Nucleon form factors and moments of GPDs in twisted mass lattice QCD

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OUTLINE

A Motivation

B Nucleon Generalized Parton Distributions

C Evaluation on the Lattice

- 2pt and 3pt functions
- Renormalization
- Cut-off and Volume effects

D Nucleon form factors

E Nucleon Moments

Motivation

- ★ Characterization of nucleon structure is considered a milestone in hadronic physics

  $\rightarrow$ many experiments have been carried out to measure form factors and structure functions.

- ★ New generation experiments using polarized beams and target are yielding high precision data spanning larger Q^2 ranges.

- ★ They provide ideal probes of the charge and magnetization , determination of shape in analogy to e.g. deuteron and other nuclei

- ★ Non-relativistically form factors are related to density distribution:

  $F(\vec{q}^2) = \int d^3x e^{-i\vec{q}.\vec{x}} < \psi|\rho(\vec{x})|\psi >$

Nucleon Generalized Parton Distributions (GPDs)

- High energy scattering: Formulate in terms of light-cone correlation functions

$$F_\Gamma(x,\xi,q^2) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{ix\lambda} \langle p'|\bar{\psi}(-\lambda n/2)\, \Gamma\, \mathcal{P} e^{ig\int_{-\lambda/2}^{\lambda/2} d\alpha\, n\cdot A(n\alpha)} \psi(\lambda n/2)|p\rangle$$

where $q = p' - p$, $\bar{P} = (p'+p)/2$, n: light-cone vector ($\bar{P}.n = 1$).

- Choices of operators:

$$\Gamma = \not{n} \quad \to \frac{1}{2}\bar{u}(p')\left[\not{n} H(x,\xi,q^2) + i\frac{n_\mu q_\nu \sigma^{\mu\nu}}{2m} E(x,\xi,q^2)\right] u(p)$$

$$\Gamma = \not{n}\gamma_5 \quad \to \frac{1}{2}\bar{u}(p')\left[\not{n}\gamma_5 \tilde{H}(x,\xi,q^2) + \frac{n.q\gamma_5}{2m} \tilde{E}(x,\xi,q^2)\right] u(p)$$

$$\Gamma = n_\mu \sigma^{\mu\nu} \quad \to \text{tensor GPDs}$$

- Expansion of the light cone operator leads to a tower of local twist-2 operators $\mathcal{O}^{\mu_1\cdots\mu_n}$
- Diagonal matrix element $\langle P|\mathcal{O}(x)|P\rangle$ (DIS) $\to$ parton distributions: $q(x)$, $\Delta q(x)$, $\delta q(x)$
- twist-2 operators

$$\mathcal{O}^{\mu_1\cdots\mu_n} = \bar{q}\gamma^{\{\mu_1} iD^{\mu_2}\ldots iD^{\mu_n\}} q \quad \overset{unpolarized}{\to} \quad \langle x^n\rangle_q = \int_0^1 dx\, x^n \left[q(x) - (-1)^n \bar{q}(x)\right]$$

$$\tilde{\mathcal{O}}^{\mu_1\cdots\mu_n} = \bar{q}\gamma_5\gamma^{\{\mu_1} iD^{\mu_2}\ldots iD^{\mu_n\}} q \quad \overset{helicity}{\to} \quad \langle x^n\rangle_{\Delta q} = \int_0^1 dx\, x^n \left[\Delta q(x) + (-1)^n \Delta\bar{q}(x)\right]$$

$$q = q_\downarrow + q_\uparrow,\ \Delta q = q_\downarrow - q_\uparrow,\ \delta q = q_T + q_\perp$$

Nucleon generalized form factors

Decomposition of matrix elements into GFFs: contain form factors, parton distributions

$\langle N(p')|\mathcal{O}_{\not{n}}^{\mu_1\cdots\mu_n}|N(p)\rangle =$

$$\bar{u}(p')\Big[\sum_{\substack{i=0\\ \text{even}}}^{n-1}\Big(A_{ni}(q^2)\gamma^{\{\mu_1} + B_{ni}(q^2)\frac{i\sigma^{\mu_1\alpha}q_\alpha}{2m}\Big)q^{\mu_2}\ldots q^{\mu_{i+1}}\overline{P}^{\mu_{i+2}}\ldots\overline{P}^{\mu_n\}}$$

$$+\delta^n_{\text{even}}C_{n0}(q^2)\frac{1}{m}q^{\{\mu_1}\ldots q^{\mu_n\}}\Big]u(p)$$

Similarly for $\mathcal{O}_{\not{n}\gamma_5}$ in terms of $\tilde{A}_{ni}(q^2)$, $\tilde{B}_{ni}(q^2)$

Special cases:

- $n = 1$: ordinary nucleon form factors

  1. vector operator $\Longrightarrow \gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m}F_2(q^2)$

     $G_E(q^2) = F_1(q^2) - \frac{q^2}{(2m)^2}F_2(q^2)$, $\quad G_M(q^2) = F_1(q^2) + F_2(q^2)$

  2. axial operator $\Longrightarrow i\left[\gamma_\mu\gamma_5 G_A(q^2) + \frac{q_\mu\gamma_5}{2m}G_p(q^2)\right]\frac{\tau^a}{2}$

- $n = 2$: moments of parton distributions

  1. vector operator $\Longrightarrow A_{20}(q^2), \quad B_{20}(q^2), \quad C_{20}(q^2)$

     $\langle x\rangle_q = A_{20}(0)$: spin independend moment

  2. axial operator $\Longrightarrow \tilde{A}_{20}(q^2), \quad \tilde{B}_{20}(q^2)$

     $\langle x\rangle_{\Delta q} = \tilde{A}_{20}(0)$: helicity moment

Evaluation on the Lattice

Issues to be addressed:

- Evaluation of three-point correlators and renormalization
- Choice of operators - avoid mixing, consider iso-vector operators $\rightarrow$ no disconnected contributions
- Cut-off effects
- Finite volume effects
- Chiral expansions - not as developed as in the light meson case $\rightarrow$ Volume and cut-off effects more difficult to assess
- Extrapolation to physical point more demanding

This work focuses on:

- $N_F = 2$ twisted mass gauge configurations (produced by ETMC)
- Nucleon form factors
- Nucleon lower moments of GPDs
- Dynamical simulations, pion mass $m_\pi \lesssim 500$ MeV, $L \gtrsim 2$ fm

Fermion and Gluon Action

$N_F = 2$ Twisted mass fermions (twisted basis)

$$S_F = a^4 \sum_x \overline{\chi}(x) \left(\frac{1}{2}\gamma_\mu (\overrightarrow{\nabla}_\mu + \overrightarrow{\nabla}^*_\mu) - \frac{ar}{2} \overrightarrow{\nabla}_\mu \overrightarrow{\nabla}^*_\mu + m_0 + i\mu_0 \gamma_5 \tau^3 \right) \chi(x)$$

physical basis at maximal twist

$$\psi = \frac{1}{\sqrt{2}}[\mathbf{1} + i\tau^3\gamma_5]\chi \qquad \bar{\psi} = \bar{\chi}\frac{1}{\sqrt{2}}[\mathbf{1} + i\tau^3\gamma_5]$$

Tree-level Symanzik improved gluons

$$S_g = \frac{\beta}{3} \sum_x \left(\frac{5}{3} \sum_{\substack{\mu,\nu=1 \\ 1\leq\mu<\nu}}^{4} \left\{ 1 - \mathrm{Re}\,\mathrm{Tr}(U^{1\times 1}_{x,\mu,\nu}) \right\} - \frac{1}{12} \sum_{\substack{\mu,\nu=1 \\ \mu\neq\nu}}^{4} \left\{ 1 - \mathrm{Re}\,\mathrm{Tr}(U^{1\times 2}_{x,\mu,\nu}) \right\} \right)$$

Ensembles

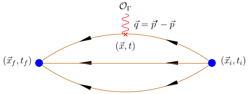
β	a (fm)	$a\mu_0$	m_π (GeV)	$L^3 \times T$
3.9	0.089	0.0030	0.260	$32^3 \times 64$
3.9	0.089	0.0040	0.298	$32^3 \times 64$
3.9	0.089	0.0040	0.302	$24^3 \times 48$
3.9	0.089	0.0064	0.376	$24^3 \times 48$
3.9	0.089	0.0085	0.430	$24^3 \times 48$
3.9	0.089	0.0100	0.468	$24^3 \times 48$
4.05	0.070	0.0030	0.293	$32^3 \times 64$
4.05	0.070	0.0060	0.404	$32^3 \times 64$
4.05	0.070	0.0080	0.465	$32^3 \times 64$
4.20	0.055	0.0020	0.262	$48^3 \times 96$
4.20	0.055	0.0065	0.476	$32^3 \times 64$

- C. Alexandrou et al. (ETM Collaboration), Phys. Rev. D83 (2011) 045010
- C. Alexandrou et al. (ETM Collaboration), Phys. Rev. D83 (2011) 094502
- C. Alexandrou et al. (ETM Collaboration), Phys. Rev. D83 (2011) 114513
- C. Alexandrou et al. (ETM Collaboration), Phys. Rev. D83 (2011) 014503

Evaluation of two-point and three-point functions

$$G(\vec{q},t) \;=\; \sum_{\vec{x}_f} e^{-i\vec{x}_f\cdot\vec{q}}\,\Gamma^4_{\beta\alpha}\,\langle J_\alpha(\vec{x}_f,t_f)\overline{J}_\beta(0)\rangle$$

$$G^{\mu\nu}(\Gamma,\vec{q},t) \;=\; \sum_{\vec{x}_f,\vec{x}} e^{i\vec{x}\cdot\vec{q}}\,\Gamma_{\beta\alpha}\,\langle J_\alpha(\vec{x}_f,t_f)\mathcal{O}^{\mu\nu}(\vec{x},t)\overline{J}_\beta(0)\rangle$$



Sequential inversion “through the sink”
$\rightarrow$ fix sink-source separation $t_f - t_i$,
final momentum $\vec{p}_f = 0$, Γ

Smearing techniques $\rightarrow$ improvement of ground state dominance in three-point correlators

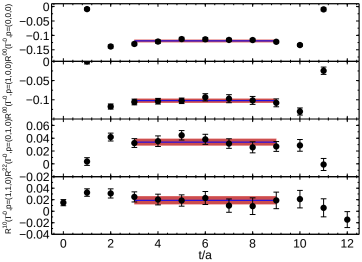
Ratios: Leading time dependence cancels

$$R(\Gamma,\vec{q},t) = \frac{G(\Gamma,\vec{q},t)}{G(\vec{0},t_f)}\sqrt{\frac{G(-\vec{q},t_f-t)G(\vec{0},t)G(\vec{0},t_f)}{G(\vec{0},t_f-t)G(-\vec{q},t)G(-\vec{q},t_f)}}$$

$$\lim_{t_f-t\to\infty}\;\lim_{t-t_i\to\infty} R(\Gamma,\vec{q},t) \rightarrow \Pi(\Gamma,\vec{q})$$

$R(\Gamma,\vec{q},t)$ depends on the indices of current insertion

Variational approach, possible improvement on plateaux $\rightarrow$ extend to $Q^2 \sim 4$ GeV^2



Renormalization Constants

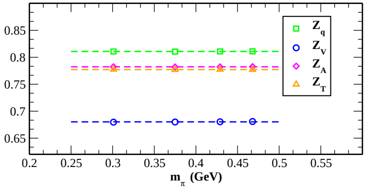
- RI'-MOM renormalization scheme
- Fix configurations to Landau gauge.

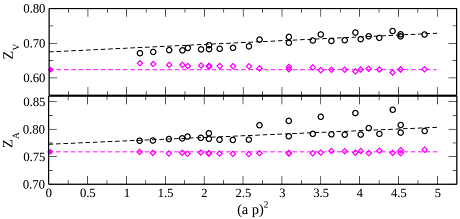
$$S^u(p) = \frac{a^8}{V} \sum_{x,y} e^{-ip(x-y)} \langle u(x)\bar{u}(y)\rangle$$

$$G(p) = \frac{a^{12}}{V} \sum_{x,y,z,z'} e^{-ip(x-y)} \langle u(x)\bar{u}(z)\mathcal{J}(z,z')d(z')\bar{d}(y)\rangle$$

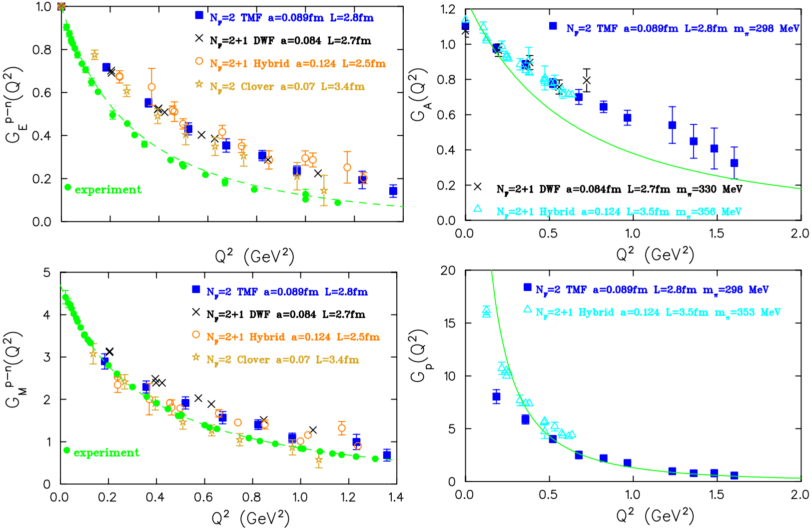
$\rightarrow$ Amputated vertex functions $\Gamma(p) = (S^u(p))^{-1}\; G(p)\; (S^d(p))^{-1}$

- Renormalization functions: Z_q and $Z_{\mathcal{O}}$
- Mass independent renormalization scheme $\rightarrow$ need chiral extrapolations
- Subtract $\mathcal{O}(a^2)$ perturbatively





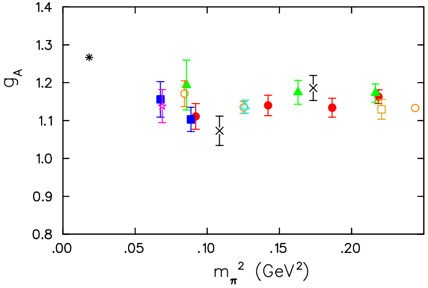
Nucleon EM and Axial form factors (m_π:300-350MeV)



Can we get results at physical point?

Nucleon axial charge

$g_A = G_A(0)$

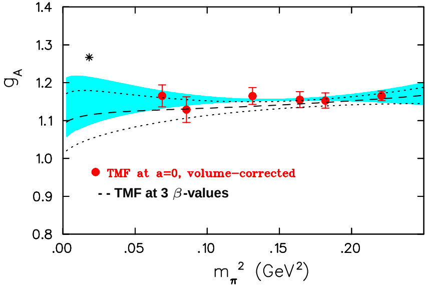


- $N_F = 2$ twisted mass fermions, ETMC
- $N_F = 2 + 1$ Domain wall fermions, RBC-UKQCD
- $N_F = 2 + 1$ hybrid action, LHPC

- Agreement among recent lattice results - all use non-perturbative Z_A
- Weak light quark mass dependence
- What can we say about the physical value of g_A?

Nucleon axial charge

$g_A = G_A(0)$

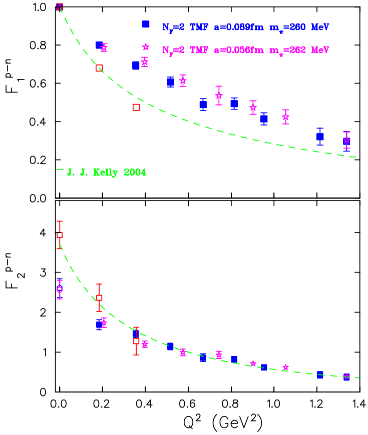


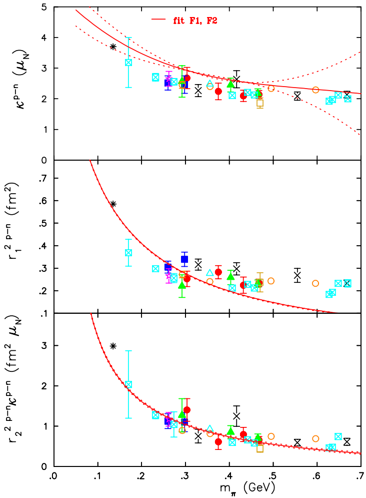
- Continuum limit of our volume corrections results
- 1-loop chiral perturbation theory in the small scale expansion (SSE)
- Fitting volume corrected and extrapolated to the continuum results $\rightarrow g_A = 1.12(7)$
- Fitting lattice results directly $\rightarrow g_A = 1.08(8)$ (black dashed line)

Chiral extrapolation of EM form factors

Baryon χPT to 1-loop, with Δ d.o.f.(SSE) and isovector $N\Delta$ coupling included in LO

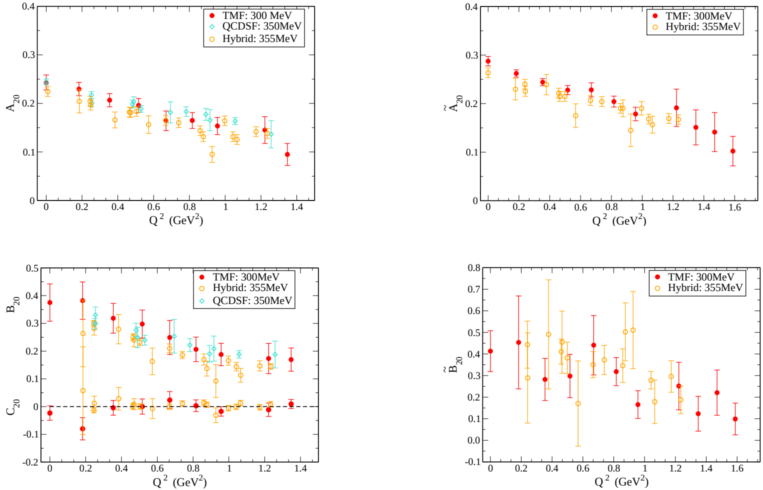
Fit $F_1(m_\pi, Q^2)$ and $F_2(m_\pi, Q^2)$ with 5 parameters: κ_v, the isovector (c_v) and axial N to Δ ($g_{N\Delta}$ or c_A) couplings and two counterterms

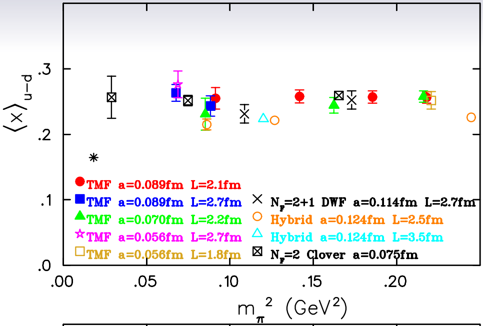




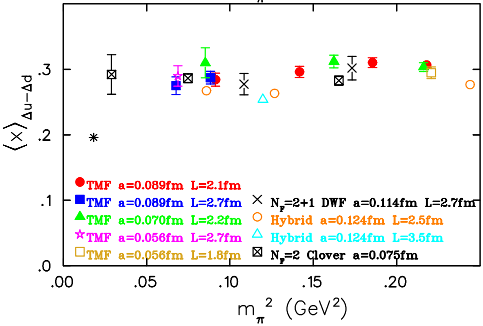
Nucleon Moments

- GFFs: $\bar{u}\gamma_{\{\mu}\overleftrightarrow{D}_{\nu\}}u - \bar{d}\gamma_{\{\mu}\overleftrightarrow{D}_{\nu\}}d$ and $\bar{u}\gamma_{\{\mu}\gamma_5\overleftrightarrow{D}_{\nu\}}u - \bar{d}\gamma_{\{\mu}\gamma_5\overleftrightarrow{D}_{\nu\}}d$
- Results given in the $\overline{MS}$ scheme at $\mu = 2$ GeV





spin-independent moment: $A_{20}(0)$



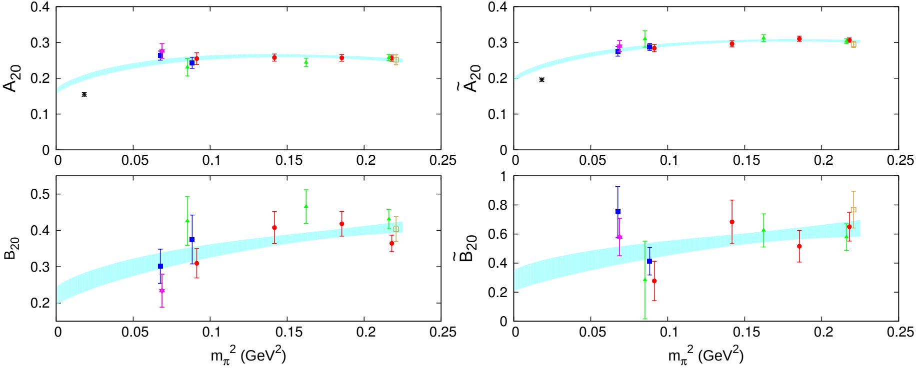
helicity moment: $\tilde{A}_{20}(0)$

Chiral extrapolation of $A_{20},\ B_{20},\ \tilde{A}_{20},\ \tilde{B}_{20}$ at $Q^2 = 0$

HBχPT: A combined fit to raw data A_{20} and B_{20} is carried out (agreement with $a \to 0$ results).

The mass of the nucleon at the chiral limit is used as input

Isovector unpolarized and polarized first moments of quark distribution



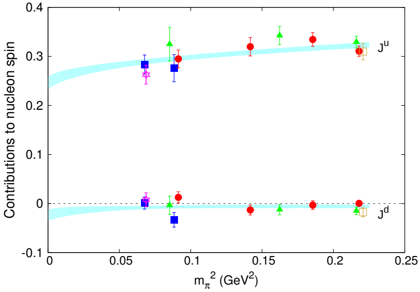
Proton Spin

One also needs the isoscalar moments A_{20}^{u+d} and B_{20}^{u+d} since the total spin of a quark is

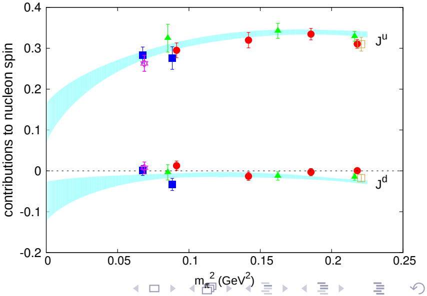
$$
\begin{aligned}
J^q &= \frac{1}{2}\left(A_{20}^q(0) + B_{20}^q(0)\right) \\
L^q &: \text{orbital angular momentum} \\
\Delta\Sigma^q &: \text{spin components} \\
J^q &= \frac{1}{2}\Delta\Sigma^q + L^q \\
\Delta\Sigma^{u+d} &= g_A^{u+d}
\end{aligned}
$$

- no disconnected contributions

CBχPT

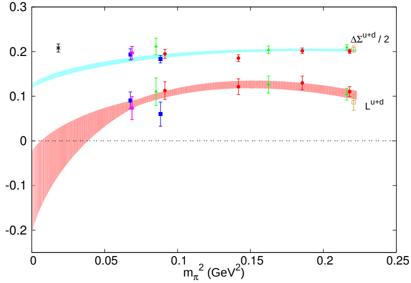


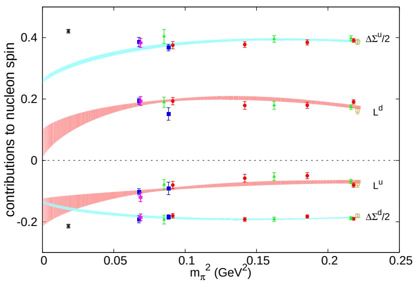
HBχPT



Chiral extrapolation using HBχPT

- left: total angular momentum and total spin component
- right: Angular momentum and spin carried by u- d- quarks





★ Physical points from HERMES 2007 analysis

Conclusions

- Nucleon form factors provide a benchmark for lattice QCD beyond hadron masses.

  Need results at both lower Q^2 $\rightarrow$ extract radii and magnetic moments and higher Q^2
- Cut-off effects small for $a \lesssim 0.1$ fm
- Finite volume corrections difficult to assess

  Within current statistical errors of ~ 3% results on G_E, G_M, G_A, $< x >_q$ and $< x >_{\Delta q}$ are consistent for $Lm_\pi \gtrsim 3.5 \rightarrow Lm_\pi = 4$

  Finite volume corrections significant for G_p
- Need to include the disconnected contributions
- Make a lattice determination of a number of couplings used as input in chiral extrapolations
- explore GPDs that yield more detailed information on both longitudinal and transverse distributions

THANK YOU