Crossing symmetry in the $\pi\pi$ D- and F-wave scattering amplitudes and new precise results for the S-wave amplitude

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No! here we do not present any spectacular and shocking results!

but Yes! we show VERY EFFICIENT, WELL ESTABLISHED and VERY DEMANDING METHOD for testing the $\pi\pi$ amplitudes using only theoretical constraints: dispersion relations with imposed crossing symmetry conditions.
Here we present results for the D and F partial waves up to effective two pion mass $m_{\pi\pi} \approx 1100$ MeV using dispersion relations with only ONE SUBTRACTION (in contrast to the well known Roy's equations with two subtractions). As was explained in [1] for the S and P waves, it gives us much smaller uncertainties of the results.

So let's present one general equation (in fact set of equations) for amplitudes $f_l^I(s)$ with spin $l = 0, 1, 2, 3$ and isospin $I = 0, 1, 2$ (please note that sum $I + l$ must be even):

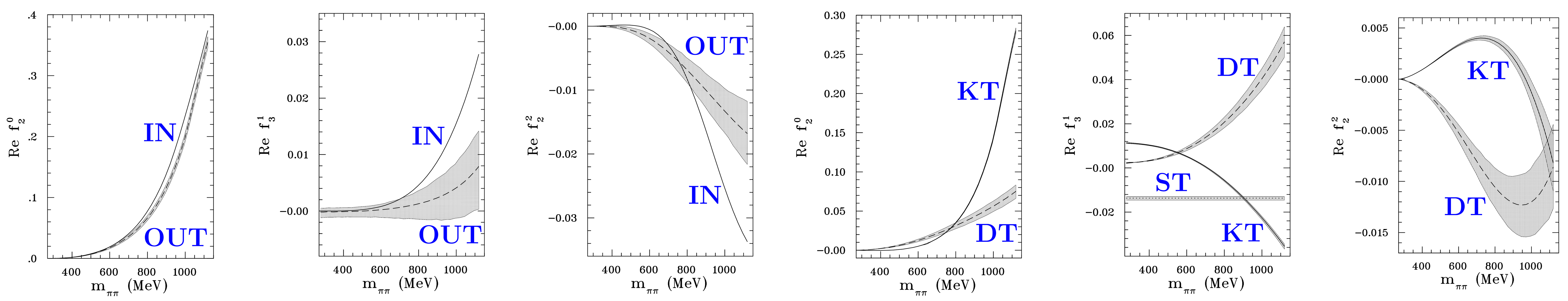
$$\mathrm{Re} f_\ell^I(s) = ST + KT(s) + DT(s) \qquad (s = m_{\pi\pi}^2), \tag{1}$$

where for the D and F partial waves:

"Subtracting term" "Kernel term" "Driving term"

$$ST = -\tfrac{1}{24}(a_0^0 - \tfrac{5}{2}a_0^2)\delta_{I1}\delta_{l3}, \qquad KT(s) = \sum_{I'=0}^{2}\sum_{\ell'=0}^{3} \fint_{4m_\pi^2}^{s'_{max}} ds' K_{\ell\ell'}^{II'}(s,s')\,\mathrm{Im}\, f_{\ell'}^{I'}(s'), \qquad DT_\ell^I(s). \tag{2}$$

Of course amplitude $f_l^I(s)$ is related with phase shifts $\delta_l^I(s)$ and inelasticities $\eta_l^I(s)$ by $f_l^I(s) = \frac{\sqrt{s}\,\eta_l^I(s) e^{i\delta_l^I(s)} - 1}{2i\sqrt{s - 4m_\pi^2}}$. Scattering lengths a_l^I are given by threshold expansion: $Re f_l^I(k) = k^{2l}\left[a_l^I + b_l^I k^2 + O(k^4)\right]$ where pion momentum $k = \sqrt{m_{\pi\pi}^2/4 - m_\pi^2}$.
On figures a), b) and c) the OUTPUTS (left side of the equation (1)) are compared with corresponding real parts of the INPUT amplitudes $Im f_\ell^I(s)$ (in the $KT(s)$ and $DT(s)$). Dashed bands represent uncertainties of the outputs.
Crossing symmetry requires $\Delta =$ (INPUT $-$ OUTPUT) $\longrightarrow 0$.
Important! although the presented amplitudes of the D and F waves WERE NOT FITTED to $\Delta = 0$, one can see that they fulfill crossing symmetry quite well up to ~ 800 MeV (some work is still needed). All input amplitudes were taken from [1].



On figures d), e) and f) we compare the subtracting, kernel and driving terms for given partial wave. Dashed bands represent their uncertainties. The nonzero subtracting term is only for the F wave. In all waves, absolute values of the kernel and driving terms are of $\sim$ similar size (except for the $Ref_0^2(s)$ above $m_{\pi\pi} \sim 800$ MeV).
BUT! notice different vertical scales of the figure d) and on figures e), f). Much larger $\mathrm{Re}f_2^0(s)$ than $\mathrm{Re}f_3^1(s)$ and $\mathrm{Re}f_2^2(s)$ is due to near $f_2(1270)$ meson.

Full analysis and derivations are in presented in [2].

But it is not the end of the story! Recently, new and very precise dispersive analysis of the S and P wave amplitudes appeared [3]. Using similar to presented here once subtracted dispersion equations (+ other dispersion relations) and very recent K_{l4} experimental resullts we have calculated set of $\pi\pi$ partial wave amplitudes fulfilling, inter alia, crossing symmetry condition. Making analytic continuation to the complex plane we have found the $f_0(600)$ pole at $(457^{+14}_{-13} - i\,279^{+11}_{-7})$ MeV and $f_0(980)$ pole at $(996 \pm 7 - i\,25^{+10}_{-6})$ MeV.

!!!These are much, much more precise results than those in e.g. PDG Tables!!!

Presented here and in [2] new dispersion relations, together with the previous ones for the S and P waves [1] form a complementary set of theoretical constraints that imposed on the experimental amplitude can define them clearly and precisely. You can easily use them!!! Please note that this analysis is based only on unitarity, analyticity and crossing symmetry!

References:

[1] *"The pion-pion scattering amplitude. IV: Improved analysis with once subtracted Roy-like equations in the 450-1100 MeV region"*, R. Garcia-Martin, R. Kamiński, J. Ruiz de Elvira, J. Pelaez and F. Yndurain, Phys. Rev. D 83, 074004 (2011).
[2] *"Dispersion relations with crossing symmetry for $\pi\pi$ D and F wave amplitudes"*, R. Kamiński, Phys. Rev. D 83, 076008 (2011).
[3] *"Precise determination of the $f_0(600)$ and $f_0(980)$ pole parameters from a dispersive data analysis"*, R. Garcia-Martin, R. Kamiński, J. Pelaez, J. Ruiz de Elvira, arXive:1107.1635, accepted for publication in Phys. Rev. Lett.