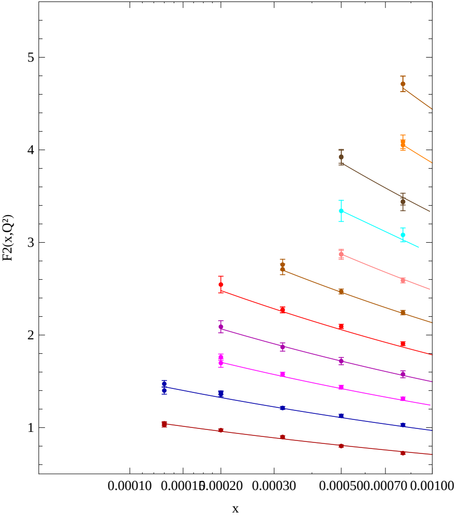
$$\chi_{\text{R.C.}}(\gamma) = \frac{\beta_0 \chi_0(\gamma)}{8N_c}\left[\frac{\partial}{\partial\gamma}\log\left((Q^2 Q_0^2)^{\gamma}\frac{h_{\gamma^* g^* \to q\bar{q}}(\gamma, 0)}{\phi_P(\gamma)}\right) - 2\ln\mu^2\right]$$

- suggests symmetric scale of running coupling: $\alpha_S(QQ_0)$ – dependence on proton scale Q_0!

- running coupling in general delicate issue in k_T-factorization: integrals such as $\int_0^\infty d\boldsymbol{k}^2 \alpha_s(\boldsymbol{k}^2)$ require modelling of $\alpha_s(\boldsymbol{k}^2)$ for $\boldsymbol{k}^2 \to 0$
- here: region $\boldsymbol{k}^2 \to 0$ manifest through γ poles of $\chi_{R.C.}$

$$\chi_{R.C.}(\gamma) = \frac{\beta_0}{8Nc}\left(\frac{a'}{\gamma} + \frac{b'}{\gamma^2} + \frac{\bar{a}'}{1-\gamma} + \frac{\bar{b}'}{(1-\gamma)^2}\right)$$

- included in resummation ➡ stabelizes running coupling, no further modelling or freezing needed



- Preliminary results!
- Use combined HERA data for F_2 with $Q^2 \geq 4.5$ GeV^2 and $x \leq 10^{-3}$
- Obtain perturbative scale for proton impact factor $Q_0^2 = 4.5$ GeV^2, $\delta = 1.32$ and $A = 0.202216$
- At first unexptected, no final conclusion on its meaning means
- $\chi^2/N_{\text{d.o.f.}} \simeq 0.85$

Extension to larger $x \simeq 10^{-2}$ requires heavy quark effects and probably full NLO corrections to photon impact factor

[Bartels, Chachamis, Colferai, Gieseke, Kyrieleis , Qiao], [Balitsky, Chirilli]

→ future work

- Definition of q_T dependent seaquark density for CASCADE
  - (•) interpolates in the sense of CCFM evolution between DGLAP and high energy limit
  - (•) verified gauge invariance of off-shell splitting and ME (at level of current conservation)
  - (•) Numerical checks: Qualtitative agreement of exact and factorized expression
  - (•) Outlook: systematically include quark emissions into parton shower $+$ combine sea- and valence-quarks $\&$ generally use off-shell quark matrix elements
- NLO BFKL gluon density from HERA fit
  - (•) constructed RG improved gluon Green's function with full running coupling corrections for DIS kinematics
  - (•) fit: very good agreement with data for $x < 10^{-3}$ – proton scale rather large
  - (•) Outlook 1: include heavy quark effects and (hopefully) NLO virtual photon impact factors $+$ construct NLO BFKL MonteCarlo
  - (•) Outlook 2: Apply to small x study of LHC observables and provide further evidence for BFKL

CASCADE: scales, masses, couplings, parton densities

Scales:

- Z-mass: $M_Z = 91.1876$ GeV

Unintegrated gluon density:

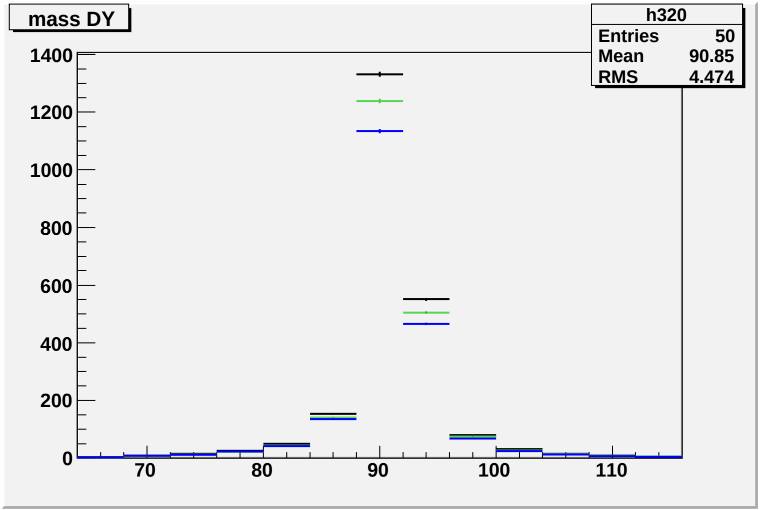
- CCFM set A0

Valence quark distribution:

- CCFM parametrization of valence quark distribution
  - starting distribution CTEQ
  - evolution with $P_{qq}(z)$ and angular ordering of emitted gluon

Coupling constants:

- $G_F = 1.166 \times 10^{-5} GeV^{-2}$
- $\alpha_s(Q^2)$ with $Q^2 = M_Z^2 + \boldsymbol{p}_Z^2$


mass DY
h320
Entries 50
Mean 90.85
RMS 4.474
1400
1200
1000
800
600
400
200
0
70
80
90
100
110

NLO BFKL fit: scales, masses, number of flavors ....

Number of active flavors

- $n_f = 4$, $m_c = 0$, no finite mass effects so far

Data used for fit

- Combined HERA data for F_2
- $x \leq 10^{-3}$, $Q^2 \geq 4.5\text{GeV}^2 \equiv$ 43 data points
- $Q^2_{\text{max}} = 45\text{GeV}^2$

Results of the fit

- $\chi^2/40 \simeq 0.85$
- $Q_0^2 = 4.5\text{GeV}^2$, $\delta = 1.32$, $A = 0.202216$