Matter swapping between two branes from the equivalence between two-brane worlds and noncommutative two-sheeted spacetimes

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We report a mathematical equivalence between certain models of universe relying on domain-walls and noncommutative geometries.
It is shown that a two-brane world made of two domain-walls can be seen as a noncommutative two-sheeted spacetime under certain assumptions.
This equivalence implies a model-independent phenomenology: Matter swapping between the two branes (or sheets) is predicted through fermionic oscillations induced by magnetic vector potentials. This phenomenon, which might be experimentally studied, could reveal the existence of extra dimensions in a new and very affordable way.

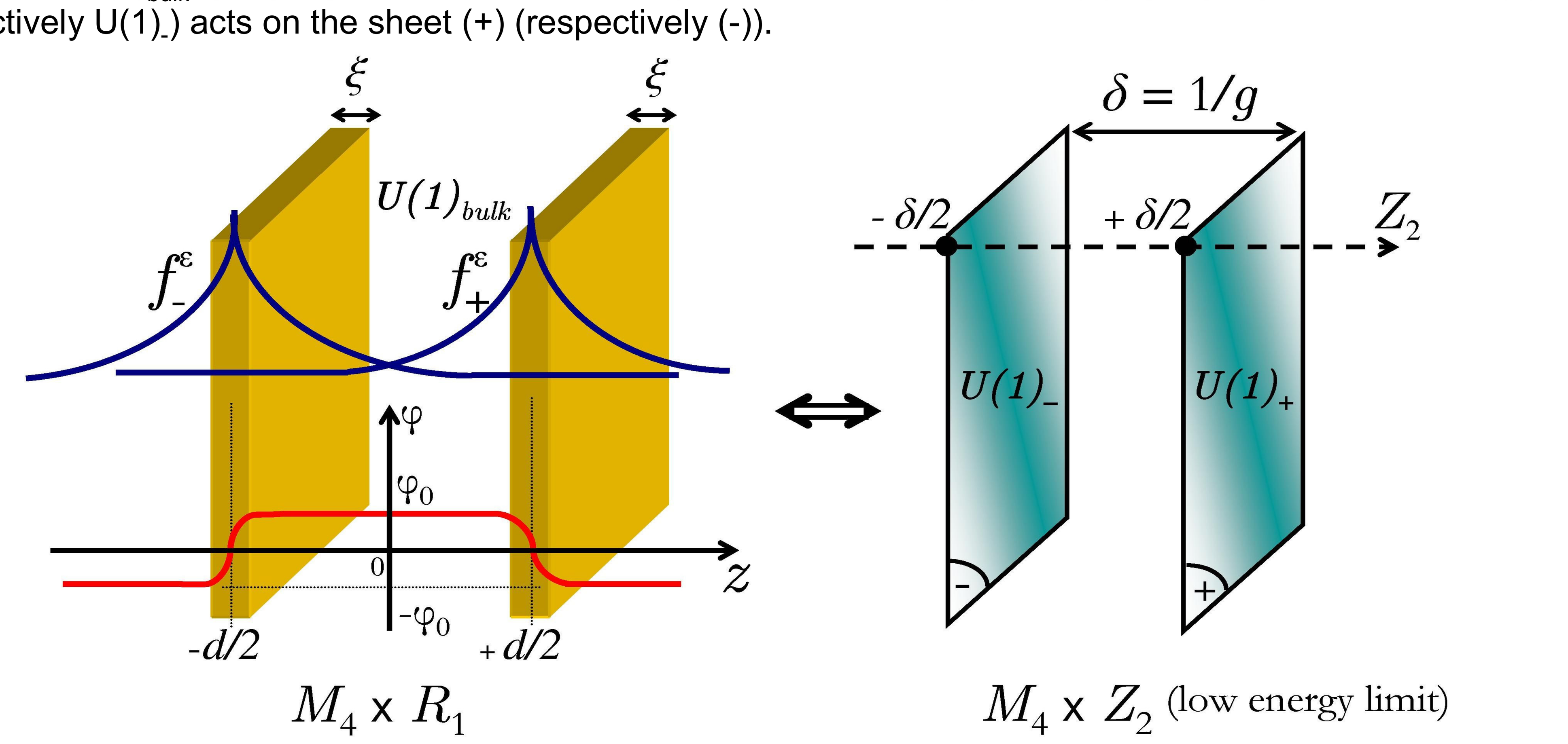
Domain-walls described by noncommutative geometries at low energy

Considering the dynamics of a fermion, a two-brane world (related to a kink-antikink system, i.e. two domain-walls [1]) is formally equivalent to a two-sheeted noncommutative spacetime (a product manifold $M_4 \times Z_2$), at low energies (there is no coupling between different mass states) [2].

-The demonstration [2] of this result is inspired by quantum chemistry and the construction of molecular orbitals, here extended to fermions on branes from the action:

$$S=\int\left[-\frac{1}{4G^2}\mathcal{F}_{AB}\mathcal{F}^{AB}+\frac{1}{2}g^{AB}\left(\partial_A\Phi\right)\left(\partial_B\Phi\right)-V(\Phi)+\overline{\Psi}\left(i\Gamma^A\left(\partial_A+i\mathcal{A}_A\right)-\lambda\Phi\right)\Psi\right]\sqrt{g}d^5x$$

- The extra dimensional extensions of the fermion wave functions $f_{\pm}^{\varepsilon}(z)$ depend on $\varepsilon=\varepsilon(V,\lambda)$.
- Two domain-walls in a $M_4 \times R_1$ geometry are then described by a $M_4 \times Z_2$ two-sheeted spacetime with an effective distance $\delta=\delta(\xi,d,\varepsilon)$.
- The effective coupling constant $g=1/\delta$ between the branes depends of the overlap of the fermion wave functions.
- The bulk $U(1)_{bulk}$ gauge group is substituted by an effective $U(1)_+ \times U(1)_-$ gauge group where $U(1)_+$ (respectively $U(1)_-$) acts on the sheet (+) (respectively (-)).



Fermion dynamics in a two-brane world

Any braneworld containing two different branes can be described by a noncommutative $M_4 \times Z_2$ two-sheeted spacetime [2]. The dynamics of a spin-1/2 fermion can be then described with a two-brane Dirac equation [2,3].

The derivative operator being:

$$D_\mu=\begin{pmatrix}\partial_\mu & 0\\ 0 & \partial_\mu\end{pmatrix},\ \mu=0,1,2,3 \text{ and } D_5=\begin{pmatrix}0 & g\\ -g & 0\end{pmatrix}$$

the Dirac equation becomes:

$$\not{D}_{\rm dirac}\Psi=(i\not{D}-M)\Psi=(i\Gamma^N D_N-M)\Psi$$
$$=\begin{pmatrix}i\gamma^\mu\partial_\mu-m & ig\gamma^5-m_c\\ ig\gamma^5-m_c^* & i\gamma^\mu\partial_\mu-m\end{pmatrix}\begin{pmatrix}\psi_+\\ \psi_-\end{pmatrix}=0$$

with: $\Gamma^\mu=\mathbf{1}_{2\times2}\otimes\gamma^\mu$ and $\Gamma^5=\sigma_3\otimes\gamma^5$

The two components of the wave function describe the fermion in each brane.

The electromagnetic gauge field can be introduced in the Dirac equation through:

$$\not{D}\rightarrow\not{D}+\not{A}$$

According to the expected U(1) x U(1) gauge group, we get:

$$\not{A}=\begin{pmatrix}iq\gamma^\mu A_\mu^+ & \gamma^5\Upsilon\\ \gamma^5\bar{\Upsilon} & iq\gamma^\mu A_\mu^-\end{pmatrix}$$

In the following, we assume that $\Upsilon\sim0$. With such a choice, we simply assume that the electromagnetic field of a brane couples only with the particles belonging to the same brane.

Pauli equation in a two-brane world

In order to underline some specific phenomena, the non-relativistic limit of the Dirac equation is derived to obtain a two-brane Pauli equation [2,3]:

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix}\psi_+\\ \psi_-\end{pmatrix}=\{\mathbf{H}_0+\mathbf{H}_{cm}+\ldots\}\begin{pmatrix}\psi_+\\ \psi_-\end{pmatrix}$$

with

$$\mathbf{H}_0=\begin{pmatrix}\mathbf{H}_+ & 0\\ 0 & \mathbf{H}_-\end{pmatrix}$$

where

$$\mathbf{H}_\pm=-\frac{\hbar^2}{2m}\left(\nabla-i\frac{q}{\hbar}\mathbf{A}_\pm\right)^2+g_s\mu\frac{1}{2}\sigma\cdot\mathbf{B}_\pm+V_\pm$$

is the usual Pauli Hamiltonian in each brane.

Moreover, a new fundamental coupling term appears [2,3]:

$$\mathbf{H}_{cm}=igg_s\mu\frac{1}{2}\begin{pmatrix}0 & -\sigma\cdot\{\mathbf{A}_+-\mathbf{A}_-\}\\ \sigma\cdot\{\mathbf{A}_+-\mathbf{A}_-\} & 0\end{pmatrix}$$

This specific term induces a coupling between the two branes through the magnetic vector potentials of each brane and the fermionic magnetic moment $g_s\mu$.

Conclusions and outlooks

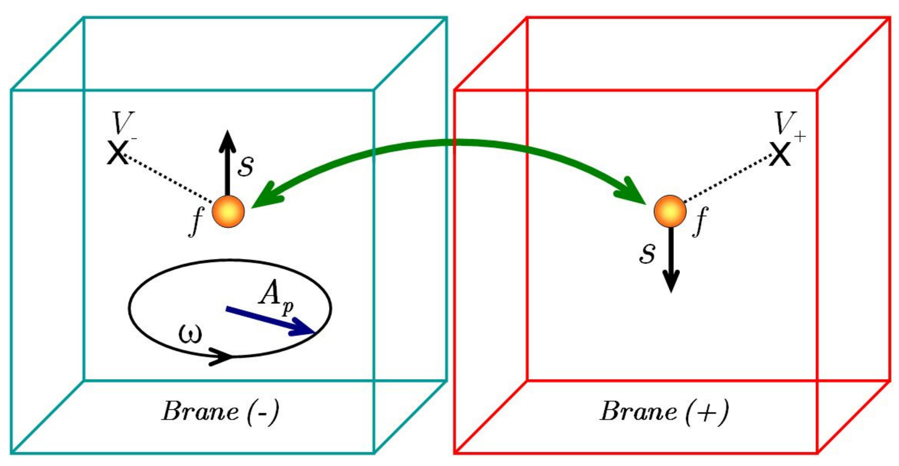
We show that a universe which contains at least two branes can be modeled by a noncommutative $M_4 \times Z_2$ two-sheeted spacetime.
Therefore, the dynamics of fermions in a two-brane world can be studied independently of any domain-wall formalism, and we show the existence of a new effect.
This effect corresponds to an exchange of fermionic matter between the two branes under the influence of relevant magnetic vector potentials. This matter swapping effect could be investigated by using current technologies.
A work is in progress to compare our results with existing experimental data.
Works are also in progress to prove the equivalence in a superstring formalism and to consider these results at high energy (coupling between different mass states). The study of the dynamics of the coupling constant g is planned.

Model-independent phenomenology in a two-brane world

The two-brane Pauli equation holds resonant solutions [4]. Considering a neutral particle with a magnetic moment initially localized in our brane and under the influence of a rotative magnetic vector potential $\boldsymbol{A}_p$ (with an angular frequency ω) localized in our brane, the probability to find the particle in the second brane is [4]:

$$P(t)=\frac{4\Omega_p^2}{(\Omega_0-\omega)^2+4\Omega_p^2}\sin^2\left((1/2)\sqrt{(\Omega_0-\omega)^2+4\Omega_p^2}t\right)$$

with $\Omega_p=gg_s\mu A_p/(2\hbar)$ and $\Omega_0=(V_+-V_-)/\hbar$ which defines the interactions of the particle with its environment.

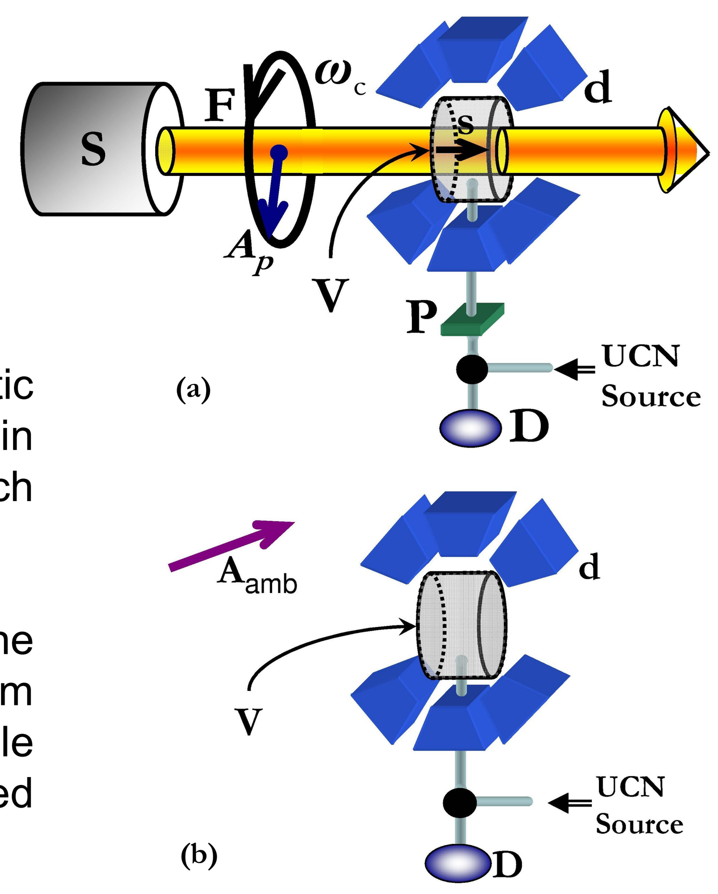


When the angular frequency of the magnetic vector potential matches the environmental confining potential, the particle is then resonantly transfered in the second brane. Experimental devices could be used to test this matter swapping effect between branes.
Two ways can be considered:

In both case, one follows the population of a neutron gas. The dispearance of neutrons can be observed through the decrease of the neutron decays (d) or from the counting of remaining neutrons in the vessel (D).

(a) - A resonant experiment [4]: (S) emits a pulsed monochromatic coherent electromagnetic radiation (F) on the ultracold neutron gas in vessel (V). The experiment is improved by the use of the Hänsch frequency comb technique used in spectroscopy [5].

(b) - A non resonant experiment [2,3]: Some works [6] underlined the existence of an astrophysical ambient magnetic vector potential from astrophysical magnetic fields. This ambient field can be responsible for non resonant swapping. This last experiment could be soon tested by teams skilled in ultracold neutrons.



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