Right Unitarity Triangles and Tri-Bimaximal Mixing from Discrete Symmetries and Unification

S. Antusch[1], S. F. King[2], C. Luhn[2], and M. Spinrath[3]

[1]Max-Planck-Institut für Physik, Föhringer Ring 6, D–80805 Munich, Germany, now at Department of Physics, University of Basel, Klingelbergstr. 82, CH-4056 Basel, Switzerland
[2]School of Physics and Astronomy, University of Southampton, SO17 1BJ Southampton, United Kingdom
[3]SISSA/ISAS and INFN, Via Bonomea 265, I-34136 Trieste, Italy

Abstract

We propose a new class of flavour models which predict both tri-bimaximal lepton mixing (TBM) and a right-angled Cabibbo-Kobayashi-Maskawa (CKM) unitarity triangle, $\alpha \approx 90°$. The ingredients of the models include a supersymmetric (SUSY) unified gauge group such as $SU(5)$, a discrete family symmetry such as A_4 or S_4, a shaping symmetry including products of Z_2 and Z_4 groups as well as spontaneous CP violation. We show how the vacuum alignment in such models allows a simple explanation of $\alpha \approx 90°$ by a combination of purely real or purely imaginary vacuum expectation values (vevs) of the flavons responsible for family symmetry breaking.

Motivation: The Quark Unitarity Triangle

Best-fits to experimental data give the unitarity triangle angle $\alpha = \left(89.0^{+4.4}_{-4.2}\right)^\circ$ [1].

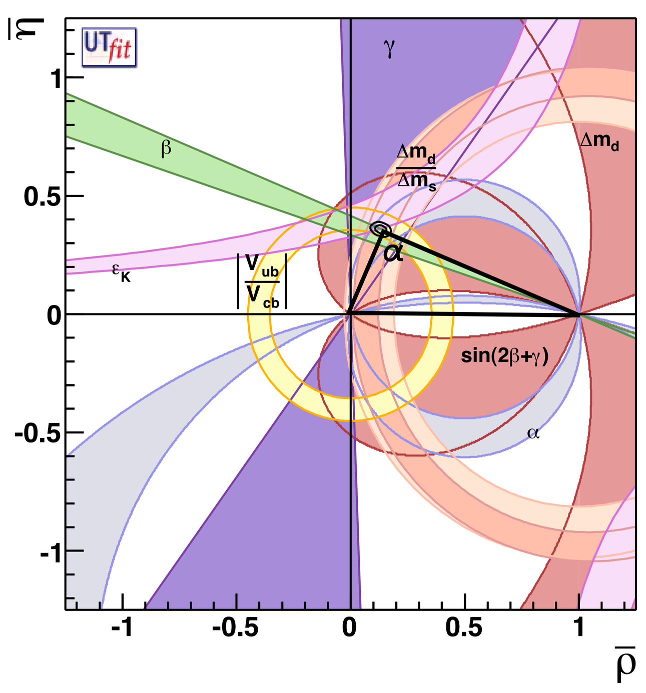

FIGURE 1: *(Taken from the UTfit collaboration* `http://www.utfit.org`*) Up to date status of the CKM unitarity triangle with the angle α sketched in.*

Is this an accident or the consequence of an underlying theory of flavour?

First Step: The Quark Phase Sum Rule [2]

Assumptions:

- Hierarchical quark mass matrices
- Texture zeros in the 1-3 elements of the mass matrices

The quark pase sum rule [2] with $\delta_{12}^{d/u}$ being the arguments of the complex 1-2 quark rotation angles reads then:

$$\alpha \approx \delta_{12}^d - \delta_{12}^u \overset{!}{\approx} 90° \,. \tag{1}$$

Consequence:

- Mass matrices with purely real/imaginary elements can predict the correct α, see also [3].

The Class of Models: Discrete Vacuum Alignment [4]

Ingredients:

- Discrete family (A_4, S_4, ...), and shaping symmetry (product of Z_n's)
- Spontaneous CP violation
- SUSY GUT ($SU(5)$, ...)

The Method:

1. Use family symmetry to align flavon vevs (only one complex parameter x left), e.g.:

$$\langle \phi \rangle \propto (0,0,x)^T \quad \text{or} \quad \langle \phi \rangle \propto (x,x,x)^T \,.$$

2. Add additional terms to the superpotential allowed by the shaping symmetry (M real):

$$P\left(\frac{\phi^n}{\Lambda^{n-2}} \mp M^2\right) \,. \tag{2}$$

3. Solving the F-term conditions, $F_P = 0$, the phase of the flavon vev is determined to be:

$$\arg(\langle\phi\rangle) = \arg(x) = \begin{cases} \frac{2\pi}{n} q \,, \quad q = 1,\ldots,n & \text{for “}-\text{” in Eq. (2)} \,, \\ \frac{2\pi}{n} q + \frac{\pi}{n} \,, \quad q = 1,\ldots,n & \text{for “}+\text{” in Eq. (2)} \,. \end{cases} \tag{3}$$

4. Use Z_2 and/or Z_4 symmetries to fix the flavon phases and then to fulfill the quark phase sum rule in Eq. (1). For concrete examples see [4,5].

Sketch of a Concrete Example: The A_4 Model from [4]

- The Symmetry: $SU(5) \times A_4 \times Z_4^4 \times Z_2^2 \times U(1)_R$
- The Matter content: $F = (\bar{\mathbf{5}}, \mathbf{3}, \ldots)$, $T_{1,2,3} = (\mathbf{10}, \mathbf{1}, \ldots)$, and $N_{1,2} = (\mathbf{1}, \mathbf{1}, \ldots)$
- The Flavons:

$$\langle\phi_1\rangle \propto \begin{pmatrix}1\\0\\0\end{pmatrix},\ \langle\phi_2\rangle \propto \begin{pmatrix}0\\-\mathrm{i}\\0\end{pmatrix},\ \langle\phi_3\rangle \propto \begin{pmatrix}0\\0\\1\end{pmatrix},\ \langle\phi_{23}\rangle \propto \begin{pmatrix}0\\1\\-1\end{pmatrix},\ \langle\phi_{123}\rangle \propto \begin{pmatrix}1\\1\\1\end{pmatrix}$$

- The Quark Yukawa matrices:

$$Y_d = \begin{pmatrix} 0 & \mathrm{i}\,\epsilon_2 & 0 \\ \epsilon_{123} & \epsilon_{23} + \epsilon_{123} & -\epsilon_{23} + \epsilon_{123} \\ 0 & 0 & \epsilon_3 \end{pmatrix} \quad \text{and} \quad Y_u = \begin{pmatrix} a_{11} & a_{12} & 0 \\ a_{12} & a_{22} & a_{23} \\ 0 & a_{23} & a_{33} \end{pmatrix}$$

- The Neutrino sector:

$$M_R = \begin{pmatrix} M_{R_1} & 0 \\ 0 & M_{R_2} \end{pmatrix},\ Y_\nu = \begin{pmatrix} 0 & a_{\nu_2} \\ a_{\nu_1} & a_{\nu_2} \\ -a_{\nu_1} & a_{\nu_2} \end{pmatrix},\ Y_e^T = -\frac{3}{2}\begin{pmatrix} 0 & \mathrm{i}\,\epsilon_2 & 0 \\ \epsilon_{123} & -3\,\epsilon_{23} + \epsilon_{123} & 3\,\epsilon_{23} + \epsilon_{123} \\ 0 & 0 & \epsilon_3 \end{pmatrix}$$

- For phenomenological implications, see box below.

Some Phenomenological Implications [2, 4, 5]

- Shaping symmetries + unification:
  - → $\alpha \approx 90°$ [2,4]
  - → Fixed leptonic CP phases (close to 0°, 90°, 180°, 270°) [4,5]
- Discrete family symmetry (A_4, S_4, ...):
  - → (Almost) tri-bimaximal mixing [4,5]
- In combination with a type II seesaw upgrade:
  - → Testable neutrino mass scale via neutrinoless double beta decay [5] (see figure below)

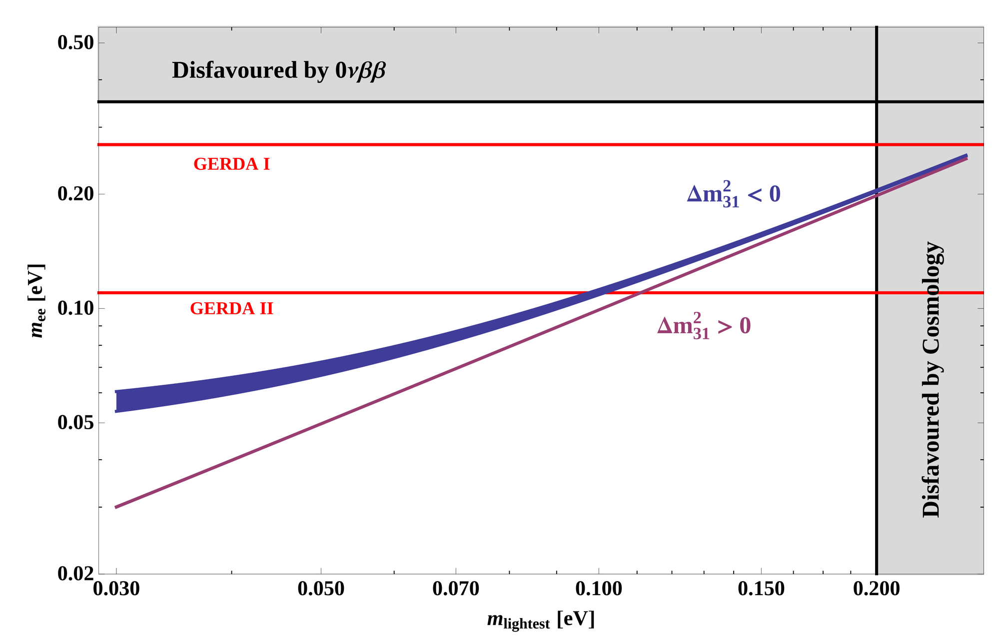

FIGURE 2: *(Taken from [5]) Prediction for effective mass m_{ee} relevant for neutrinoless double beta decay as a function of the mass of the lightest neutrino, m_{lightest}, for an inverted neutrino mass ordering ($\Delta m^2_{31} < 0$, upper line) and for a normal mass ordering ($\Delta m^2_{31} > 0$, lower line). The bands represent the experimental uncertainties of the mass squared differences. The mass bounds from cosmology [6] and from the Heidelberg-Moscow experiment [7] are displayed as grey shaded regions. The red lines show the expected sensitivities of the GERDA experiment in phase I and II [8].*

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