Hadron production in hot and dense nuclear matter

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Abstract

We study the hadron production at finite values of temperature and baryon density by means of an effective relativistic mean-field model with the inclusion of the full octet of baryons, the Δ-isobar degrees of freedom and the lightest pseudoscalar and vector mesons. These last particles are considered in the so-called one-body contribution, taking into account of an effective chemical potential and an effective mass depending on the self-consistent interaction between baryons. The analysis is performed by requiring the Gibbs conditions on the global conservation of baryon number, electric charge fraction and zero net strangeness. In this context, we study the influence of the Δ-isobars degrees of freedom in the behavior of different hadron ratios and strangeness production.

Relativistic mean field model

The relativistic mean-field (RMF) model, first introduced by Walecka and Boguta-Bodmer in the mid-1970s [1,2], is widely successful used for describing the properties of finite nuclei as well as hot and dense nuclear matter [3,4]. In this context, the total baryon Lagrangian density can be written as

$$\mathcal{L}_B = \mathcal{L}_{\rm octet} + \mathcal{L}_\Delta + \mathcal{L}_{\rm qpm}\,, \tag{1}$$

where $\mathcal{L}_{\rm octet}$ stands for the full octet of baryons (p, n, Λ, Σ^+, Σ^0, Σ^-, Ξ^0, Ξ^-), $\mathcal{L}_\Delta$ corresponds to the degree of freedom for the Δ isobars (Δ^{++}, Δ^+, Δ^0, Δ^-) and $\mathcal{L}_{\rm qpm}$ is related to a quasi-particle gas of the lightest pseudoscalar and vector mesons with effective chemical potentials and effective masses (see below for details).
The RMF model for the full octet of baryons ($J^P = 1/2^+$) was originally studied by Glendenning with the following standard Lagrangian [5]

$$\begin{aligned}\mathcal{L}_{\rm octet} \;=\; & \sum_k \overline{\psi}_k\,[i\,\gamma_\mu\,\partial^\mu - (M_k - g_{\sigma k}\,\sigma) - g_{\omega k}\,\gamma_\mu\,\omega^\mu - g_{\rho k}\,\gamma_\mu\,\vec{t}\cdot\vec{\rho}^{\,\mu}]\,\psi_k \\ & +\frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) - \frac{1}{3}a\,(g_{\sigma N}\,\sigma)^3 - \frac{1}{4}\,b\,(g_{\sigma N}\,\sigma^4) \\ & +\frac{1}{2}\,m_\omega^2\,\omega_\mu\omega^\mu + \frac{1}{4}\,c\,(g_{\omega N}^2\,\omega_\mu\omega^\mu)^2 + \frac{1}{2}\,m_\rho^2\,\vec{\rho}_\mu\cdot\vec{\rho}^{\,\mu} \\ & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}\vec{G}_{\mu\nu}\vec{G}^{\mu\nu}\,,\end{aligned} \tag{2}$$

In the framework of the non-linear Walecka model it is predicted that there is a phase transition from nucleonic matter to Δ-exited nuclear matter and the occurrence of this transition depends on the coupling constants [6]. It has been pointed out that the existence of degree of freedom related to Δ isobar can be very relevant in relativistic heavy ion collisions and in the core of neutron stars [7,8]. The Lagrangian density concerning the Δ isobar can be expressed as

$$\mathcal{L}_\Delta \;=\; \bar{\Delta}_\mu\,[i\gamma_\alpha^{\mu\nu}(\partial^\alpha + ig_{\omega\Delta}\omega^\alpha + ig_{\rho\Delta}\,\vec{t}\cdot\vec{\rho}^{\,\alpha}) - (M_\Delta - g\sigma_\Delta\sigma)\gamma^{\mu\nu}]\Delta_\nu\,, \tag{3}$$

where Δ_μ is the Rarita-Schwinger spinor, $\vec{t}$ is the isospin operator for the Δ baryon.
In the RMF approach baryons are considered as Dirac quasiparticles moving in classical meson fields and the field operators are replaced by their expectation values. The effective mass of k-th baryon particle is given by

$$M_k^\star = M_k - g_{\sigma k}\sigma\,. \tag{4}$$

In the meson fields equations appear the couplings with all considered baryons (octet and Δs) and the baryon and the scalar (ρ_i^B and ρ_i^S) densities of the baryon particle of index i. They are given by

$$\rho_i^B = \gamma_i \int \frac{{\rm d}^3k}{(2\pi)^3}\,[f_i(k) - \overline{f}_i(k)]\,, \tag{5}$$

$$\rho_i^S = \gamma_i \int \frac{{\rm d}^3k}{(2\pi)^3}\,\frac{M_i^\star}{E_i^\star}\,[f_i(k) + \overline{f}_i(k)]\,, \tag{6}$$

where $\gamma_i = 2J_i + 1$ is the degeneracy spin factor of the i-th baryon and $f_i(k)$ and $\overline{f}_i(k)$ are the standard fermion particle and antiparticle distributions defined in terms of the baryon effective energy $E_i^\star(k) = \sqrt{k^2 + M_i^{\star 2}}$ and of the effective chemical potentials μ_i^*

$$\mu_i^* = \mu_i - g_{\omega i}\,\omega - g_{\rho i}\,t_{3i}\,\rho\,, \tag{7}$$

where t_{3i} is the third component of the isospin of the i-th baryon.
Because of we are going to describe finite temperature and density nuclear matter with respect to strong interaction, we have to require the conservation of three "charges": baryon number, electric charge and strangeness number. The systems is described by three independent chemical potentials: μ_B, μ_C and μ_S, respectively, the baryon, the electric charge and the strangeness chemical potentials of the system. Therefore, the chemical potential of particle of index i can be written as

$$\mu_i = b_i\,\mu_B + c_i\,\mu_C + s_i\,\mu_S\,, \tag{8}$$

where b_i, c_i and s_i are, respectively, the baryon, the electric charge and the strangeness quantum numbers of the i-th hadronic species.
At low baryon density and high temperature, the contribution of the lightest pseudoscalar and vector mesons to the total thermodynamical potential becomes very relevant. From a phenomenological point of view, we can take into account of these contributions by incorporating such mesons by adding to the thermodynamical potential their one-body contribution, i.e. the contribution of an ideal Bose gas with an effective chemical potential.
Following Ref.[8], the values of the **meson effective chemical potentials** $\mu_j^\star$ are fixed from the "bare" chemical potentials and writing them in terms of the corresponding baryon effective chemical potentials, respecting the strong interaction. For example, for pions (and rho mesons) we have that $\mu_{\pi^+} = \mu_{\rho^+} = \mu_C \equiv \mu_p - \mu_n$ and its effective chemical potential can be written as

$$\mu_{\pi^+}^* = \mu_{\rho^+}^* \equiv \mu_p^* - \mu_n^*\,. \tag{9}$$

For the other mesons, we have

$$\mu_{K^+}^* = \mu_{K^{*+}}^* \equiv \mu_p^* - \mu_{\Lambda(\Sigma^0)}^*\,, \tag{10}$$

$$\mu_{K^0}^* = \mu_{K^{*0}}^* \equiv \mu_n^* - \mu_{\Lambda(\Sigma^0)}^*\,, \tag{11}$$

while the others non-strange neutral mesons have a vanishing chemical potential. Thus, the effective meson chemical potentials are coupled with the meson fields related to the interaction between baryons. As seen in the next section, this assumption represents a crucial feature in the EOS at finite density and temperature and can be seen somehow in analogy with the hadron resonance gas within the excluded-volume approximation. There the hadronic system is still regarded as an ideal gas but in the volume reduced by the volume occupied by constituents (usually assumed as a phenomenological model parameter), here we have a (quasi free) mesons gas but with an effective chemical potential which contains the self-consistent interaction of the meson fields.

As well as the effective meson chemical potentials has been obtained from a difference between the effective baryon chemical potentials respecting the Gibbs conditions and the strong interaction, so we postulate that the **meson effective masses** can be expressed as a difference of the effective baryon masses respecting the strong interaction and the main processes of meson production/absorption involving *different* baryons. More explicitly, concerning pions, being the Δ-isobar one of the most prominent feature of πN dynamics, the main process involving different baryons can be identified with $\Delta \leftrightarrow \pi N$. The energy balance can be expressed in terms of a mass difference of the involved particles plus an additional kinetic energy $\Delta E_{\Delta N}$: $m_\pi = M_\Delta - M_N - \Delta E_{\Delta N}$, where m_π is the vacuum mass of the pion. Assuming $\Delta E_{\Delta N}$ to be almost the same in the free space and in nuclear medium ($\Delta E_{\Delta N}^* \simeq \Delta E_{\Delta N}$), we can write the effective pion mass as

$$m_\pi^* = M_\Delta^* - M_N^* - \Delta E_{\Delta N}^* \simeq m_\pi - (x_{\sigma\Delta} - 1)g_{\sigma N}\,\sigma\,, \tag{12}$$

where the last equivalence follows from Eq.(4) and $x_{\sigma\Delta} = g_{\sigma\Delta}/g_{\sigma N}$.
Concerning the other strangeless mesons, they can be considered as correlated states of pions in the nuclear medium having the corresponding dependence of the effective meson masses. For example, for the ρ meson: $\rho \leftrightarrow 2\,\pi$ and for the ω meson: $\omega \leftrightarrow 3\,\pi$. Therefore, we have, respectively,

$$m_\rho^* = m_\rho - 2\,(x_{\sigma\Delta} - 1)g_{\sigma N}\,\sigma\,, \tag{13}$$

$$m_\omega^* = m_\omega - 3\,(x_{\sigma\Delta} - 1)g_{\sigma N}\,\sigma\,. \tag{14}$$

On the other hand, because η meson is not allowed to strongly decay into pions, for η and η' mesons we consider their respective vacuum masses.
Following the above scheme and considering strong interaction only, the effective kaon masses will be related to the difference of the effective hyperons and nucleons masses mainly by means of two different channels: the associate production/absorption due to pion conversion modes on a single nucleon

$$\pi N \leftrightarrow \Lambda K\,,\quad \pi N \leftrightarrow \Sigma K\,,\quad \pi N \leftrightarrow \Xi K K\,, \tag{15}$$

and the channel due to non-pionic modes on two nucleons

$$NN \leftrightarrow N\Lambda K\,,\quad NN \leftrightarrow N\Sigma K\,,\quad NN \leftrightarrow N\Xi K K\,, \tag{16}$$

and any conjugate processes involving the same type of particle/antiparticle.
In literature there is uncertainty about which of the two above channels is dominant in nuclear medium at different temperatures and densities. Taking into account that we are going to study our EOS in regime of hot nuclear matter, where mesons become dominant on the baryon degrees of freedom, for simplicity in the following we will limit our considerations to the first channel only.
Considering the processes indicated in Eq.(15), kaons are always related to the presence of hyperons, therefore, being the scalar σ field less attractive for hyperons than for nucleons ($x_{\sigma Y} < 1$), we aspect an increase in the effective kaon mass m_K^*. Following the above criterion, we can set the kaon effective mass as follows

$$m_K^* = m_K + \left[x_{\sigma\Delta} - \frac{x_{\sigma\Lambda} + x_{\sigma\Sigma}}{2}\right]\,g_{\sigma N}\,\sigma\,, \tag{17}$$

where we have taken the average contribution between the first two modes of Eq.(15), neglecting the last one involving multistrange production/absorption and we have used Eq.(12) for the effective pion mass.
Moreover, K^* meson can be viewed as a strongly correlated state of K and π ($K^* \leftrightarrow K\pi$) and its effective mass will be expressed as

$$m_{K^*}^* = m_{K^*} + \left[1 - \frac{x_{\sigma\Lambda} + x_{\sigma\Sigma}}{2}\right]\,g_{\sigma N}\,\sigma\,, \tag{18}$$

where we have made explicit the effective pion and kaon masses given in Eq.s (12) and (17), respectively. Finally, according to the Zweig rule, ϕ meson decays mainly into two kaons ($\phi \leftrightarrow 2\,K$), therefore, we can assume that its effective mass to be related to the effective kaons meson mass as follows

$$m_\phi^* = m_\phi + 2\,\left[x_{\sigma\Delta} - \frac{x_{\sigma\Lambda} + x_{\sigma\Sigma}}{2}\right]\,g_{\sigma N}\,\sigma\,, \tag{19}$$

where we have used the effective kaon mass given in Eq.(17).

Effective hadronic equation of state and particle ratios

The numerical evaluation of the above thermodynamical quantities can be performed if the meson-nucleon, meson-Δ and meson-hyperon coupling constants are known. Concerning the meson-nucleon coupling constants they are determined to reproduce properties of equilibrium nuclear matter such as the saturation densities, the binding energy, the symmetric energy coefficient, the compression modulus and the effective Dirac mass at saturation. The set marked TM1 is from Ref.[4] and GM3 is from Glendenning and Moszkowski [5]. The implementation of hyperon degrees of freedom comes from determination of the corresponding meson-hyperon coupling constants that have been fitted to hypernuclear properties [8,9].
In Fig. 1, we report for different temperatures (in units of MeV) the pressure as a function of the baryon chemical potential. As expected, the value of the pressure grows sensibly at higher temperature also at low baryon chemical potentials, in agreements with statistical thermal model predictions.

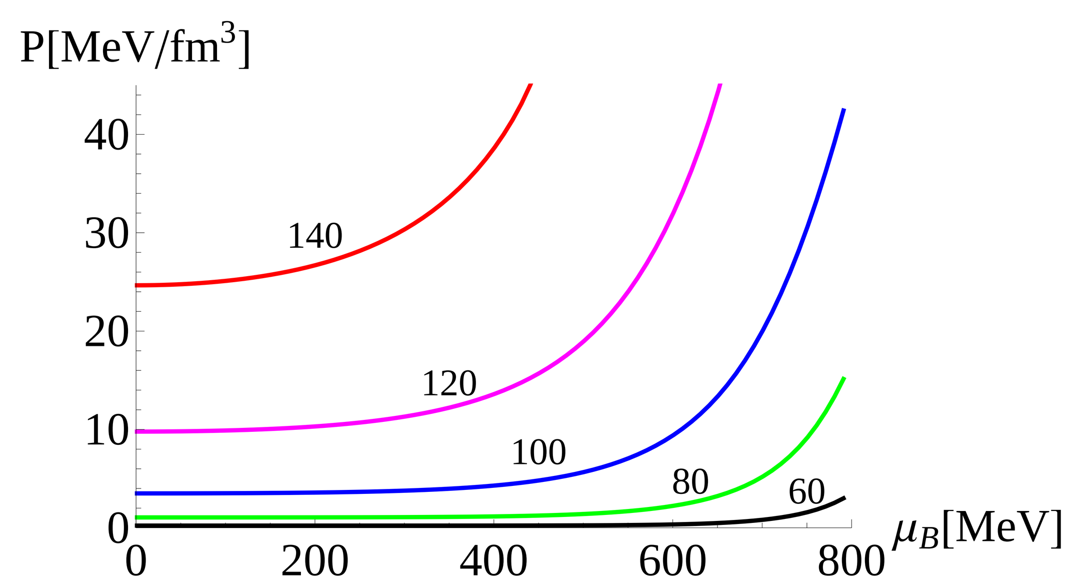

Fig. 1 - Pressure as a function of the baryon chemical potential μ_B at different values of temperature (in units of MeV).

In Fig. 2, we report, for the most relevant mesons, the behavior of the effective particle chemical potentials μ_i^*, the effective masses m_i^* as a function of the baryon chemical potential at different temperatures (solid lines: $T = 60$ MeV, long dashed lines: $T = 100$ MeV, short dashed lines: $T = 140$ MeV). We can observe that effective μ_i^* and m_i^* result to be significantly altered respect to the case of a free meson gas and, consequently, the particle densities too.
In the following, we study the influence of the Δ-isobars degrees of freedom in the behavior of different particle ratios and strangeness production.

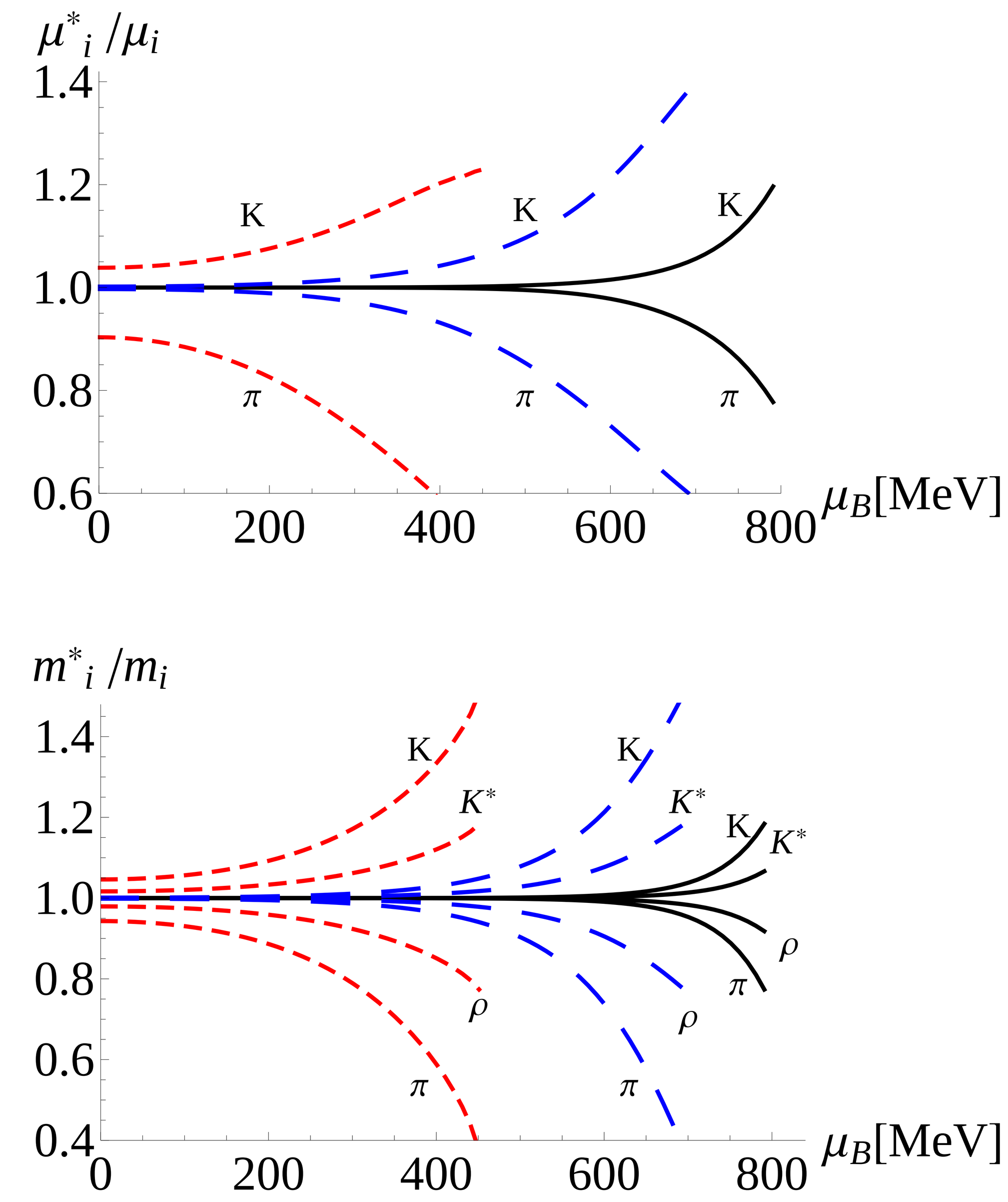

Fig. 2 - Meson effective chemical potentials μ_i^* and effective masses m_i^* (in units of the respective values corresponding to a free meson gas) versus the baryon chemical potential for different temperatures (solid lines: $T = 60$ MeV, long dashed lines: $T = 100$ MeV, short dashed lines: $T = 140$ MeV).

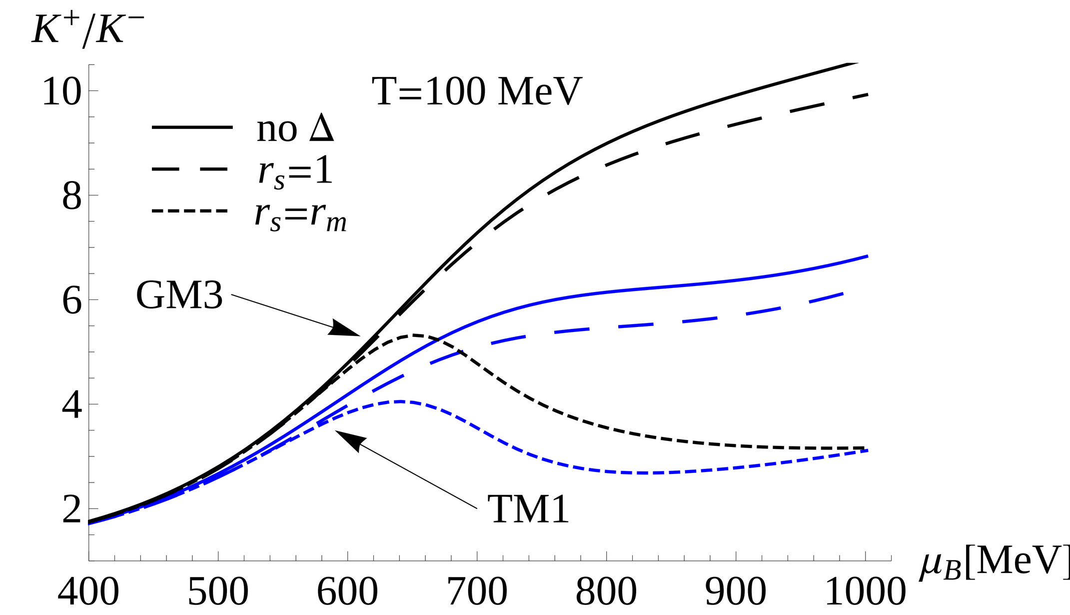

Fig. 3 - Variation of the K^+/K^- ratio with respect to baryon chemical potential at fixed temperature $T = 100$ MeV. The value r_m corresponds to the average value $r_m = 1.30$ for TM1 and $r_m = 1.46$ for GM3 parameter sets ($r_s = g_{\sigma\Delta}/g_{\sigma N}$).

Finally, it is interesting to investigate the study of the EOS also at high temperatures and low baryon chemical potential regime. At this scope, in Fig. 4, we report the results of various particle-antiparticle ratios and K^+/π^+ ratio as a function of the $\overline{p}/p$ ratio for different values of temperature. The ratios are reported for the GM3 parameter set, however, we have verified that very close results are obtained for the other two parameter sets. Also in this case we can observe good agreement with the results obtained in the framework of statistical-thermal models and with experimental SPS and RHIC data [10].

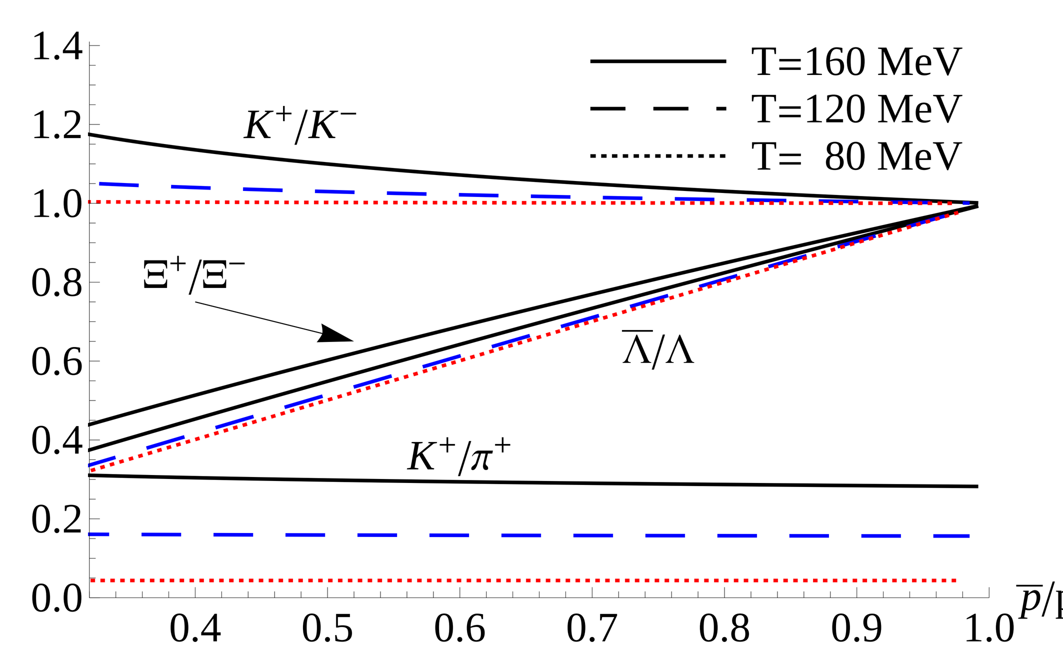

Fig. 4 - Particle-antiparticle and K^+/π^+ ratios as a function of the $\overline{p}/p$ ratio for different temperatures. The Δ coupling ratios are fixed to $r_s = r_v = 1$. The ratios of Ξ^+/Ξ^- at $T = 80$, 120 MeV are not reported because they are very strictly to the $\overline{\Lambda}/\Lambda$ ones.

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