Center-symmetric effective theory for two-color QCD with massive quarks at nonzero chemical potential

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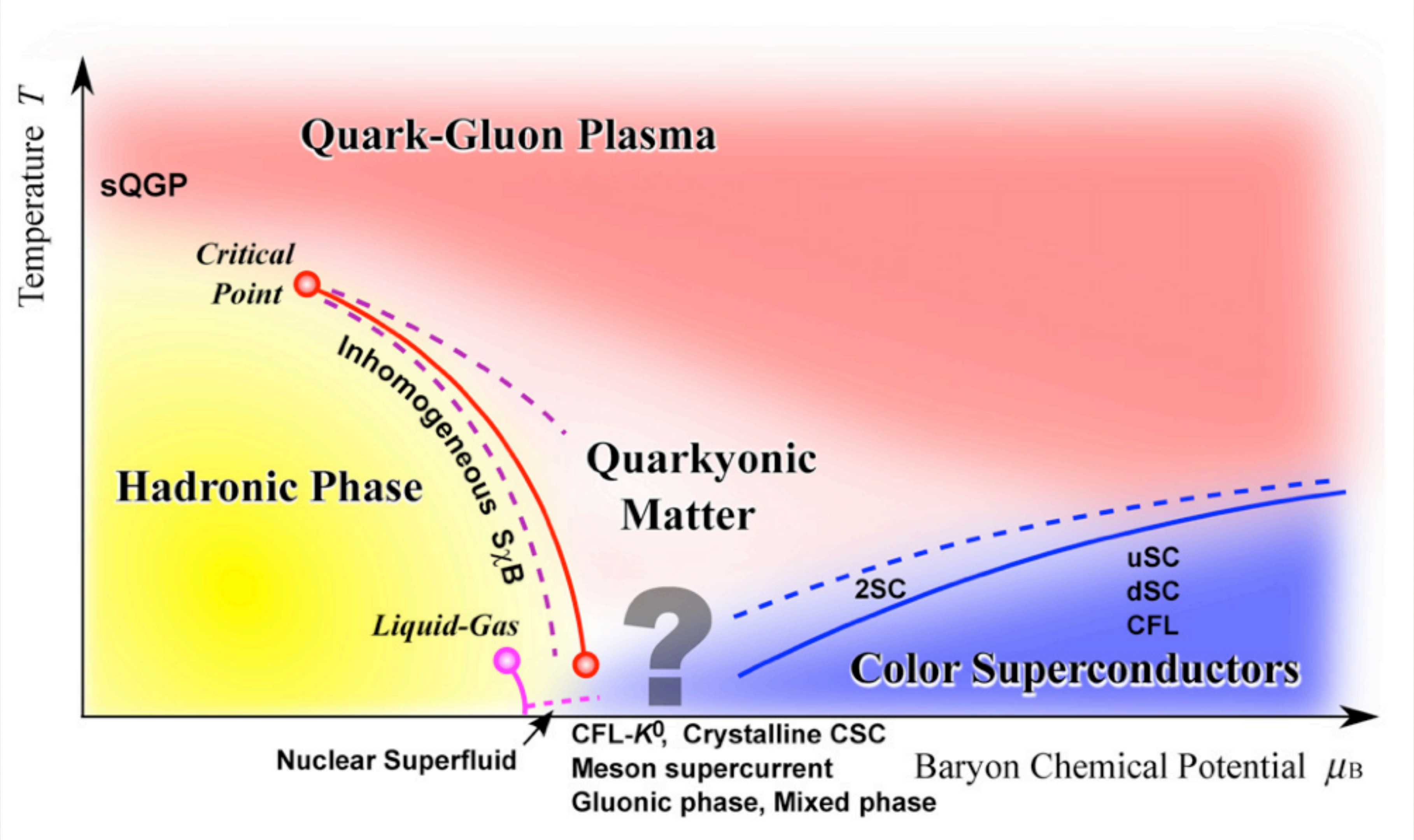
QCD phase diagram



Fukushima, Hatsuda, Rept. Prog. Phys. 74 (2011)

- High temperature: lattice QCD, resummed PT, EFT.
- High density: phenomenological models.

Dimensional reduction at high T

4d (Euclidean) quantum field theory at high temperature reduces to a 3d theory of the zero Matsubara mode.

- Heavy modes: "hard mass" $\omega_n = 2\pi n T,\ n \neq 0$.
- Light modes: "soft mass" $\propto gT$ by loop corrections.
- Dimensionally reduced theory of QCD: EQCD.
- Deg-s of freedom: 3d gauge field $\boldsymbol{A}_a$ + adjoint scalar $A^0{}_a$.

$$\mathscr{L}_{\mathrm{EQCD}} = \frac{1}{4}(F^a_{ij})^2 + \frac{1}{2}(\mathscr{D}_i A^a_0)^2 + \frac{1}{2} m_E^2 (A^a_0)^2 + \frac{1}{8}\lambda_E (A^a_0 A^a_0)^2$$

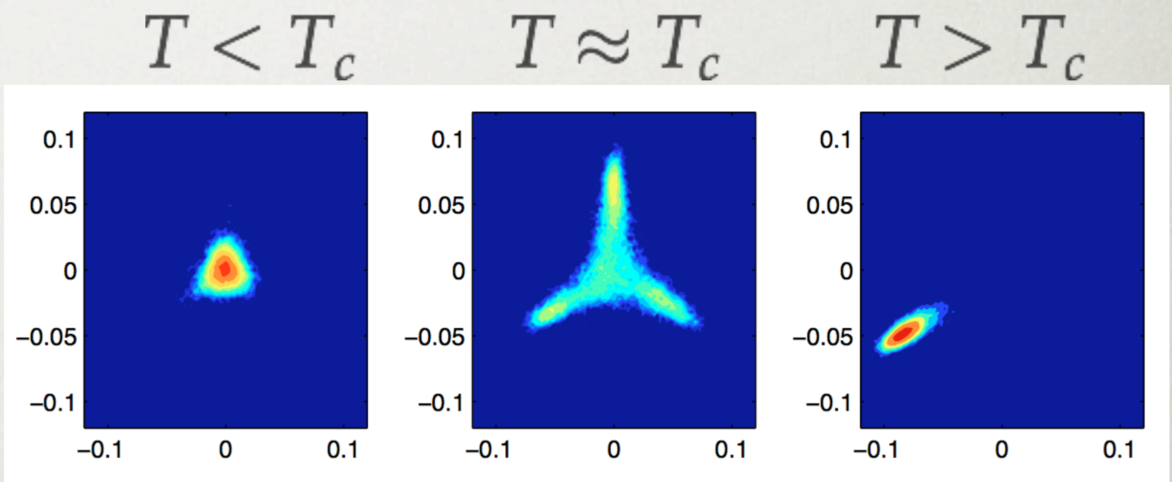
Braaten, Nieto, Phys. Rev. D 53 (1996)

- The EFT determines physics on length scales $\propto 1/gT$.

Center symmetry

- SU(N) Yang–Mills theory has global Z_N symmetry.
- Symmetry changes in the (de)confinement transition.
- Order parameter: expectation value of Polyakov loop.

$$\Omega(\boldsymbol{x}) = \mathrm{tr}\left\{ \mathscr{P} \exp\left[ig \int_0^\beta d\tau\, A_0(\tau, \boldsymbol{x}) \right] \right\}$$

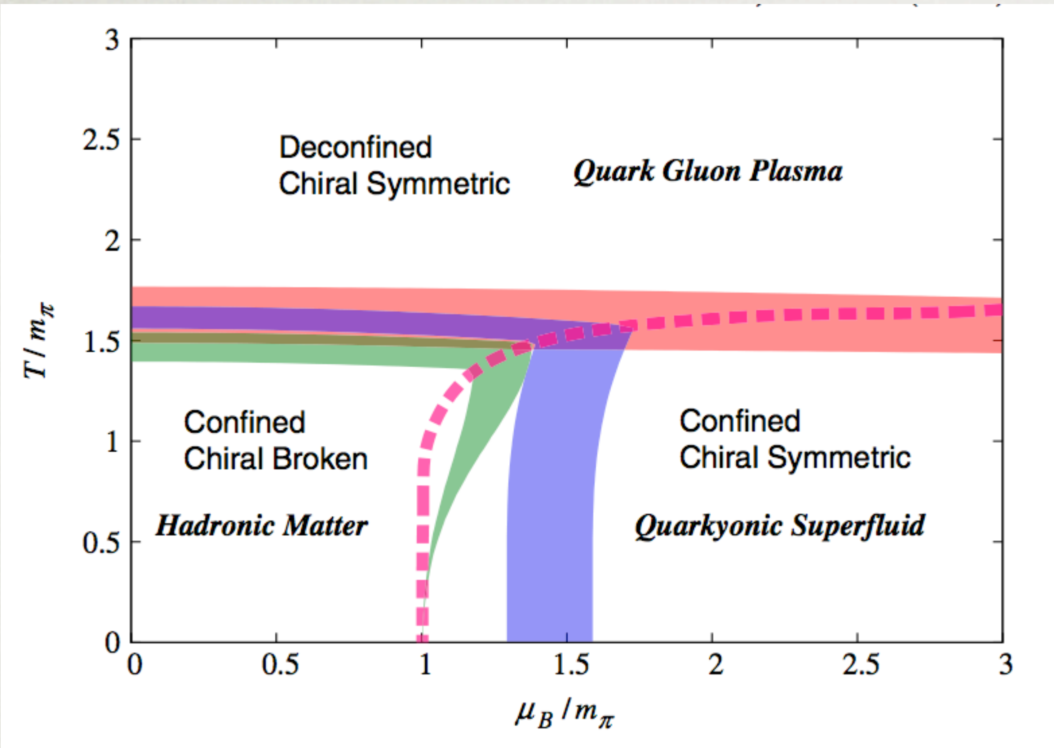


Kurkela, Vienna (2009)

- Dynamical fermions break center symmetry explicitly.
- EQCD breaks Z_N explicitly by expanding around one of the N degenerate minima.

Two-color QCD

- Toy world where baryons as well as mesons are bosons.
- BEC of scalar diquarks at sufficient high μ_B.
- Allows for *ab initio* calculations of the phase diagram.
- Cold dense matter: useful hints about (de)confinement.
- Hot dilute matter: analytic check of lattice techniques.



TB, Fukushima, Hidaka, Phys. Rev. D 80 (2009)
Andersen, TB, Phys. Rev. D 81 (2010)
Zhang, TB, Rischke, JHEP 06 (2010)

Center-symmetric effective theory

- Construct an effective field theory that:
  - ⋆ Preserves the Z_N center symmetry.
  - ⋆ Reduces to EQCD at high temperature.
  - ⋆ Is superrenormalizable.
- Fix parameters by matching to (E)QCD.

- Worked out for SU(3) Yang–Mills:
  Vuorinen, Yaffe, Phys. Rev. D 74 (2006)
- Worked out for SU(2) Yang–Mills:
  de Forcrand, Kurkela, Vuorinen, Phys. Rev. D 77 (2008)
- Goal of this work: add dynamical quarks.

Construction of the EFT

- Degrees of freedom:
  - ⋆ 3d spatial (magnetic) gluon field, $\boldsymbol{A_a(x)}$.
  - ⋆ Coarse-grained Polaykov loop field:

$$\mathcal{Z}(\boldsymbol{x}) = \frac{1}{2}\left[\Sigma(\boldsymbol{x}) + i\sigma_a \Pi_a(\boldsymbol{x})\right]$$

- Gauge and center symmetry of the theory:

$$\mathcal{Z}(\boldsymbol{x}) \to U(\boldsymbol{x})\mathcal{Z}(\boldsymbol{x})U^\dagger(\boldsymbol{x})$$
$$A(\boldsymbol{x}) \to U(\boldsymbol{x})[A(\boldsymbol{x}) + i\boldsymbol{\nabla}]U^\dagger(\boldsymbol{x})$$

$$\mathcal{Z}(\boldsymbol{x}) \to \pm\mathcal{Z}(\boldsymbol{x})$$

- Most general Lagrangian respecting the symmetries:

$$\mathcal{L}_{\text{EFT}} = g_3^{-2}\left[\tfrac{1}{2}\operatorname{tr} F_{ij}^2 + \operatorname{tr}(\mathcal{D}_i\mathcal{Z}^\dagger\mathcal{D}_i\mathcal{Z}) + V(\mathcal{Z})\right]$$
$$V(\mathcal{Z}) = b_1\Sigma^2 + b_2\Pi_a^2 + c_1\Sigma^4 + c_2(\Pi_a^2)^2 + c_3\Sigma^2\Pi_a^2 + d_1\Sigma^3 + d_2\Sigma\Pi_a^2$$

Perturbative matching

- 1 heavy mode (mass $\propto T$): Σ, integrated out.
- 3 light modes (mass $\propto gT$): Π_a, matched to $A^0{}_a$ of EQCD.
- 5 of 7 parameters determined by matching to the perturbative one-loop Weiss effective potential.

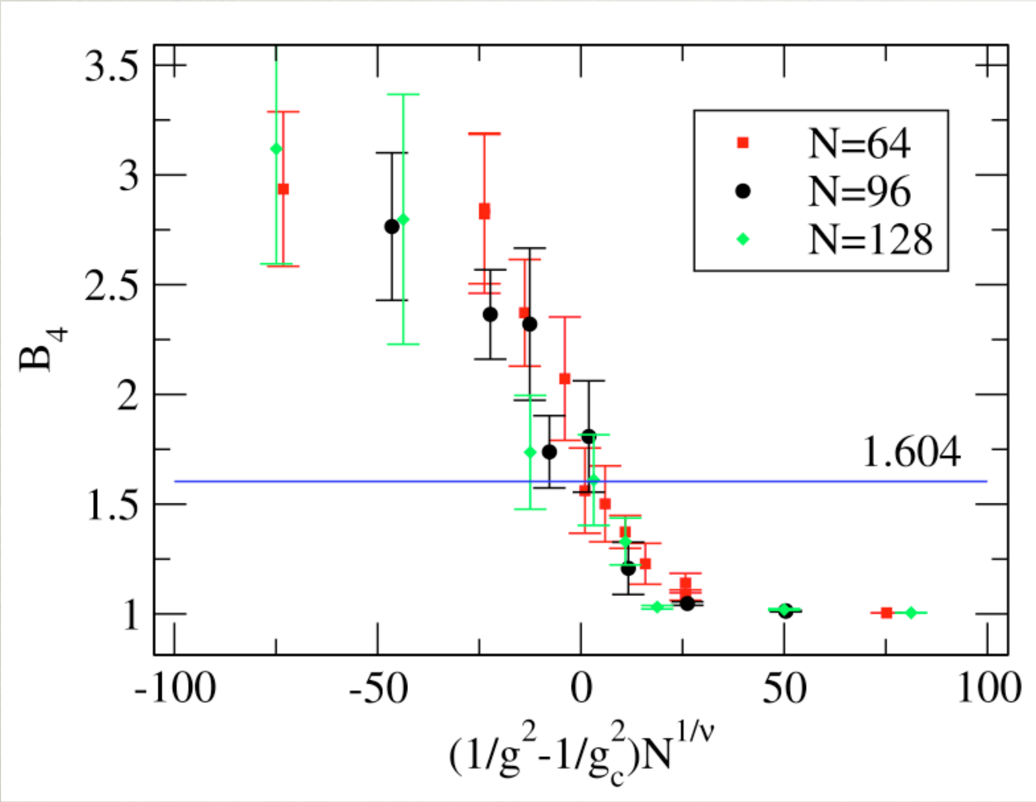
$$V_{\rm eff}(\Pi_a) = \frac{4}{3}\pi^2 T^4 \left\langle \frac{g|\mathbf{\Pi}|}{2\pi T} \right\rangle^2 \left(1 - \left\langle \frac{g|\mathbf{\Pi}|}{2\pi T} \right\rangle\right)^2 - \frac{4T^2}{\pi^2} \sum_{j=1}^{N_f} m_j^2 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} K_2(n\beta m_j) \cosh(n\beta\mu_j) \cos\frac{ng|\mathbf{\Pi}|}{2T}$$

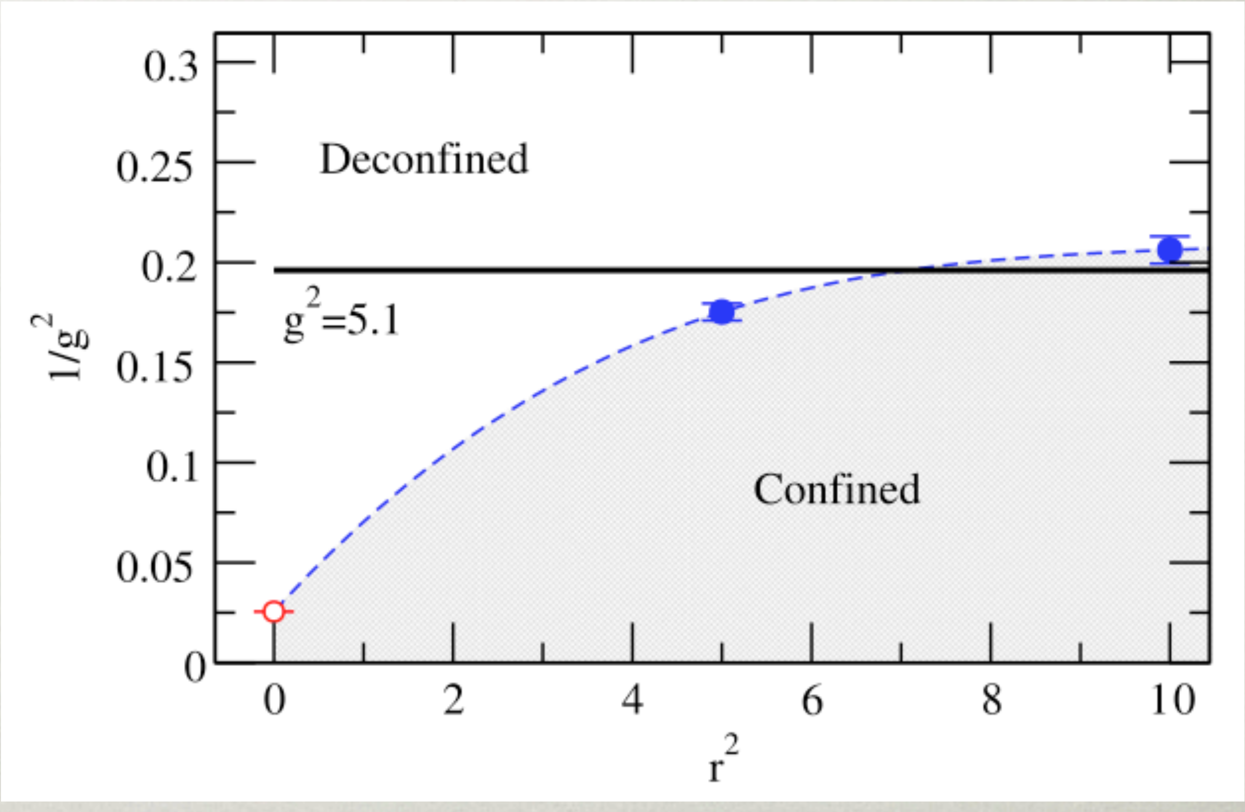
- Remaining parameters are related to the heavy mode Σ.
- They must be found by nonperturbative simulation.

de Forcrand, Kurkela, Vuorinen, Phys. Rev. D 77 (2008)

Predictions of the theory I

- Numerical lattice simulation to determine the critical temperature within the effective theory.
- Binder cumulant $B_4=\langle\Sigma^4\rangle/\langle\Sigma^2\rangle^2$ to check universality of the 3-dimensional phase transition with $\nu=0.63$.

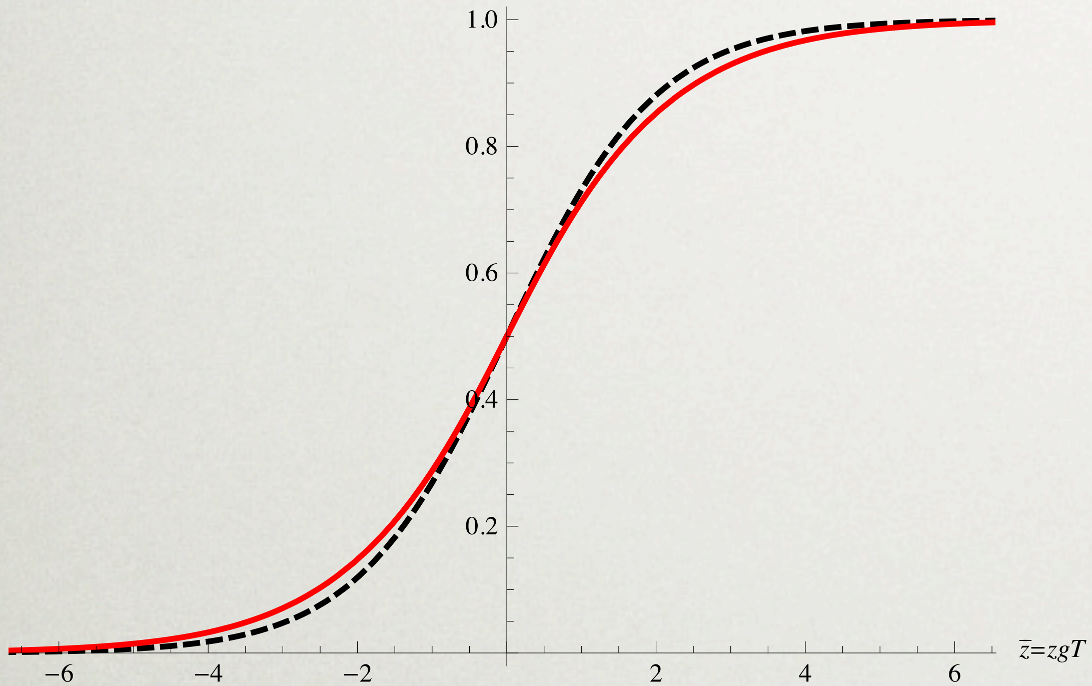


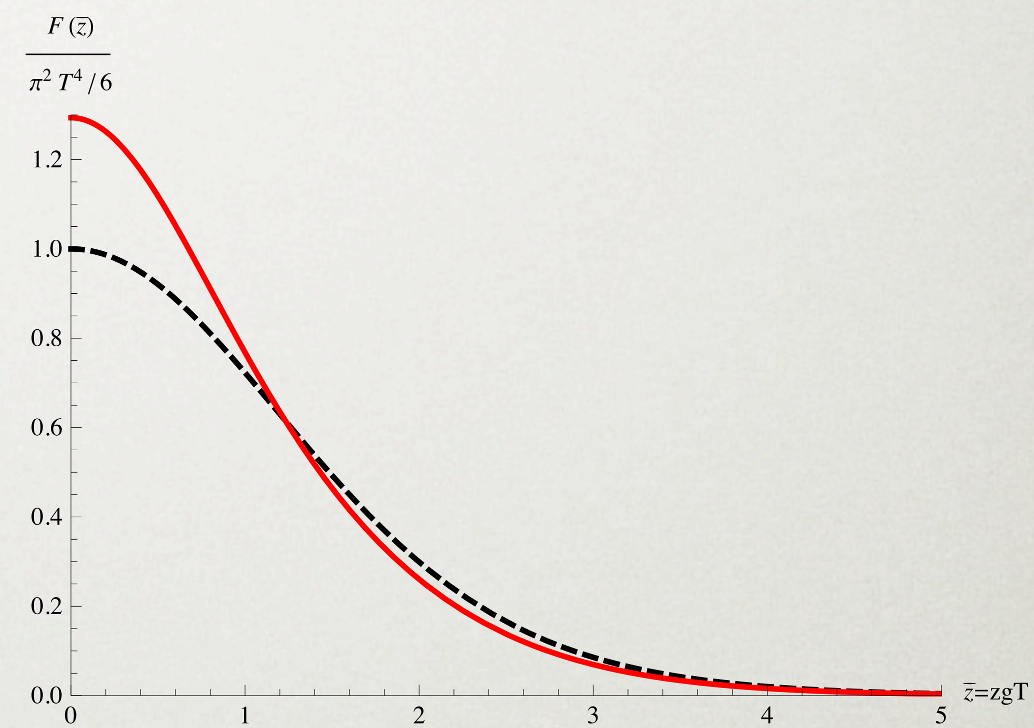


de Forcrand, Kurkela, Vuorinen, Phys. Rev. D 77 (2008)

Predictions of the theory II

- Domain wall solution (in absence of Z_2 breaking).
- Wall tension differs by 8% from the Yang–Mills value.





- In presence of Z_2 breaking: bubble solutions.

$$R_c = \left(\frac{2}{3}\right)^{3/2} \frac{\pi^2}{\kappa}, \qquad S_{\text{bubble}} = \frac{2^{13/2}\pi^7}{3^{11/2} g^3 \kappa^2}, \qquad \kappa = \frac{4}{\pi^2} \sum_{j=1}^{N_f} (\beta m_j)^2 K_2(\beta m_j) \cosh(\beta \mu_j)$$

- To do: thermodynamics at nonzero chemical potential.

Summary and outlook

- Constructed a low-energy EFT for two-color QCD.
- Quarks with arbitrary masses and chemical potentials.
- All but 2 parameters fixed by perturbative matching.
- Including center symmetry gets us closer to T_c.
- Domain wall structure of QCD correctly reproduced.
- Use the EFT to make predictions:
  - ⋆ Thermodynamics at moderate T and low μ.
  - ⋆ Phase structure at imaginary chemical potential.
  - ⋆ Test convergence of Taylor expansions in μ.